7 Trigonometry

Jacob wants to purchase a piece of land. The land is in the shape of a triangle. The price is based on the area of the land. Jacob has hired a surveyor to help him determine the area.

1. a) What measurements will the surveyor need to create a scale diagram of the land?

b) Will the surveyor need to take each measurement described in part a), or are there any measurements that can be determined using other measurements?

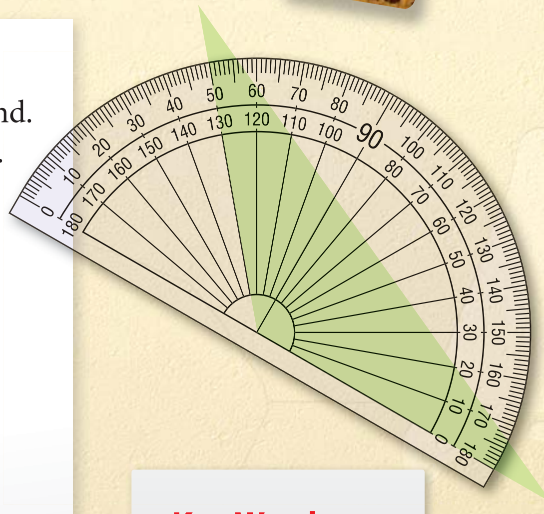
c) What role will precision and accuracy play in the surveyor's task?

2. a) What measuring tools will the surveyor need?

b) What does each tool measure?

3. a) How will the surveyor calculate the area of the land?

b) What unit of measurement should be used to calculate the area? Why?

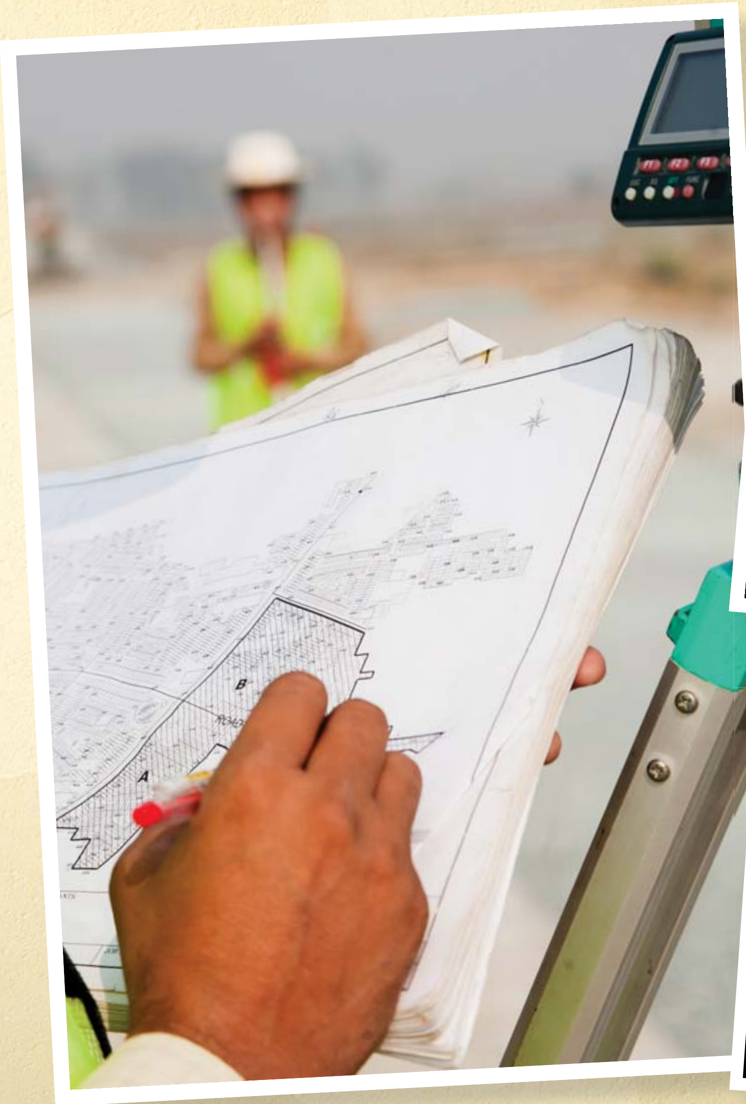


Key Words

oblique triangle
acute triangle
obtuse triangle
sine law
cosine law

Career Link

Andrea is a land surveyor. She uses measuring instruments to find or retrace the official boundaries of land for legal or governmental purposes. She works as part of a group called a survey party. Surveyors measure distances, angles, and directions. They also draw diagrams, plans, and maps.



7 Get Ready

Trigonometric Ratios

1. Evaluate each trigonometric ratio. Express your answer to three decimal places.

> Make sure your calculator is in degree mode.
>
> Use this key sequence to calculate sin 45°.
>
> Press **C** **SIN** **45** **=**.
>
> Your calculator may use slightly different key strokes. Experiment to find out which ones work.

a) sin 37° **b)** cos 63°

c) tan 78° **d)** cos 8°

e) tan 40° **f)** sin 62°

2. Use each trigonometric ratio to determine the size of the angle, to the nearest degree.

> Make sure your calculator is in degree mode.
>
> Use this key sequence to calculate $\sin^{-1}(0.408)$.
>
> Press **C** **2nd** **SIN^{-1}** **0.408** **=**.

a) $\sin X = 0.602$

b) $\tan W = 0.309$

c) $\cos B = 0.951$

d) $\sin V = 0.978$

e) $\tan A = 2.246$

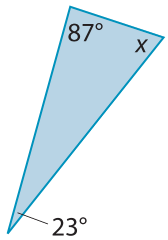
f) $\cos P = 0.445$

Unknown Angles in Triangles

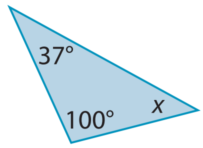
3. Determine the size of $\angle x$.

> Sum of angles in a triangle = 180°

a)

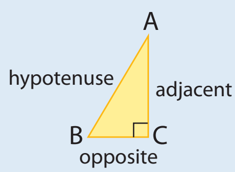


b)

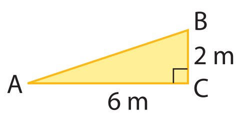


4. Use trigonometric ratios to determine $\angle$A, to the nearest degree.

> $\tan A = \dfrac{\text{opposite}}{\text{adjacent}}$
>
> $\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}}$
>
> $\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}}$



a)



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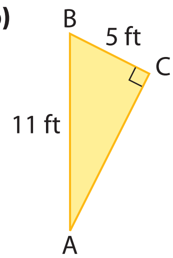
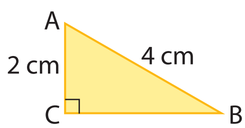


c)



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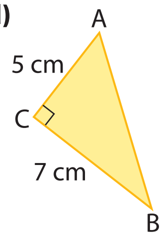
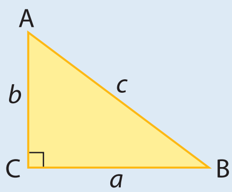


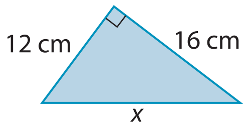
Unknown Side Lengths in Right Triangles

5. Determine the length of side x, to the nearest tenth of a unit.

The Pythagorean relationship is $c^2 = a^2 + b^2$, where c is the hypotenuse and a and b are the legs.



a)



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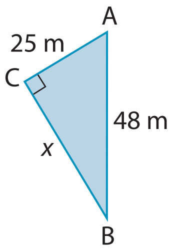

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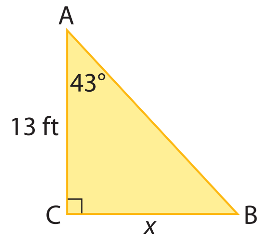
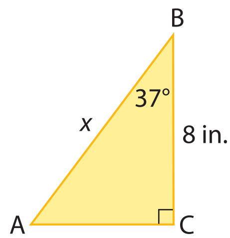
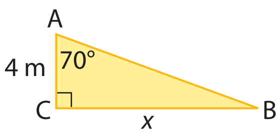


d)

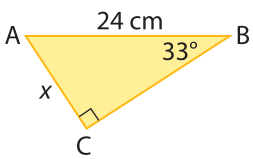


6. Use trigonometric ratios to determine the length of side x, to the nearest tenth of a unit.

a)



b)



Proportions

7. Solve for each unknown quantity. Round to two decimal places, if necessary. Estimate to check your answer.

Example:

$$\frac{3}{17} = \frac{15}{x}$$

$$(17x)\frac{3}{17} = (17x)\frac{15}{x}$$

$$3x = 255$$

$$\frac{3x}{3} = \frac{255}{3}$$

$$x = 85$$

The answer 85 makes sense because $17 \approx 6 \times 3$ and $85 \approx 6 \times 15$.

a) $\dfrac{5}{20} = \dfrac{24}{x}$

b) $\dfrac{11.58}{2} = \dfrac{x}{30}$

c) $\dfrac{16}{2.2} = \dfrac{x}{3.4}$

d) $\dfrac{7.8}{25} = \dfrac{9.1}{x}$

7.1 The Sine Law

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Focus On . . .

- recognizing and describing an oblique triangle
- applying the sine law
- explaining how to use the sine law
- describing how the sine law is used in problem situations

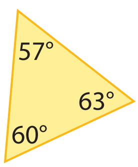
Bishop's Mitre is a mountain in northern Labrador. It has an elevation of 1113 m (3652 ft). To find the height of a mountain, surveyors use a theodolite, which is a tool or instrument that measures angles. Once surveyors know the measure of certain angles and distances, they can use trigonometry to calculate other angles and distances that cannot be measured or are difficult to measure.

oblique triangle

- a triangle that does not contain a right angle; it can be acute or obtuse

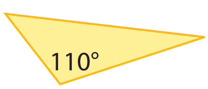
acute triangle

- a triangle with three acute interior angles (between 0° and 90°)



obtuse triangle

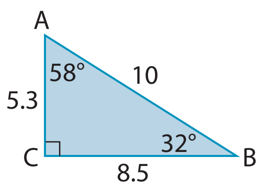
- a triangle with one obtuse interior angle (between 90° and 180°)



Explore the Sine Law

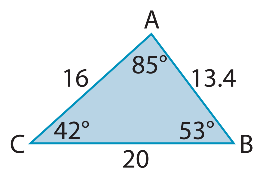
Use what you know about right triangles to develop a rule, or law, that can be used to determine unknown sides and angles in an **oblique triangle**, whether it is an **acute triangle** or an **obtuse triangle**.

1. a) Calculate sin A, sin B, and sin C for the right triangle shown. Round to the nearest hundredth, if necessary.



$\sin \theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$

b) Repeat part a) for the oblique triangle shown.



c) Copy and complete a chart like the one below for the two triangles.

Triangle	a	b	c	sin A	sin B	sin C	$\frac{a}{\sin A}$	$\frac{b}{\sin B}$	$\frac{c}{\sin C}$
Right									
Oblique									

Side a is opposite ∠A.
Side b is opposite ∠B.
Side c is opposite ∠C.

Strategy

Look for a Pattern

sine law

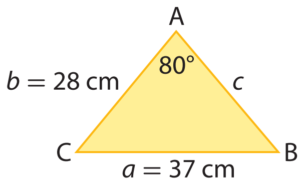
- the sides of a triangle are proportional to the sines of the opposite angles

2. Reflect Examine the values in the chart.

a) What do you notice about the three ratios, $\frac{a}{\sin A}$, $\frac{b}{\sin B}$, and $\frac{c}{\sin C}$?

b) The relationship you described in part a) is called the **sine law**. How could you represent the relationship mathematically?

c) Could the sine law be used to determine side length c in △ABC below? Explain.

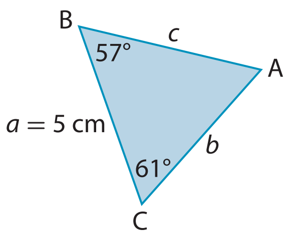


d) Could the sine law be used to determine ∠B? Explain.

