5

Probability

In the summer of 2005, over $75 000 000 000 damage was done to communities in the southern United States by Hurricanes Katrina and Rita.

Scientists use satellites to watch the movement of hurricanes and to predict where they might cause damage. Based on these predictions, residents of Texas and Louisiana were advised to move inland to safety. Unfortunately, many people underestimated the power of the hurricanes and stayed where they were.

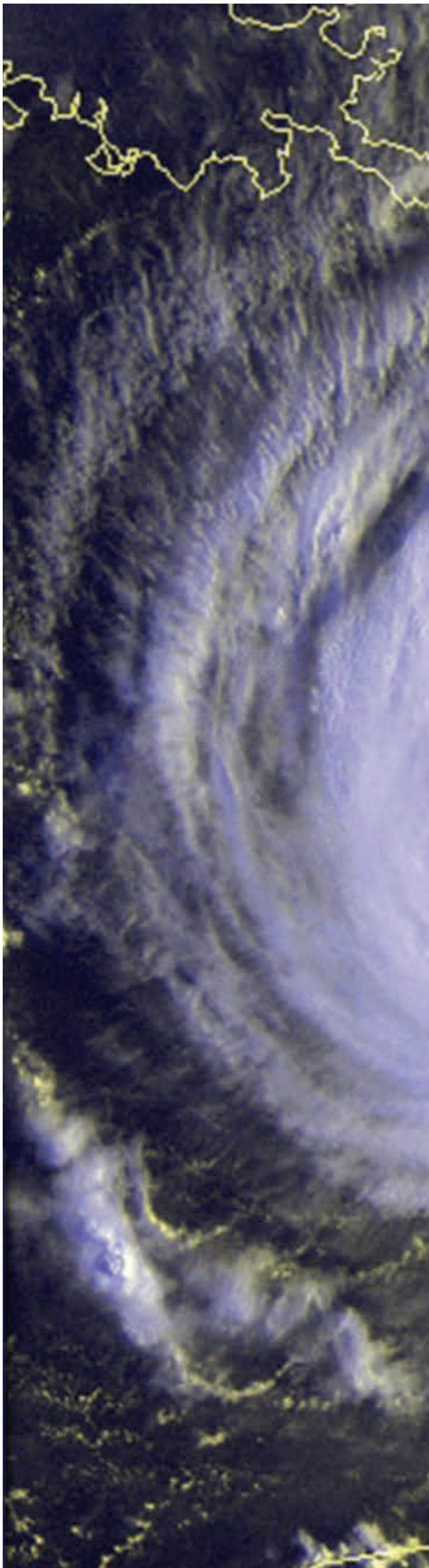
This is just one example of probability being used to help protect people. What other examples can you think of that use probability to help keep people safe?

What You Will Learn

- to use tables and diagrams to organize outcomes
- to calculate probability
- to compare experimental and theoretical probability
- to predict the probability of events

Key Words

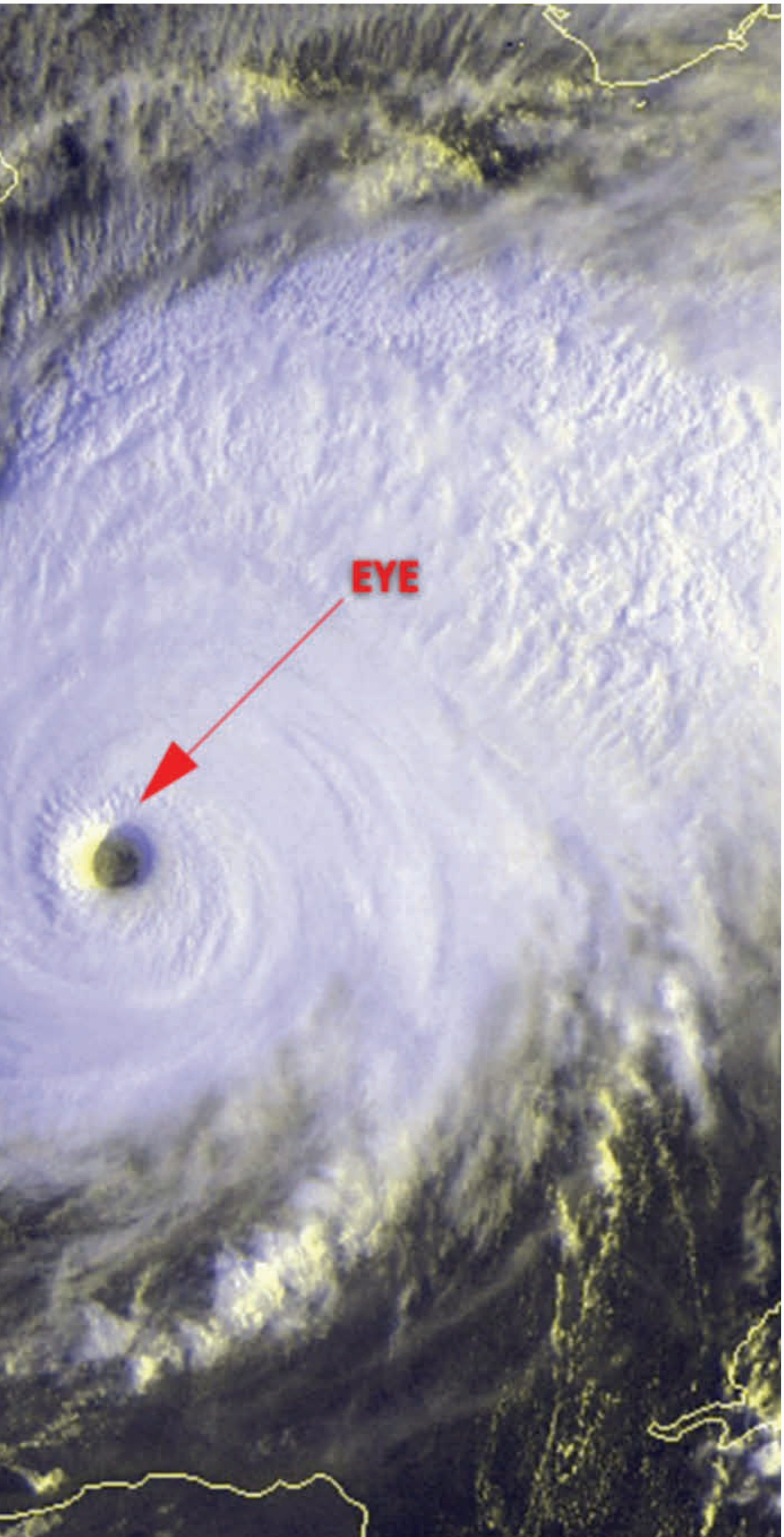
probability
outcome
favourable outcome
independent events
sample space
tree diagram
random
experimental probability
theoretical probability



MATH LINK

In this chapter you will work with probability. Luck is part of most games, but you can increase your chances of winning by understanding the probability of the numbers you get when you roll dice.

By the end of the chapter, you will create a game that requires players to understand and use probability to win.

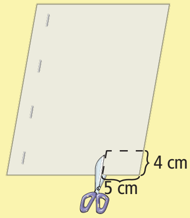


FOLDABLES™
Study Tool

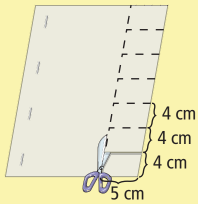
Make the following Foldable to organize what you learn in Chapter 5.

Step 1 Staple eight sheets of notebook paper together along the left edge.

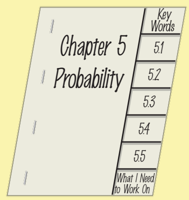
Step 2 Make a mark five lines up from the bottom of the top page. Cut through the top seven sheets about 3 cm in from the right edge up to this mark and across to the right edge. Do not cut the last page, which is your back cover.



Step 3 Cut through the top six sheets of paper, up five more lines and across to the right edge. As you do this, you will form tabs along the right side of the foldable.



Step 4 Continue to cut tabs in this way until you have seven tabs.



Step 5 Label the tabs as shown.

Literacy Link

As you work through Chapter 5, take notes under the appropriate tab. Include information about the key words, examples, and key ideas.

5.1

Probability

Focus on...

After this lesson, you will be able to...

- ❑ find the probability of an event in several different ways
- ❑ give answers as probabilities from 0% to 100%

The picture shows a certain event and an impossible event. It is certain that the girl's ball will go through the hoop. The **probability** of her being successful is 100%. Her younger brother has an impossible task. The beach ball will not fit through the hoop. He has a 0% chance of making a basket. What other examples of certain and impossible events can you think of from your own life?

probability

- the likelihood or chance of an event occurring
- Probability $= \frac{\text{favourable outcomes}}{\text{possible outcomes}}$
- can be expressed as a ratio, fraction, or percent

Materials

- ruler

Discuss the Math

How can you compare probabilities?

1. Copy the following number line into your notebook. Make it long enough to cross your entire notebook page.

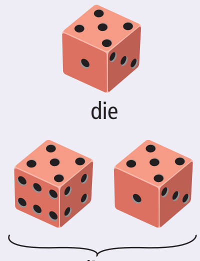
Fraction	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
Decimal	■	0.25	0.5	■	■
Percent	0%	■	■	75%	100%

2. Complete the equivalent decimals and percents below each fraction on the number line.

3. Estimate each of the following probabilities. Mark and label each event where it should be on your copy of the number line.
   A. You flip a Canadian penny. It lands with the maple leaf facing up.
   B. You roll an 8 on a six-sided die numbered 1 to 6.
   C. A bag contains 8 red markers. You reach into the bag and pull out a red marker.
   D. You spin a spinner with 4 equal sections marked chocolate, fruit, frozen yogurt, and ice cream. The pointer stops on fruit.
   E. A bag contains the letters A, B, C, D, E, F, G, H. You reach into the bag and pull out an H.
   F. A bag has 3 silver marbles and 1 gold marble. You put in your hand and pull out the gold marble.
   G. The next baby to be born in your town will be a boy.

Literacy Link

The word dice is the plural form of die.



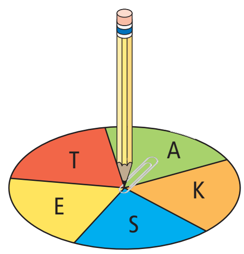
Reflect on Your Findings

4. a) Describe an event that you think has a probability of 100%.
   b) Describe an event that you think would have a $\frac{1}{7}$ chance of occurring. Record it on your number line.
   c) Describe an event that you think would have a probability between 60% and 90%. Record it on your number line.
   d) Share your answers to a), b), and c) with a classmate.
   e) How did you arrive at your answers?
   f) Why can all possible probabilities be shown on a number line between zero and one?

