Scale Factors and Similarity

The Canadian Museum of Civilization in Gatineau, Québec, was designed by architect Douglas Cardinal. This Canadian of Aboriginal heritage was born in Red Deer, Alberta. Cardinal is famous for designing buildings with smooth, flowing lines that reflect the landscape. The building has been recognized, in Canada and internationally, as a world-class structure.

The scale model shown here is an exact replica of the actual building. It has the same shape but not the same size. What scale models have you seen? How is a scale model useful?

In this chapter, you will learn about scale models and their relationship to scale factors and similarity.

> **⦾ Did You Know?**
>
> The form of the Canadian Museum of Civilization represents four natural features. These include the Canadian Shield, the glaciers, the streams formed by the melting glaciers, and the plains that stretched before the melting glaciers. Try to identify each of these elements.

WWW Web Link

For more information about Douglas Cardinal and his designs, go to www.mathlinks9.ca and follow the links.

What You Will Learn

- to draw enlargements and reductions to scale
- to identify scale diagrams and interpret the scale factor
- to determine the scale factor from scale diagrams
- to determine similar triangles and similar polygons
- to solve problems using the properties of similar triangles and similar polygons

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Key Words

|	
enlargement	proportion
scale factor	corresponding angles
reduction	corresponding sides
scale	similar
scale diagram	polygon

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⚬⚬ Literacy Link

A spider map can help you understand and connect new terms and concepts. This spider map is designed to be used throughout the chapter.

Create a spider map in your math journal or notebook. As you work through the chapter, complete the map.

Define each term using words, a diagram, or a mathematical expression. In your definition of *polygon*, mention the sum of the angles of a polygon.

scale   enlargement

scale diagram   scale factor

proportion   reduction

Scale Diagrams

Enlargements and Reductions

Scale Factors and Similarity

corresponding angles

corresponding sides

Similar Triangles

Similar Polygons

similar   polygon
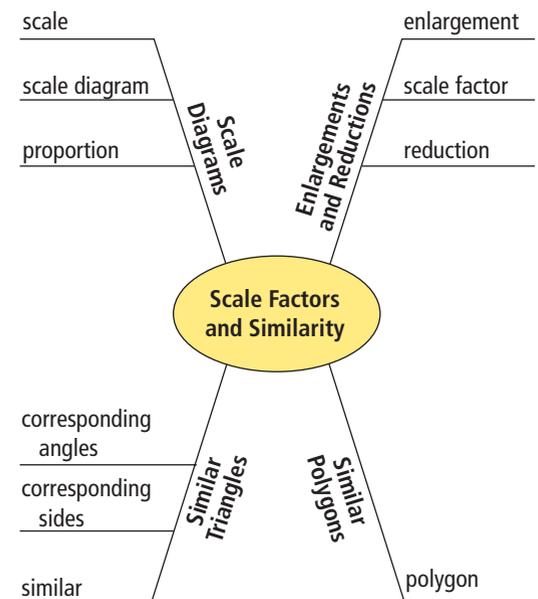
Materials

- sheet of 11 × 17 paper
- ruler
- two sheets of 8.5 × 11 paper
- scissors
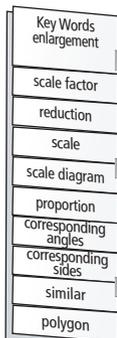- two sheets of 8.5 × 11 grid paper
- stapler

Step 1

Fold the long side of a sheet of 11 × 17 paper in half. Pinch it at the midpoint. Fold the outer edges of the paper to meet at the midpoint. Label it as shown.

Chapter 4
Scale Factors and Similarity

What I Need to Work On
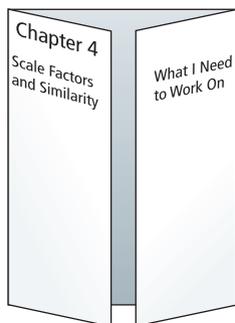
Step 2

Fold the short side of a sheet of 8.5 × 11 paper in half. On one side, use a ruler to draw a line 5.5 cm from the top. Then, draw eight more lines at 2.5-cm intervals. Cut along the lines through one thickness of paper, forming ten tabs. Label the tabs as shown.

Key Words
enlargement
scale factor
reduction
scale
scale diagram
proportion
corresponding angles
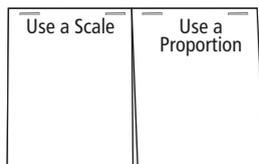corresponding sides
similar
polygon

Step 3

Fold the long side of a sheet of 8.5 × 11 grid paper in half. Fold in half the opposite way. Make a cut through one thickness to make a two-tab book. Label the outside of the left tab Enlargements. Label the outside of the right tab Reductions. Open the two-tab book. Label the inside of the tabs as shown here.

Use Grid Paper | Use Grid Paper

Use a
Scale Factor | Use a
Scale Factor
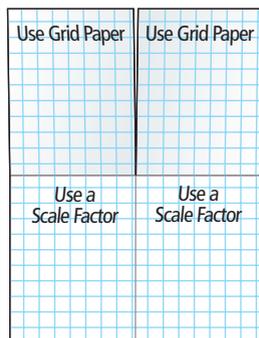
Repeat Step 3, using a plain sheet of 8.5 × 11 paper, to make another two-tab book. Label the outside of it as shown.
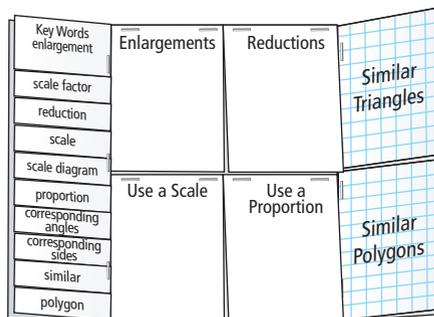
Use a Scale | Use a
Proportion

Step 4

Fold the short side of a sheet of 8.5 × 11 grid paper in half. Fold in half the opposite way. Make a cut through one thickness of paper, forming two tabs. Label the tabs as shown below.

Step 5

Staple the four booklets you made into the Foldable from Step 1 as shown.

Key Words
enlargement
scale factor
reduction
scale
scale diagram
proportion
corresponding angles
corresponding sides
similar
polygon

Enlargements | Reductions

Similar Triangles

Use a Scale | Use a Proportion

Similar Polygons

Using the Foldable

As you work through Chapter 4, define the Key Words beneath the tabs on the left. Beneath the tabs at the top of the centre panel, record notes about enlargements and reductions. Beneath the tabs at the bottom of the centre panel, record notes about using a scale factor and proportions to solve problems. Beneath the tabs on the right, record notes about the properties of similar triangles and similar polygons.

On the back of the Foldable, record ideas for the Math Link: Wrap It Up! On the back of the right flap of the Foldable, make notes under the heading What I Need to Work On. Check off each item as you deal with it.

Math Link

Designers

Many occupations require people to design projects using models or diagrams. Some examples include architecture, fashion and furniture design, web design, automotives, and tourism.

For example, architects create plans also for homes. These plans are called blueprints. Architects work with ratios and proportions to produce floor plans that represent accurate dimensions of the various areas of a home. The floor plan helps people judge if the proposed design is suitable for their lifestyle.
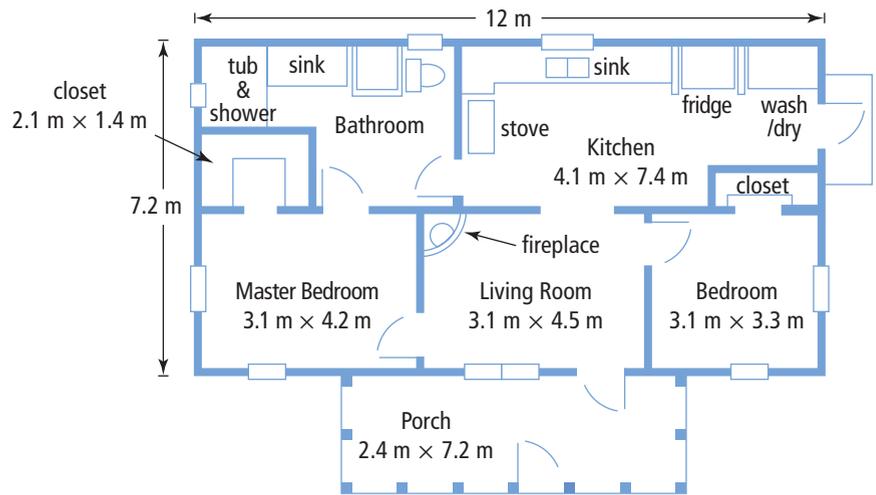
Use the floor plan to answer the following questions.

1. **a)** What is the area of the actual house?
   b) What is the area of the house on the blueprint?

2. **a)** What is the area of the actual living room?
   b) What is the area of the living room on the blueprint?

3. **a)** What is the ratio of the area of the actual house to the area of the blueprint house?
   b) What is the ratio of the area of the actual living room to the area of the blueprint living room?
   c) Compare the two ratios. What can you conclude about the ratios?
   d) What ratio do you expect for the areas of the actual and blueprint master bedrooms? Explain why.

4. **a)** Why do you think accuracy is important in developing a floor plan?
   b) Why is it important to maintain the same proportions for the dimensions of an actual object and its image?

5. Discuss with a partner other examples in which ratios are used to compare objects in daily life.

In this chapter, you will learn skills to draw diagrams that are proportional to the actual objects. You will also plan and complete your own design project.

Did You Know?

The term *blueprint* refers to a detailed technical drawing. The drawings originally got their name from the special blue paper on which the prints were made. The lines appeared in white. These traditional prints have been replaced by modern printing methods and digital displays.

Literacy Link

A *ratio* compares quantities measured in the same units.

4.1 Enlargements and Reductions

A microscope magnifies objects that are too small to be seen by the naked eye. This picture shows an enlarged view of cells in onion skin.