3. Extend Your Understanding Use what you have learned about the ratios $\frac{a}{\sin A}$, $\frac{b}{\sin B}$, and $\frac{c}{\sin C}$ to determine each side length in △ABC below.

a) side length b

b) side length c

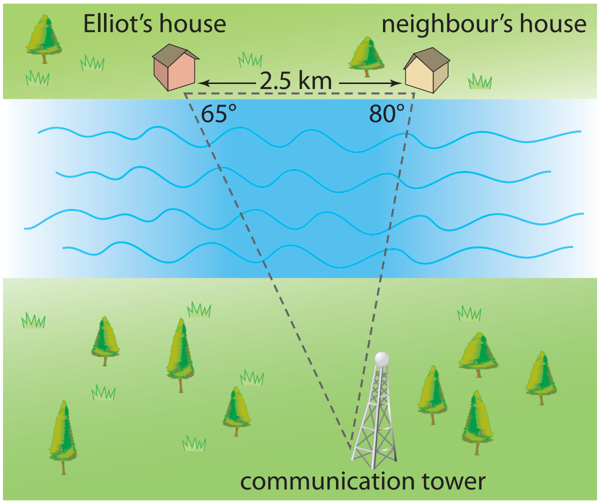


How can you use what you know about the sum of the angles in a triangle to help you?

On the Job 1

Determine a Side Length Using the Sine Law

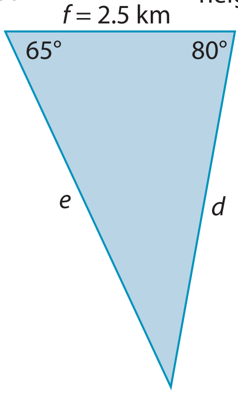
Elliot's family owns a house on the Gander River in Newfoundland. Their closest neighbour's house is 2.5 km away. A surveyor uses a theodolite to measure the angles between the houses and a nearby communication tower, as shown in the diagram. Use the sine law to determine the distance from Elliot's house to the tower, to the nearest tenth of a kilometre.



Solution

The locations of Elliot's house, the neighbour's house, and the tower form the vertices of an oblique triangle with two known angles and one known side length.

Draw a diagram to model the situation.



Tools of the Trade

A theodolite is a precision tool used by surveyors to measure horizontal and vertical angles. A transit is a specialized and more convenient type of theodolite that does not measure to the same level of precision.

To find out more about the tools used by surveyors, go to www.mcgrawhill.ca/books/mathatwork12 and follow the links.

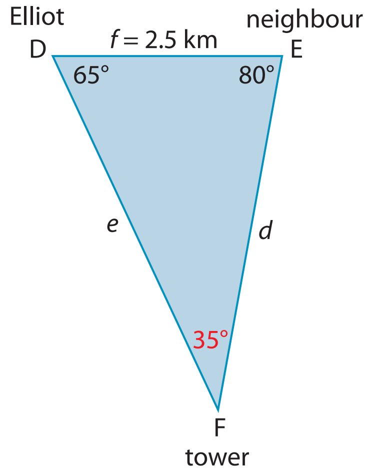
Strategy

Draw or Model

Calculate the unknown angle, $\angle F$:

$\angle F = 180° - 80° - 65°$
$= 35°$

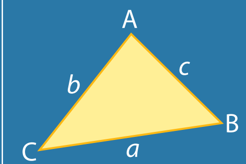
Sum of angles in a triangle $= 180°$



F.Y.I.

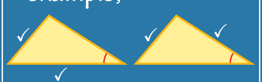
The version of the sine law used to find an unknown side length is

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

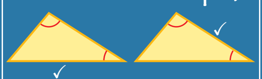


You can use the sine law to determine an unknown side length in two situations:

- when two sides and an angle opposite one of them (SSA) are known. For example,



- when two angles and any side (AAS) are known. For example,



Choose a pair of ratios from the sine law, $\frac{d}{\sin D} = \frac{e}{\sin E} = \frac{f}{\sin F}$, to solve for e.

What pair of ratios has e as the only unknown in the proportion?

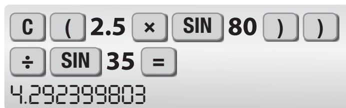
$$\frac{e}{\sin E} = \frac{f}{\sin F}$$

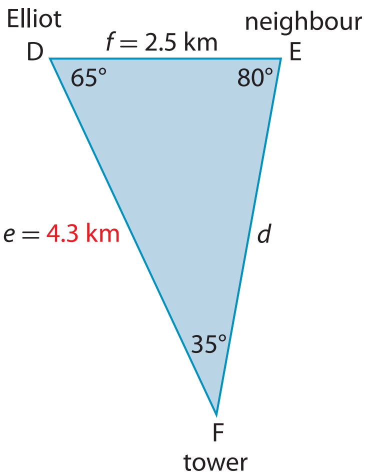
$$\frac{e}{\sin 80°} = \frac{2.5}{\sin 35°}$$

$$(\sin 80°)\left(\frac{e}{\sin 80°}\right) = (\sin 80°)\left(\frac{2.5}{\sin 35°}\right)$$

$$e = \frac{2.5(\sin 80°)}{\sin 35°}$$

$$e = 4.293\ldots$$





The answer 4.3 km makes sense because I can see that the distance from Elliot's house to the tower is less than twice the distance between the houses, which is 2×2.5 km $= 5$ km.

The distance from Elliot's house to the tower is approximately 4.3 km.

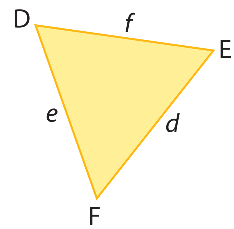
Your Turn

Determine the distance from the neighbour's house to the tower, to the nearest tenth of a kilometre. Estimate to check your answer.

Check Your Understanding

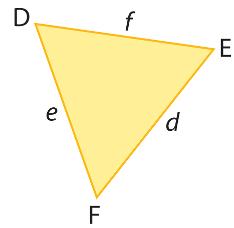
Try It

1. Use the sine law to write two ratios that are equivalent to $\dfrac{d}{\sin \text{D}}$ in $\triangle$DEF.



2. Imagine that you are going to use the sine law to determine each side length listed below in $\triangle$DEF. Which of the following measurements would you need to know?

Choose from side lengths d, e, and f, and angle measures $\angle$D, $\angle$E, and $\angle$F



a) side length d

b) side length e

c) side length f

3. Solve for each unknown side, to the nearest tenth of a unit.

a) $\dfrac{a}{\sin 28°} = \dfrac{20 \text{ cm}}{\sin 50°}$

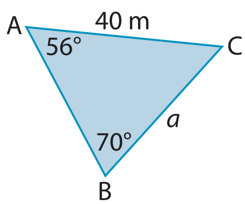
b) $\dfrac{65 \text{ m}}{\sin 73°} = \dfrac{c}{\sin 45°}$

c) $\dfrac{a}{\sin 34°} = \dfrac{44 \text{ in.}}{\sin 88°}$

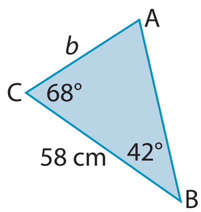
d) $\dfrac{31 \text{ ft}}{\sin 62°} = \dfrac{c}{\sin 45°}$

4. Determine the length of each side, to the nearest tenth of a unit. Estimate to check your answer.

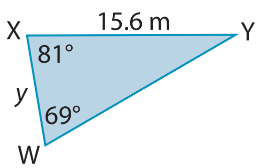
a) side length a



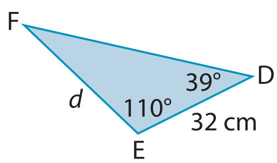
b) side length b



c) side length y



d) side length d



Tools of the Trade

Think about how these occupations use triangles:

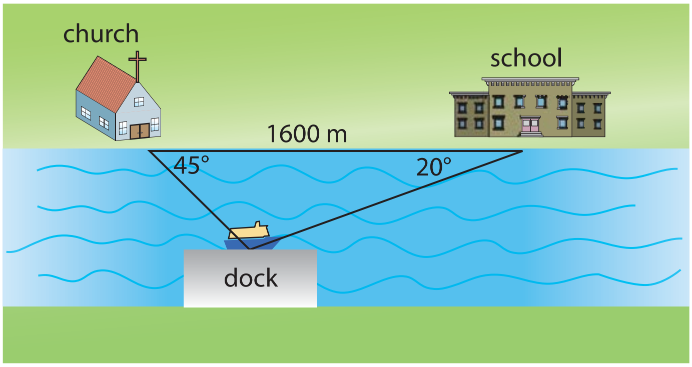
- carpenters
- machinists
- tool and die makers
- navigators and pilots
- artists

To learn more about how knowledge of triangles is essential in many trades and careers, go to www.mcgrawhill.ca/books/mathatwork12 and follow the links.

Apply It

Round all side lengths to the nearest tenth of a unit. Estimate to check your answers.

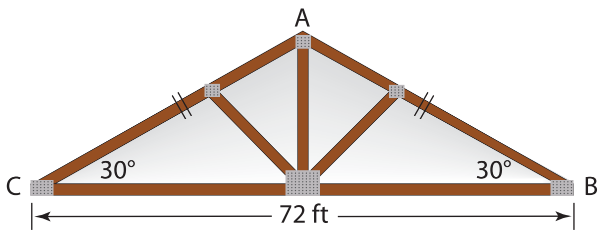
5. Two historical buildings, a school and a church, are 1600 m apart on a riverbank. Across the river is the dock for the ferry that brings tourists to these buildings.



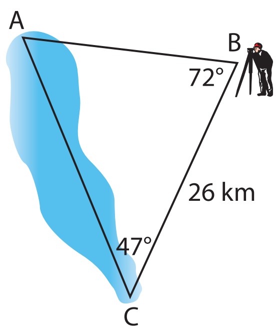
a) How far is the dock from the church?

b) How far is the dock from the school?

6. A carpenter is making a truss to support the roof of a building. The truss will be in the shape of an isosceles triangle with a base length of 72 ft and a pitch of 30°. What length should the carpenter make the other two sides of the truss?



7. Jason is planning a sailboat race near The Pas, Manitoba and needs to know the length of the lake. He hires a surveyor to measure some of the angles and distances.

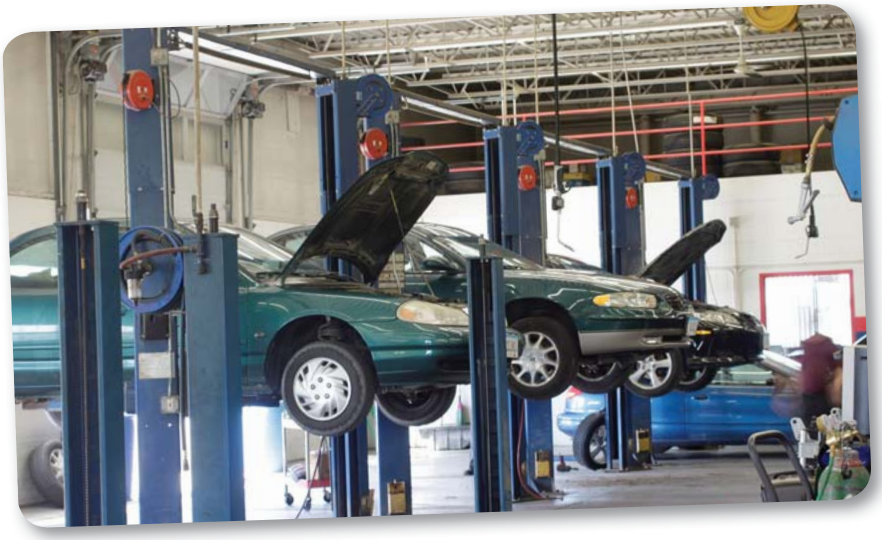


a) Jason figures that once he calculates ∠A, he can solve $\frac{a}{\sin A} = \frac{c}{\sin C}$ to determine the length of the lake. What would you tell him to help him understand his error?

b) Determine the length of the lake.