Example 1: Represent Probabilities

A spinner is divided into 5 equal sections. The spinner is spun once. Find the following probabilities. Write each answer as a fraction, a ratio, and a percent.

a) What is the probability of spinning an A?
b) What is the probability of spinning a vowel?
c) What is the probability of spinning a Q?



Literacy Link

The alphabet has 26 letters. The five vowels are a, e, i, o, and u. The other letters are called consonants.

outcome

- one possible result of a probability experiment

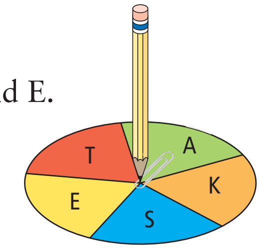
favourable outcome

- a successful result in a probability experiment

Solution

a) There are five possible **outcomes**: S, K, A, T, and E.

There is only one **favourable outcome**, A.



$$\text{Probability} = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$

$$\begin{aligned} P(\text{A}) &= \frac{1}{5} \\ &= 0.2 \\ &= 0.2 \times 100\% \\ &= 20\% \end{aligned}$$

C 1 ÷ 5 × 100 = 20.

P(A) is a short way to write "the probability of A occurring."

$\frac{1}{5}$ can be written as the ratio 1 : 5.

The probability of spinning an A is $\frac{1}{5}$, 1 : 5, or 20%.

b) There are two vowels, A and E, on the spinner. So, there are two favourable outcomes.

$$\begin{aligned} P(\text{A } or \text{ E}) &= \frac{\text{favourable outcomes}}{\text{possible outcomes}} \\ &= \frac{2}{5} \\ &= 0.4 \\ &= 0.4 \times 100\% \\ &= 40\% \end{aligned}$$

C 2 ÷ 5 × 100 = 40.

$\frac{2}{5}$ can be written as the ratio 2 : 5.

The probability of spinning a vowel is $\frac{2}{5}$, 2 : 5, or 40%.

c) The letter Q is not represented on the spinner. So, there are no favourable outcomes.

$$\begin{aligned} P(\text{Q}) &= \frac{\text{favourable outcomes}}{\text{possible outcomes}} \\ &= \frac{0}{5} \\ &= 0\% \end{aligned}$$

This is an impossible event.

The probability of spinning a Q is $\frac{0}{5}$, 0 : 5, or 0%.

Show You Know

Letter tiles for the word SNOW are placed in a bag.

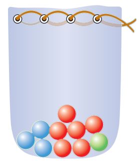
a) What is the probability of drawing a letter W from the bag?

b) What is the probability of drawing a consonant from the bag?

c) What is the probability of drawing a letter B from the bag?

Example 2: Determine Probabilities

A bag contains 10 marbles. Show the probability of the following events as a fraction, a ratio, and a percent.



a) selecting a red marble
b) selecting a red or blue or green marble
c) *not* selecting a red marble

Solution

a) There are 10 possible outcomes. There are six red marbles, so there are six favourable outcomes.

$$
\begin{aligned}
P(\text{red}) &= \frac{\text{favourable outcomes}}{\text{possible outcomes}} \\
&= \frac{6}{10} \\
&= 60\%
\end{aligned}
$$

The probability of selecting a red marble is $\frac{6}{10}$, 6 : 10, or 60%.

b)
$$
\begin{aligned}
P(\text{red or blue or green}) &= \frac{10}{10} \\
&= 1 \\
&= 100\%
\end{aligned}
$$

This is a certain event.

The probability of selecting a red, blue, or green marble is $\frac{10}{10}$, 10 : 10, or 100%.

c) The probability of *not* selecting a red marble is the same as the probability of selecting a blue or green marble.

$$
\begin{aligned}
P(\textit{not}\ \text{red}) &= P(\text{blue or green}) \\
&= \frac{4}{10} \\
&= 40\%
\end{aligned}
$$

The probability of selecting a marble that is not red is $\frac{4}{10}$, 4 : 10, or 40%.

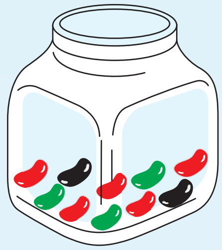
Show You Know

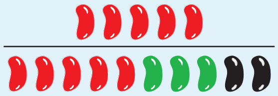
Letter tiles for the word MOUNTAIN are placed in a bag.

a) What is the probability of drawing a letter N from the bag?
b) What is the probability of drawing a consonant from the bag?
c) What is the probability of drawing a letter from the bag?

Key Ideas

- Probability $= \dfrac{\text{favourable outcomes}}{\text{possible outcomes}}$



$P(\text{red}) =$ 

- Probability can be written as a fraction, a ratio, or a percent.

  $P(\text{red}) = \frac{5}{10}$ or $5:10$ or 50%

- The probability of an impossible event is 0 or 0%.

  $P(\text{yellow}) = \frac{0}{10}$ or $0:10$ or 0%

- The probability of a certain event is 1 or 100%.

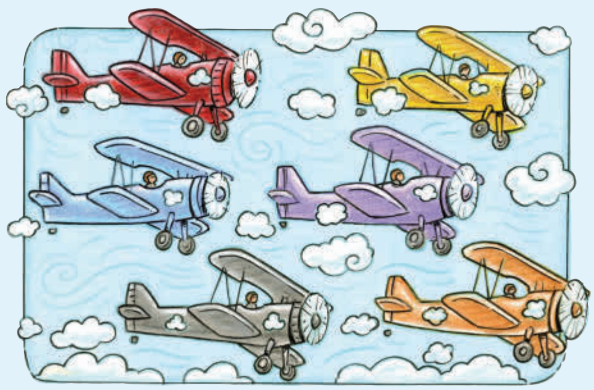
  $P(\text{jellybean}) = \frac{10}{10}$ or $10:10$ or 100%

Communicate the Ideas

1. Six airplanes are waiting to land at an air show. Show each of the following probabilities as a fraction, a ratio, and a percent.

a) the first plane to land will be blue or purple

b) the first plane to land will be silver



2. a) Give an example of an event from your life that is 100% certain.

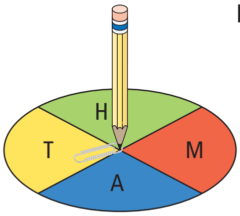
b) Give an example of an event that is impossible.

Practise

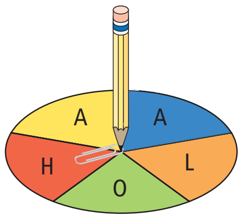
For help with #3 and #4, refer to Example 1 on pages 159–160.

3. What is the probability of each of the following spinners landing on A? Write your answer as a fraction, a ratio, and a percent.

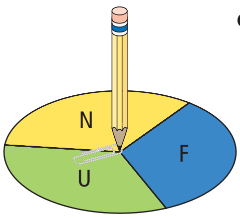
a)



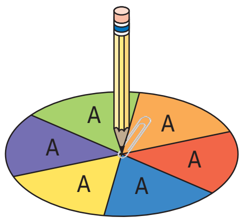
b)



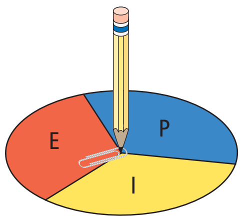
c)



d)



4. A spinner with three equal sections is spun once.

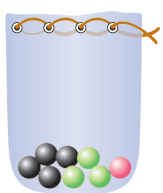


a) How many outcomes are possible?

b) What is the probability of spinning a vowel. Express your answer as a fraction, a ratio, and a percent.

For help with #5 and #6, refer to Example 2 on page 161.

5. A bag contains 8 marbles. One marble is chosen from the bag. Write each answer as a fraction, a ratio, and a percent.

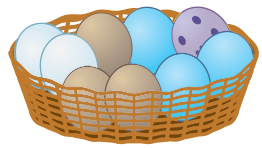


a) What is *P*(green)?

b) What is *P*(green *or* pink)?

c) What is the probability that the pink marble is *not* selected?

6. A basket contains 9 eggs. One egg is chosen from the basket. Write each answer as a fraction, a ratio, and a percent.



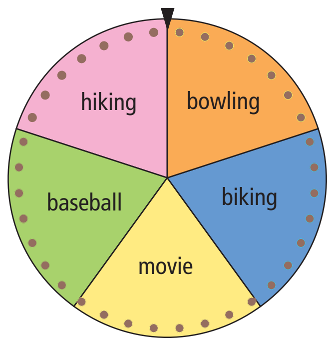
a) What is *P*(blue egg)?

b) What is *P*(speckled egg)?

c) What is the probability that a white egg is *not* chosen?

Apply

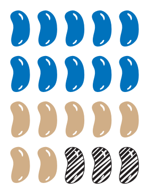
7. At Ben's birthday party, he spins the wheel to decide the afternoon's activity.



a) What is the probability that Ben's spin will land on *hiking*, *baseball*, or *movie*?

b) What is the probability that Ben's spin will *not* land on *bowling*?

8. In a jar of jellybeans, there are



- 10 blue belly blasters
- 7 tan tonsil twisters
- 3 zebra-striped zappers

You reach into the jar and pull out one jellybean without looking.

a) What is the probability of selecting a tan tonsil twister? Write your answer in fraction and decimal form.

b) What is *P*(zebra-striped zapper)?

9. Mrs. Sweet has a die with 20 sides. Each student has a number. When students' numbers are rolled, they have to show that they have everything needed for class. If they do, they receive a bonus point.

a) If there are 20 students in Marianne's class, what is the probability that she will be picked?

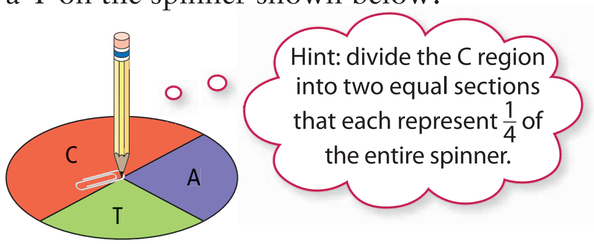
b) Girls are assigned even numbers, and boys are assigned odd numbers. What is the probability that a boy will be chosen?

c) What is the probability that a multiple of 5 will be rolled?

Did You Know?

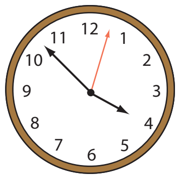
A 20-sided die is called an icosahedron.

10. What is the probability of spinning a C or a T on the spinner shown below?

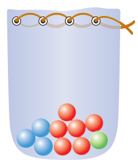


Extend

11. A clock is dropped and stops working. What is the probability that the second hand is stopped between the 12 and 1? Show how you solve the problem.



12. How many marbles would you have to select, without replacing them, until you could be guaranteed of having at least one marble of each of the three colours in this bag? Show the steps you use to solve the problem.



MATH LINK

Six-sided dice are the ones you are most familiar with. There are other types of dice with different numbers of sides. For example, a four-sided die has four triangular faces that are all the same size.

a) How many different dice shapes can you build or draw with up to 12 faces? Use a table to help keep track of your shapes.