To calculate the factor that the onion cells are magnified by, multiply the magnification of the eyepiece by the magnification of the objective lens. The objective lens is the lens you choose to look at the object with.
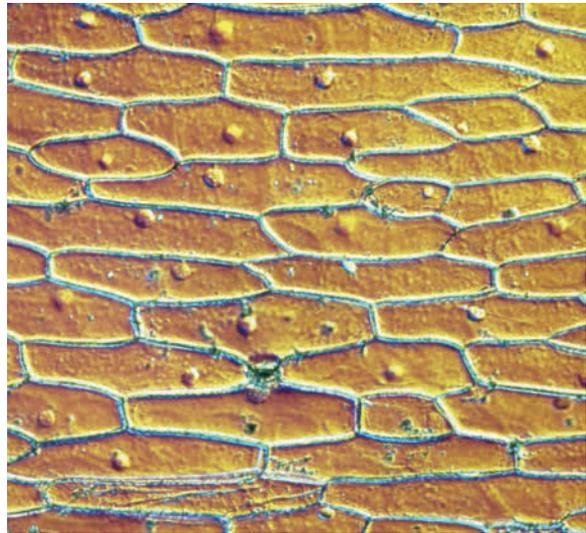
Magnification of eyepiece: 10x

Magnification of objective lens: 40x

Total magnification: $10 \times 40 = 400$x

The onion cells are enlarged by 400 times their original size.

How do you think the enlarged view is the same as the actual piece of onion skin? How is it different?

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> **Did You Know?**
>
> One of the most powerful microscopes used in high schools today can enlarge an object 1500 times.

Materials

• centimetre grid paper

• tracing paper

• ruler

Explore How to Enlarge an Image

1. Brainstorm with a classmate how you might enlarge the onion cell. What different strategies can you develop?

2. Try out at least one of your strategies and draw an image that is twice as large as this onion cell. What will be the ratio of the lengths of the sides of the enlargement to the original?
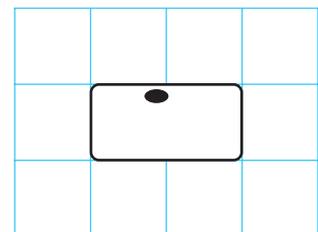
3. Compare your diagram(s) with one of a classmate. Which strategy for making an **enlargement** do you prefer? Explain why.

Reflect and Check

4. a) What method might you use to check that the enlarged image is twice as large as the original? Try your method.

   b) How are the enlargement and the original the same? How are they different?
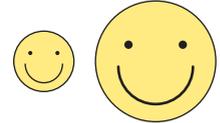
Link the Ideas

Example 1: Draw an Enlargement

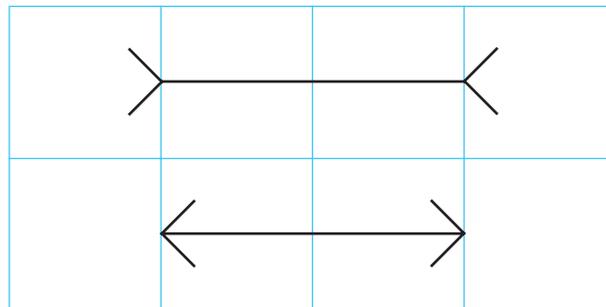Draw a picture with dimensions that are twice as large as the original.
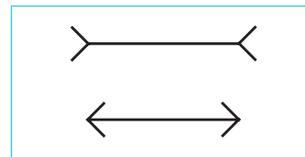
Solution

Method 1: Use Grid Paper
Trace the picture on centimetre grid paper.
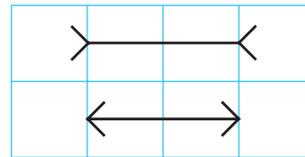
> How could you use 1-cm grid paper to draw the enlargement?

Draw the contents of each grid square into the corresponding region on a piece of 2-cm grid paper.

> A map grid names the regions between grid lines. Try using a map grid to help copy the information. The first arrow has been drawn.

Method 2: Use a Scale Factor
Measure the length of each line segment.

2 cm

0.3 cm
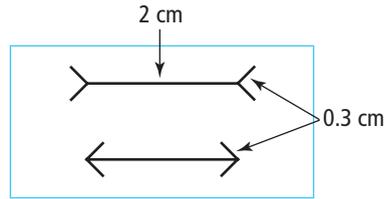
scale factor

- the constant factor by which all dimensions of an object are enlarged or reduced in a scale drawing
- the dimensions of this rectangle are multiplied by 3, so the scale factor is 3

Multiply each measurement by a **scale factor** of 2.
$2 \times 2 = 4$
$0.3 \times 2 = 0.6$

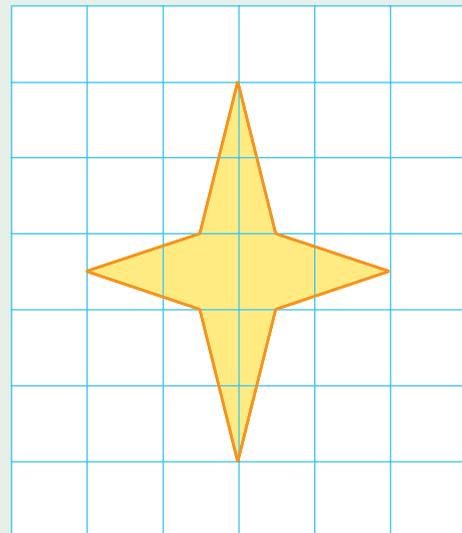The line lengths for the enlargement are 4 cm and 0.6 cm.

Use the new lengths to draw the enlargement.
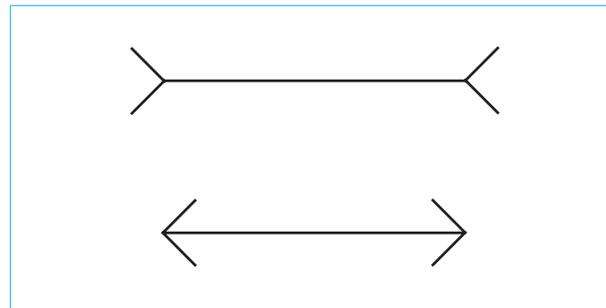
> How do you know the scale factor is 2?

> How else might you enlarge the picture?

Show You Know

Use two methods for drawing a picture with dimensions three times as large as this original.

Example 2: Draw a Reduction

Draw a **reduction** that is half as large as the original.
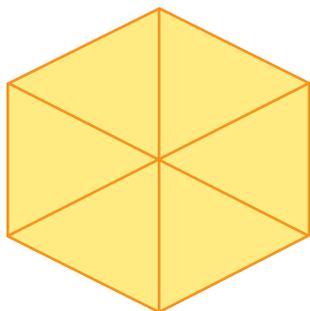
Solution

Method 1: Use Grid Paper

Trace the picture on centimetre grid paper.
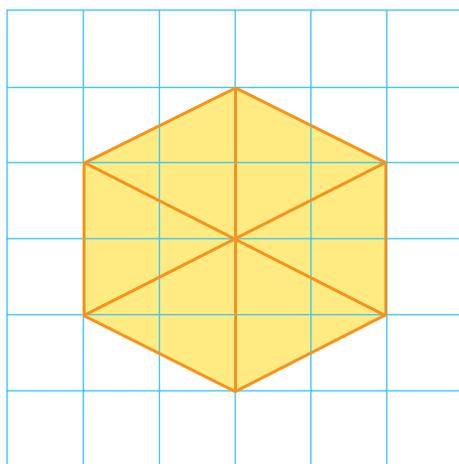
Draw the contents of each grid square into the corresponding area on a piece of 0.5-cm grid paper.
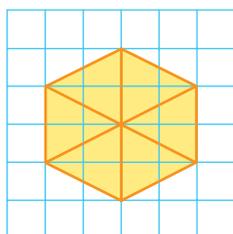
How could you use 2-cm grid paper and 1-cm grid paper to draw this reduction? What if you have only 1-cm grid paper?
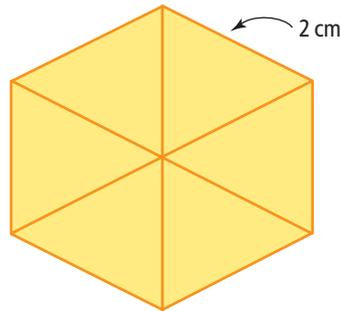
Method 2: Use a Scale Factor
Measure the length of each line segment.

2 cm

Tech Link

You can use a drawing program to enlarge or reduce an image using a scale factor. Or, you can drag an object to the size you want.

Multiply the length by 0.5.
$2 \times 0.5 = 1$

The length of each line segment for the reduction is 1 cm.

Use the new length of each line segment to draw the reduction.

The scale factor indicates whether an object is enlarged or reduced. What does each statement tell you?
• a scale factor greater than 1
• a scale factor less than 1
• a scale factor equal to 1

Which method do you prefer for drawing a reduction? Explain.
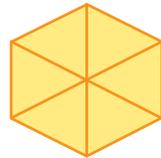
Show You Know

Use a method of your choice and a scale factor of 0.5 to draw a reduction of this shape.
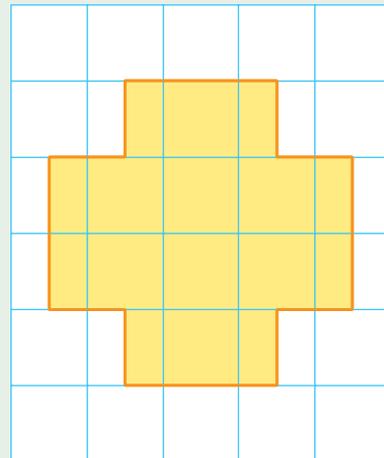
Key Ideas

- An enlargement results in an image that is the same shape but proportionally larger than the original.

- A reduction results in an image that is the same shape but proportionally smaller than the original.

- The scale factor is the constant amount that all dimensions of an object are multiplied by to draw an enlargement or reduction.
  - A scale factor greater than 1 indicates an enlargement.
  - A scale factor less than 1 indicates a reduction.
- You can use grid paper and a scale factor to draw enlargements and reductions.
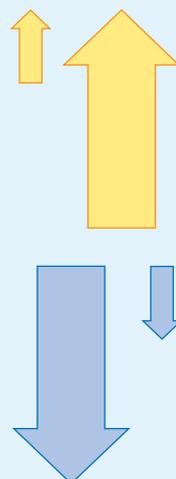
# Check Your Understanding

Communicate the Ideas

1. Jesse thinks many photographs in this student resource are reductions. Is he correct? Justify your reasoning.

2. Mary used a scale factor of 3 to enlarge a rectangle.

   3 X 3 = 9

   The length of each side for the enlargement is 9 cm.

   Is she correct? If so, explain how you know. If she is incorrect, explain her mistake. Discuss your answer with a partner.