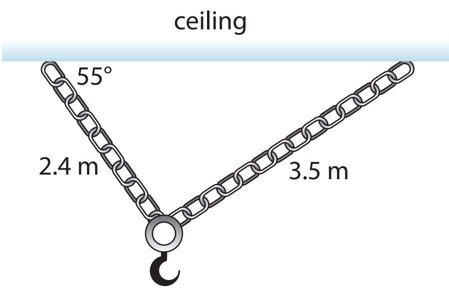
On the Job 2

Determine an Angle Measure Using the Sine Law



A pulley winch in an automotive shop is suspended from a ceiling by two support chains. One of the chains is 2.4 m long and forms an angle of 55° with the ceiling. The second chain is 3.5 m long.

Use the sine law to determine the size of the angle that the second chain makes with the ceiling, to the nearest degree.

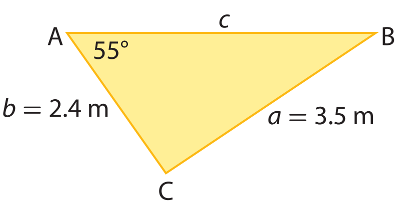


Solution

The winch and the points where the chains are attached to the ceiling form the vertices of an oblique triangle with two known side lengths and one known angle.

Strategy
Draw or Model

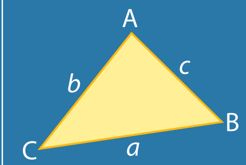
Draw a diagram to model the situation.



F.Y.I.

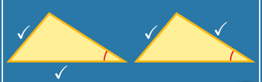
A different version of the sine law can be used to find an unknown angle:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



This version makes the calculations easier to do.

You can use the sine law to determine an unknown angle when two sides and an angle opposite one of them (SSA) are known. For example,



Choose a pair of ratios from the sine law, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$, to solve for ∠B.

Choose a pair of ratios that has ∠B as the only unknown in the proportion.

Solve the proportion for sin B:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 55°}{3.5} = \frac{\sin B}{2.4}$$

$$(2.4)\left(\frac{\sin 55°}{3.5}\right) = (2.4)\left(\frac{\sin B}{2.4}\right)$$

$$\frac{2.4(\sin 55°)}{3.5} = \sin B$$

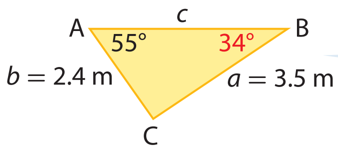
$$0.561\ldots = \sin B$$

$$\sin^{-1} 0.561\ldots = \angle B$$

$$34.173\ldots = \angle B$$

C (2.4 × SIN 55)) ÷ 3.5 =
0.561704258

2nd SIN⁻¹ 0.561704258 =
34.17374066

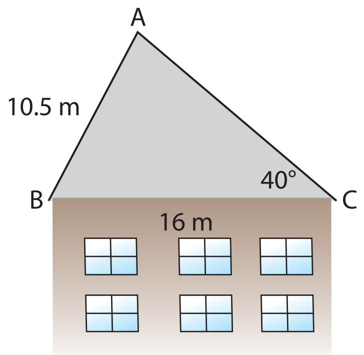


This answer makes sense because the size of ∠B is just a bit greater than half of ∠A, or 55° ÷ 2, which is about 28°.

The other chain makes an angle of approximately 34° with the ceiling.

Your Turn

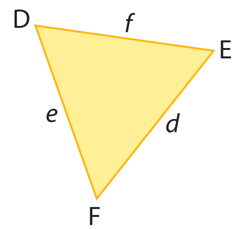
An artist creates a triangular mural that will fit under the rafters at the top of a building. The roof is 16 m wide at its base, the pitch of the rafter on the right side is 40°, and the left rafter is 10.5 m long. The artist needs to know the size of the angle at the peak. Calculate ∠A to the nearest degree. Estimate to check your answer.



Check Your Understanding

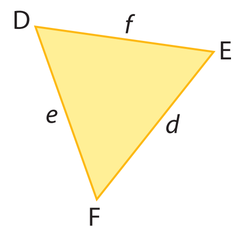
Try It

1. Use the sine law to write two ratios that are equivalent to $\frac{\sin D}{d}$ in $\triangle DEF$.



2. Imagine that you are going to use the sine law to determine each angle listed below in $\triangle DEF$ above. Which of the following measurements would you need to know?

   Choose from side lengths d, e, and f, and angle measures $\angle D$, $\angle E$, and $\angle F$



   a) $\angle D$

   b) $\angle E$

   c) $\angle F$

3. Solve for each unknown angle, to the nearest degree.

   a) $\frac{\sin A}{12} = \frac{\sin 52}{60}$

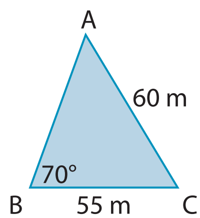
   b) $\frac{\sin 61}{30} = \frac{\sin B}{25}$

   c) $\frac{\sin B}{16} = \frac{\sin 50}{42}$

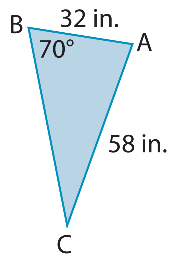
   d) $\frac{\sin 70}{75} = \frac{\sin C}{39}$

4. Determine the size of each angle, to the nearest degree. Estimate to check your answer.

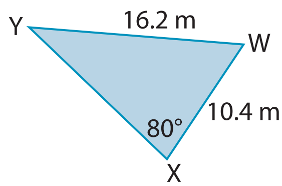
   a) $\angle A$



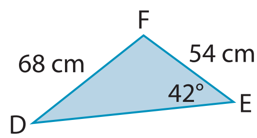
   b) $\angle C$



   c) $\angle Y$

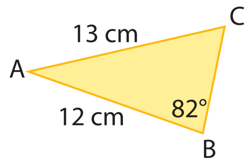


   d) $\angle D$

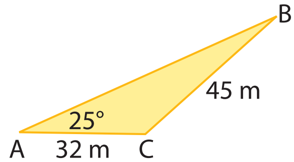


5. Determine the two unknown angles in each triangle, to the nearest degree.

   a) $\angle A$ and $\angle C$



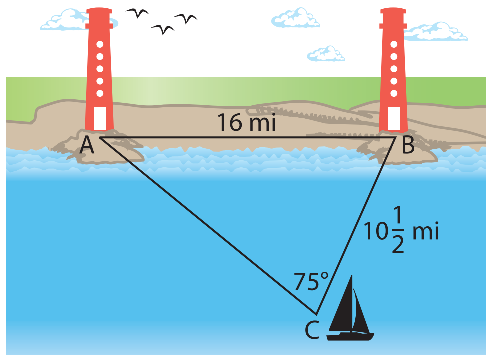
   b) $\angle B$ and $\angle C$



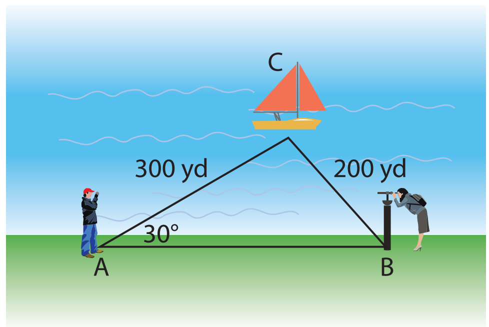
Apply It

Round all angles to the nearest degree. Estimate to check your answers.

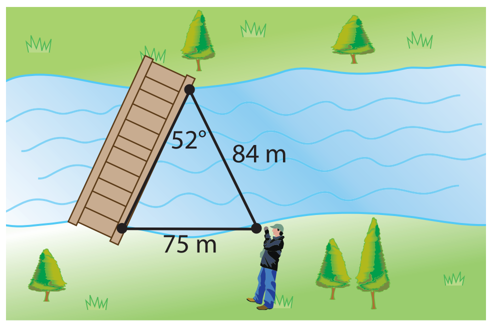
6. The captain of a boat spots two lighthouses on the shore, as shown in the diagram. Determine the two unknown angle measures.



7. Observer A spots a boat 300 yards away. His line of sight to the boat makes an angle of 30° with the shoreline. Observer B spots the same boat 200 yards away. What is the size of the angle with the shoreline from Observer B to the boat?



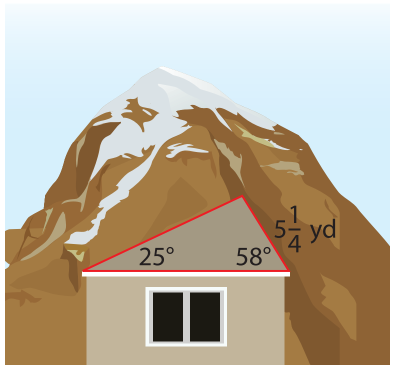
8. A surveyor measures his distance from each end of the bridge. He is 84 m from the north side of the bridge and 75 m from the south side. What is the size of the angle between the surveyor's two sight lines?



Work With It

Round all angles to the nearest degree and all side lengths to the nearest tenth of a unit. Estimate to check your answers.

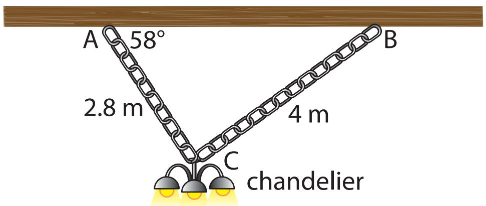
1. The roof of a ski chalet has an irregular shape. The pitch of one roof line is 25°. The pitch of the other roof line is 58°. The length of the shorter roof line is $5\frac{1}{4}$ yards.



a) What is the length of the longer roof line?

b) How wide is the roof at the bottom?

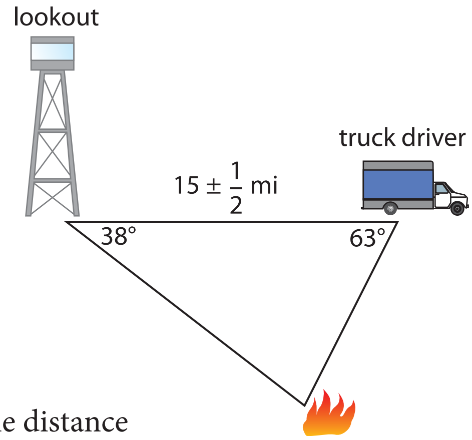
2. A chandelier in a hotel lobby is suspended from a horizontal beam by two support chains. One of the chains is 2.8 m long and forms an angle of 58° with the beam. The second chain is 4 m long.



a) What angle does the second chain make with the beam?

b) How long is the beam from A to B?

3. Michael, a forest ranger, sees smoke from his lookout tower. A truck driver east of the lookout tower also sees the smoke. He uses his GPS to determine the fire's position. The diagram shows the readings taken by the truck driver and the forest ranger.



a) What is the tolerance in the distance between the truck driver and the lookout?

b) What are the maximum and minimum distances between the lookout and the fire?

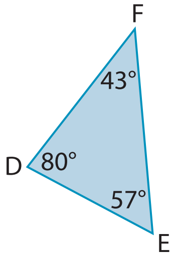
F.Y.I.

To "solve a triangle" means to find all the unknown side lengths and angle measures in the triangle.

