

CHAPTER 6 Analyse Linear Relations
6.4 Parallel and Perpendicular Lines
Slope Relations of Parallel and Perpendicular Lines

Example:

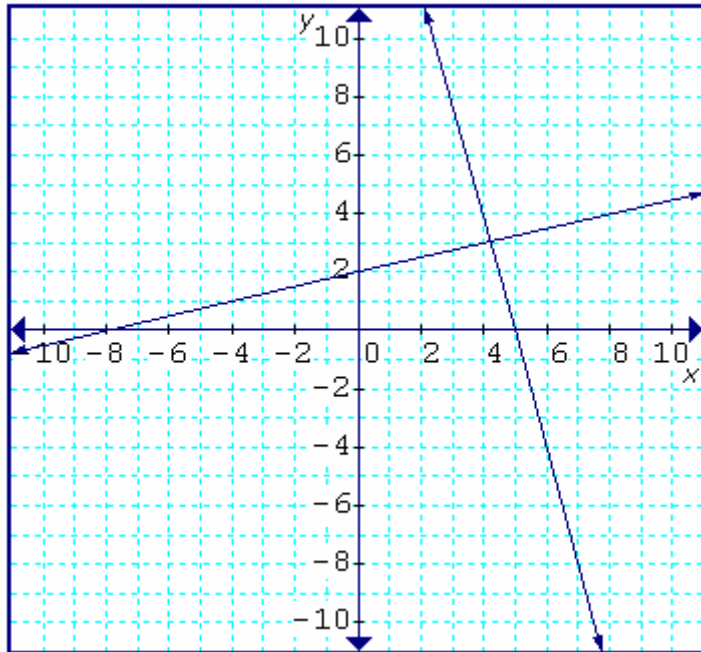
a) A line has the equation $y = \frac{2}{3}x + 1$. Another line is parallel to this line. What is the slope of the other line?

b) A line has the equation $y = -\frac{3}{5}x + 2$. Another line is perpendicular to this line. What is the slope of the other line?

c) A line has the equation $2x + 3y - 6 = 0$. Another line is parallel to this line. What is the slope of the other line?

d) A line has the equation $4x - 3y - 12 = 0$. Another line is perpendicular to this line. What is the slope of the other line?

e) Consider the lines shown. Provide evidence that these two lines are perpendicular.



Solution:

a) The slope of the given line is $\frac{2}{3}$. Since the lines are parallel, the slope of the other line must also be $\frac{2}{3}$.

b) The slope of the given line is $-\frac{3}{5}$. Since the lines are perpendicular, the slope of the other line must be the negative reciprocal, which is $\frac{5}{3}$.

c) Change the equation to slope y -intercept form.

$$\begin{aligned}2x + 3y - 6 &= 0 \\2x + 3y - 6 - 2x + 6 &= 0 - 2x + 6 \\3y &= -2x + 6 \\ \frac{3y}{3} &= \frac{-2x}{3} + \frac{6}{3} \\ y &= -\frac{2}{3}x + 2\end{aligned}$$

The slope of the given line is $-\frac{2}{3}$. Since the lines are parallel, the slope of the other line must also be $-\frac{2}{3}$.

d) Change the equation to slope y -intercept form.

