CHAPTER 7 Geometric Relationships
7.3 Angle Relationships in Polygons
Interior and Exterior Angle Relationships in Polygons

The sum of the exterior angles of a convex polygon is 360º.
For a polygon with n sides, the sum of the interior angles, in degrees, is $180(n-2)$.

Example:

a) A gazebo is built in the shape of a regular polygon with 10 sides. Find the measure of each of the exterior angles.

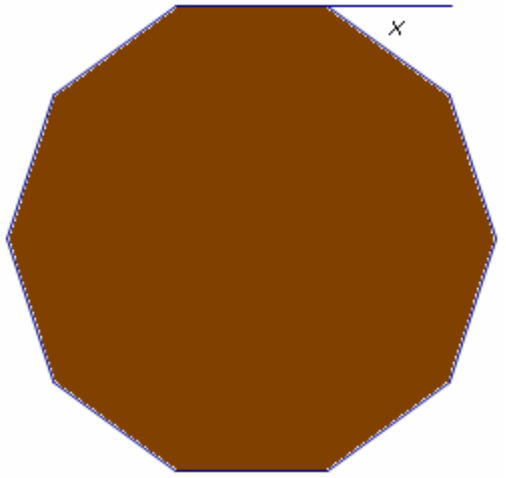
b) A swimming pool is made in the shape of a regular polygon with 18 sides. Find the sum of the interior angles.

c) Find the measure of each of the interior angles of the swimming pool in part b).

d) The sum of the interior angles of a polygon is 2160º. Find the number of sides.

Solution:

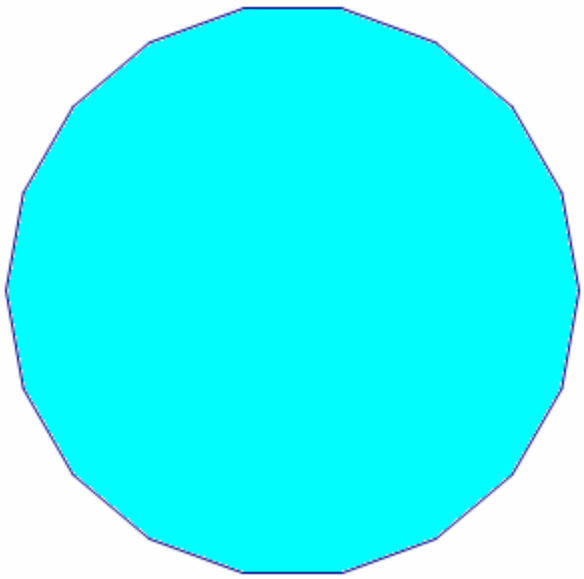
a) The sum of the exterior angles of a convex polygon is 360º. There are 10 exterior angles in the gazebo. Let the measure of each be represented by x.



$$x = \frac{360^\circ}{10}$$
$$= 36^\circ$$

Each exterior angle measures 36º.

b) Sum of Interior Angles $= 180(n-2)$
$= 180(18-2)$
$= 180(16)$
$= 2880$



The sum of the interior angles for a polygon with 18 sides is 2880º.

c) Let the measure of each interior angle be represented by x.

$$x = \frac{2880^\circ}{18}$$
$$= 160^\circ$$

Each interior angle measure 160º.

d) Sum of Interior Angles $= 180(n-2)$

$$\begin{aligned} 2160 &= 180(n-2) \\ 2160 &= 180n - 360 \\ 2160 + 360 &= 180n - 360 + 360 \\ 2520 &= 180n \\ \frac{2520}{180} &= \frac{180n}{180} \\ 14 &= n \end{aligned}$$

The polygon has 14 sides.

Practice:

1. a) A garden pond is built in the shape of a regular polygon with 15 sides. Find the measure of each of the exterior angles.

b) A patio is made in the shape of a regular polygon with 24 sides. Find the sum of the interior angles.

c) Find the measure of each of the interior angles of the patio in part b).

d) The sum of the interior angles of a polygon is 2700°. Find the number of sides.

Answers:

1. a) 24° b) 3960° c) 165° d) 17