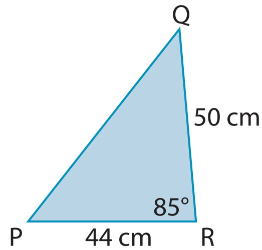
Discuss It

4. Explain why the sine law cannot be used to solve each triangle.

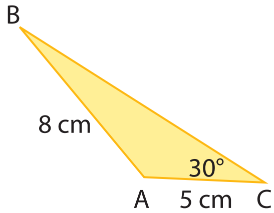
a)

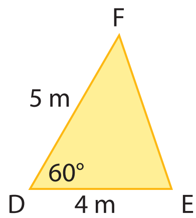


b)

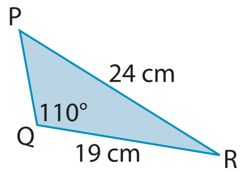


5. Ling says that you can solve △ABC using the sine law, but not △DEF. Is she right? Explain.





6. Stacey tried to calculate ∠P for △PQR and got an error message.



$$\frac{\sin P}{p} = \frac{\sin Q}{q}$$

$$\frac{\sin P}{24} = \frac{\sin 110^\circ}{19}$$

$$\frac{24(\sin P)}{24} = \frac{24(\sin 110^\circ)}{19}$$

$$\sin P = 1.186...$$

$$P = \sin^{-1} 1.186...$$

$$P = \text{ERROR}$$

C (24 × SIN 110)) ÷ 19 =
1.186980153

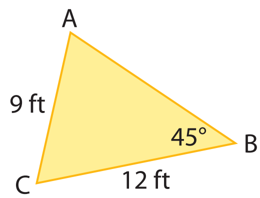
2nd SIN⁻¹ 1.186980153 =
ERROR

a) Where did Stacey go wrong?

b) What could she have done before she started calculating to prevent this error?

7. a) Use the sine law $\frac{\sin A}{a} = \frac{\sin B}{b}$ to determine ∠A.

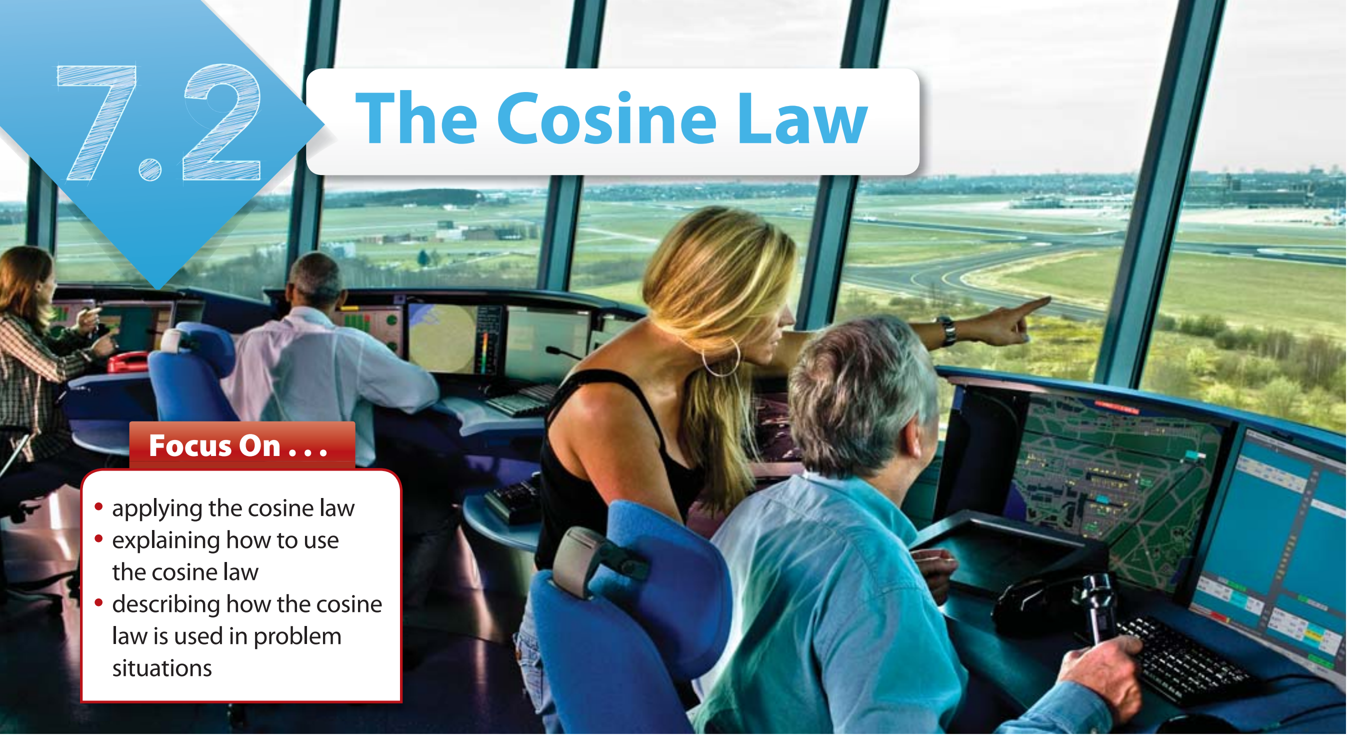
b) Now, use the other version of the sine law, $\frac{a}{\sin A} = \frac{b}{\sin B}$, to determine ∠A.



c) Which version of the sine law did you prefer for determining ∠A? Why?

d) Which version of the sine law would you use to determine side length *c*? Why?

7.2 The Cosine Law



Focus On . . .

- applying the cosine law
- explaining how to use the cosine law
- describing how the cosine law is used in problem situations

Web Link

To learn more about air traffic controllers, go to www.mcgrawhill.ca/books/mathatwork12 and follow the links.

In the previous section, you learned about the sine law. This section is about another law that can be used when the sine law will not work. For example, you can determine the distance between two planes if you know the distance from the air traffic control tower to each plane and the angle between those distances.

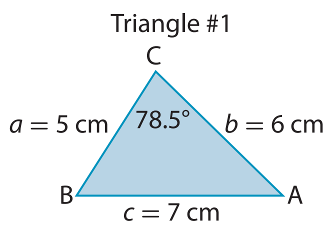
Explore the Cosine Law

Round all values to the nearest whole number.

1. a) Calculate the value of c^2 in oblique Triangle #1.

b) Calculate the value of $a^2 + b^2$.

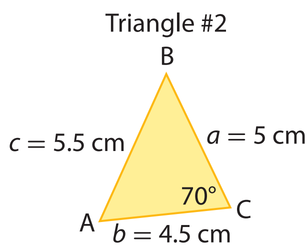
c) Calculate the value of $2ab \cos C$.



2. a) Calculate the value of c^2 in oblique Triangle #2.

b) Calculate the value of $a^2 + b^2$.

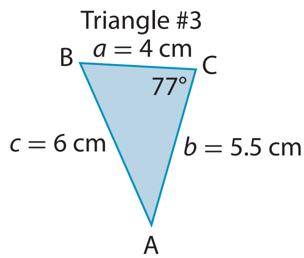
c) Calculate the value of $2ab \cos C$.



3. **a)** Calculate the value of c^2 in oblique Triangle #3.

b) Calculate the value of $a^2 + b^2$.

c) Calculate the value of $2ab \cos C$.



4. Copy this chart in your notebook and record your results from steps 1, 2, and 3.

Triangle	Triangle Side Lengths (cm)	c^2	$a^2 + b^2$	$2ab \cos C$
#1	$a = 5,\quad b = 6,\quad c = 7$			
#2	$a =$, $b =$, $c =$			
#3	$a =$, $b =$, $c =$			

Remember to round all values to the nearest whole number.

Strategy

Look for a Pattern

cosine law

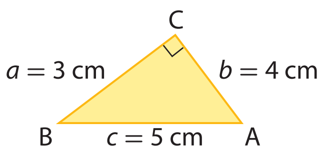
- a law that relates all three side lengths of a triangle with the cosine of one of the angles

5. **Reflect** Examine the values in your chart.

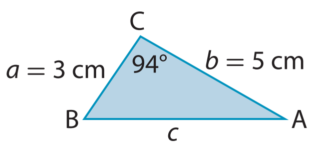
a) What do you notice about the relationship between the values for c^2, $a^2 + b^2$, and $2ab \cos C$?

b) The relationship you described in part a) is called the **cosine law**. Write an equation to represent the cosine law.

c) Calculate the values of c^2, $a^2 + b^2$, and $2ab \cos C$ for $\triangle ABC$ below, to see if the cosine law works for right triangles. What do you notice?



6. **Extend Your Understanding** Examine $\triangle ABC$ below.



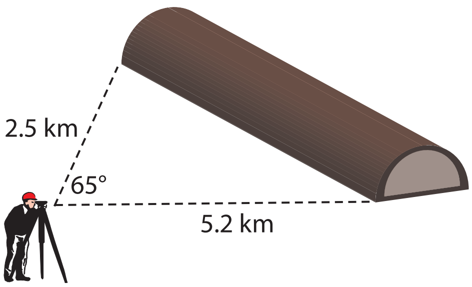
a) Is it possible to calculate the length of c in $\triangle ABC$ using the sine law? Explain.

b) Describe how you could use the cosine law from step 5b) to determine c.

On the Job 1

Determine a Length Using the Cosine Law

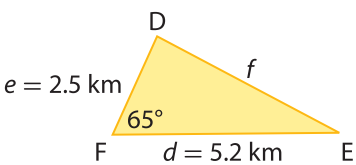
A tunnel for a new highway is built in a straight line through a mountain in Northern B.C. To determine the length of the tunnel, surveyors choose a point where they can see both ends of the tunnel. Their measurements are shown below. Use the cosine law to determine the length of the tunnel, to the nearest tenth of a kilometre.



Solution

The location of the surveying crew and the two ends of the tunnel form the vertices of an oblique triangle with two known side lengths and the angle in between them.

Draw a diagram to model the situation.



Strategy
Draw or Model

F.Y.I.

There are three versions of the cosine law used to determine an unknown side length:

$a^2 = b^2 + c^2 - 2bc \cos A$

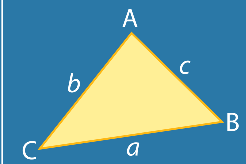
($\angle$A is opposite a)

$b^2 = a^2 + c^2 - 2ac \cos B$

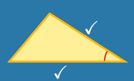
($\angle$B is opposite b)

$c^2 = a^2 + b^2 - 2ab \cos C$

($\angle$C is opposite c)



Each version of the cosine law can be used to determine an unknown side length, if you know the other two sides and the angle between them (SAS).



Choose the version of the cosine law that will determine side length f.

$d^2 = e^2 + f^2 - 2ef \cos D$
$e^2 = d^2 + f^2 - 2df \cos E$
$f^2 = d^2 + e^2 - 2de \cos F$

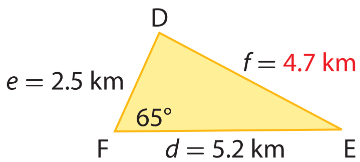
Make sure the side length you are calculating is on its own on one side of the equation, and that you know the values of all the variables on the other side.

Substitute the known values into the equation and evaluate:

$f^2 = d^2 + e^2 - 2de \cos F$
$f^2 = 5.2^2 + 2.5^2 - 2(5.2)(2.5) \cos 65°$
$f^2 = 22.301\ldots$
$f = \sqrt{22.301\ldots}$
$f = 4.722\ldots$

C 5.2 x^2 + 2.5 x^2 − 2 × 5.2 × 2.5 × COS 65 =
22.30192519

2nd √ 22.30192519 =
4.722491418

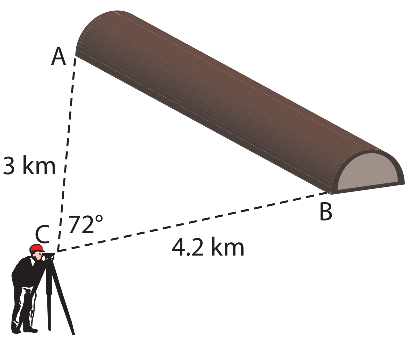


The answer 4.7 km makes sense because f is a bit shorter than the 5.2-km side and quite a bit longer than the 2.5-km side.

The length of the tunnel is approximately 4.7 km.

Your Turn

A second survey crew is bidding on the same job. Find the length of the tunnel they choose to survey from a different point, as shown by the diagram. Check to see if they will arrive at the same measurement for the length of the tunnel.