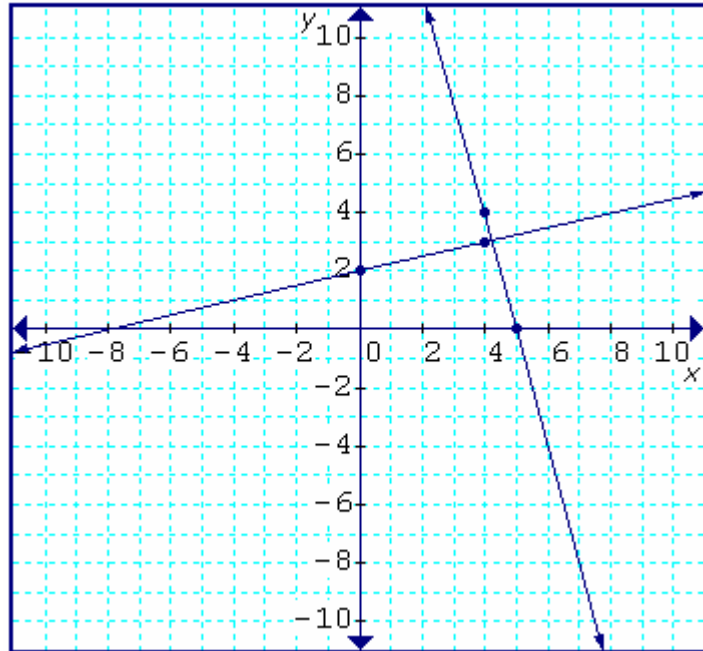
$$\begin{aligned}4x - 3y - 12 &= 0 \\4x - 3y - 12 - 4x + 12 &= 0 - 4x + 12 \\-3y &= -4x + 12 \\ \frac{-3y}{-3} &= \frac{-4x}{-3} + \frac{12}{-3} \\ y &= \frac{4}{3}x - 4\end{aligned}$$

The slope of the given line is $\frac{4}{3}$. Since the lines are perpendicular, the slope of the other line must be the negative reciprocal, which is $-\frac{3}{4}$.

e) Select two points on each line. Use the slope formula to calculate the slope of each line.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{3 - (2)}{4 - (0)} \\ &= \frac{1}{4} \end{aligned}$$

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{0 - (4)}{5 - (4)} \\ &= -\frac{4}{1} \\ &= -4 \end{aligned}$$



The slopes of the lines are negative reciprocals. Therefore, these two lines are perpendicular.

Practice:

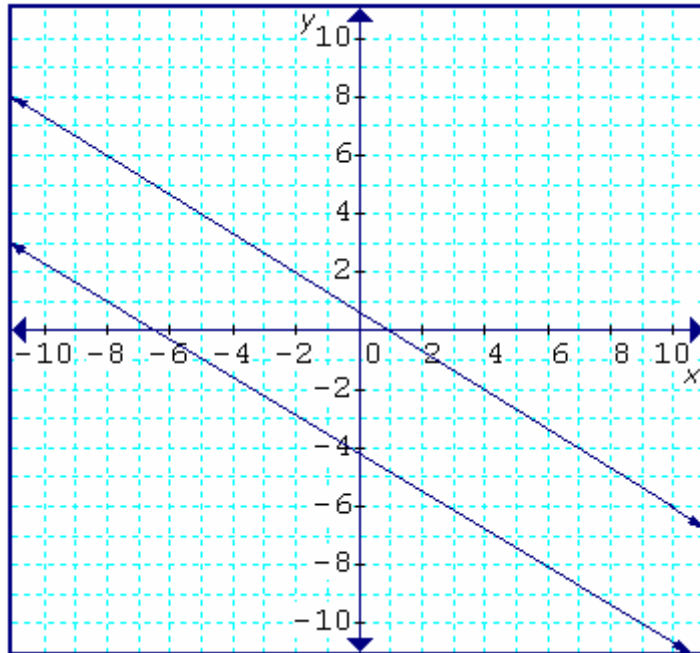
1. a) A line has the equation $y = -\frac{4}{5}x + 3$. Another line is parallel to this line. What is the slope of the other line?

b) A line has the equation $y = \frac{2}{7}x + 5$. Another line is perpendicular to this line. What is the slope of the other line?

c) A line has the equation $3x + 5y - 15 = 0$. Another line is parallel to this line. What is the slope of the other line?

d) A line has the equation $7x - 2y - 14 = 0$. Another line is perpendicular to this line. What is the slope of the other line?

e) Consider the lines shown. Provide evidence that the lines are parallel.



Answers:

1. a) $-\frac{4}{5}$ **b)** $-\frac{7}{2}$ **c)** $-\frac{3}{5}$ **d)** $-\frac{2}{7}$

e) Show that the lines have the same slope, $-\frac{2}{5}$.