Number Relations

1. The Golden Rectangle is often used in art and architecture, since this proportion is pleasing to the eye. To make a golden rectangle, you multiply the width by a number y to satisfy the relation $y^2 - y - 1 = 0$.

a) Use your calculator or computer to determine the value of y using trial and error. Continue until you obtain accuracy to four decimal places.

b) If an architect wants to design a building with a "golden window" of width 2.4 m, what is the required height of the window?

c) Measure the height and width of several heads, including your own. Divide the height by the width. Use a table to record the results. What do you observe?

d) Use the Internet or the library to research the use of golden rectangles in art and architecture. Find one example of each. Hint: Check Leonardo da Vinci for art and the Parthenon in Greece for architecture.

To learn more about the Golden Rectangle, follow the web links on the same page where you found this file on the *MathLinks 8* Online Learning Centre.

To learn about the Golden Proportion in the human body, follow the web links on the same page where you found this file on the *MathLinks 8* Online Learning Centre.

2. A chessboard consists of an area of eight squares by eight squares of alternating colours.

a) Pick a rectangular area of n squares by m squares. Choose integers greater than 0 and less than 9 for m and n.

b) How do the areas of the two colours in your rectangle compare?

c) Is it possible to pick m and n such that the areas of the two colours in your rectangle are equal? If so, can this be done in more than one way? Justify your answer. If not, explain why.
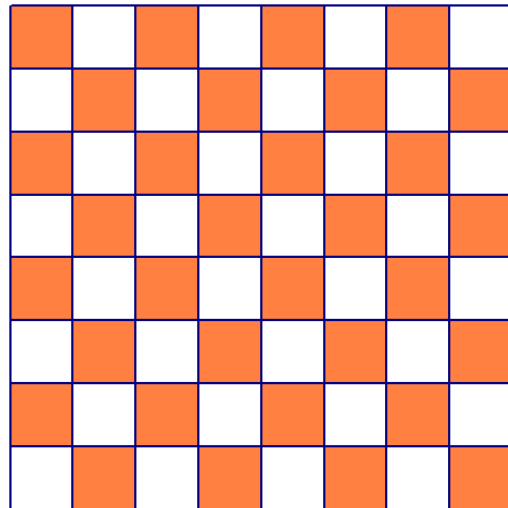
3. House numbers are groups of integers such as 67, 234, or 7293.
a) Consider the house numbers on one page of a telephone book. Suppose that you counted how many times each digit from 0 to 9 occurred and then, created a graph with the digits along the x-axis and the number of times each occurred along the y-

axis. What would the graph look like? Make a sketch and justify your choice of graph.

b) Choose a page of a telephone book at random. Record all the house numbers that occur on the page. Count how many times each digit occurred. Use grid paper and create a graph to display your results.

c) How does your graph compare to the sketch you made.

d) Use the Internet or the library to research Benford's Law. How does Benford's Law apply to your results?

For information about which type of graph to use for displaying data, follow the web links on the same page where you found this file on the *MathLinks 8* Online Learning Centre.

For information about Benford's Law, follow the web links on the same page where you found this file on the *MathLinks 8* Online Learning Centre.

4. Not everyone in the world tells time in the same way. People who speak Swahili measure time in a different way than you do. For example, 7 a.m. is considered as 1 a.m. on a Swahili clock. Use the Internet or the library to investigate why this is so.

a) Where do most people who speak Swahili live?

b) Why would a Swahili clock be oriented to sunrise? Explain your answer in geographical terms.

c) Draw a picture of a Swahili clock showing the placement of the hours.

d) How can you convert the time on a standard clock to the time on a Swahili clock? Explain why this works.

For information about the Swahili clock, follow the web links on the same page where you found this file on the *MathLinks 8* Online Learning Centre.

5. Data are often estimated by taking a sample. This always introduces some error.

a) How does the error depend on the size of the sample? State your hypothesis and provide reasons why you think your hypothesis is true.

b) Obtain three 500 mL measuring cups, a bag of dried beans (use large beans), and a 25 mL measuring spoon. Pour 100 mL of beans into the first cup, 250 mL into the second cup, and 500 mL into the third cup.

c) Take a 100 mL sample from each of the cups in turn. Count how many beans are in the sample. Then, use the count to estimate the number of beans in the cup.

d) Count the actual number of beans in each cup.

e) Use a table to organize your data. Compare the errors in each of the three cases as percents. Hint: Error ÷ actual number × 100. How does the percent error depend on the sample size? How well does the data support your hypothesis in part a)?