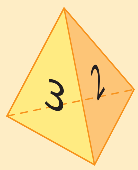
Number of Sides on Each Face	Number of Faces	Sketch of Die
3	4	
3	8	
4	6	

b) What is the probability of rolling a 7 using an eight-sided die?

c) What is the probability of rolling a number less than 7 using a 12-sided die?

Did You Know?

A four-sided die is called a tetrahedron.



5.2 Organize Outcomes

Focus on...

After this lesson, you will be able to...

- ❑ explain how to identify an independent event
- ❑ determine the outcomes of two independent events
- ❑ organize the outcomes of two independent events using tables and tree diagrams

Maryam offers to play a game with her brother, Payam. She has four coins in a cup. They are a quarter, a dime, a nickel, and a penny.

Maryam says Payam can shake out one coin from the container, put it back, and then shake out another coin. If his coins add up to an odd number of cents, she will do Payam's chores for a week. If his coins have an even sum, he has to do Maryam's chores for a week.

If you were Payam, would you agree to these conditions?

Discuss the Math

How can you organize outcomes?

To find the probability that Payam will have to do Maryam's chores, you need to organize and count the possible outcomes.

1. Create a table in your notebook to help organize the outcomes.

Value of First Coin	Value of Second Coin	Sum
25¢	25¢	50¢
25¢	10¢	35¢

2. a) How many possible combinations are there?
   b) How many combinations have an even sum?
   c) How many combinations have an odd sum?

3. a) What is the probability that Payam will have to do Maryam's chores?
   b) What is the probability that Maryam will have to do Payam's chores?

Reflect on Your Findings

4. a) How do you know that the table includes all possible combinations of coins?
   b) How does identifying all the possible outcomes help you determine the probability of the favourable outcome?
   c) Should Payam agree to Maryam's conditions? What advice would you give Payam about playing this game with his sister?

Example 1: Represent Outcomes With a Table

A coin is flipped and a six-sided die is rolled. These two events are called **independent events**.

a) Use a table to list all the possible outcomes.
b) How many possible outcomes are there?
c) Write the **sample space** for this combination of events.

independent events
- the outcome of one event has no effect on the outcome of another event

sample space
- all possible outcomes of an experiment

Solution

a)

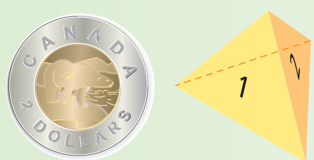
		Die					
		1	**2**	**3**	**4**	**5**	**6**
Coin Flip	**Heads (H)**	H, 1	H, 2	H, 3	H, 4	H, 5	H, 6
	Tails (T)	T, 1	T, 2	T, 3	T, 4	T, 5	T, 6

b) From the table, there are 12 possible outcomes.

c) The sample space is (H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6).

Show You Know

A toonie is flipped and a four-sided die labelled 1, 2, 3, 4 is rolled.



a) List all the possible outcomes.
b) How many possible outcomes are there?
c) Write the sample space for this combination of events.

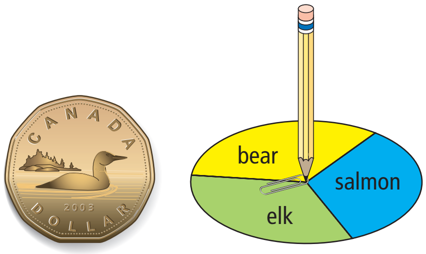
Example 2: Represent Outcomes With a Tree Diagram

A coin is flipped and the spinner is spun once.

a) Create a **tree diagram** that shows all of the possible outcomes.

b) List the sample space for these two events.

c) Think of another diagram that could be used to show the outcomes.

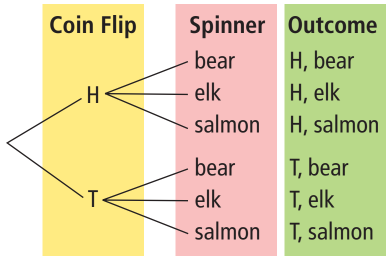


tree diagram

- a diagram used to organize outcomes
- contains a branch for each possible outcome of an event

Solution

a)



b) The sample space is (H, bear), (H, elk), (H, salmon), (T, bear), (T, elk), (T, salmon).

Write each outcome in the sample space as an ordered pair.

c) One possible diagram is shown. This diagram is called a "spider diagram."

Literacy Link

Reading Tree Diagrams

Read tree diagrams from left to right.

- The branches on the left of the tree show the outcomes for the coin flip.
- The branches on the right show the outcomes for the spinner.
- The column on the far right of the diagram lists the combined outcomes.

Show You Know

a) Trudy, Shana, Saira, Kendra, and Tracie are on basketball team A. Jordan, Michael, Terrance, Sean, and Suni are on team B. Use a tree diagram to show all the possible outcomes if the coach assigns each person on team B to defend against a player on team A.

b) Write the sample space.

Key Ideas

- Two events are independent if the outcome of one event has no affect on the outcome of the other event.

- You can create tables, tree diagrams, and other diagrams to organize the outcomes for two independent events.

Communicate the Ideas

1. Decide whether each pair of events are independent or not independent. Explain your reasoning.

a) Choose a student from grade 7 and choose a student from grade 8.

b) Choose one marble from a bag and then choose a second marble from the bag without replacing the first marble.

c) Choose an apple from one basket and then choose an apple from another basket.

2. Pretend a friend missed today's lesson. Roll a six-sided die and flip a coin. Teach your friend how to use a tree diagram to organize the outcomes. Then, show your friend how to identify the sample space.

3. Sharon created this table to list the possible outcomes from tossing a coin twice.

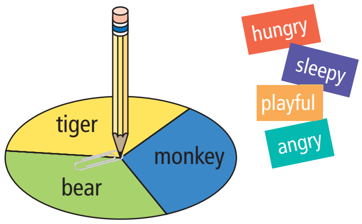
		Second Toss	
		Heads (H)	**Tails (T)**
First Toss	**Heads (H)**	H, H	H, T
	Tails (T)	H, T	T, T

Kevin says that the outcome in the bottom left corner should be (T, H). Who is correct? Why?

Practise

For help with #4 and #5, refer to Example 1 on page 166.

4. Jeremy chooses a tile and spins the spinner.



a) Organize the outcomes of these two events in a table.

b) What is the sample space for this experiment?

c) Are the outcomes independent? Explain why.

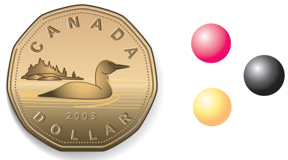
5. Clarise flips a disk that is black on one side and white on the other. Then she chooses one card from the five cards shown here.

a) Organize the outcomes of these two events in a table.

b) What is the sample space for this experiment?

For help with #6 and #7, refer to Example 2 on page 167.

6. Alan flips a coin and chooses one of three marbles: black (B), yellow (Y), and red (R).



a) Draw a tree diagram to organize the outcomes of these two events.

b) What is the sample space for this experiment?

7. The wheel is spun twice.

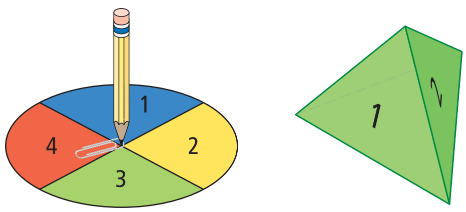
a) Make a tree diagram to organize the outcomes of the two spins.

b) What is the sample space?

c) Create a different diagram to show the outcomes of two spins of the wheel.

Apply

8. Georgina spins a spinner with four equal sections as shown and rolls a four-sided die labelled 1, 2, 3, and 4.

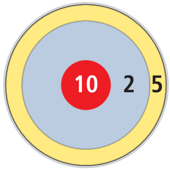


a) Use a tree diagram to organize the outcomes of these two events.

b) In an extra column, determine the sums of the two outcomes along each branch.

c) What is the most common sum?

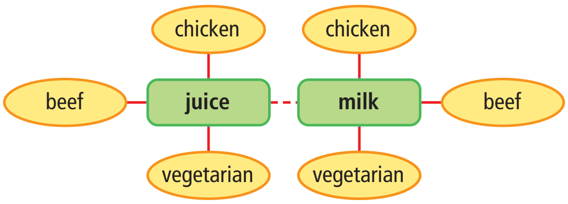
9. Jake throws two darts at this dart board.



a) What is the sample space for this experiment? Use a tree diagram to represent the sample space. Assume every dart hits the board.

b) Is each outcome equally likely? Explain.

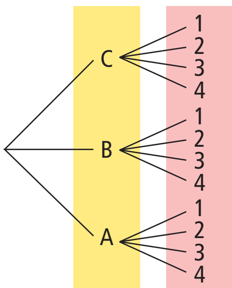
10. A birthday menu at Timmy's Taco Shop offers two drink choices and three kinds of taco. The diagram shows the possible combinations.



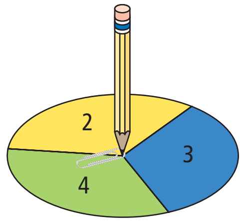
a) What is the sample space?

b) Create a table or tree diagram to organize all of the outcomes in a different way.

11. Describe two events that would result in this tree diagram.



12. A spinner with three equal sections is spun twice.



a) Create a tree diagram to show the sample space.

b) What is the product of each pair of outcomes?

c) What is the probability of having an odd product when you multiply the outcomes of the two spins?

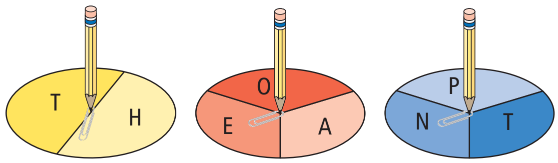
Extend

13. A coin is flipped three times. Either a head (H) or tail (T) appears face up.

a) Use a tree diagram to show the sample space of these three events.

b) What is the sample space?

14. Three spinners are divided into equal sections as shown. Each spinner is spun once.



a) Organize the outcomes.

b) Why did you use the type of organizer that you did?

c) Write the sample space. Circle all of the letter arrangements that form words.

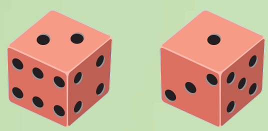
MATH LINK

1. You roll a pair of six-sided dice.

a) Use a tree diagram or table to show the sample space.

b) What sum appears most frequently?

c) What sum appears least frequently?



2. Repeat #1 with a pair of four-sided dice.

5.3

Probabilities of Simple Independent Events

Focus on...

After this lesson, you will be able to...

- ❑ solve probability problems involving two independent events

In the fairytale *Goldilocks and the Three Bears,* Goldilocks enters the bears' house while they are out. During her visit, she samples their porridge and their chairs.