   5 cm

   3 cm

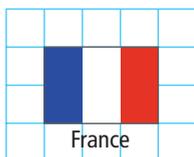3. This logo was designed for a film club. Describe two different methods to enlarge the logo for a poster.
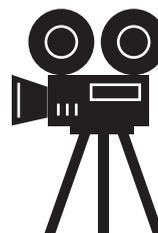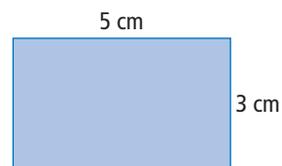
Practise

For help with #4 and #5, refer to Example 1 on pages 131–132.

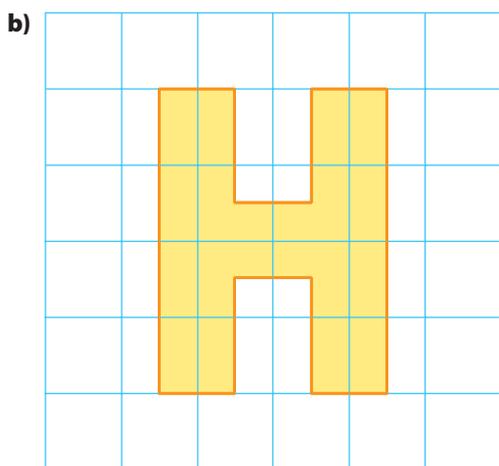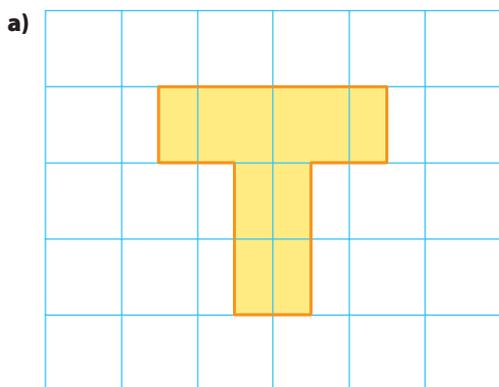**4.** Use a scale factor of 2 to enlarge each letter.

a)

b)
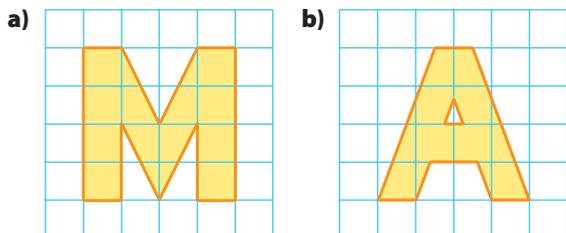
5. Draw an enlargement of the flag using a scale factor of 4.

France

For help with #6 to #8, refer to Example 2 on pages 133–134.

6. Use a scale factor of 0.5 to draw a reduction of each letter.

a)

b)

7. For the image on the right in each pair of pictures, indicate if the scale factor is
- greater than 1
- less than 1
- equal to 1

Explain how you know.
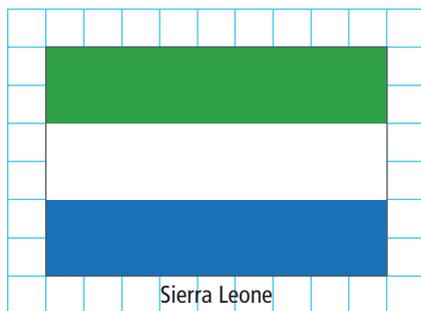
a)

b)

c)

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@ **Web Link**

To explore enlargements and reductions of images graphically or numerically by using a scale factor, go to www.mathlinks9.ca and follow the links.
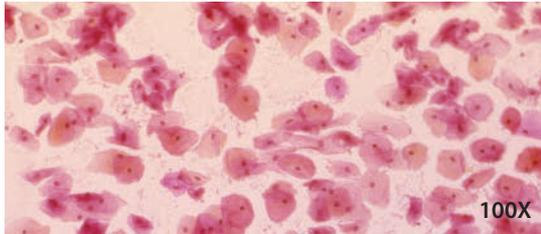
**8.** Draw an image of the flag using a scale factor of $\frac{1}{3}$.

Sierra Leone

Apply

9. Melissa is observing a slide of human cheek cells under the microscope.

a) Is this an enlargement or a reduction? Explain your reasoning.

b) What is the scale factor? Explain its meaning.

100X

10. Hassan and Mia made posters for the *Festival du Voyageur*. What is the scale factor on Mia's poster compared to Hassan's poster? Explain your reasoning.
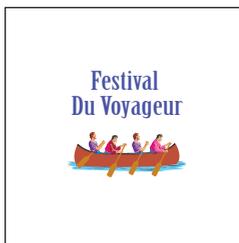

Hassan

Mia

> ### ⊙⊙ Did You Know?
>
> In 1969, the *Festival du Voyageur* was founded in Saint Boniface. The event has grown from three days held in Winnipeg's French Quarter to a ten-day, province-wide celebration every February. This festival celebrates the *joie de vivre* of the fur traders who established the Red River colony and the growing French-Canadian community in western Canada. The Festival encourages people to appreciate the beauty of winter by participating in historical and entertaining activities.
>
>

11. How can you determine if Figure B is a true reduction of Figure A?


Figure A   Figure B
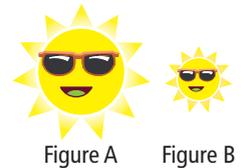
12. The ratio of the length to the width of the Canadian flag is 2 : 1. Assume that you have a flag that is 9 cm wide.


a) What are the dimensions of a flag that has a scale factor of 3?

b) What are the dimensions of a flag that has a scale factor of 0.5?

13. For the Heritage Fair, Chloe wants to sew miniature replicas of traditional hunting dress. The pattern piece below is for making pants. She wants to make three different sizes of the pants using the pattern. Use a scale factor of 0.5, 2, and 3 to draw each size.
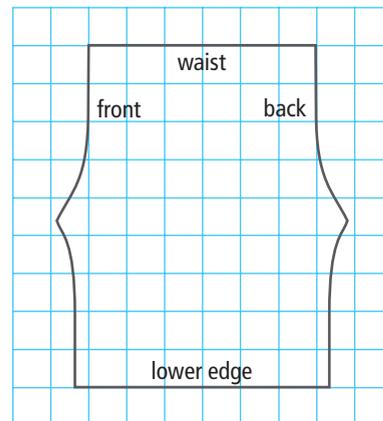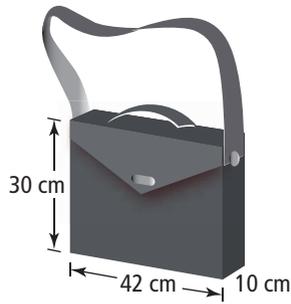

Extend

**14.** Draw an enlargement of the quadrilateral on grid paper using a scale factor of 2.
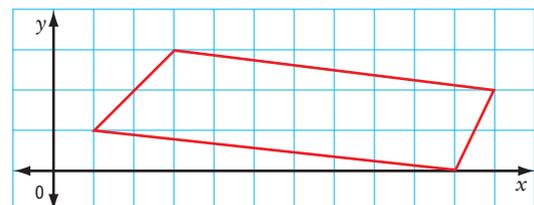
15. Keita made a new bag for her laptop. Her cousin would like the pattern so she can make one. Draw a pattern using the actual measurements. You do not have to include the flap or the strap. Then, reduce the pattern so it will fit on a piece of notebook paper.

30 cm

42 cm → 10 cm

16. Create a scale diagram of your classroom.

   a) Measure the dimensions of the classroom and items that can be seen in a top view, including desks, tables, cupboards, and shelves.

   b) Choose a scale factor and draw the scale diagram on grid paper.

   c) What changes would you make to the layout of your classroom? Where would you place desks or tables? Draw a scale diagram of your new classroom layout.

17. Draw an image so that each line segment is

   a) 40% of the original length
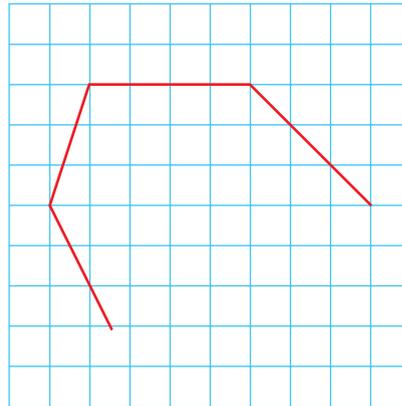
   b) 2.5 times the original length
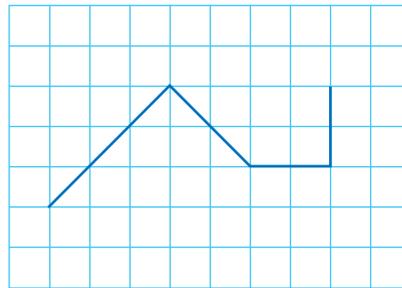
Math Link

Use what you have learned to design a project that requires a scale diagram. You may wish to choose one of the following projects:
- Design at least four different hopscotch patterns for a local recreational area.
- Design or enlarge a pattern for an outfit to wear at your school's fashion show. Assume that you have the instructions and the skills needed to construct the outfit.
- Design a modification of a car's blueprints for a project in your automotive course.
- Design a miniature version of a landmark in your province for display in a tourism project.
- Design a web page featuring a topic and related visuals of your choice. For example, you might feature contemporary drum designs.

a) What design project will you choose?

b) Research your project using the library or the Internet. Obtain or develop an initial design or drawing.

c) Using grid paper, draw an enlargement or a reduction of your design to scale.

Haida hand drum design

Scale Diagrams

scale

• a comparison between the size of an object's diagram and the actual size of the object

• can be expressed as a ratio, as a fraction, as a percent, in words, or in a diagram

• the scale 1 : 32 means that 1 cm on the diagram represents 32 cm on the actual car

Materials

• ruler
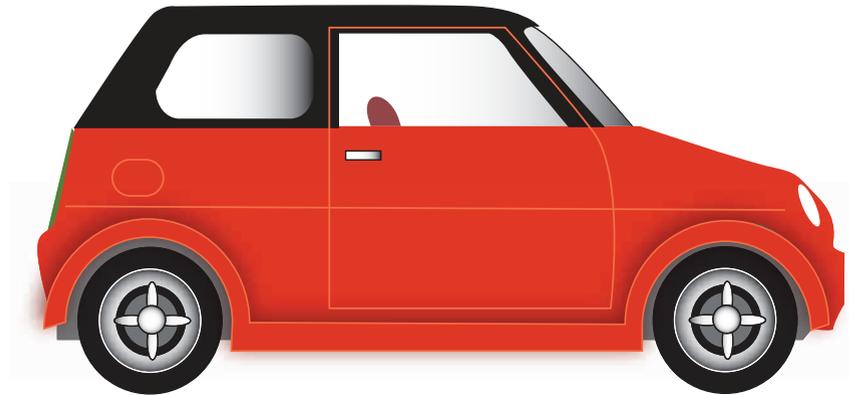
Car manufacturers create scale drawings that show what a new car will look like.

