CHAPTER 9: Optimizing Measurements
9.3 Minimize the Surface Area of a Square-Based Prism
Minimizing the Surface Area of a Square-Based Prism

For a square-based prism with a given volume, the minimum surface area occurs when the prism is a cube.
Given a volume, you can find the dimensions of a square-based prism with minimum surface area by solving for $s$ in the formula $V = s^3$, where $V$ is the given volume and $s$ is the length of a side of the cube.

**Example:**

a) Farmer Macdonald is planning to build a new grain storage bin in the shape of a square-based prism, with a desired capacity of 1000 $m^3$. In order to minimize construction costs, he would like to minimize the surface area of the bin. Show that this occurs when he uses a cubical bin with a side length of 10 m.

b) A new strawberry-kumquat fruit drink is to be sold in 1 L cardboard packages with square bases and a minimum surface area. Find the dimensions of the package required.

**Solution:**

a) Use pencil and paper, a graphing calculator, a spreadsheet, or other aid to construct a table of possible dimensions. Look for a minimum surface area with a constant volume of 1000 $m^3$. This occurs for a cube with a side length of 10 m.

| Base (m) | Height (m) | Volume ($m^3$) | Surface Area ($m^2$) |
|---|---|---|---|
| 7.0 | 20.4 | 1000 | 669.43 |
| 8.0 | 15.6 | 1000 | 628.00 |
| 9.0 | 12.3 | 1000 | 606.44 |
| 10.0 | 10.0 | 1000 | 600.00 |
| 11.0 | 8.3 | 1000 | 605.64 |
| 12.0 | 6.9 | 1000 | 621.33 |
| 13.0 | 5.9 | 1000 | 645.69 |

b) The volume of the box is 1 L = 1000 $cm^3$. The minimum surface area occurs when the shape is a cube.

$$\begin{aligned} s &= \sqrt[3]{V} \\ &= \sqrt[3]{1000} \\ &= 10 \text{ cm} \end{aligned}$$

The side length of the cube is 10 cm.

**Practice:**

1. a) Peanuts are sold at the ball park in lidless boxes with square bases. If each box holds 300 $cm^3$ of peanuts. Find the dimensions that minimize the surface area.

b) A new office building is to be designed in the shape of a square-based prism for an interior volume of 27 000 $m^3$. If the architect wants a minimum surface area, what dimensions should the building have?

**Answers:**

1. a) $b$ = 8.5 cm, $h$ = 4.2 cm  b) cube, $s$ = 30 m