Discuss the Math

How do you determine probabilities of simple independent events?

1. Draw a tree diagram in your notebook to organize all the possible combinations of porridge and chairs.

2. How many possible outcomes are there?

3. Goldilocks chooses the smallest porridge bowl and the smallest chair. How many favourable outcomes are there?

4. What fraction shows the probability that Goldilocks will choose, at **random**, the smallest porridge bowl and the smallest chair?

random

- an event in which every outcome has an equal chance of occurring

Reflect on Your Findings

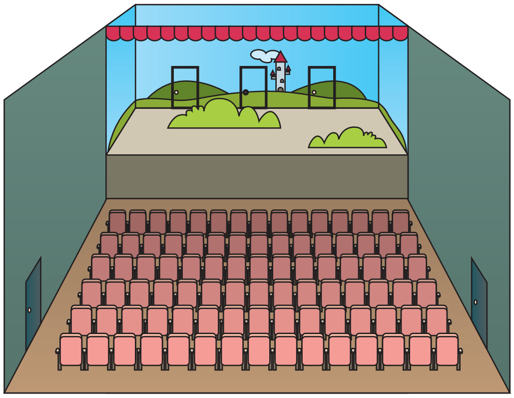
5. a) Do you think that Goldilocks really chose her favourite porridge and chair at random? Explain your answer.
   b) If Goldilocks did not choose at random, what is the probability of her choosing the smallest chair and smallest porridge? Discuss your opinion.

WWW Web Link

Stories change over time as they are told and retold by different people. To find examples of stories from different cultures with math in them, go to www.mathlinks7.ca and follow the links.

Example 1: Use a Tree Diagram to Determine Probabilities

A school gym has three doors on the stage and two back doors. During a school play, each character enters through one of the five doors. The next character to enter can be either a boy or a girl.



a) Draw a tree diagram to show the sample space.

b) What is P(boy, centre stage door)? Show your answer as a fraction and as a percent.

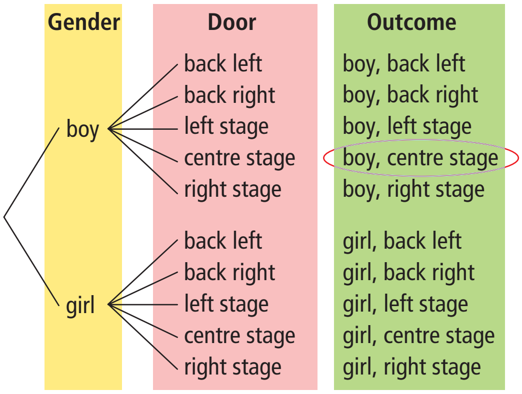
Solution

Strategies

Make an Organized List or Table

Refer to page xvii.

a)



b) There are 10 possible outcomes. There is 1 favourable outcome.

$$\text{Probability} = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$$

$$P(\text{boy, centre stage door}) = \frac{1}{10}$$
$$= 0.1$$
$$= 10\%$$

C 1 ÷ 10 × 100 = 10.

The probability of a boy entering through the middle door is $\frac{1}{10}$ or 10%.

Show You Know

a) Create a tree diagram to show all the possible outcomes when a coin is flipped and a spinner with five equal sections labelled *run*, *skip*, *jump*, *twirl*, and *twist* is spun.

b) What is the probability a student would flip a head and spin the spinner to land on *jump*?

Example 2: Use a Table to Determine Probabilities

A marble is randomly selected from a bag containing one blue, one red, and one green marble. Then, a four-sided die labelled 1, 2, 3, and 4 is rolled.

a) Create a table to show the sample space.

b) What is the probability of choosing any colour, and rolling any number but 3?

c) What is P(blue *or* green, a number greater than 1)?

d) What is P(black, 1)?

e) What is the probability that a red *or* green *or* blue marble is selected and the die displays a 4?

Strategies

Make an Organized List or Table

Refer to page xvii.

Solution

a)

		Die			
		1	**2**	**3**	**4**
Marble	**Blue (B)**	B, 1	B, 2	B, 3	B, 4
	Red (R)	R, 1	R, 2	R, 3	R, 4
	Green (G)	G, 1	G, 2	G, 3	G, 4

Literacy Link

You can use short forms of words in probability diagrams and tables. Here, blue, red, and green have become B, R, and G. You might make up your own abbreviations for an organizer, but write the full words for your final answers.

b) To find each probability, count the favourable outcomes and divide by the total number of outcomes.

$$\begin{aligned} P(\text{any colour, any number but 3}) &= \frac{9}{12} \\ &= 0.75 \\ &= 75\% \end{aligned}$$

C 9 ÷ 12 × 100 = 75.

c)
$$\begin{aligned} P(\text{blue } \textit{or} \text{ green, greater than 1}) &= \frac{6}{12} \\ &= 0.5 \\ &= 50\% \end{aligned}$$

C 6 ÷ 12 × 100 = 50.

d) There is no black marble.

$$\begin{aligned} P(\text{black, 1}) &= \frac{0}{12} \\ &= 0 \\ &= 0\% \end{aligned}$$

This is an impossible event.

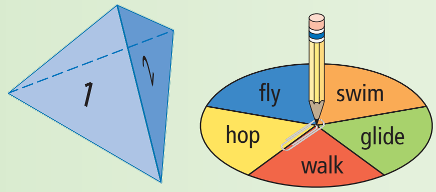
e)
$$\begin{aligned} P(\text{red } \textit{or} \text{ green } \textit{or} \text{ blue, 4}) &= \frac{3}{12} \\ &= 0.25 \\ &= 25\% \end{aligned}$$

C 3 ÷ 12 × 100 = 25.

Show You Know

A four-sided die is labelled 1, 2, 3, and 4 and a spinner is divided into 5 equal sections as shown.

a) Create a table to show all the possible outcomes when the die is rolled and the spinner is spun.

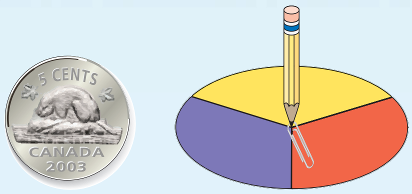


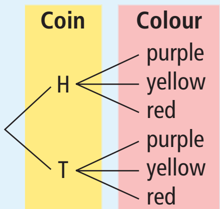
b) What is $P(3, \text{swim})$?

c) What is $P(\text{odd number, hop})$?

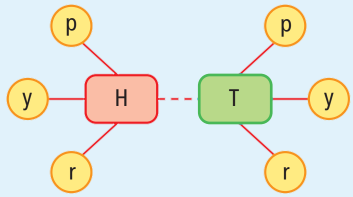
Key Ideas

- You can use a tree diagram, table, or other organizer to help determine probabilities.
- Count the favourable outcomes and divide by the total number of outcomes to find the probability.





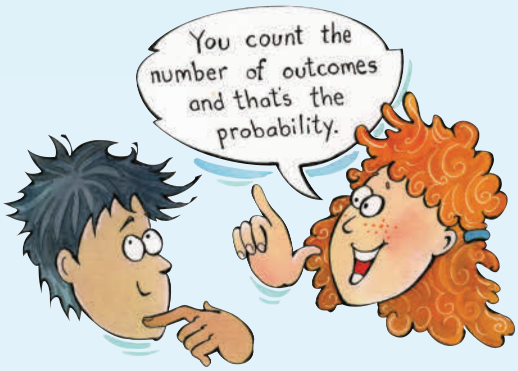
	Purple	**Yellow**	**Red**
Heads	H, purple	H, yellow	H, red
Tails	T, purple	T, yellow	T, red



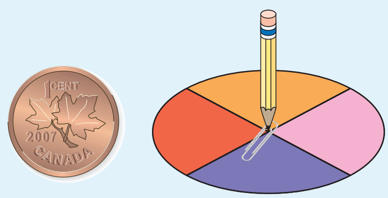
$P(\text{heads, purple}) = \frac{1}{6}$

Communicate the Ideas

1. Kimmy is explaining to Jason how to use a table to determine the probability of an event occurring.
   - **a)** Is Kimmy correct?
   - **b)** How could you improve on her explanation?



2. How would you explain to a classmate who missed today's class how to find the probability of a flipped penny landing with the maple leaf up and red or purple being spun on this spinner?



Practise

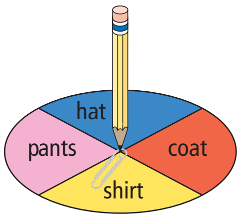
For help with #3 to #5, refer to Example 1 on page 172.

3. In a board game, a player flips a small card that says *back* on one side and *forward* on the other side. Then the player spins a 10-section spinner labelled 1 to 10 to see how many spaces to move on the board.

a) Draw a tree diagram to show the sample space.

b) What is the probability that the player will have to move 6 spaces back?

4. a) Draw a tree diagram to show the sample space for the coin and spinner.



b) What is P(H, hat *or* coat)?

5. a) Draw a tree diagram for flipping a card with an *A* on one side and a *B* on the other side and spinning a spinner with 5 equal sections labelled A, B, C, D, and E.

b) How many possible outcomes exist?

c) What is P(A, A)?

For help with #6 and #7, refer to Example 2 on page 173.

6. Joey randomly picks a marble from a bag containing one red, one green, one yellow, one purple, and one black marble and spins a spinner with five equal sections labelled 1, 2, 3, 4, and 5.

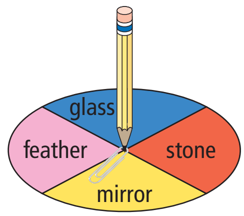
a) Create a table to organize the outcomes for these two events.

b) What is P(green, 1)?

c) What is P(yellow, 2 *or* 3)?

d) What is the probability of selecting a green marble and spinning a number that is less than 3?

7. Charlie randomly takes a block from the bag and spins the spinner.



a) Create a table or diagram to show the sample space.

b) What is P(black, stone)?

c) What is P(red *or* blue, mirror *or* glass)?

Apply

8. Mark keeps his shirts and shorts in separate drawers. He randomly pulls one piece of clothing out of each drawer.

a) How could you organize the possible outcomes? Show your method.

b) What is P(striped orange shirt, purple polka-dotted shorts)?

9. Greta flips a nickel and rolls a six-sided die.

a) Draw a table to organize the results.

b) What is P(H, 6)?

c) What is the probability of having the nickel land tails and rolling a number larger than 2?

10. How would you describe two events that might result in the eight outcomes in the following table?

H, 1	H, 2	H, 3	H, 4
T, 1	T, 2	T, 3	T, 4

11. Carlo flips two cards that are each black on one side and white on the other side. They land with either black or white facing up.

a) Draw a table to show the possible outcomes.

b) What is *P*(black, black)?

c) What is the probability that one card lands with white facing up and the other card lands with black facing up?