An actual car measures 339.2 cm in length and 163.2 cm in height. It is drawn to a scale of 1 : 32. Is the drawing an accurate representation of the actual model? What different strategies can you develop to find out?

Explore the Accuracy of a Diagram

1. What measurements would help you compare the diagram of the car to the actual car? Take the measurements.

2. Compare the measurements. What conclusions can you make?

Reflect and Check

3. a) How did you set up your calculations to determine if the diagram accurately represents the actual car?

b) What information did you need to determine whether the diagram is an accurate representation of the actual car?

4. a) Choose an object and draw one view of it. Estimate the scale between your drawing and the actual object.

b) Use the method you developed to determine how accurately the drawing represents the actual object.

5. Compare your method with the one used by a classmate. How are the methods similar? How are they different? Which method seems more efficient? Explain.

Link the Ideas

scale diagram

- a drawing that is similar to the actual figure or object
- may be smaller or larger than the actual object, but must be in the same proportions

Example 1: Use the Scale to Determine the Actual Length of an Object

The **scale diagram** of a skateboard uses a scale of $1:14$. What is the actual length of the skateboard?

5.5 cm
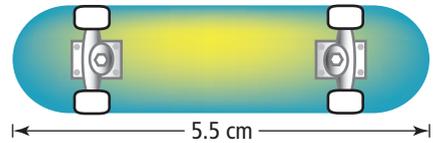
Solution

Method 1: Use the Scale
The scale $1:14$ means that the actual dimensions of the skateboard are 14 times those of the diagram. Multiply the length of the skateboard in the diagram by 14.
$5.5 \times 14 = 77$

The actual length of the skateboard is 77 cm.

Method 2: Use a Proportion
Set up a proportion using the scale and the measurement that is given.

$$\text{scale} = \frac{\text{diagram measurement}}{\text{actual measurement}}$$

The scale is $1:14$. The diagram measures 5.5 cm. The actual measurement is unknown.

$$\frac{1}{14} = \frac{5.5}{y}$$

$\times\ 5.5$

$$\frac{1}{14} = \frac{5.5}{77}$$

$\times\ 5.5$

The actual length of the skateboard is 77 cm.

> **Literacy Link**
>
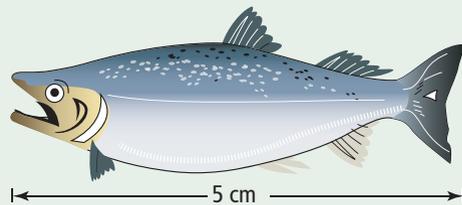> A *proportion* is a relationship that shows two ratios are equal. It can be written in fraction or ratio form.
>
> For example, the ratio 1 girl to 4 students is the same as 5 girls to 20 students. As a proportion, write:
> $\frac{1}{4} = \frac{5}{20}$ or $1:4 = 5:20$
>
> The corresponding parts of each ratio are in the same units.

Show You Know

The scale for the diagram of the chinook salmon is $1:9.2$.

5 cm

Calculate the actual length of the salmon.

Example 2: Determine the Scale Factor

An actual Canadian quarter has a diameter of 23.88 mm. Calculate the scale factor used to create the diagram of the quarter. Express the answer to the nearest tenth.

Solution

Measure the diameter of the diagram of the quarter. It measures 1.4 cm.

> The diagram is a reduction. The scale factor will be less than 1.

Set up a proportion for the scale and the measurements.

$$\text{scale} = \frac{\text{diagram measurement}}{\text{actual measurement}}$$

$$\frac{1}{x} = \frac{14}{23.88}$$

$$\div 14$$

$$\frac{1}{1.7} = \frac{14}{23.88}$$

$$\div 14$$

> To compare items using a ratio, the units must be the same. The actual measurement is 23.88 mm. The diagram measures 1.4 cm, which is 14 mm.

Strategies
Solve an Equation

Divide to determine the scale factor.
$$1 \div 1.7 \approx 0.588\ldots$$
$$\approx 0.6$$

The scale factor is approximately 0.6.

This means that the quarter in the diagram is approximately 0.6 times as large as the actual quarter.
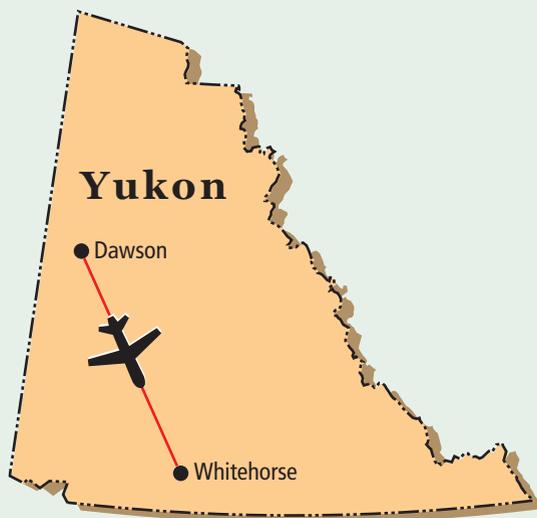

Show You Know

The flying distance from Dawson City to Whitehorse is 540 km. The distance shown on the map is 3 cm.

a) Complete the following to express the map scale in words.

scale: 1 cm represents ■ km

b) What is the scale factor?
Hint: 1 km = 100 000 cm.

Yukon

● Dawson

● Whitehorse

Key Ideas

- A scale diagram is a proportionally smaller or larger representation of an actual object.

A map is a scale diagram.

A • Alphaville

B • Betaville   • C Centretown

- The scale is a ratio between two sets of measurements.
  The scale compares a distance on the map to the actual distance.
  If 1 cm represents 12 km, then 1 cm represents $12 \times 100\,000$ cm.
  The scale is $1 : 1\,200\,000$.
  The scale factor is $\dfrac{1}{1\,200\,000}$.
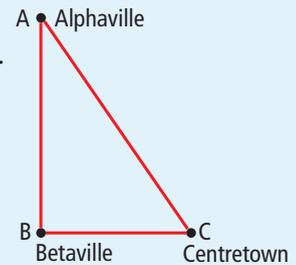
> 1 km = 1000 m and 1 m = 100 cm, so 1 km = 100 000 cm.

- You can solve problems involving scale diagrams using different methods.

  - Use a scale.
    The distance from A to B on the map is 3 cm. Determine the actual distance.
    $3 \times 1\,200\,000 = 3\,600\,000$
    The actual distance is 3 600 000 cm or 36 km.

  - Use a proportion.
    The distance from A to C on the map measures 4 cm. Determine the actual distance.
    $$\text{scale} = \frac{\text{diagram measurement}}{\text{actual measurement}}$$
    $$\frac{1}{1\,200\,000} = \frac{4}{\blacksquare}$$
    The actual distance is 4 800 000 cm or 48 km.

Check Your Understanding

Communicate the Ideas

1. Joseph is unsure about how to determine the actual length of an object using a scale diagram. List the steps to solve a problem of your choice. Discuss the steps with a classmate.

2. Kira plans to ride 150 km on her bike. This distance is 10 cm on a map. Express the scale of the map
   a) in words
   b) as a ratio

3. How can you check that the larger image of the airliner is proportional to the dimensions in the original photo? Try out your method. Describe your results.

original

image

Practise

4. State whether you would multiply or divide to determine the missing value.

a) $\dfrac{1}{3} = \dfrac{\blacksquare}{144}$ **b)** $\dfrac{1}{\blacksquare} = \dfrac{5.2}{117}$

5. Determine the missing value in each proportion.

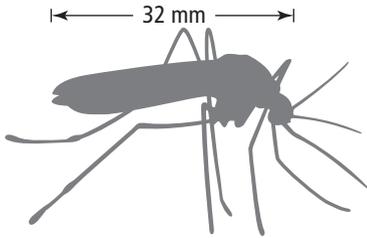**a)** $\dfrac{1}{9} = \dfrac{\blacksquare}{117}$ **b)** $\dfrac{1}{12} = \dfrac{10.5}{\blacksquare}$

6. Calculate the actual length of each object.
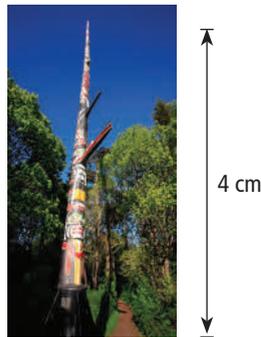
a) The scale for the image of the school bus is $1:302.5$.

b) The scale for the enlarged image of a mosquito is $1:0.5$.

7. Determine the actual length of each object.

a) The scale for the image of Victoria's tallest totem pole is $1:972.5$.

b) The scale for the model of the humpback whale is $1:280$.
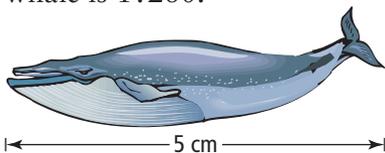
8. What is the scale factor?

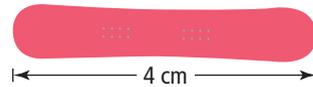**a)** $\blacksquare = \dfrac{30}{200}$ **b)** $\blacksquare = \dfrac{21}{12.5}$

**9.** Determine the scale factor.

a) $\blacksquare = \dfrac{0.5}{25}$ **b)** $\blacksquare = \dfrac{1.6}{3.2}$

10. What scale factor was used to create the image of the snowboard if its actual length is 166 cm? Express your answer to the nearest hundredth.

11. At the time his photo was taken for the hockey card, Ken was 152.4 cm tall. Calculate the scale factor used to create Ken's image on the hockey card. Express the answer to the nearest hundredth.