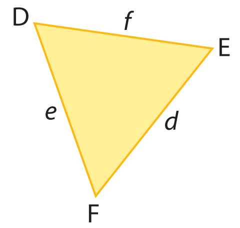
Check Your Understanding

Try It

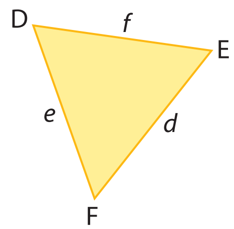
1. Write the cosine law you would use to determine these side lengths in △DEF.

   a) side length *e*

   b) side length *f*



2. Imagine that you are going to use the cosine law to determine each side length listed below in △DEF.



   Which of the following measurements would you need to know?

   Choose from side lengths *d*, *e*, and *f*, and
   angle measures ∠D, ∠E, and ∠F

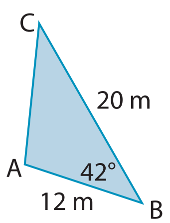
   a) side length *d*

   b) side length *e*

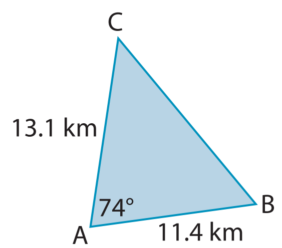
   c) side length *f*

3. Determine the length of the unknown side of each triangle, to the nearest tenth of a unit. Estimate to check.

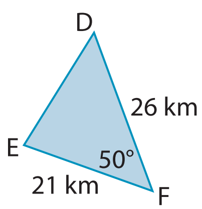
   a)



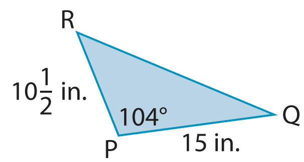
   b)



   c)



   d)

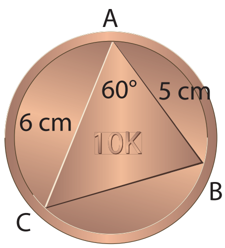


Apply It

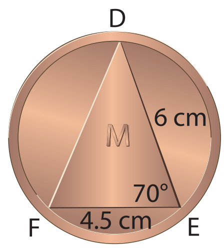
Round all side lengths to the nearest tenth of a unit. Estimate to check your answers.

4. Kyle creates two designs for medals for a sports competition. He wants both medals to be a circle with a triangle inside. Each has a different triangle.

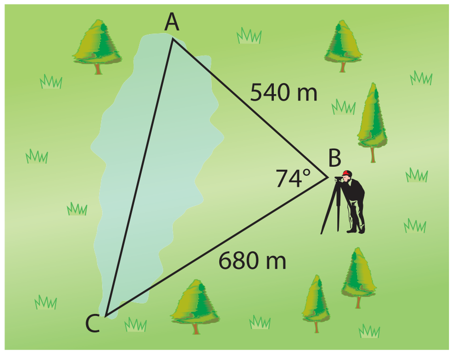
a) What is the length of the unknown side?



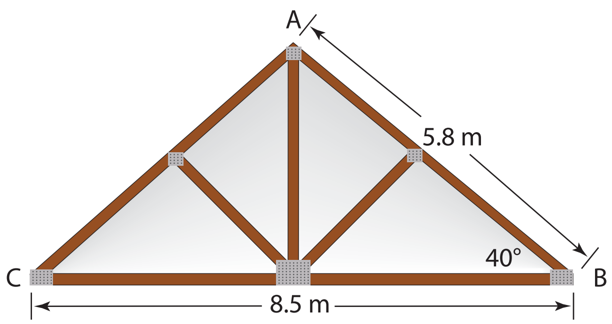
b) What is the length of the unknown side?



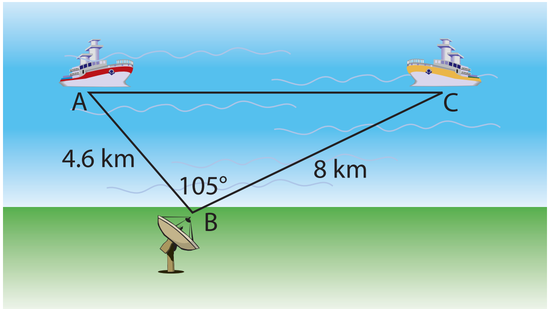
5. A surveyor measures the distance to one end of a marsh in St. Peter's Lake Run, P.E.I. as 540 m, and the distance to the other end as 680 m. The angle between the lines of sight is 74°. What is the length of the marsh?



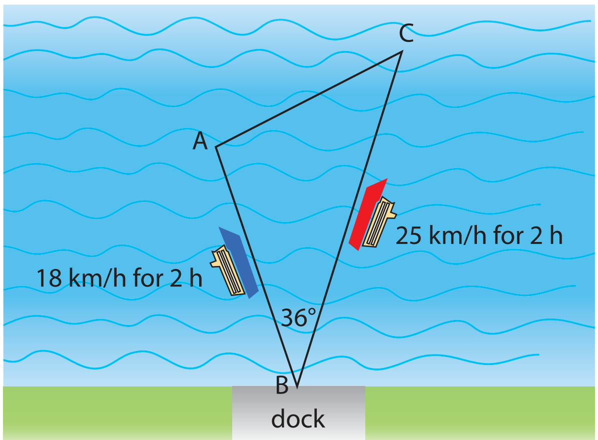
6. A house roof truss needs to span 8.5 m. One piece of the truss is 5.8 m and is set at an angle of 40°. A roofer needs to determine the length of the other piece of the truss. How long is the other piece of the truss?



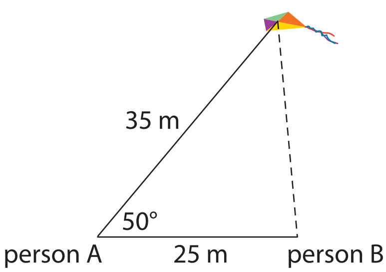
7. A radar station tracks two ships. The distance to one ship is 4.6 km, and the distance to the other ship is 8 km. The angle at the radar station 105°. How far apart are the two ships?



8. Two boats leave a dock, sailing at 18 km/h and 25 km/h. The angle between their directions of travel from the dock is 36°. How far apart will the boats be after 2 h?
Hint: To determine side lengths *c* and *a*, calculate how far each boat travelled in 2 h.

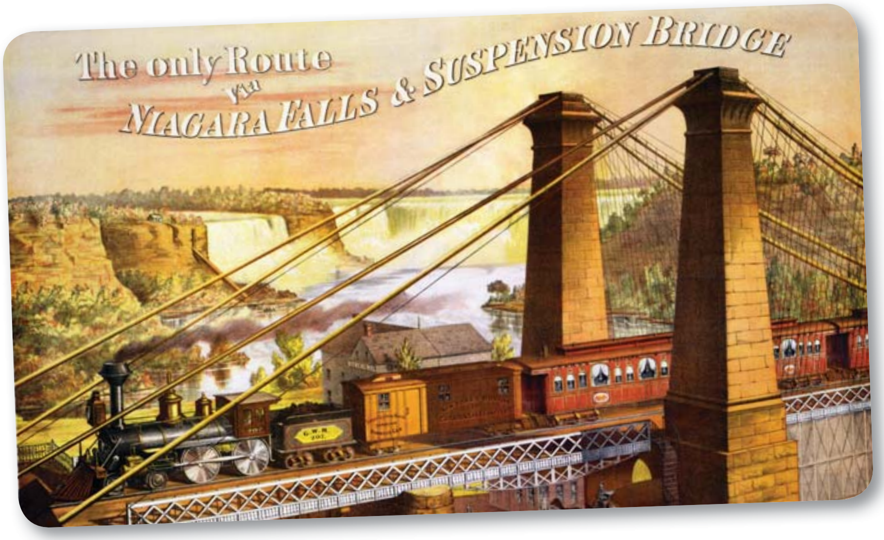


9. A 35-m long kite string makes a 50° angle with the ground. Person A is holding the string. Person B is 25 m away. How far is person B from the kite?



On the Job 2

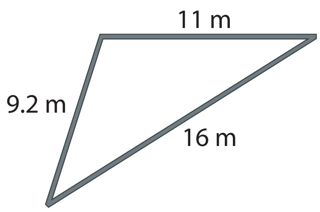
Determine an Angle Using the Cosine Law



Advertisement for Great Western Railway's Niagara Falls Suspension Bridge
"The only route via Niagara Falls & Suspension Bridge"
(then two separate villages, combined in 1892 as the city of Niagara Falls)
Date circa 1876.

The Niagara Falls Suspension Bridge was the world's first working railway suspension bridge. It was shut down over a century ago. Bridges like this are strengthened by triangular braces.

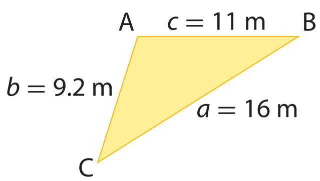
Suppose a triangular brace has side lengths of 11 m, 16 m, and 9.2 m. Use the cosine law to determine the size of the angle opposite to the 11-m side, to the nearest degree.



Solution

The brace is in the form of an oblique triangle. The three side lengths are known. Draw a diagram to model the situation.

Strategy
Draw or Model



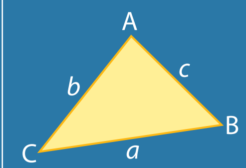
F.Y.I.

There are three versions of the cosine law that are used to determine an unknown angle:

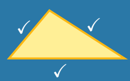
$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$
($\angle A$ is opposite a)

$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$
($\angle B$ is opposite b)

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$
($\angle C$ is opposite c)



Each version of the cosine law can be used to determine any unknown angle, if you know all three side lengths (SSS).



Choose the version of the cosine law that will determine $\angle C$.

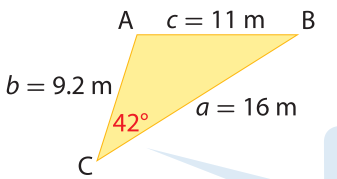
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Substitute the known values into the equation and evaluate:

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$
$$\cos C = \frac{16^2 + 9.2^2 - 11^2}{2(16)(9.2)}$$
$$\cos C = 0.746\ldots$$
$$\angle C = \cos^{-1}(0.746\ldots)$$
$$\angle C = 41.749\ldots$$

C (16 x^2 + 9.2 x^2 − 11 x^2) ÷ (2 × 16 × 9.2) =
0.746059783

2nd $\cos^{-1}$ 0.746059783 =
41.74979284

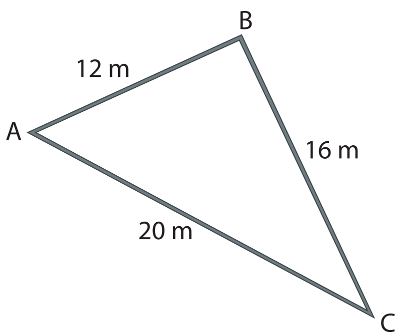


42° makes sense because it appears to be about half a right angle, or very close to 45°.

The angle opposite the 11-m side is approximately 42°.

Your Turn

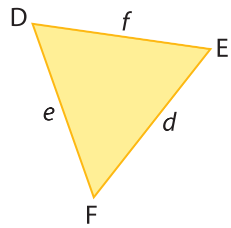
Another triangular brace has side lengths 12 m, 16 m, and 20 m. Determine the size of the angle opposite to the 16-m side, to the nearest degree. Estimate to check that your answer makes sense.



