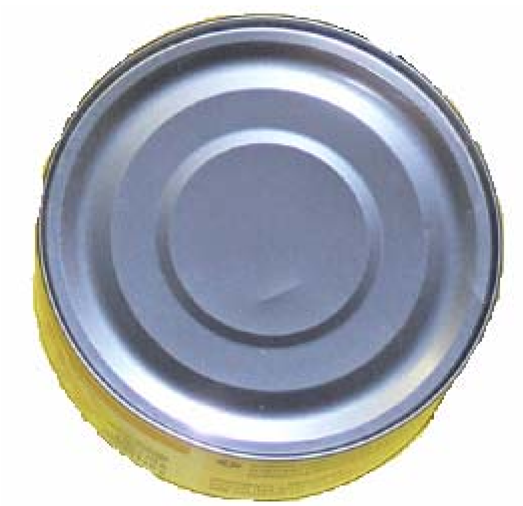
CHAPTER 9: Optimizing Measurements
9.5 Maximize the Volume of a Cylinder
Maximizing the Volume of a Cylinder with a Given Surface Area

The maximum volume for a given surface area of a cylinder occurs when its height equals its diameter.
The dimensions of the cylinder with maximum volume for a given surface area can be found by solving the formula $SA = 6\pi r^2$ for r. The height will be $2r$.

Example:



a) Outdoors Unlimited sells a line of trail foods packaged in cylindrical cans. Each can is made from 360 cm^2 of metal. The design of the can maximizes the volume. Find the radius of the can.

b) What is the maximum amount of food that each can holds?

Solution:

a)
$$\begin{aligned} SA &= 6\pi r^2 \\ 360 &= 6\pi r^2 \\ \frac{360}{6\pi} &= \frac{6\pi r^2}{6\pi} \\ 19.1 &= r^2 \\ 4.4 &= r \end{aligned}$$

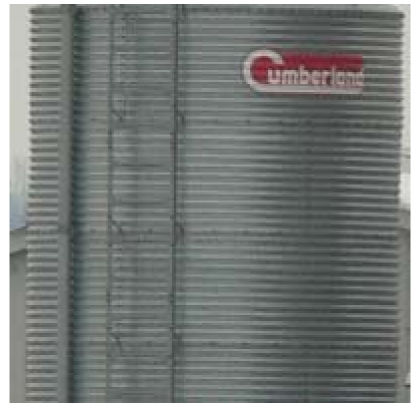
The radius of the can with maximum volume is 4.4 cm.

b)
$$\begin{aligned} V &= \pi r^2 h \\ &= \pi r^2 (2r) \\ &= 2\pi r^3 \\ &= 2\pi (4.4)^3 \\ &= 535.2 \end{aligned}$$

The maximum amount of food that each can holds is 535.2 cm^3.

Practice:

1. a) A cylindrical silo was constructed using 75 m^2 of corrugated steel. The dimensions were chosen such that the volume was a maximum. Find the radius of the silo.



b) Find the volume of the silo.

Answers:

1. a) 2.0 m b) 50.3 m^3