Digital rights not available.
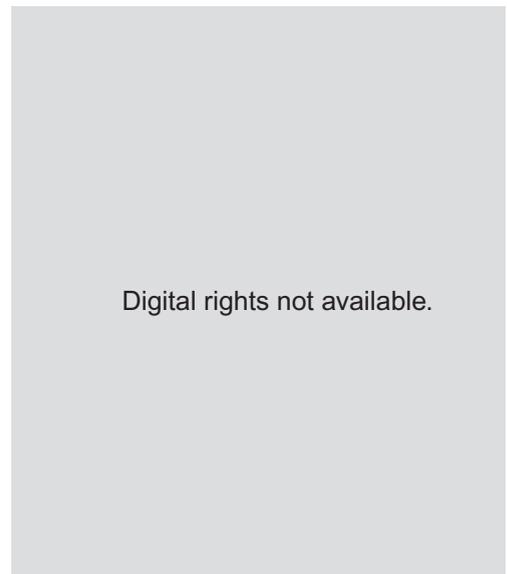
12. A flying distance is 800 km. If this distance on a map is 5 cm, what is the scale factor? Hint: 1 km = 100 000 cm.

Apply

13. A Ukrainian decorated egg is called a pysanka. A giant version of a pysanka is located in Vegreville, Alberta. The length of the egg is 9.4 m.

a) On a scale diagram of the pysanka, what would the length be, if you used a scale of 1:150?

b) Could your result represent the length of an actual egg? Explain.

14. The footprint of an adult male polar bear measures 30 cm across.

a) What is the scale factor of this drawing?

b) What is the actual length of the polar bear's footprint? Show how you know.

c) Measure your hand span by spreading your hand on a piece of paper. Write the ratio of your hand span to the span of the polar bear's footprint. What conclusion can you make?

15. Viruses are much smaller than bacteria. Some viruses measure 0.0001 mm in diameter. An artist's diagram of a virus shows the diameter as 5 mm. Determine the scale factor used.

16. For the science fair, Leanne plans to build a scale model of a communications tower that is actually 250 m in height. The model has to fit in the foyer of the school, which has a floor-to-ceiling height of 3 m. If Leanne uses a scale of 1:100 to build the model, will it fit into the foyer? Show your work.

17. A model train is a scale model created from actual measurements. The scale factor for HO or Half Zero model trains is 1:87. A typical engine, such as the one shown, is 50 mm in height and 200 mm in length. Determine the actual dimensions of the train engine.

18. Determine the scale factor for each enlargement or reduction.

a) from A to B **b)** from A to C

c) from B to C **d)** from C to A

e) from C to B
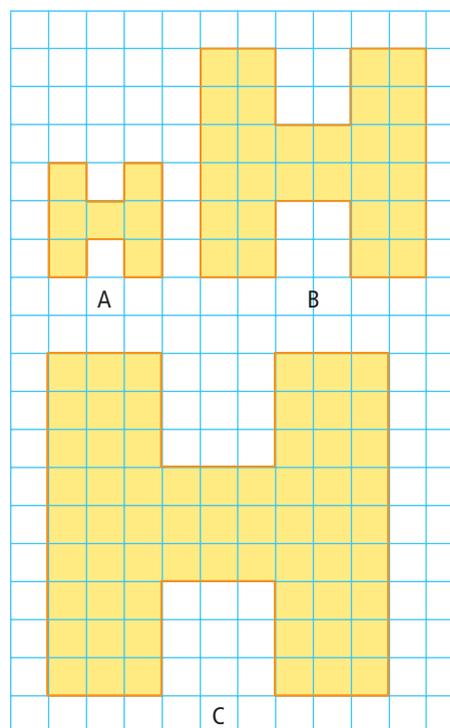
**19.** Tracy took a picture of a wind turbine at the wind farm in Cowley Ridge, Alberta. The height of the turbine is 45 m.

2.5 cm

a) What scale factor was used to make this reduction?

b) What is the length of a wind turbine blade?

Extend

20. △ABC has coordinates A(4, 3), B(4, 0), and C(7, 0). △DEF has coordinates D(0, −1), E(0, −2), and F(1, −2).

a) Draw the triangles on grid paper.

b) Are the two triangles proportional to each other? Justify your answer.

c) What is the scale factor of △ABC to △DEF?

d) Determine the scale factor of △DEF to △ABC.

e) Calculate the area of each triangle.

f) What is the ratio of the area of △ABC to the area of △DEF? of the area of △DEF to the area of △ABC?

g) How does the scale factor of the side lengths compare to the scale factor of the areas?

21. Elk Valley Coal uses trucks such as the one shown. The man in the picture is 1.69 m tall.

Digital rights not available.
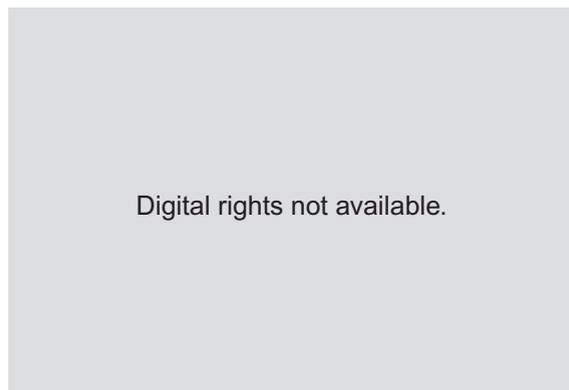
a) What is the height of the wheel of the truck?

b) What is the height of the truck?

> **Did You Know?**
>
> Elk Valley Coal operates five open-pit coal mines. The mines are in southeastern British Columbia and in west-central Alberta.

22. A rectangle has sides measuring 12 cm and 16 cm. An enlarged, similar rectangle has an area of 1200 cm².

a) What is the scale factor between
- the smaller and the larger rectangle?
- the larger and the smaller rectangle?

b) Is one method better than the other to express this scale factor? Explain your reasoning.

Math Link

a) Determine the scale factor for the enlargement or reduction of the design you drew for the Math Link on page 138. Show your work.

b) Choose a new feature to add to your design.
- Draw it on your scale diagram.
- Calculate the actual dimensions of the new feature.

c) Explain how you know the scale diagram is proportional to the actual design.

4.3 Similar Triangles

Focus on…

After this lesson, you will be able to…

- **determine similar triangles**
- **determine if diagrams are proportional**
- **solve problems using the properties of similar triangles**
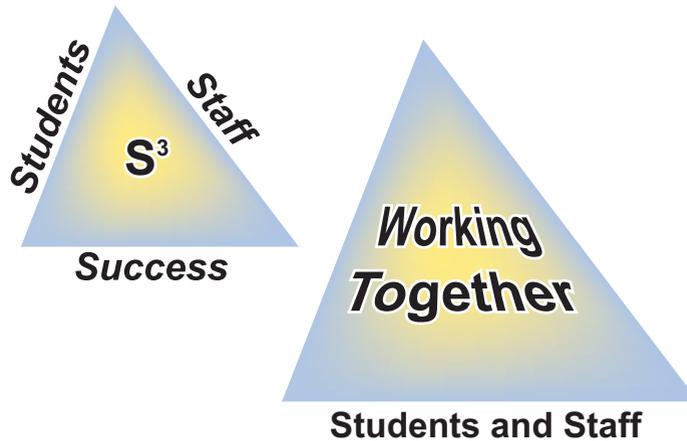
Bonnie and Justin created these logos for the Student Council.

Their advisor tells them that the triangles are similar. How can she tell? What do you know about similar figures? What strategies can you develop to determine if triangles are similar?

Explore How to Identify Similar Triangles

1. Trace each logo on separate pieces of tracing paper.

2. a) Measure the angles in each logo. What do you notice about the **corresponding angles**?

b) Measure the side lengths in each logo. What do you notice about the ratios of the **corresponding sides** of the triangles?

Reflect and Check

3. a) What conclusions can you make about the corresponding angles of the two triangles?

b) What conclusions can you make about the corresponding sides of the two triangles?

4. a) What conditions do you think are necessary in order for two triangles to be similar?

b) Test the conditions on a different set of two triangles. Are the triangles similar? Discuss with a classmate why you think the triangles are, or are not, similar.

Materials

- tracing paper
- ruler
- protractor

corresponding angles
corresponding sides

- have the same relative position in geometric figures

A

B   C

D

E   F
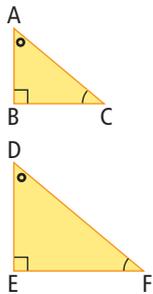
corresponding angles:
∠A and ∠D
∠B and ∠E
∠C and ∠F

corresponding sides:
AB and DE
BC and EF
AC and DF

Link the Ideas

Example 1: Identify Similar Triangles

Determine if △ABC is **similar** to △EFG.
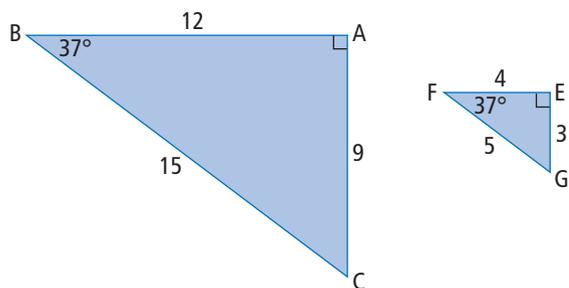
similar (figures)

- have the same shape but different size
- have equal corresponding angles and proportional corresponding sides

Solution

Similar triangles have corresponding angles that are equal in measure and corresponding sides that are proportional in length.

Compare corresponding angles:
$\angle A = 90°$ and $\angle E = 90°$
$\angle B = 37°$ and $\angle F = 37°$
$\angle C = 53°$ and $\angle G = 53°$

The sum of the angles in a triangle is 180°. If you know the measures of two pairs of angles are equal, then what can you conclude about the third pair of angles?

The corresponding angles are equal.

Compare corresponding sides:

$$\frac{AB}{EF} = \frac{12}{4} \qquad \frac{BC}{FG} = \frac{15}{5} \qquad \frac{AC}{EG} = \frac{9}{3}$$
$$\quad = 3 \qquad\qquad = 3 \qquad\qquad = 3$$

The corresponding sides are proportional with a scale factor of 3.
△ABC ∼ △EFG

Literacy Link

The symbol $\sim$ means is similar to.

△ABC ∼ △EFG means triangle ABC is similar to triangle EFG.

Literacy Link

Angles can be named in two ways:

- Use three capital letters. The middle letter is the vertex of the angle.
- Use only the middle letter identifying the vertex. Use a single letter when there is only one angle at a vertex.