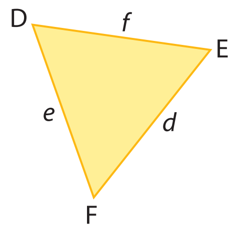
Check Your Understanding

Try It

1. Write the cosine law you would use to determine each angle in $\triangle$DEF.
Hint: For example, to determine $\angle$D, you would use $\cos D = \dfrac{e^2 + f^2 - d^2}{2ef}$.
   a) $\angle$E
   b) $\angle$F

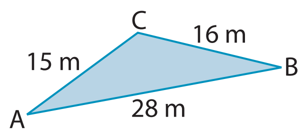


2. Imagine that you are going to use the cosine law to determine each angle listed below in $\triangle$DEF. Which of the following measurements would you need to know?
Choose from side lengths d, e, and f, and angle measures $\angle$D, $\angle$E, and $\angle$F
   a) $\angle$D
   b) $\angle$E
   c) $\angle$F

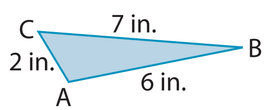


3. Solve for the size of the unknown angle, to the nearest degree.
   a) $\cos C = \dfrac{11^2 + 5^2 - 6.5^2}{2(11)(5)}$
   b) $\cos C = \dfrac{42^2 + 28^2 - 21^2}{2(42)(28)}$

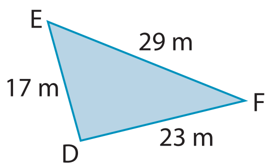
4. Determine the size of each angle, to the nearest degree. Estimate to check.
   a) $\angle$B



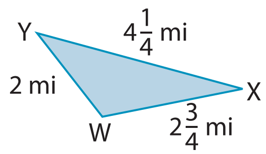
   b) $\angle$C



   c) $\angle$E



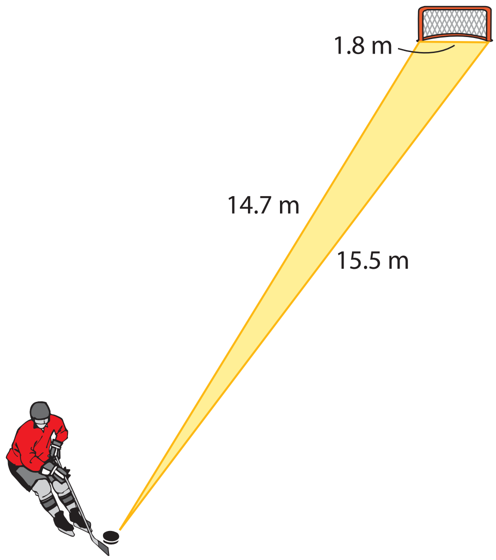
   d) $\angle$Y



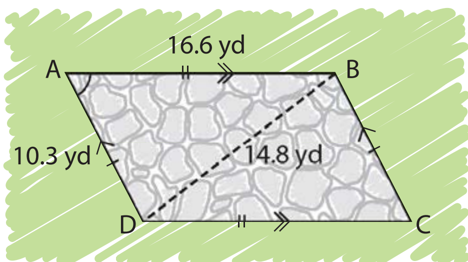
Apply It

Round all angles to the nearest degree. Estimate to check your answers.

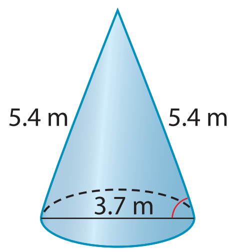
5. At a hockey tournament in St. John's, NL, a player shoots the puck from a point 15.5 m from one goal post and 14.7 m from the other post. The net is 1.8 m wide. Within what angle must he shoot to hit the net?



6. Gillian, a landscaper, is building a frame for a concrete patio in the shape of a parallelogram. What size must she make ∠A for the design and measurements shown?



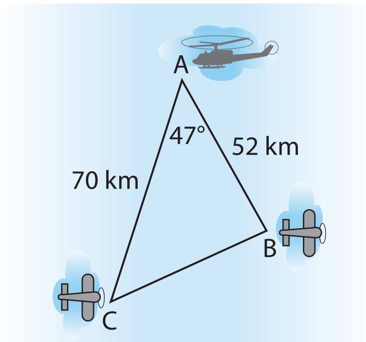
7. The tops of the rocket boosters used to launch the space shuttle are cone shaped. The diameter is 3.7 m and the slant height is 5.4 m. What is the size of the angle that the curved surface of the cone makes with the diameter?



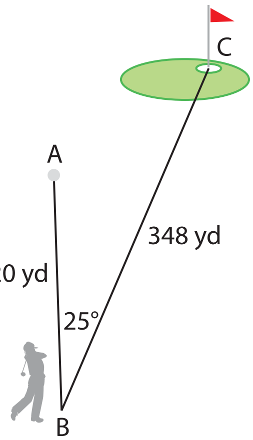
Work With It

Round all angles to the nearest degree and all side lengths to the nearest tenth of a unit. Estimate to check your answers.

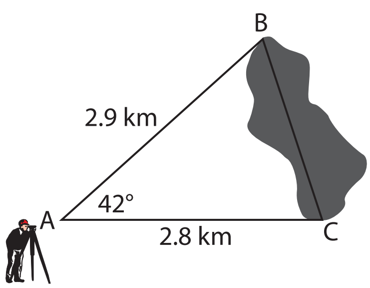
1. An aircraft-tracking station determines the distances from a helicopter to two airplanes. One airplane is 52 km from the helicopter and the other is 70 km. The angle between these two distances is 47°. What is the distance between the two airplanes?



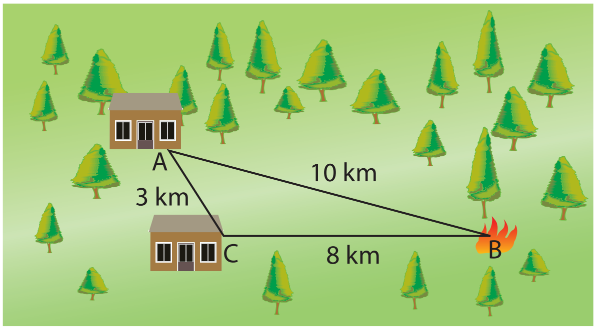
2. The fourth hole at a golf course is a 348-yd straightaway. Stella hits the ball. It travels 25° to the left of the line directly from the tee to the hole. The ball stops 220 yd from the tee. How far is the ball from the hole?



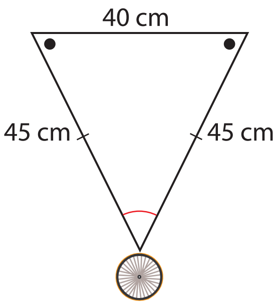
3. Many oil spills in Canada have caused long-term ecological damage. After an oil spill, a surveyor is hired to measure the length of an oil spill to estimate the area it covers. What is the length of the oil spill?



4. A forest ranger spots a fire about 10 km from her station. A second station is 3 km southeast of the first station. The ranger there estimates that the fire is about 8 km away. To give directions to a water bomber, the rangers must know the angles of the triangle. Determine all three angles.



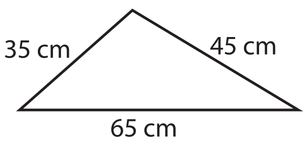
5. Wheelchairs are often custom designed for individual needs. Think about how specialized Steven Hawking's wheelchair is and why. Assume that you are designing a triangle brace for a wheelchair given the specifications shown. What are the measures of the three angles?



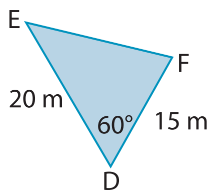
F.Y.I.

Professor Stephen Hawking is a famous theoretical physicist. A computer screen on his wheelchair displays icons that allow him to control his wheelchair, doors, and appliances in his house. Hawking has enough muscle control to operate the wheelchair's computer by pressing a button with his right hand.

6. Use the cosine law to determine if it is possible to build an acute triangular frame with these dimensions.
Hint: An acute triangle has three angles less than 90°.



7. What is the perimeter of △DEF?



Discuss It

8. Gita rewrote each cosine law to determine side length *g* and ∠G in △GHI. What would you tell her to help her correct each mistake?

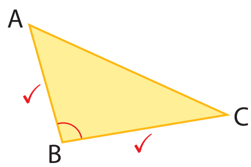
a) $\cos G = \dfrac{g^2 + h^2 - i^2}{2gh}$ **b)** $g^2 = h^2 + i^2 - 2hi \cos H$

9. Alicia knows the length of two sides of a triangle. What other measurement(s) does she need to know to be able to use the cosine law for each situation? Explain your reasoning.

a) to determine the third side length

b) to determine any one of the angles

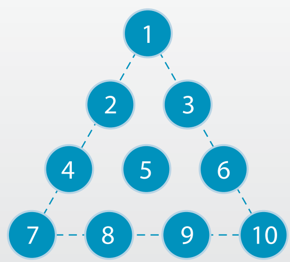
10. In △ABC, the sides with the red checkmarks and the angle with the red arc are known measures. Describe the steps you would follow to solve △ABC using the cosine law.



11. Create a problem involving an unknown angle or side length in an acute triangle that the cosine law could be used to solve. Include only the measurements necessary to solve the problem. How do you know that the cosine law will work?

Puzzler

The sum of the numbers on each side of the triangle is equal to the length of that side.



a) What type of triangle is represented by the puzzle: equilateral, isosceles, or scalene?

b) Use trigonometry to find the measures of the three angles. What do you notice? Why does this make sense?

c) Rearrange the numbers so that the numbers along each side result in three side lengths that form a triangle. What type of triangle is it?

7.3 Solving Trigonometric Problems

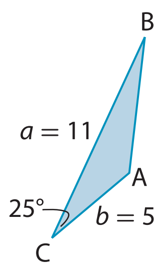
Focus On . . .

- deciding whether to use the sine law or cosine law to solve a problem
- solving a problem using the sine law or the cosine law

Many engineers, designers, architects, and landscapers create designs for homeowners, contractors, and business owners. Their designs must include all measurements. As you learned in the previous sections, you can use trigonometry and known measurements to determine unknown measurements.

Explore the Sine Law and the Cosine Law

1. What do you know about $\triangle ABC$? What do you not know?



2. a) Substitute all the known values for $\triangle$ABC into each version of the sine law.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

b) The sine law cannot be used to solve for any unknown angles or side lengths in $\triangle$ABC. Why?

3. a) Substitute all the known values for $\triangle$ABC into each version of the cosine law.

$$a^2 = b^2 + c^2 - 2bc \cos A \qquad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$b^2 = a^2 + c^2 - 2ac \cos B \qquad \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$c^2 = a^2 + b^2 - 2ab \cos C \qquad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

b) How do you know that you can use the cosine law to solve for an unknown measure in $\triangle$ABC? What is the unknown measure?

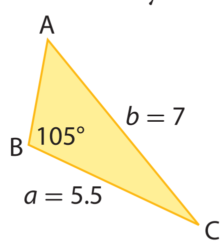
4. Reflect Imagine that you want to determine an unknown measure in a triangle. What measures do you need to know to be able to use each of these laws?

a) the sine law

b) the cosine law

5. Extend Your Understanding

a) To solve $\triangle$ABC, what measure would you find first? Which law would you use? Why?



b) Determine the measure you identified in part a).

c) Which measure would you find next? How would you determine it?

d) Determine the measure you identified in part c).

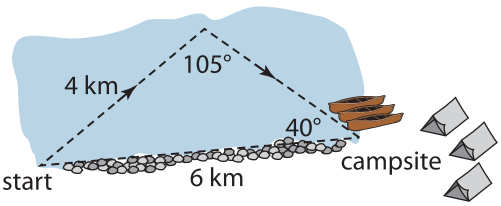
e) Which measure is left to determine? How would you determine it? Why?

f) Determine the measure you identified in part e).

On the Job 1

Choose the Sine Law or Cosine Law to Solve a Problem

Jade works at a summer camp for children on Lake of the Woods, MN. She takes small groups of children for canoe trips. The lake has a stretch of shoreline that is rocky, so she tries to avoid it. To do this, she takes an indirect route to get to the first campsite. Her route is shown on the map. By how much does she go out of her way by following this route, to the nearest tenth of a kilometre?

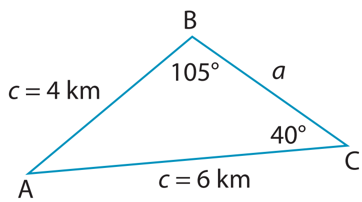


Solution

The starting point, the route's turning point, and the campsite form the vertices of an oblique triangle. Two side lengths and two angles are known.