12. Two dice each have the words *raven, osprey, eagle, hawk, falcon*, and *crow* on them. Game players roll both dice at the same time.

a) Create a diagram or table to show the possible outcomes.

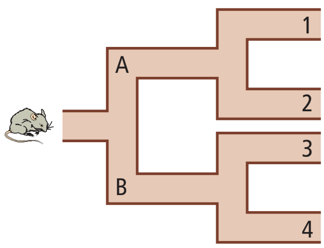
b) List the sample space.

c) What is *P*(raven, crow)?

d) What is *P*(eagle, eagle)?

e) What is the probability of rolling the name of a bird on both dice?

13. A mouse enters a maze and continues forward without turning back. The mouse is equally likely to travel along any pathway. His trip ends at 1, 2, 3, or 4.



a) What is the probability that the mouse takes path A?

b) What is the probability that the mouse takes path B and exits at 3?

c) Create a tree diagram that shows all possible outcomes.

d) What is *P*(A, 3)? Explain.

Extend

14. For sports day, each student will spin two spinners to find out their first and second activity.

a) Use the information in this table of outcomes to help draw the two spinners.

	Floor Hockey	Dodge Ball	Trampoline
Volleyball	v, fh	v, db	v, t
Basketball	b, fh	b, db	b, t
Softball	s, fh	s, db	s, t
Football	f, fh	f, db	f, t

b) Draw a different diagram to show the sample space.

c) Jen wants to play football and floor hockey. What is the probability she will get her wish?

d) What is the probability that Amir will get to play a ball game?

e) What is the probability that Suzi will get to spend time on the trampoline?

15. The last two digits of a phone number are smudged. Walter remembered that there was an even number followed by an odd number.

a) What is the sample space?

b) What is the probability that Walter will dial the number with the correct pair the first time?

c) The first smudged digit is either a six or an eight. List the new sample space. What is the new probability that Walter will dial the correct number the first time?

5.4 Applications of Independent Events

Focus on…

After this lesson, you will be able to…

- ❑ use tree diagrams, tables, and other graphic organizers to solve probability problems

In the game of Sit and Save, you try to collect more points than your opponents in five rounds of play.

- At the beginning of the round you stand up next to your chair.
- In each round, two dice are rolled. As long as a six does not appear on the face of either die, you may collect the sum of the numbers facing up.
- After each roll you must decide whether to continue standing, or to sit down and save all the points you have so far from that round.
- Each round ends when a six is rolled on one or both dice. If you are still standing when a six is rolled, you lose all of your collected points for that round.

Web Link

To play a similar game, called Piggy, on the computer, go to www.mathlinks7.ca and follow the links.

Explore the Math

Materials

- 2 six-sided dice

How can you win at the game of Sit and Save?

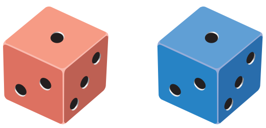
Here is a sample chart for a player named May.

Round 1	Round 2	Round 3	Round 4	Round 5
$3 + 4 = 7$	$1 + 2 = 3$	$1 + 4 = 5$	$3 + 4 = 7$	$2 + 1 = 3$
$3 + 5 = 8$	$2 + 3 = 5$	$3 + 3 = 6$	$5 + 2 = 7$	A six was rolled and May was still standing.
$1 + 2 = 3$	A six was rolled and May was still standing.	$5 + 5 = 10$	$4 + 5 = 9$	
$2 + 2 = 4$		Sat down	$2 + 4 = 6$	
Sat down			Sat down	
22	0	21	29	0

Game Total: $22 + 21 + 29 = 72$

1. What was the highest score May obtained in
   a) a single roll? **b)** a single round?

2. Play several games of Sit and Save with a group until you understand how frequently a six is rolled on either die.



3. Complete a table in your notebook to show all of the possible outcomes for rolling two dice.

4. Predict the probability of a six being rolled on either die in a single roll. What is the probability of a six on both dice?

WWW Web Link

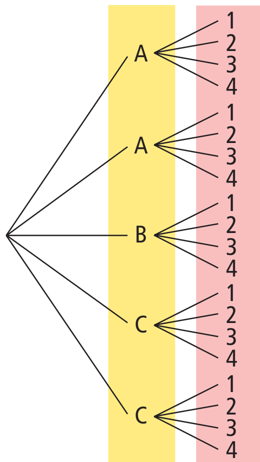
A guessing game called Lahal involves six-player teams that hide sets of bones in their hands. To learn more about this game, played by Aboriginal people on the west coast of Canada, go to www.mathlinks7.ca and follow the links.

Reflect on Your Findings

5. **a)** Explain a strategy to maximize your points in a game of Sit and Save.
   b) Test your strategy by playing the game again and report on how well you think it worked.

Example 1: Interpret Outcomes in a Tree Diagram

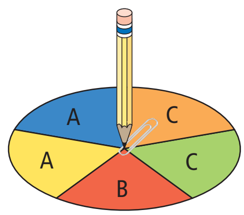
Look at the tree diagram.



a) Describe or draw a spinner and a die that would produce the possible outcomes shown.
b) What is $P(\text{B}, 2)$?
c) What is the probability of getting an A and a 3?
d) What is the probability of getting a C and a number less than 4?

Solution

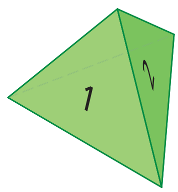
a) The tree diagram shows outcomes for something with 5 sections and something with 4 sections. A spinner divided into five equal sections and 4-sided die would work.



b) By counting the branches in the right column, there are 20 possible outcomes. All 20 outcomes are equally likely.

There is only 1 favourable outcome.

$$P(\text{B}, 2) = \frac{1}{20}$$
$$= 0.05$$
$$= 5\%$$



c) There are 2 favourable outcomes.

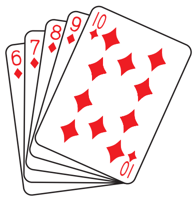
$$\begin{aligned} P(\text{A}, 3) &= \frac{2}{20} \\ &= 0.1 \\ &= 10\% \end{aligned}$$

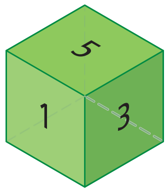
d) There are 3 numbers that are less than 4: 1, 2, and 3.
For each of these numbers, there are 2 possible regions labelled C.
By counting, there are 6 favourable outcomes.

$$\begin{aligned} P(\text{C, less than } 4) &= \frac{6}{20} \\ &= 0.3 \\ &= 30\% \end{aligned}$$

Example 2: Interpret Outcomes in a Table

A card is chosen at random and a die labelled 1 to 6 is rolled.





a) Organize the outcomes in a table.
b) What is the probability of getting only one 6?
c) What is the probability of getting at least one 6?
d) What is the probability of the two numbers having a sum of 10?
e) What is the probability of the two numbers having a sum of 10 or more?

Solution

a)

		Six-Sided Die					
		1	**2**	**3**	**4**	**5**	**6**
Number Cards	**6**	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6
	7	7, 1	7, 2	7, 3	7, 4	7, 5	7, 6
	8	8, 1	8, 2	8, 3	8, 4	8, 5	8, 6
	9	9, 1	9, 2	9, 3	9, 4	9, 5	9, 6
	10	10, 1	10, 2	10, 3	10, 4	10, 5	10, 6

b) There are 9 favourable outcomes: (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (7, 6), (8, 6), (9, 6), (10, 6).

$$\begin{aligned} P(\text{one } 6) &= \frac{9}{30} \\ &= 0.3 \\ &= 30\% \end{aligned}$$

C 9 ÷ 30 × 100 = 30.

(6, 6) is not included because it has two 6s.

c) There are 10 favourable outcomes: (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6), (7, 6), (8, 6), (9, 6), (10, 6).

$$P(\text{at least one } 6) = \frac{10}{30}$$
$$\approx 0.333333$$
$$\approx 33.3\%$$

C 10 ÷ 30 × 100 = 33.33333333

d) There are 4 favourable outcomes: (9, 1), (8, 2), (7, 3), (6, 4).

These outcomes form a diagonal line in the table.

$$P(\text{sum of } 10) = \frac{4}{30}$$
$$\approx 0.133333$$
$$\approx 13.3\%$$

C 4 ÷ 30 × 100 = 13.33333333

e) By counting, there are 24 favourable outcomes.

$$P(\text{sum of 10 or more}) = \frac{24}{30}$$
$$= 0.8$$
$$= 80\%$$

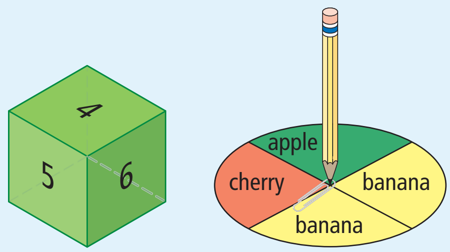
C 24 ÷ 30 × 100 = 80.

Key Ideas

- Tables and tree diagrams can be useful tools for organizing the outcomes of complex independent events.

Communicate the Ideas

1. Maggie rolls a die labelled 1 to 6 and spins the spinner.



a) Discuss the outcomes with a classmate. Before making any diagrams or tables, predict the probability of getting an even number and *banana*.

b) Create a table or diagram to show the sample space with all possible outcomes. Why did you choose the organizer that you did?

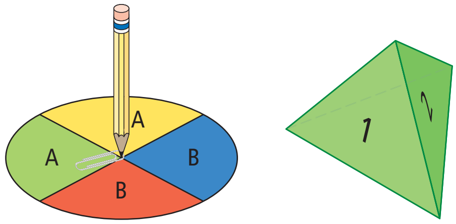
c) What is P(even, banana)? How close was your prediction to your calculation?

2. Make up a probability problem with two independent events. Explain how you know the events are independent. Trade with a friend and try to solve each other's problems.

Practise

For help with #3 and #4, refer to Example 1 on pages 178–179.

3. Annetta spins the spinner and rolls a four-sided die labelled 1 to 4.



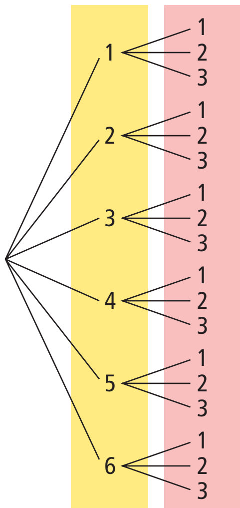
a) Create a tree diagram to organize the sample space.

b) What is P(A, 2)?

4. This tree diagram shows the outcomes when a die is rolled and a spinner is spun once.

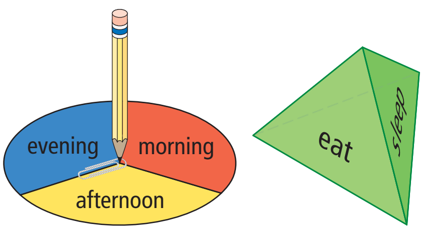
a) Draw a diagram of the die and the spinner.

b) What is the probability of a 2 appearing on both the die and the spinner?