For example, the angle at vertex L can be named $\angle$KLM or $\angle$L.
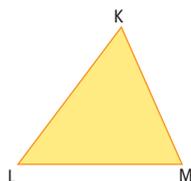
Show You Know

Determine if each pair of triangles is similar. Show how you know.

a)

b)
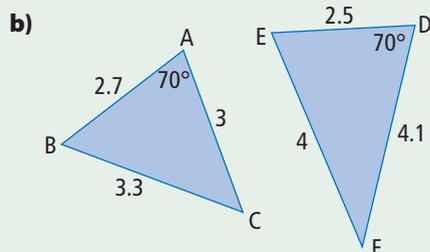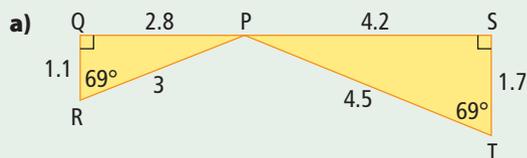
Example 2: Use Similar Triangles to Determine a Missing Side Length

Kyle is drawing triangles for a math puzzle. Use your knowledge of similar triangles to determine

a) if the triangles are similar
b) the missing side length

Solution

a) Check that $\triangle$KLM is similar to $\triangle$TUV.

The sum of the angles in a triangle is 180°.

$\angle K = 180° - 50° - 85°$
$\quad = 45°$
$\angle U = 180° - 85° - 45°$
$\quad = 50°$

Compare corresponding angles:
$\angle K = 45°$ and $\angle T = 45°$
$\angle L = 50°$ and $\angle U = 50°$
$\angle M = 85°$ and $\angle V = 85°$
All pairs of corresponding angles are equal.

> It is not necessary to prove both conditions for similarity. One is sufficient.

Therefore, $\triangle KLM \sim \triangle TUV$.

b) You can compare corresponding sides to determine the scale factor.

$$\frac{LM}{UV} = \frac{24}{8} \qquad \frac{KM}{TV} = \frac{21}{7} \qquad \frac{KL}{TU} = \frac{x}{10.5}$$
$$= 3 \qquad\qquad = 3 \qquad\qquad = \blacksquare$$

The scale factor is 3. You can solve for the unknown length.

Method 1: Use a Scale Factor

Since the triangles are similar, you can use the scale factor to determine the missing length.

$$\frac{x}{10.5} = 3$$
$$x = 31.5$$

> How would you solve for *x*?

The missing side length is 31.5 units.

Method 2: Use a Proportion

Since the triangles are similar, you can use equal ratios to set up a proportion.

$$\frac{KM}{TV} = \frac{KL}{TU}$$

$\times 1.5$

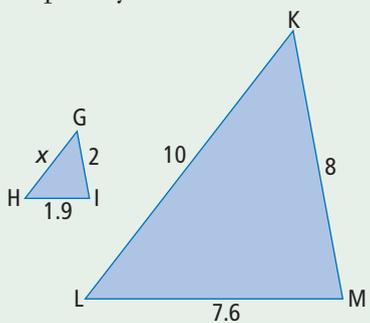$$\frac{21}{7} = \frac{x}{10.5}$$

> $10.5 \div 7 = 1.5$

$\times 1.5$

$$x = 31.5$$

The missing side length is 31.5 units.

Show You Know

Solve using a method of your choice.

a) △GHI is similar to △KLM. What is the missing side length? Express your answer to the nearest tenth.

b) △ABC is similar to △EFC. Determine the missing side length. Express your answer to the nearest tenth.
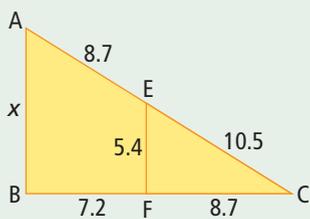
Key Ideas

- Triangles are similar if one of the following conditions holds true:
  - corresponding angles are equal in measure
  - corresponding sides are proportional in length

    △DEF is similar to △ABC.
    △DEF is not similar to △PQR.
    $\angle D = \angle A, \angle E = \angle B, \angle F = \angle C$

    $\dfrac{DE}{AB} = \dfrac{3}{1.5}$ $\dfrac{EF}{BC} = \dfrac{2.2}{1.1}$ $\dfrac{DF}{AC} = \dfrac{2.6}{1.3}$
    $\quad = 2 \qquad\qquad = 2 \qquad\qquad = 2$

- You can solve problems related to similar triangles using different methods.
  - Use a scale factor.
  - Use a proportion.
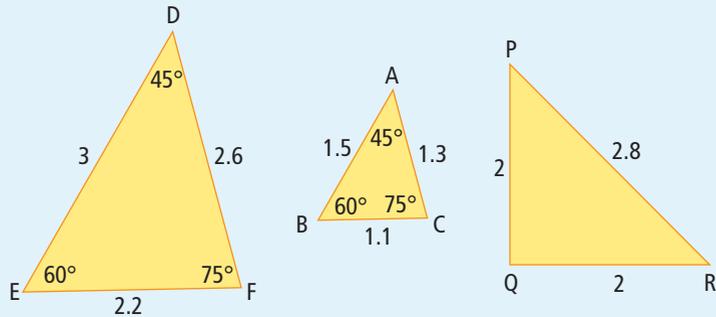
Check Your Understanding

Communicate the Ideas

1. If two triangles are similar, what can you say about the angles of the triangles? the side lengths of the triangles?

2. Amanda is unclear about similar triangles. She drew these two triangles and states they are similar. Is she correct? Explain.

3. Are two triangles that have equal angles and equal sides similar? Use an example to support your answer.
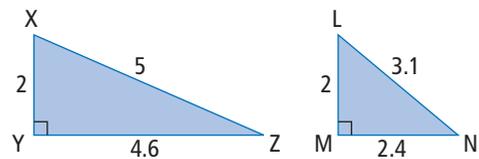
Practise

For help with #4 to #8, refer to Example 1 on page 147.

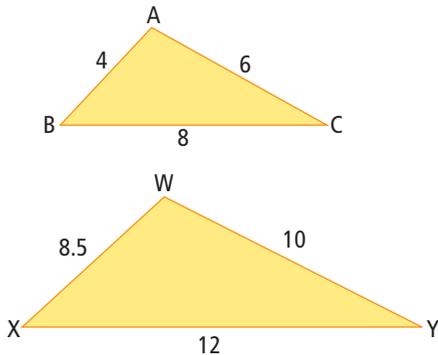4. List the corresponding angles and the corresponding sides for △PQR and △TUV.
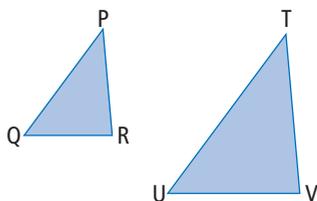
5. What are the corresponding angles and the corresponding sides in this pair of triangles?
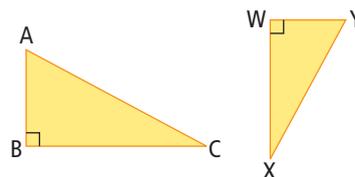
6. Are the triangles similar? Show how you know.
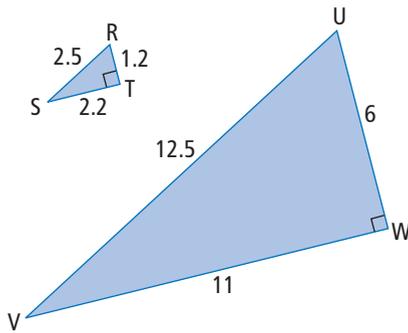
**7.** Determine if the triangles are similar. Show how you know.

**8.** Determine which pairs of triangles are similar. Use a sketch to help explain how you know.

Triangle	Angles	Sides
△ABC	$\angle A = 90°$ $\angle B = 45°$ $\angle C = 45°$	AB = 6 BC = 8.4 AC = 6
△EFG	$\angle E = 90°$ $\angle F = 45°$ $\angle G = 45°$	EF = 3 FG = 4.2 EG = 3
△HIJ	$\angle H = 90°$ $\angle I = 60°$ $\angle J = 30°$	HI = 9.2 IJ = 18.4 HJ = 15.9
△KLM	$\angle K = 90°$ $\angle L = 45°$ $\angle M = 45°$	KL = 9 LM = 12.6 KM = 9

For help with #9 to #11, refer to Example 2 on pages 148–149.

9. △STR is similar to △UWV. Determine the missing side length.
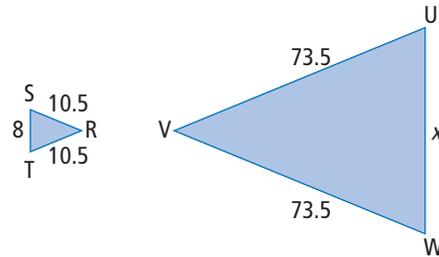
**10.** △CDE is similar to △GFE. What is the missing side length?
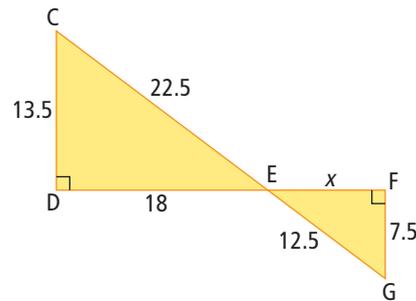
11. Draw a triangle that is similar to the one shown. Label the measurements for angles and sides on your similar triangle.
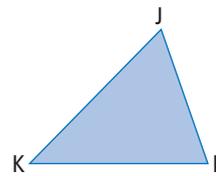

Apply

12. Sam built a ramp to a loading dock. The ramp has a vertical support 2 m from the base of the loading dock and 3 m from the base of the ramp. If the vertical support is 1.2 m in height, what is the height of the loading dock?
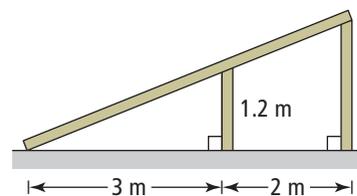
13. Two extension ladders are leaning at the same angle against a vertical wall. The 3-m ladder reaches 2.4 m up the wall. How much farther up the wall does the 8-m ladder reach?
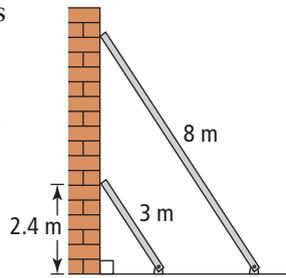
14. Erin, who is 1.60 m tall, casts a shadow that is 1.25 m long. Her shadow extends to the end of a tree's shadow when she stands 4.75 m from the tree. What is the height of the tree?
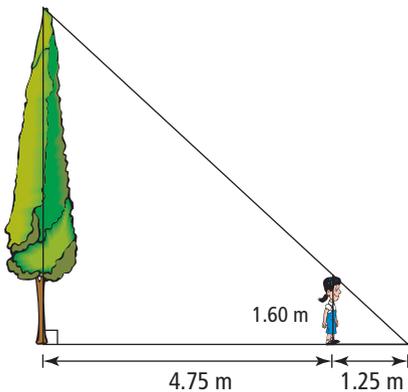
15. Sara was helping her father assemble a slide for the local park. He decides to reinforce the slide with an extra support beam. How long should the extra support beam be?
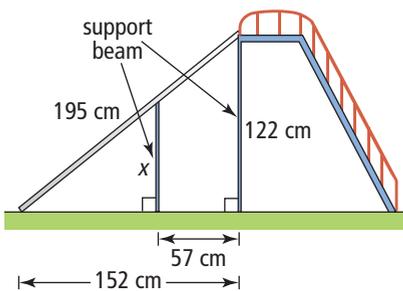
16. Peter, who is 168 cm tall, casts a 45-cm shadow. Michael, who is standing beside him, casts a 40-cm shadow. Can you tell who is taller? Use a diagram to help explain why or why not.