Strategy
Draw or Model

Draw a diagram to model the situation:



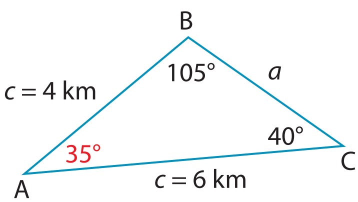
List what is known and what needs to be determined:

- You know four measures: $b = 6$ km, $c = 4$ km, $\angle C = 40°$, and $\angle B = 105°$.
- You need to determine side length a to calculate the difference between the direct route, which is 6 km, and the route Jade took, which is 4 km + a.

Calculate ∠A:

$$\angle A = 180° - 105° - 40°$$
$$= 35°$$

To determine side length a using the cosine law or the sine law, first calculate ∠A.



Strategy

Develop a Strategy

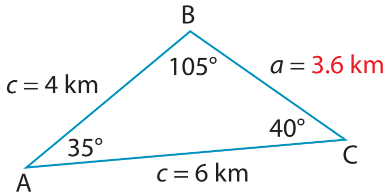
What other ratio pair could you use for Method 1?

Calculate side length a:

Method 1: Use the Sine Law

The sine law will work because two angles (∠A and ∠B) and a side length (b) are known (AAS).

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{a}{\sin 35°} = \frac{6}{\sin 105°}$$
$$(\sin 35°)\left(\frac{a}{\sin 35°}\right) = (\sin 35°)\left(\frac{6}{\sin 105°}\right)$$
$$a = (\sin 35°)\left(\frac{6}{\sin 105°}\right)$$
$$a = 3.560\ldots$$



The answer 3.6 km makes sense because it appears to be a bit shorter than the 4-km side length in the diagram.

Side length a is approximately 3.6 km.

Method 2: Use the Cosine Law

The cosine law will also work because two side lengths (b and c) and the angle between them (∠A) are known (SAS).

$a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = (6)^2 + (4)^2 - 2(6)(4) \cos 35°$

$a^2 = 12.680\ldots$

$a = \sqrt{12.680\ldots}$

$a = 3.560\ldots$

Side length a is approximately 3.6 km.

Calculate the difference between the route Jade took and the direct route:

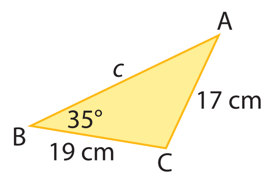
$(4 + 3.6) - 6 = 1.6$

Jade went 1.6 km out of her way.

Your Turn

a) Determine ∠A to the nearest degree in △ABC. Which law did you use and why?

b) Determine side length c to the nearest tenth of a centimetre. Which law did you use and why?



Strategy

Develop a Strategy

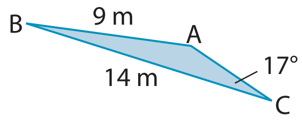
Could you determine the length of side c a different way?

Check Your Understanding

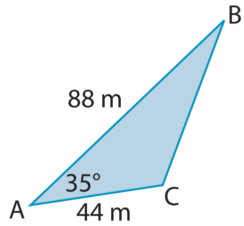
Try It

1. Can you use the sine law to determine each measure?
Note: You do not need to calculate the unknown measure.

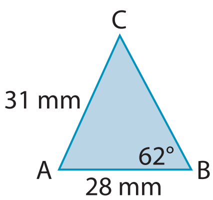
a) b in $\triangle$ABC



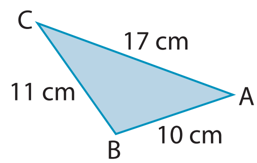
b) a in $\triangle$ABC



c) $\angle$C in $\triangle$ABC

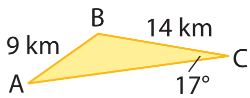


d) $\angle$B in $\triangle$ABC

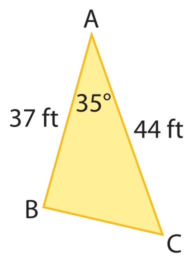


2. Can you use the cosine law to determine each measure?
Note: You do not need to calculate the unknown measure.

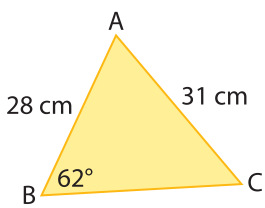
a) b in $\triangle$ABC



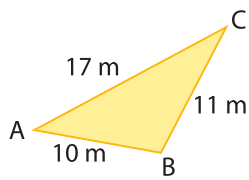
b) a in $\triangle$ABC



c) $\angle$C in $\triangle$ABC

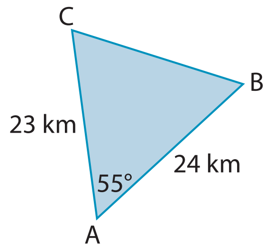


d) $\angle$A in $\triangle$ABC

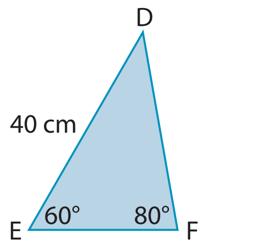


3. Determine each side length or angle. Round all angles to the nearest degree and all side lengths to the nearest tenth of a unit. Estimate to check your answers.

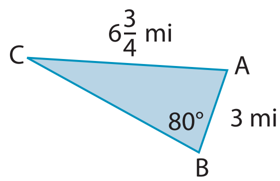
a) side length a in $\triangle ABC$



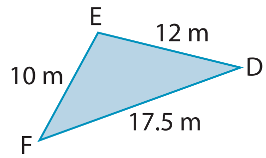
b) side length e in $\triangle DEF$



c) $\angle C$ in $\triangle ABC$



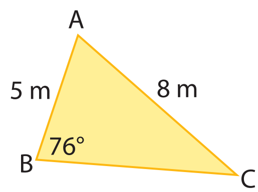
d) $\angle D$ in $\triangle DEF$



Apply It

Round all angles to the nearest degree and all side lengths to the nearest tenth of a unit. Estimate to check your answers.

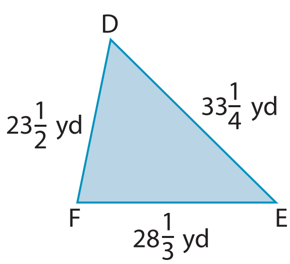
4. A carpenter is building a frame for a sign using the diagram shown. He needs to know the perimeter of the triangle to estimate the length of wood he needs, but some of the measurements in the diagram are missing.



a) Determine $\angle C$.

b) What total length of wood will the carpenter need for the frame?

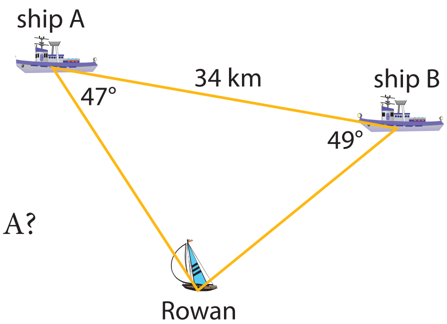
5. The owner of an art gallery is carpeting the floor of a triangular room. She gives the carpet installer this diagram of the floor. To cut the carpet, the carpet installer must know the size of each angle. Solve the triangle by determining the three unknown angles.



Work With It

Round all angles to the nearest degree and all side lengths to the nearest tenth of a unit. Estimate to check your answers.

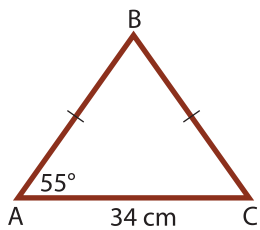
1. Rowan loves to sail. On one of her trips, she has trouble with one of her sails. Two search and rescue ships are out looking for her, as shown.



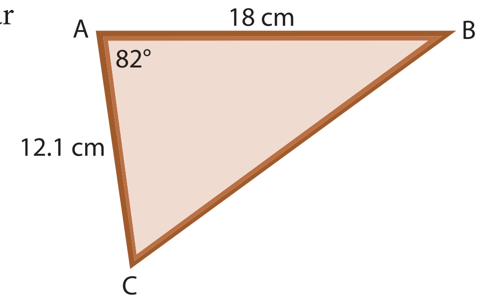
a) How far is Rowan from ship A?

b) How far is she from ship B?

2. Janine designs stained glass windows. For support and protection, each window is surrounded by a wooden frame. Her plan for the frame of her next project is shown. It is in the shape of an isosceles triangle. Determine the perimeter of the frame to estimate how much wood Janine needs to build it.



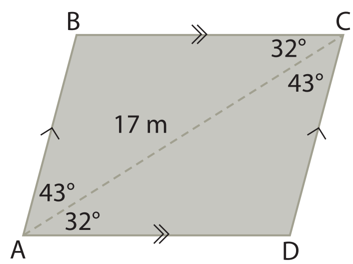
3. Justin is designing a triangular photo frame.



a) Determine ∠B.

b) What is the perimeter of the frame?

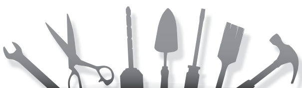
4. A landscaper is planning a patio in the shape of a parallelogram, as shown in the diagram.



a) Determine side length CD.

b) Determine side length AD.

c) What is the perimeter of the patio?



Tools of the Trade

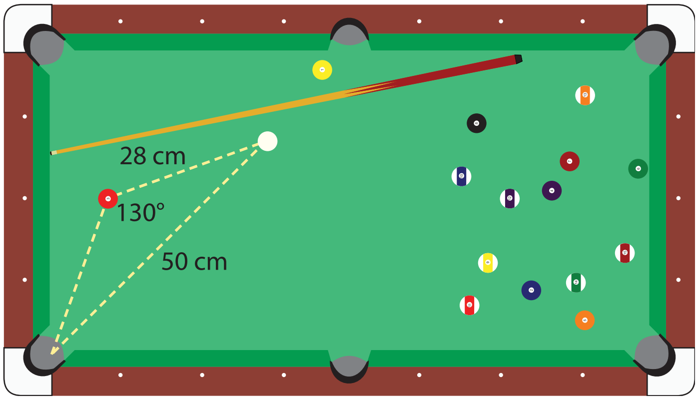
A picture framer uses a special tool called a mitre box to cut very precise angles for frames. After making a cut, a shooting board is used to fine tune the angle of the cut.

To learn more about framing and woodworking tools, go to www.mcgrawhill.ca/books/mathatwork12 and follow the links.

5. In a game of pool, one player strikes the white ball so that it travels 28 cm. It then hits a red ball that travels at a 130° angle from the path of the white ball before it sinks into a pocket. The white ball starts out 50 cm away from the pocket.

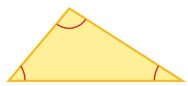
a) Determine the angle at the white ball.

b) How far did the red ball travel?

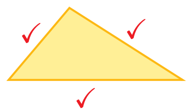


Discuss It

6. a) Chris says that if you know all the angles in an oblique triangle, you can calculate the length of any side. Do you agree? Use an example to help you explain.

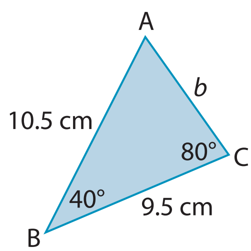


b) He also says that if you know all the side lengths, you can calculate the size of any angle. Do you agree? Use an example to help you explain.



7. a) Oliver says that you can calculate side length b using the cosine law. Annette says you should use the sine law. Who is right? Explain.

b) How would you determine side b? Why?



8. a) Create a problem involving an oblique triangle that can be solved using the sine law but not the cosine law.

b) Revise your problem from part a) so that the cosine law can be used to solve it instead.

Note: Make sure each problem provides only the required information for the law to work.

7 Skill Check

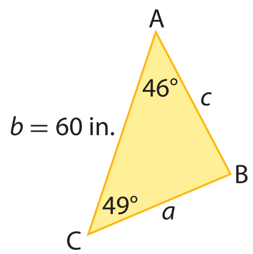
What You Need to Know

Section	After this section, I know how to . . .
7.1	■ recognize and describe an oblique triangle ■ apply the sine law ■ explain how to use the sine law ■ describe how the sine law is used in problem situations
7.2	■ apply the cosine law ■ explain how to use the cosine law ■ describe how the cosine law is used in problem situations
7.3	■ decide whether to use the sine law or the cosine law to solve a problem ■ solve a problem using the sine law or the cosine law

If you are unsure about any of these questions, review the appropriate section or sections of this chapter.

