

CHAPTER 7 Geometric Relationships
7.4 Midpoints and Medians in Triangles
Midpoint and Median Properties in a Triangle

A line segment joining the midpoints of two sides of a triangle is parallel to the third side and half as long. The height of the triangle formed by joining the midpoints is half the height of the original triangle.

The medians of a triangle bisect its area.

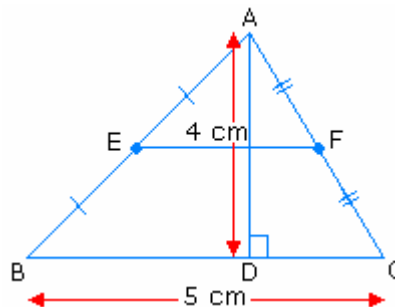
A counter-example can disprove a conjecture or hypothesis.

Example:

a) $\triangle ABC$ has a base $BC = 5$ cm and a height $AD = 4$ cm. A line segment EF is drawn joining the midpoints of sides AB and AC . Find the length of this line segment.

b) Find the height of $\triangle AEF$.

c) A median is drawn from point A to G , the midpoint of BC . Find the area of $\triangle ABG$.



d) $\triangle PQR$ is an equilateral triangle. Anil used dynamic geometry software to bisect $\angle P$. He found that this bisector went through the midpoint of side PQ . Anil conjectured that the angle bisector in any triangle goes through the midpoint of the opposite side. Use dynamic geometry software to provide a counter-example, showing that Anil is not correct.

Solution:

a) A line segment joining the midpoints of two sides of a triangle is half as long as the third side. Therefore, the length of EF is 2.5 cm.

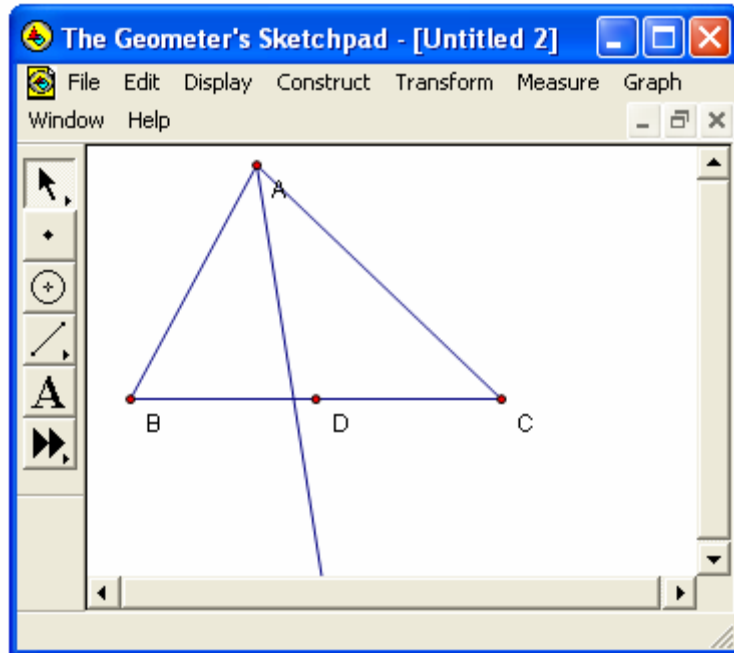
b) The height of the triangle formed by joining the midpoints is half the height of the original triangle. The height of $\triangle AEF$ is 2 cm.

c) The medians of a triangle bisect its area.

$$\begin{aligned}\text{Area of } \triangle ABG &= \frac{1}{2} \text{Area of } \triangle ABC \\ &= \frac{1}{2} \times \frac{1}{2} \times BC \times AD \\ &= \frac{1}{2} \times \frac{1}{2} \times 5 \times 4 \\ &= 5 \text{ cm}^2\end{aligned}$$

The area of $\triangle ABG$ is 5 cm^2 .

d) Draw a triangle ABC that is not an equilateral triangle. Construct the bisector of $\angle A$. Construct the midpoint of side BC. Note that the bisector of $\angle A$ does not pass through the midpoint.



Practice:

1. a) $\triangle XYZ$ has a right angle at Y. The height XY measures 6 cm, and the base YZ measures 8 cm. A line segment ST is drawn joining the midpoints of sides XY and XZ. Find the length of this line segment.
- b) Find the height of $\triangle YST$.
- c) A median is drawn from point Y to U, the midpoint of XZ. Find the area of $\triangle YXU$.
- d) $\triangle GHI$ is a scalene triangle. Baldur used dynamic geometry software to bisect $\angle G$. He found the midpoint of side HI, and noted that the angle bisector did not pass through the midpoint. Baldur conjectured that the angle bisector in any triangle will never pass through the midpoint of the opposite side. Use dynamic geometry software to provide a counter-example, showing that Baldur is not correct.

Answers:

1. a) 4 cm b) 3 cm c) 12 cm²
- d) Answers will vary. One possible counter-example is to use an equilateral triangle.