17. Develop a word problem that can be solved using similar triangles. Include a diagram.

Extend

18. A tourist wants to estimate the height of an office tower. He places a mirror on the ground and moves away to sight the top of the tower in the mirror.
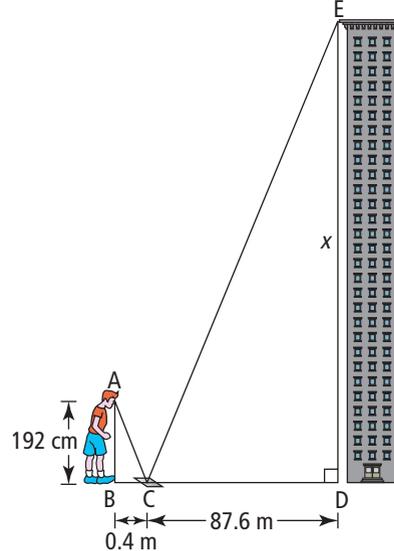
a) How tall is the tower?

b) In this situation, why is the mirror reflection a better way to indirectly measure the tower than by using shadows?

19. Is it possible for the two triangles described below to be similar? Explain your reasoning.

a) Two angles of one triangle measure 60° and 70°. Two angles of the other triangle measure 50° and 80°.

b) Two angles of one triangle measure 45° and 75°. Two angles of the other triangle measure 45° and 60°.

20. The sides of a triangle measure 3 cm, 5 cm, and 6 cm. If the side of a similar triangle corresponding to 3 cm measures 8 cm,

a) determine the lengths of the other sides

b) determine the ratio of the perimeter of the smaller triangle to the perimeter of the larger triangle

21. Using a measuring tape, your shadow, and yourself, how can you determine the height of your school without actually measuring it?

22. △WXY is similar to △ZWY. Calculate ZY to the nearest tenth.

W
7 cm
X
Z
Y
10 cm
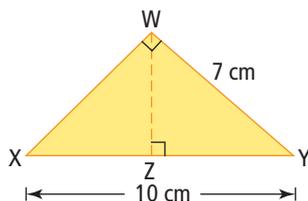
23. Use two different sets of measurements to determine the area of △KLM.

K
25 cm
h
L
20 cm
M
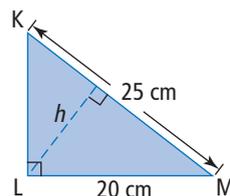
Math Link

For your design project report, include a signature logo that features your name.

a) On a sheet of 8.5 × 11 paper, design your logo. Include a triangle that is similar to the one shown. Measure all the angles and side lengths.

b) Draw a scale diagram of the logo to fit on your design project. Identify the scale factor you used.
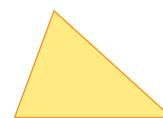
Tech Link

Similarity and Scale Factors

In this activity, you can use dynamic geometry software to explore similarity and scale factors. To use this activity, go to www.mathlinks9.ca and follow the links.

Explore

1. Slide point X along line segment AB and describe what happens to the image drawing.

2. How do the measures of the corresponding sides of the drawing change relative to each other? Explain.

3. Compare the scale factor to the lengths of the sides of the original drawing and the image drawing. Create and complete a table similar to the one below with measurements taken at different locations. Discuss your findings with a classmate. Hint: In the table, m means *the measure of*.

GF = 3.61 cm   F'G' = 1.39 cm
FE = 4.12 cm   F'E' = 1.58 cm
EG = 3.16 cm   E'G' = 1.22 cm
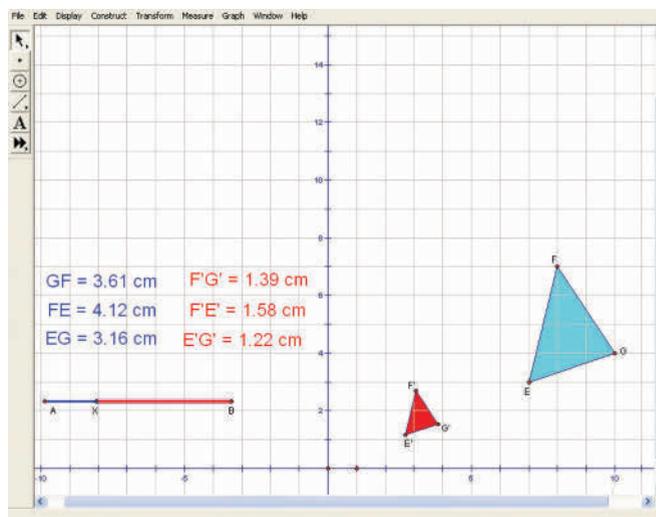
mFE	mF'E'	mAX mXB

4.4 Similar Polygons

Focus on...

After this lesson, you will be able to…

- identify similar polygons and explain why they are similar
- draw similar polygons
- solve problems using the properties of similar polygons

The single star in a Lakota star quilt is made from fabric cut into diamond shapes and pieced together in eight sections. When the sections are joined together, an eight-pointed star is formed.
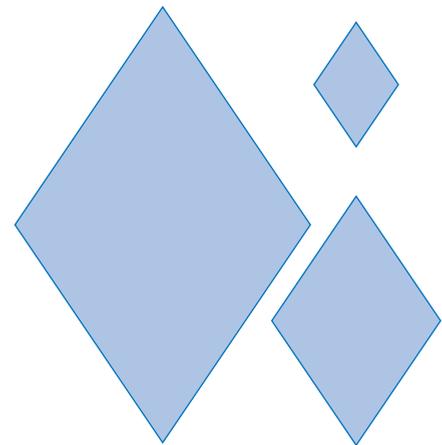
Are the different-sized diamonds formed on the quilt similar? What strategies might you use to find out?

Materials
- tracing paper
- protractor
- ruler

Explore How to Identify Similar Polygons

1. Trace each diamond on separate pieces of tracing paper.

2. a) Organize your data about corresponding angles and corresponding sides.

b) What do you observe about the corresponding angles?

c) What do you observe about the ratios of the corresponding sides?

Reflect and Check

3. What conclusions can you make about the three diamonds?

4. a) What conditions do you think are necessary in order for two polygons to be similar?

b) Test the conditions on a different set of two polygons. Are the polygons similar? Discuss with a classmate why you think the polygons are, or are not, similar.

polygon

- a two-dimensional closed figure made of three or more line segments

Link the Ideas

Similar polygons have corresponding angles that are equal and corresponding side lengths that are proportional.

Example 1: Identify Similar Polygons

The two quadrilaterals look similar. Is M′A′T′H′ a true enlargement of MATH? Explain.
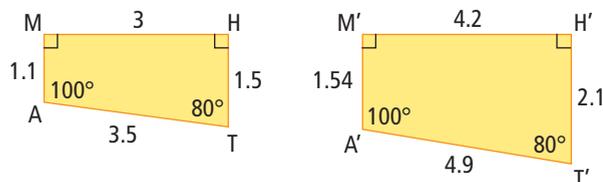
Solution

Compare corresponding angles:
$\angle M = 90°$ and $\angle M' = 90°$
$\angle A = 100°$ and $\angle A' = 100°$
$\angle T = 80°$ and $\angle T' = 80°$
$\angle H = 90°$ and $\angle H' = 90°$

> What is the sum of the interior angles in a quadrilateral?

Compare corresponding sides:

$$\frac{M'A'}{MA} = \frac{1.54}{1.1} \qquad \frac{A'T'}{AT} = \frac{4.9}{3.5}$$
$$= 1.4 \qquad\qquad = 1.4$$

$$\frac{H'T'}{HT} = \frac{2.1}{1.5} \qquad \frac{M'H'}{MH} = \frac{4.2}{3}$$
$$= 1.4 \qquad\qquad = 1.4$$

The corresponding side lengths are proportional with a scale factor of 1.4.

M′A′T′H′ is a true enlargement of MATH by a scale factor of 1.4.

Show You Know

Determine if the two trapezoids are similar. Explain how you know.

Did You Know?

Polygons can be divided into non-overlapping triangles. The sum of the interior angles in a triangle is 180°. You can determine the sum of the interior angles in a polygon by multiplying the number of triangles by 180.

To draw the triangles, start with any vertex of the polygon, and from there draw a line to connect to each of the other vertices. The pentagon can be divided into three triangles.
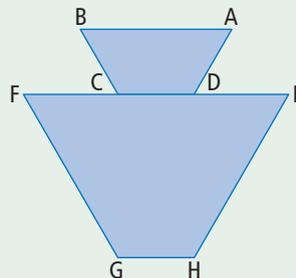
$3 \times 180° = 540°$
The sum of the interior angles in a pentagon is 540°.

WWW Web Link

To learn more about the properties of similar polygons, go to www.mathlinks9.ca and follow the links.