For help with #5 and #6, refer to Example 2 on pages 179–180.

5. Maurice spins the spinner and rolls the four-sided die labelled *eat*, *work*, *play*, and *sleep*.

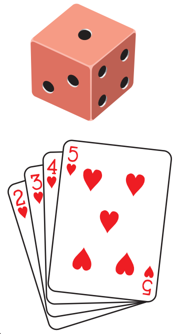


a) Use a table to organize the outcomes.

b) What is P(sleep, morning)?

c) What is P(eat or play, afternoon)?

6. The die is rolled and one card is chosen at random.



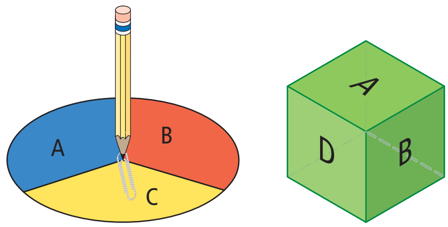
a) Draw a table to organize the outcomes.

b) What is the probability that the same number will appear on the die and the card?

c) What is the probability that the sum of the numbers is less than 6?

Apply

7. Margot spins the spinner and rolls the cube labelled A, B, C, D, E, and F.

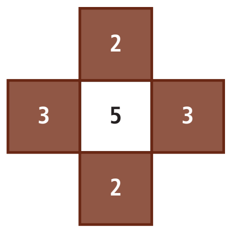


a) Create a tree diagram to organize the sample space.

b) What is the probability of spinning an A and rolling an A?

c) What is the probability of spinning and rolling the same letter?

8. Two darts are thrown and land randomly on the dart board.

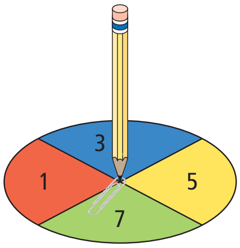


a) Draw a table to organize the outcomes.

b) What is the probability that the score will be the same for each throw?

c) What is the probability that the sum of the two numbers will be more than 5?

9. The following spinner is spun twice.



a) What is the probability that the sum of the numbers is even?

b) What is the probability that the product is even?

c) What is the probability that the positive difference between the two numbers is 2?

Extend

10. Lesley throws two 6-sided dice each labelled 1 to 6. What is the probability that

a) the first die is odd and the second die is even?

b) the first die is prime and the second die is composite?

c) the sum is greater than 6?

11. Monte has an MP3 player with only five songs on it. Two of these songs are the same song: "Pink Pants" by the band Western Canucks! He hits the shuffle option and listens to one song, then hits the shuffle option again and listens to a second song.

a) Organize the possible outcomes.

b) What is the probability that he hears "Pink Pants" twice in a row?

MATH LINK

Play the game Crunch Time with a partner or small group.

Step 1: Each player rolls one die. The player with the highest roll gets to choose a target sum from the Crunch Time game board.

Step 2: Take turns choosing numbers, one at a time, from the game board. Each player should print their initials on the game board at the end of the row of circles beside the chosen number.

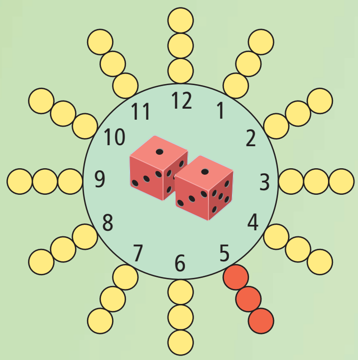
Step 3: Take turns rolling both dice. Add the numbers shown and place a coloured chip on the bubble beside the sum that is rolled. The player whose initials are beside the first sum to have all three bubbles covered is the winner.

Write a report explaining how probability affects who wins in Crunch Time. Include the following information:

- Which sum has the lowest probability of being rolled?
- Which sums have the highest probability of being rolled?
- What strategies might you use to increase your chances of winning Crunch Time? Explain why these strategies might work.

Materials

- integer chips or coins
- two dice
- Crunch Time gameboard



5.5

Conduct Probability Experiments

Focus on...

After this lesson, you will be able to...

- ❑ conduct a probability experiment and organize the results
- ❑ compare experimental probability with theoretical probability

Materials

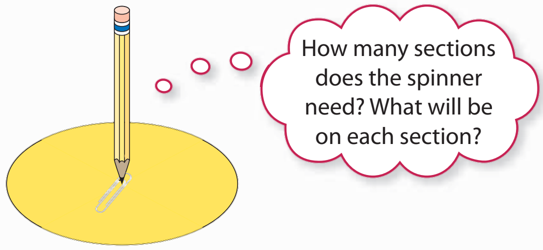
- paper clip
- compass or circular object to trace around

Katie is on the school volleyball team. The team's records show that Katie has a 75% or 3 in 4 or $\frac{3}{4}$ chance of successfully serving within the boundaries. What is the probability that she will make two successful serves in a row? How can you find out?

Explore the Math

How can you use experiments to test probabilities?

1. Katie is successful in 3 out of 4 serves. Create a spinner to show this.



2. How can you use the spinner to test how successful Katie is when she tries two serves in a row?

3. Use your spinner to test 10 sets of serves. Use a tally chart like this one to help you keep track of your results.

Trial	First Attempt (yes or no)	Second Attempt (yes or no)	Did she make both attempts? (yes or no)
1			
2			

4. What fraction of the 10 trials was successful on both attempts? Convert this fraction to a percent.

5. a) Use a tree diagram, table, or other organizer to show the sample space from spinning the spinner in #1 twice.

b) What is the probability of two successful attempts?

Reflect on Your Findings

experimental probability

- the probability of an event occurring based on experimental results

theoretical probability

- the expected probability of an event occurring

6. In #4 the **experimental probability** for two independent events is determined. In #5 the **theoretical probability** for the same two independent events is determined.

a) How do your experimental results compare with the theoretical results?

b) Compare your experimental results with those of several of your classmates. Are they the same or different? Explain why.

c) Compare your theoretical results with those of several of your classmates. Are they the same or different? Explain why.

Example 1: Compare Theoretical and Experimental Probability

At summer camp, the counsellors have created an obstacle course. Each camper must travel through the obstacle course one at a time. Halfway through the course is a fork in the path. Campers must choose to go either left or right. Halfway along each fork is another fork. Campers must again choose to go either left or right. Andrew flips a coin twice to model the possible choices of any camper. A head indicates left and a tail indicates right. The following chart shows the results for 100 pairs of coin flips.

Coin Outcomes	Outcomes	Experimental Results
head, head	two lefts	22
head, tail	left, right	24
tail, head	right, left	27
tail, tail	two rights	27

a) From the data, what is the experimental probability of taking two left turns?

b) What is the theoretical probability of taking two left turns?

c) Compare the experimental probability with the theoretical probability.

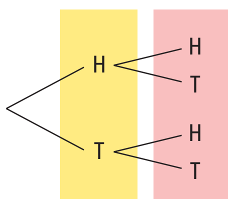
Solution

a) $P(2\text{ lefts}) = \frac{22}{100}$

$= 0.22$

$= 22\%$

b)



$P(2\text{ lefts}) = \frac{1}{4}$

$= 0.25$

$= 25\%$

c) $25\% > 22\%$. The theoretical probability is greater than the experimental probability.

Show You Know

Repeat the experiment in Example 1 using two coins that you flip 100 times. Use a tally chart to keep track of your results.

a) What is your experimental probability of making two right turns?

b) What is the theoretical probability of making two right turns?

c) Compare the experimental probability with the theoretical probability.

Example 2: Compare Experimental and Theoretical Probability Using Technology

A group of medical students wanted to determine the probability of having a girl and a boy in a two-child family. They used a random number generator to give them results for 20 families.

a) What is the experimental probability of getting children of two different genders?

b) What is the theoretical probability of getting children of two different genders?

c) Compare the experimental probability with the theoretical probability.

2 child families

	A	B	C
1	girl-0, boy-1	First Child	Second Child
2	Family 1	0	0
3	Family 2	0	1
4	Family 3	0	1
5	Family 4	0	0
6	Family 5	1	1
7	Family 6	1	1
8	Family 7	1	0
9	Family 8	1	1
10	Family 9	1	0
11	Family 10	1	1
12	Family 11	0	1
13	Family 12	1	0
14	Family 13	1	1
15	Family 14	1	1
16	Family 15	1	0
17	Family 16	0	1
18	Family 17	0	1
19	Family 18	1	0
20	Family 19	1	1
21	Family 20	1	0
22			

Sheet1 / Sheet2 / Sheet3

Tech Link

A random number generator on a computer or calculator can be used to generate a large number of outcomes for a probability experiment.

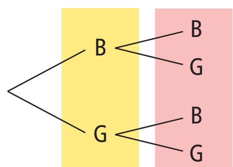
Solution

a) On the spreadsheet, a family with two different genders appears as either 0, 1 or 1, 0.

$$\begin{aligned}\text{Experimental } P(\text{boy and girl}) &= \frac{11}{20}\\ &= 0.55\\ &= 55\%\end{aligned}$$

C 11 ÷ 20 × 100 = 55.

b)



$$\begin{aligned}\text{Theoretical } P(\text{boy and girl}) &= \frac{10}{20}\\ &= 0.50\\ &= 50\%\end{aligned}$$

c) 55% > 50%. The experimental probability is greater than the theoretical probability.

Did You Know?

The ratio of boys to girls born in the world is hardly ever exactly 50%. Currently, the probability that a boy will be born is about 0.52.

Show You Know

Repeat the experiment in Example 2 using a random number generator to get results for 100 families.

a) What is your experimental probability of getting two boys?

b) What is the theoretical probability of getting two boys?

c) Compare the experimental and the theoretical probabilities.

Key Ideas

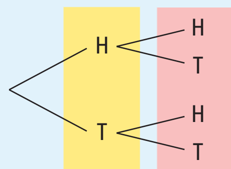
- The probability of an event determined from experimental outcomes is called experimental probability.

Flip two pennies 10 times.

- Experimental outcomes are usually collected in a tally chart and counted at the end of the experiment.

Coin Outcomes	Experimental Results	Number of Results
H, H	𝍷	2
H, T	𝍸 𝍷	6
T, H	𝍷	1
T, T	𝍷	1

- The probability of an event determined from a list of all possible outcomes is called theoretical probability.
- Experimental probability and theoretical probability are not always the same.



Experimental $P(\text{T, T}) = \frac{1}{10}$
$= 0.10$ or 10%

Theoretical $P(\text{T, T}) = \frac{1}{4}$
$= 0.25$ or 25%

Communicate the Ideas

1. Dhara flipped a coin 18 times and recorded the outcomes shown. Does the chart provide information about experimental probability or theoretical probability? Explain how you know.