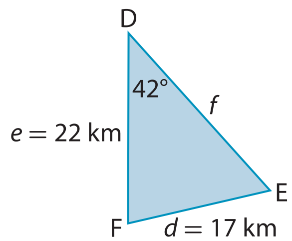
Round all angles to the nearest degree and all side lengths to the nearest tenth of a unit. Estimate to check your answers.

7.1 The Sine Law, pages 326–337



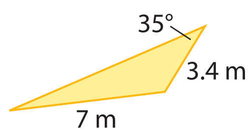
1. Determine each measure in △ABC.

a) ∠B **b)** side length a **c)** side length c



2. Determine each measure in △DEF.

a) ∠E **b)** ∠F **c)** side length f



3. Daria is a landscaper. She is designing a garden in the shape a triangle.

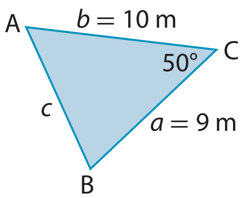
a) Determine the size of the angle that is opposite the 3.4-m side.

b) What is the length of the third side of the garden?

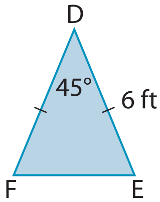
7.2 The Cosine Law, pages 338–351

4. Find the length of the third side of each triangle.

a)

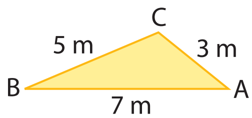


b)

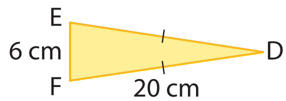


5. Solve each triangle by determining the size of all three angles.

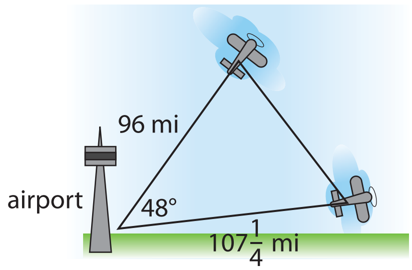
a)



b)



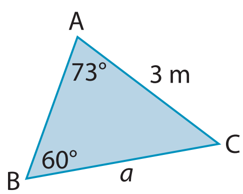
6. Two planes leave an airport at the same time. Each travels in a straight line but in a different direction. The angle between their courses is 48°. One plane travels 96 mi, and the other travels $107\frac{1}{4}$ mi. How far apart are the two planes now?



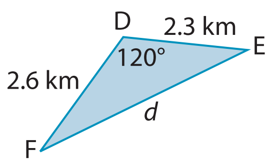
7.3 Solving Trigonometric Problems, pages 352–359

7. Would you use the sine law or the cosine law to find each measure? Why?

a) side length a in $\triangle$ABC



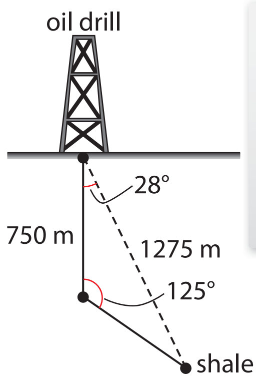
b) side length d in $\triangle$DEF



8. Hydraulic fracturing is used to extract oil and gas from shale rock. A shaft is drilled straight down for 750 m. Then, it turns 125° and continues to the shale layer. The direct distance to the shale layer from the drill site is about 1275 m.

a) Determine the total length of the shaft.

b) Describe how you solved the problem. Could you have solved it a different way?



Web Link

To find out about hydraulic fracturing, go to www.mcgrawhill.ca/books/mathatwork12 and follow the links.

7 Test Yourself

Round all angles to the nearest degree and all side lengths to the nearest tenth of a unit. Estimate to check your answers.

For #1 to #5, select the best answer.

1. Which statement is *always* true about the sine law?

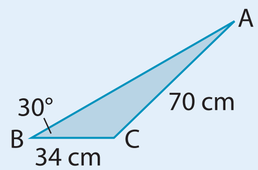
A You can use the sine law when you know the measures of all three angles.

B You can use the sine law when you know the measures of all three sides.

C You can use the sine law when you know the measures of two angles and one side.

D You can use the sine law when you know the measures of one angle and two sides.

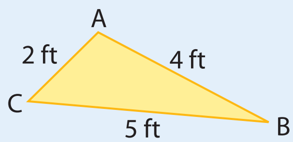
2. Which equation can you use to find the size of ∠A in △ABC?



A $\dfrac{\sin A}{70} = \dfrac{\sin 30°}{34}$ **B** $\dfrac{\sin A}{34} = \dfrac{\sin 30°}{70}$

C $\cos A = \dfrac{70^2 + 34^2 - 28^2}{2(70)(34)}$ **D** $(\angle A)^2 = \dfrac{70^2 + 34^2 - 28^2}{2(70)(34)}$

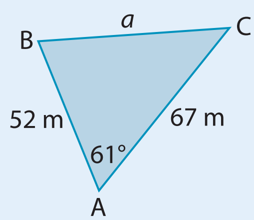
3. What is ∠B in △ABC?



A 22° **B** 49°

C 50° **D** 108°

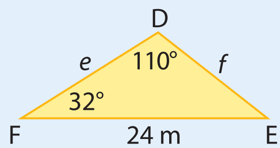
4. What is side length a in △ABC?



A 38.8 m **B** 39.9 m

C 59.9 m **D** 61.8 m

5. Which statement is *not* true about △DEF?



A You can use the sine law to determine f.

B You can use the sine law to determine e.

C You can use the sine law to determine ∠E.

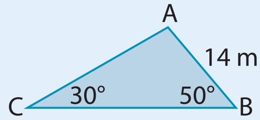
D You can use the sum of the angles of a triangle to determine ∠E.

6. Solve △ABC by following these steps.

a) Determine side length b using the sine law.

b) Determine ∠A.

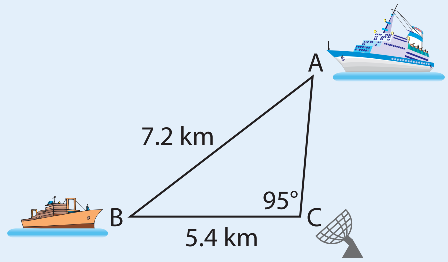
c) Determine side length a using the cosine law.



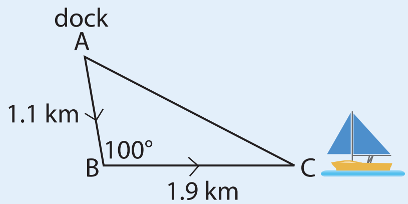
7. A radar station locates a fishing trawler at a distance of 5.4 km and a cruise ship 7.2 km from the trawler. At the station, the angle between the two boats is 95°.

a) What is the angle that the triangle makes at the cruise ship, A?

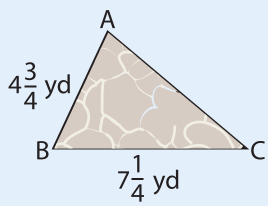
b) How far is the radar station from the cruise ship?



8. A sailboat leaves the dock and travels 1.1 km. It then turns left at a 100° angle, and travels another 1.9 km. How far is the boat from the dock now?



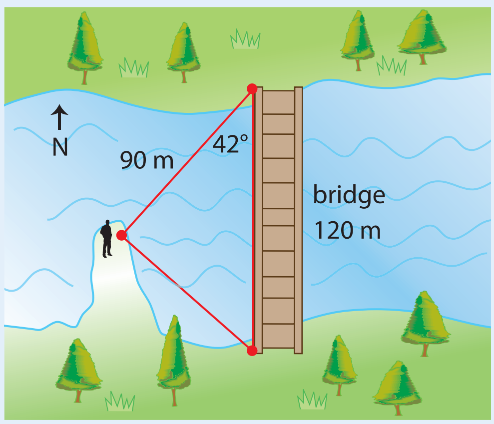
9. Bishnu is designing a triangular patio. Two sides of the patio are $4\frac{3}{4}$ yd and $7\frac{1}{4}$ yd. The angle between them is 65°. What is the perimeter of the patio?



10. A bridge is being built across a river. A surveyor spots the point where the bridge meets the north bank of the river from a position 90 m away. He determines that the bridge must be 120 m long to reach the south bank. The angle of sight at the north bank is 42°.

a) How far is the surveyor from the south point on the river bank?

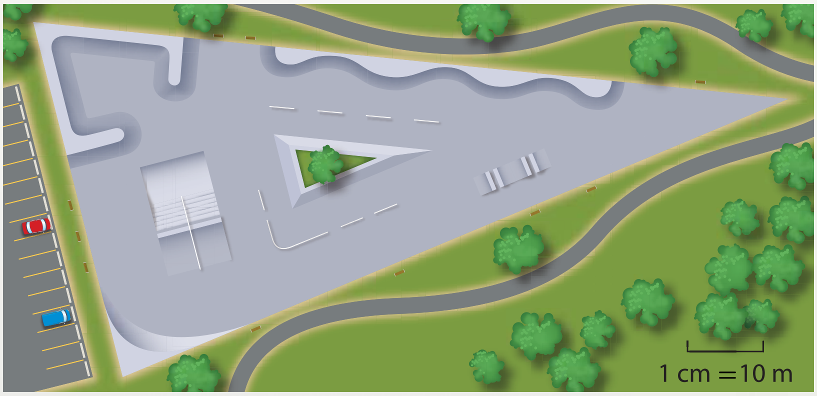
b) What is the angle between the surveyor's sight lines?



7 Chapter Project

Create a Scale Model

Create a scale diagram of a skateboard park in the shape of an oblique triangle. Then, use the diagram to determine its actual area.



Materials
- ruler
- compass
- grid paper (optional)
- Chapter Project Rubric

1. Draw a scale diagram of the park:
- Decide on a set of three side lengths for your park.
- Decide on a scale for the diagram. For example, 1 cm could represent 1 km, or 1 cm could represent 100 m.
- Use your scale to draw the diagram. Include the scale on the diagram.
- Label the diagram with the actual side lengths of the park.

Use measurements that are whole numbers or rounded to the nearest tenth of a unit.

2. Use trigonometry to determine the size of each angle, to the nearest degree. Show all your calculations.

Estimate to check.

3. **a)** Draw a line from one vertex of your triangle to the side opposite to it, making sure it is perpendicular to that side. This line represents the height of your triangle.

b) Measure the height of the triangle in your diagram, to the nearest tenth of a unit.

c) Use your scale to determine the actual height. Label the diagram with the actual height.

4. Calculate the area of your park. Show your calculations.

$A = b \times h \div 2$

7 GAMES AND PUZZLES

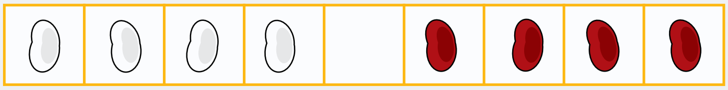
Jumping Beans

This game is designed to be played by one player.

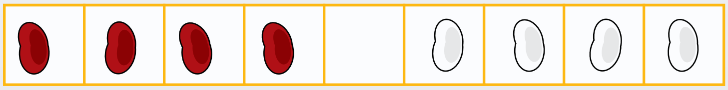
Materials
- 10 beans or counters, 5 of each colour

Goal:
- The goal is to move each set of four beans to the other end of the board in as few moves as possible.

Board at the start of the game



Board at the end of the game



Rules:
- Beans can move one square into any empty space.
- Beans can jump over another bean into an empty space.

1. Sketch the game board and place the eight counters in their start positions.
2. Play the game several times. Record the number of moves it takes each time.
3. Compare and discuss your results from step 2 with a classmate.
4. Is there a strategy for minimizing the number of moves?
5. What is the minimum number of moves needed to move the four beans at each end to the other end?
6. Predict the minimum number of moves if there are five of each colour of counter on a game board that is 11 spaces long.