Coin Outcomes	Experimental Results
H, H	𝍷𝍷𝍷𝍷
H, T	𝍸 𝍷𝍷
T, H	𝍷𝍷𝍷𝍷
T, T	𝍷𝍷𝍷

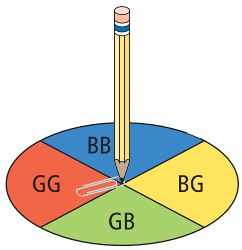
2. Explain the difference between experimental probability and theoretical probability.

3. Is it possible for experimental probability and theoretical probability to be the same? Justify your thinking.

Practise

For help with #4 and #5, refer to Example 1 on page 184.

4. Spencer uses a spinner to check the experimental probability of having a two-child family with two girls, two boys, a boy and then a girl, or a girl and then a boy.



He spins the spinner 100 times. Here are his results.

Spinner Outcome	Number of Results
GG	26
BB	27
BG	22
GB	25

a) What does BB represent?

b) What is the experimental probability of a family having two girls?

c) Calculate the theoretical probability that a family has two girls.

5. Spencer continues to analyse his experimental outcomes.

a) What is the experimental probability of a family having two boys?

b) Calculate the theoretical probability that a family has two boys.

c) Compare the experimental probability and theoretical probability.

For help with #6 and #7, refer to Example 2 on page 185.

6. The captain of a baseball team wants to determine the probability of getting two heads on two coin flips. She uses a random number generator to get results for 20 pairs of coin flips.

	A	B	C	D
1	head-1, tail-0	First Flip	Second Flip	
2	First 2 flips	0	0	
3	Second 2 flips	0	0	
4	Third 2 flips	1	1	
5	Fourth 2 flips	1	1	
6	Fifth 2 flips	0	1	
7	Sixth 2 flips	0	0	
8	Seventh 2 flips	0	0	
9	Eighth 2 flips	1	1	
10	Ninth 2 flips	0	1	
11	Tenth 2 flips	1	0	
12	Eleventh 2 flips	1	1	
13	Twelfth 2 flips	1	1	
14	Thirteenth 2 flips	0	1	
15	Fourteenth 2 flips	0	0	
16	Fifteenth 2 flips	1	0	
17	Sixteenth 2 flips	0	1	
18	Seventeenth 2 flips	0	0	
19	Eighteenth 2 flips	1	1	
20	Nineteenth 2 flips	0	1	
21	Twentieth 2 flips	1	1	
22				

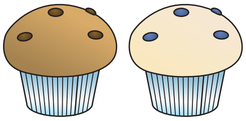
Sheet1 / Sheet2 / Sheet3

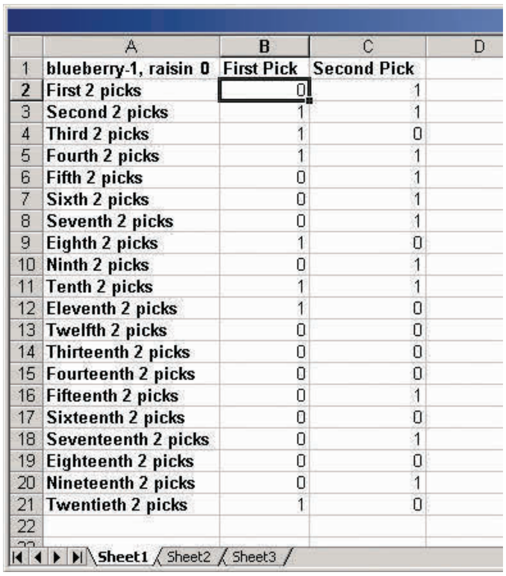
a) What is the experimental probability of getting two heads?

b) What is the theoretical probability of getting two heads?

c) Compare the experimental probability with the theoretical probability.

7. Grandpa has baked muffins to share. He puts a bran raisin muffin and a blueberry muffin in one bag, and then a bran raisin muffin and a blueberry muffin in a second bag. You get to pick one muffin from each bag. Use a random number generator to check the probability of picking two different muffins.





	A	B	C	D
1	blueberry-1, raisin 0	First Pick	Second Pick	
2	First 2 picks	0	1	
3	Second 2 picks	1	1	
4	Third 2 picks	1	0	
5	Fourth 2 picks	1	1	
6	Fifth 2 picks	0	1	
7	Sixth 2 picks	0	1	
8	Seventh 2 picks	0	1	
9	Eighth 2 picks	1	0	
10	Ninth 2 picks	0	1	
11	Tenth 2 picks	1	1	
12	Eleventh 2 picks	1	0	
13	Twelfth 2 picks	0	0	
14	Thirteenth 2 picks	0	0	
15	Fourteenth 2 picks	0	0	
16	Fifteenth 2 picks	0	1	
17	Sixteenth 2 picks	0	0	
18	Seventeenth 2 picks	0	1	
19	Eighteenth 2 picks	0	0	
20	Nineteenth 2 picks	0	1	
21	Twentieth 2 picks	1	0	
22				

Sheet1 Sheet2 Sheet3

a) What is the experimental probability of picking a muffin of each type?

b) What is the theoretical probability of picking a muffin of each type?

c) Compare the experimental probability with the theoretical probability.

Apply

8. a) Build a spinner and conduct the probability experiment described in #4. Organize your results in a tally chart.

b) According to your results, what is the experimental probability of having a family with two children of different genders?

c) What is the theoretical probability of having this type of family?

d) Compare the experimental probability and theoretical probability.

9. Scientists are working with a parrot. The parrot knows it has to push one button on the left and then one button on the right to get into the food bin. To open the door, the parrot has to push C, 2. What are the chances of the parrot choosing these buttons randomly?

Scientists collect the experimental results for the first 50 tries in a tally chart.

	1	2
S	|	||
N	卌	||||
A	卌 |||| |	|||
C	卌 |	卌 ||
K	卌 ||	||||

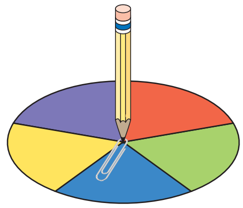
a) The correct combination is C, 2. How many times did the parrot open the food bin?

b) What is the experimental probability of selecting C, 2?

c) Show the sample space for this probability event. What is the theoretical probability of selecting C, 2?

d) Compare the experimental and theoretical probability. What might this suggest?

10. **a)** Use a coin and a spinner, to redo the experimental outcomes for #9. Show your results in a tally chart.



b) The correct combination is C, 2. According to your experimental data, how many times did the parrot get food?

c) What is your experimental probability of selecting C, 2?

d) Compare your experimental probability and the theoretical probability.

11. Around the campfire at summer camp, each camper has to flip a coin twice. The chart below tells what their rolls mean. Bianca, although really funny, is a poor singer. Everyone is hoping she will get two poems to recite instead of two songs to sing.

Coin Outcome	Outcome	Number of Results
H, H	recite two poems	125
H, T	recite a poem, then sing a song	130
T, H	sing a song, then recite a poem	140
T, T	sing two songs	135

a) The table shows the results for the past five years of campers. What is the experimental probability that Bianca will have to recite two poems?

b) What is the theoretical probability of Bianca getting two poems?

c) The campers think that the second-best option would be for Bianca to flip a head first, and then a tail. They will stay to hear her poem, but might be able to leave for her song. What is the experimental probability that Bianca will flip this combination?

Extend

12. **a)** Use a random number generator to redo the experimental outcomes for #11.

b) According to your outcomes, what is the experimental probability of Bianca singing at least one song?

c) What is the theoretical probability of her singing at least one song?

d) Compare the experimental probability and the theoretical probability.

e) How can you use the information from b) and c) to determine the probability of Bianca reciting at least one poem? Explain.

13. **a)** As a group, brainstorm ways you might get the experimental results in #12 closer to the theoretical results.

b) Try some of your ideas and see if they work.

WWW Web Link

Computers are used in many different ways to study probabilities. For links to various probability experiments, go to www.mathlinks7.ca and follow the links.

5 Chapter Review

Key Words

For #1 to #3, copy the statement and fill in the blanks. Use some of these words.

experimental	*favourable*	*independent*
possible	*random*	*sample space*
theoretical	*tree diagram*	

1. Probability is the number of ▮▮▮▮▮▮▮▮▮▮ outcomes divided by the number of ▮▮▮▮▮▮▮▮ outcomes.

2. The probability of an event occurring based on experimental results is called ▮▮▮▮▮▮▮▮▮▮▮▮ probability.

3. A ▮▮▮▮ ▮▮▮▮▮▮▮ and a table are two ways of organizing outcomes.

4. Rearrange the circled letters in #1 to #3 to find one of the remaining key words. Define this word.

5.1 Probability, pages 158–164

5. A tool box contains three screwdrivers and two wrenches. An electrician's helper chooses a tool at random. What is the probability she has grabbed a wrench? Write your answer as a fraction, a ratio, and as a percent.

6. Melée chooses a card at random from the following set. Write each probability as a fraction, a ratio, and a percent.

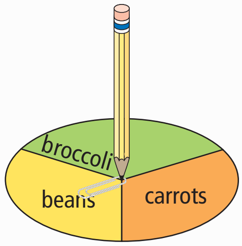
a) What is P(red)? **b)** What is P(3 *or* 4)?

c) What is the probability of choosing a number that is a multiple of 3?

d) What is P(less than 7)?

5.2 Organize Outcomes, pages 165–170

7. A chef tapes the word *salad* to one side of a coin and *cooked* to the other side. He makes a spinner with regions for broccoli, carrots, and beans. He flips the coin once and spins the spinner once to choose the vegetable for the night's special at Café Chef. List the sample space for this experiment.



8. At a restaurant, Carrie decides to close her eyes and randomly point to one dinner and one dessert on the menu.

a) What is the sample space? Draw a diagram or table that shows all of the possible combinations.

b) How many dinner and dessert combinations are possible?

5.3 Probabilities of Simple Independent Events, pages 171–176

9. A coin is flipped twice.

a) What is the sample space? Draw a tree diagram to show the possible outcomes.

b) What is the probability of flipping two tails?

10. A tool box contains a hammer, a screwdriver, a pair of pliers, and a tape measure. A pail contains 1 nail, 4 screws, and 2 hooks. You randomly choose one item from the tool box and one item from the pail.

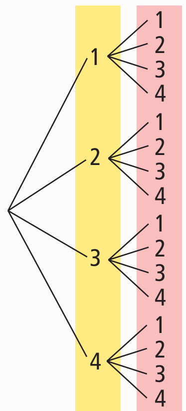
a) Create a table to organize the possible outcomes.

b) What is the probability of getting a hammer from the tool box and a nail from the pail ?

c) What is P(tape measure, screwdriver)?

5.4 Applications of Independent Events, pages 177–182

11. a) Describe or draw two possible independent events that could be represented by this tree diagram.

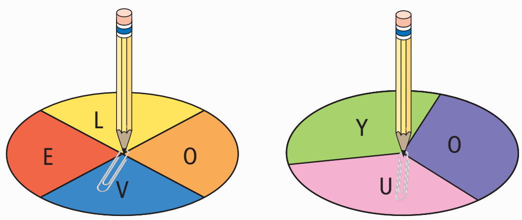


b) What is the probability that both numbers are the same?

c) What is the probability that the sum of the numbers is 5?

d) What is the probability that the number for the first event is smaller than the number for the second event?

12. Each spinner is spun once.



a) Draw a tree diagram to organize the sample space.

b) What is the probability of getting two letter Os?

c) What is the probability that the letter will *not* be the same on both spinners?

5.5 Conduct Probability Experiments, pages 183–189

13. A spinner with 4 equal regions labelled A, B, C, D is spun 20 times. The following tally chart shows the experimental outcomes. Write any probabilities in fraction form.

A	B	C	D
IIII	卌 I	卌 II	III

a) From the tally chart, what is the experimental probability of spinning C?

b) What is the theoretical probability of spinning C?

c) Explain why the answers for a) and b) are *not* the same.

14. Anya is tossing a red and white algebra tile. She wants to determine the probability of getting two different colours on two tile flips. She uses a random number generator to get results for 20 pairs of flips.

2 tile flips

	A	B	C	D
1	red-1, white-0	First Flip	Second Flip	
2	First 2 flips	0	1	
3	Second 2 flips	1	0	
4	Third 2 flips	0	0	
5	Fourth 2 flips	1	1	
6	Fifth 2 flips	1	1	
7	Sixth 2 flips	1	1	
8	Seventh 2 flips	1	1	
9	Eighth 2 flips	1	1	
10	Ninth 2 flips	0	0	
11	Tenth 2 flips	1	1	
12	Eleventh 2 flips	1	1	
13	Twelfth 2 flips	0	1	
14	Thirteenth 2 flips	1	1	
15	Fourteenth 2 flips	1	0	
16	Fifteenth 2 flips	1	0	
17	Sixteenth 2 flips	1	0	
18	Seventeenth 2 flips	1	0	
19	Eighteenth 2 flips	1	0	
20	Nineteenth 2 flips	1	1	
21	Twentieth 2 flips	0	0	
22				

Sheet1 / Sheet2 / Sheet3

a) What is the experimental probability of getting a red and a white, in any order?

b) What is the theoretical probability of getting a red and a white, in any order?

c) Compare the experimental probability with the theoretical probability.

5 Practice Test

For #1 to #5, choose the best answer.

A bag contains 10 balls. One ball is chosen at random. Use the diagram to answer #1 to #3.

1. What is the probability that a Fire ball is chosen?

A $\frac{1}{5}$ **B** $\frac{2}{5}$ **C** $\frac{1}{10}$ **D** $\frac{3}{10}$

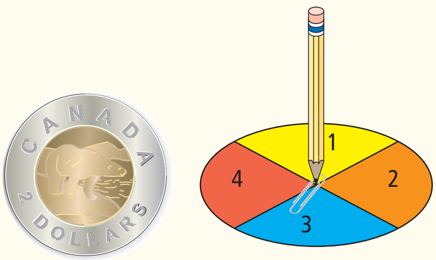
2. What is the probability that an Air or Water ball is chosen?

A $\frac{7}{10}$ **B** $\frac{2}{5}$ **C** $\frac{1}{5}$ **D** $\frac{1}{2}$

3. What is the probability that an Earth ball is *not* chosen?

A 10% **B** 40% **C** 60% **D** 90%

Use this diagram to answer #4 and #5.



4. The coin is flipped once. The spinner is spun once. What is the total number of possible outcomes?

A 2 **B** 4 **C** 6 **D** 8

5. The following tally chart shows the results of a probability experiment for 20 spins of the spinner. Which number has a higher experimental probability than would be expected?

1	2	3	4
IIII	𝍸 II	𝍸	IIII

A 1 **B** 2 **C** 3 **D** 4

Short Answer

6. The A-Plus company will print yellow, green, and orange T-shirts with the day of the week. Draw a tree diagram to display the possible outcomes.

7. Customers at Fresh Wrap Restaurant can choose a single item from each section of the menu. Create a table to show the possible outcomes.

Menu

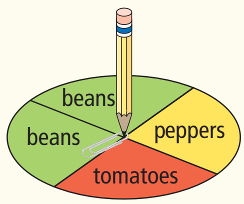
Drinks

Milk
Apple Juice

Meals

Chicken Wrap
Cheese Pizza
Caesar Salad

8. The following spinner is spun twice.



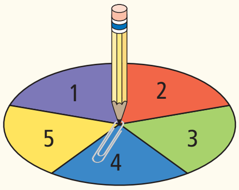
a) Are the outcomes of the spinner equally likely? Explain.

b) Create a tree diagram that shows the possible outcomes.

c) What is the probability that *beans* is spun both times?

d) What is the probability that the same vegetable is not repeated on the second spin?

9. A spinner with 5 equal regions is spun twice.



a) Create a table to show the sample space.
b) What is the probability that the same number appears on both spins? Write your answer as a fraction and as a percent.
c) What is the probability that the sum of the two spins is 3? Write your answer as a percent.
d) What is the probability that the first spin is a larger number than the second spin? Write your answer as a fraction.

Extended Response

10. Anthony has been fouled in a basketball game and will now have two free throws. The team statistics show that he has a 4 in 5 chance of making both free throws.
a) How could you use a spinner to show the possibility of Anthony making both free throws?
b) Develop a table of the possible outcomes. Circle the favourable outcomes.
c) Star used a spinner to check the experimental probability of Anthony making both free throws. She did 100 trials. Her results showed Anthony succeeding 75 times. Show this experimental probability as a percent.
d) Determine the theoretical probability of Anthony making both free throws.
e) Compare the experimental probability and the theoretical probability.

WRAP IT UP!

Work with a partner to create your own simple game that uses a pair of dice.

Play your game several times.

Write a report about your game including the following information:

- What are the rules for your game?
- What are all the possible outcomes?
- How does a player win the game?
- What probabilities are important to know? Justify your response.
- Compare the theoretical probabilities of the game to the experimental probabilities you experienced while playing.
- What strategies might you use to increase your chances of winning? Justify your response.

Math Games

Is It a Match?

In a fair game of chance, each player has an equal probability of winning. If there are two players, the probability of each player winning is $\frac{1}{2}$. This can also be written as 1 : 2 or 50%. Play the game Match or No Match with a partner to find out if this game is fair.

Materials

- paper bag
- red and blue counters of the same shape and equal sizes

1. a) Read the following rules. Before you start playing, decide who will be the match player and who will be the no match player.
- One player places two red counters and one blue counter in a paper bag. The other player shakes the bag to mix the counters.
- One player takes a counter from the bag without looking. Then the other player takes one of the remaining counters from the bag without looking.
- Compare counters. If the colours match, the match player gets a point. If the colours do not match, the no match player gets a point.
- Return the counters to the bag and play another round. The first player to reach 10 points wins.

b) Play the game and see who wins.

2. a) Use a tree diagram to record the possible outcomes for one round of Match or No Match.

b) What is the probability of the match player winning a point?

c) What is the probability of the no match player winning a point?

d) Is the game fair? Explain.

3. a) Could the game be fair if there were only two counters in the bag? Explain.

b) Could the game be fair if there were four counters in the bag? Explain using a tree diagram.

4. a) Modify the game to make it fair. Then play the game several times and see who wins.

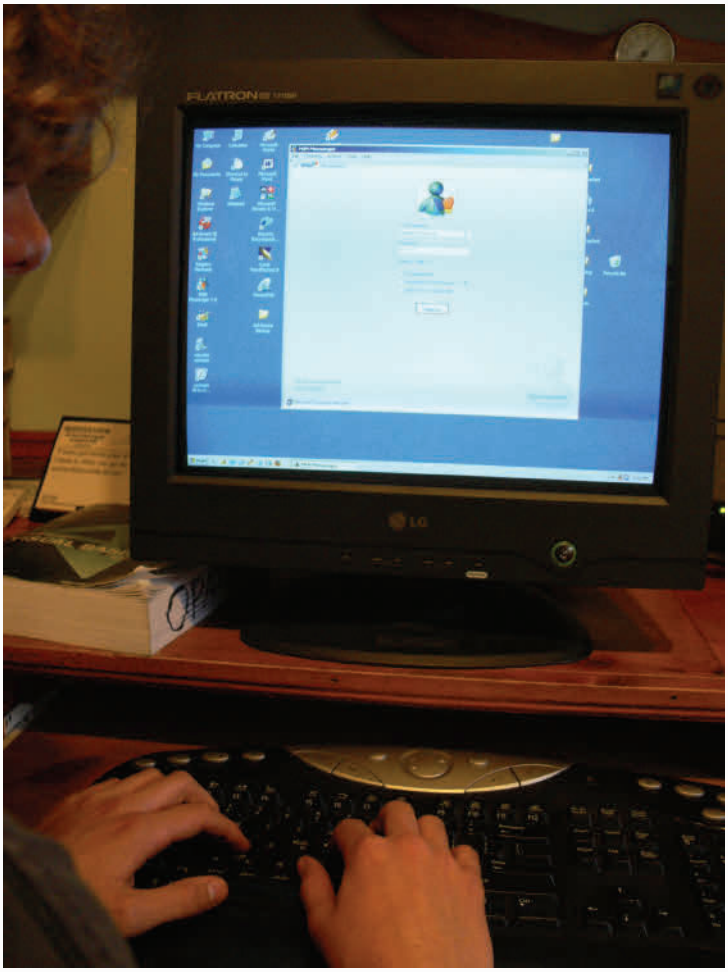
b) Suppose you play the fair game a very large number of times. How should the number of wins of the two players compare? Explain.

Challenge in Real Life

Crack the Code

Most computer accounts require people to have a password to ensure privacy and security.

Be the computer analyst!
Work in a group to develop a new password system for your class's computer network. The system you create will enable students to choose a password with two characters. Here are three possible systems:



- **System 1:** Students pick one letter from A to J and one number from 1 to 3.
- **System 2:** Students pick a 2-digit number from 10 to 45.
- **System 3:** Students use their grade level in school and then a letter from A to Z.

a) For each system, discuss the following questions with your group:
- What is the sample space?
- What is the probability of guessing a password in one try if you are aware of how the system works?

b) Which system do you recommend? Give at least one reason why you prefer this system.

c) Create a system of your own that maximizes security for the students and minimizes the memory required. Limit the sample space to no more than 36 outcomes. Show the sample space and the probability of guessing the password in one try.