Example 2: Determine a Missing Side Length

Jason wants to make an enlargement of the flag of Nunavut. He knows that the two rectangles JKLM and PQRS are similar. What is the missing side length of rectangle JKLM?
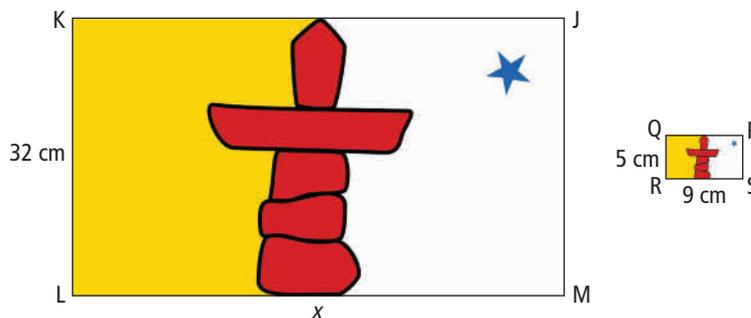
Solution

Since the rectangles are similar, the side lengths are proportional. Use corresponding sides to set up a proportion.

$$\frac{KL}{QR} = \frac{LM}{RS}$$

$$\frac{32}{5} = \frac{x}{9}$$
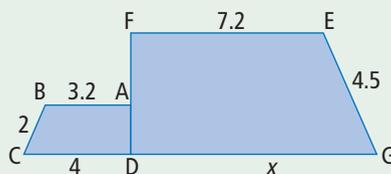
$$6.4 = \frac{x}{9}$$

$$x = 57.6$$

The missing side length is 57.6 cm.

Strategies

Solve an Equation

> What different method could you use to solve the problem? Try it.

Show You Know

The two trapezoids shown are similar. Determine the missing side length. Show your work.

Key Ideas

- Polygons are similar if both of the following conditions hold true:
  - corresponding angles are equal in measure
  - corresponding side lengths are proportional
- You can use similar polygons to determine unknown side lengths or angle measures.

The trapezoids HIJK and LMNO are similar.
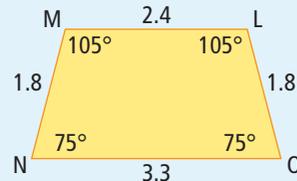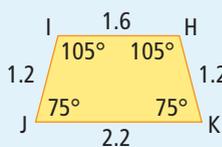
> ∠H = ∠L, ∠I = ∠M, ∠J = ∠N, ∠K = ∠O

> Each side length of LMNO is 1.5 times the corresponding side length of HIJK.

Check Your Understanding

Communicate the Ideas

1. Develop an example and a solution to help explain how to determine a missing side length in a pair of similar polygons.

2. a) Use grid paper to draw a parallelogram that is similar to the one shown. Explain how you know the two are similar.

   b) Compare your similar parallelogram with the one of a classmate. Are your parallelograms similar to each other? Explain.
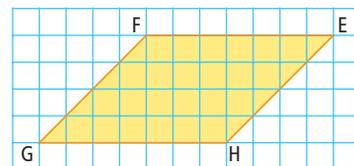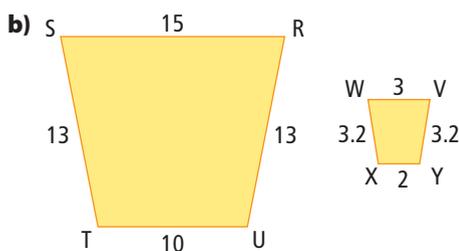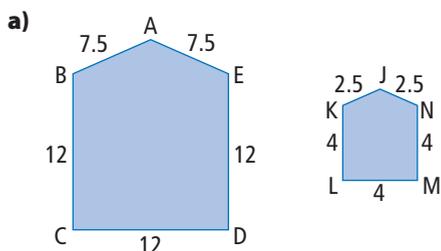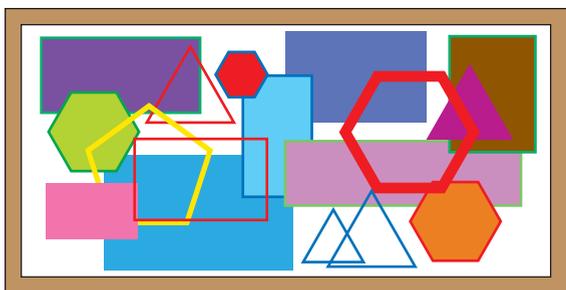
Practise

For help with #3 and #4, refer to Example 1 on page 155.
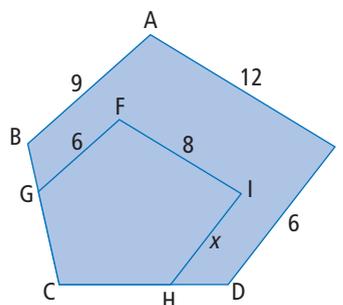
3. Decide if each pair of polygons is similar. Show your reasoning.

   a)

   b)

4. Identify all sets of similar polygons in this collage. You might trace the image and colour code sets of similar polygons.

For help with #5 and #6, refer to Example 2 on page 156.

5. Use the two similar pentagons to help determine the missing side length. Show your work.

6. The sides of rectangle A measure 22.4 m and 14.7 m. One side of a similar rectangle B measures 4.3 m. The measure for the other side is missing. Rectangle A is an enlargement of rectangle B with a scale factor of 5.2. What is the missing measurement, to the nearest tenth?

Apply

7. William made the statement, "All quadrilaterals with sides the same length are similar." Is he correct? Explain.

8. Chicken wire is often used for building fences. It is made of flexible wire with gaps that are shaped like hexagons.

   a) Use grid paper to draw and label:
   - two hexagons that are similar to one shown in the picture
   - two hexagons that are not similar to one shown

   b) For each pair of hexagons, explain how you know they are similar or not similar.
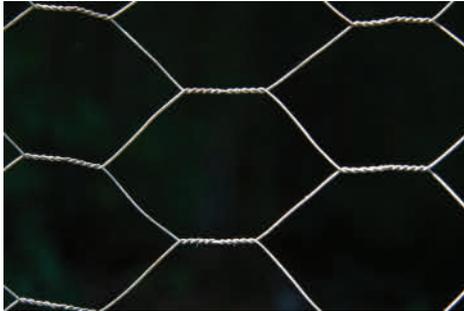
9. Michelle plans to make a game board that is a reduction of an actual baseball diamond. A baseball diamond is a square with sides that measure 27.4 m (2740 cm). Draw Michelle's game board using a scale of $1:182.5$.

10. a) Rachel's family is making a cement deck around a pool that is a regular octagon. They want the cement deck to keep the same shape as the pool, but with sides 1.5 times as long as the pool. What do the lengths of the cement forms along the sides of the outer octagon need to be in order to pour the cement?
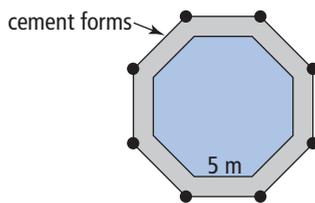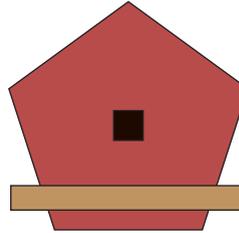

   b) What is the sum of the interior angles in an octagon? Show how you know.

**11.** The pattern shows the front of a birdhouse. Chris enlarged the pattern using a scale factor of 3. He needs to make it twice as large as that.

   a) Draw the correct size.

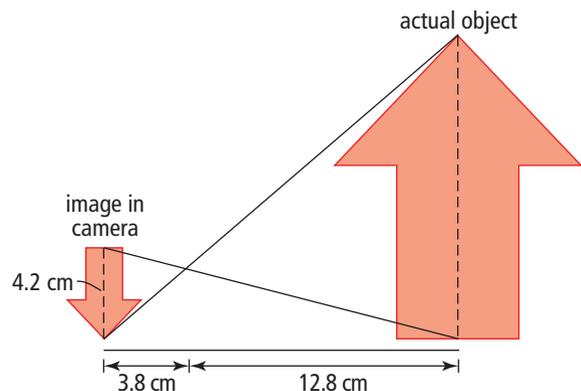   **b)** Explain how you know the enlargement is similar to the original pattern.

12. A piece of cardboard is cut showing the inner and outer boundaries of a pair of similar quadrilaterals. Calculate the perimeter of the smaller quadrilateral.
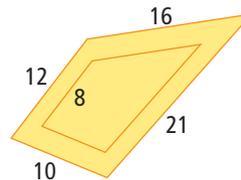

Extend

**13.** In a camera, similar figures occur as shown. Calculate the actual height of the arrow.

14. Eliza is building a model of the canvas tent her family uses in Behchoko, NWT. The model will have a peak height of 12 cm. The actual tent floor measures 2.4 m by 3 m. The walls are 1.5 m high and the peak height is 2.4 m.

  a) What scale factor will Eliza need to use for her model?

  b) The front of the tent is a pentagon. Calculate the dimensions of this polygon on the model.

  c) Calculate the other dimensions of the tent model.

Digital rights not available.
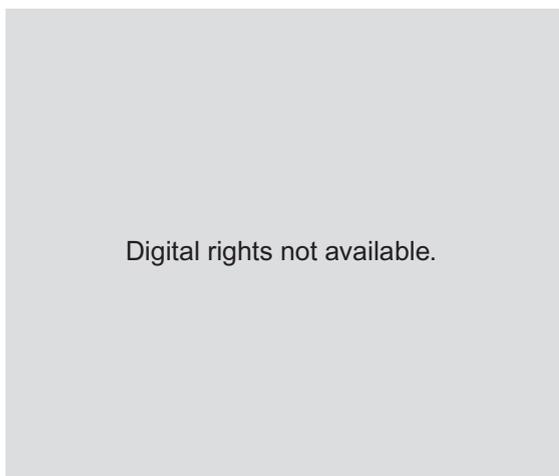
15. An old rectangular tank with length 0.3 m could hold 154 L of water. A new similar tank has a length of 1.5 m. What is the capacity of the new tank?

16. How do the ratios of areas compare to the ratios of corresponding side lengths in similar polygons? Use pairs of similar polygons to help explain your answer.

17. Develop an argument showing that if two prisms have corresponding side lengths in the ratio of $3:1$, then their volumes are in the ratio of $27:1$.

18. a) Identify the similar polygons shown in the tessellation.

  b) Describe the pattern verbally. Use your description to create your own tessellation that features similar polygons.

  c) Sketch each different set of similar polygons in your tessellation. Label the dimensions of each set.
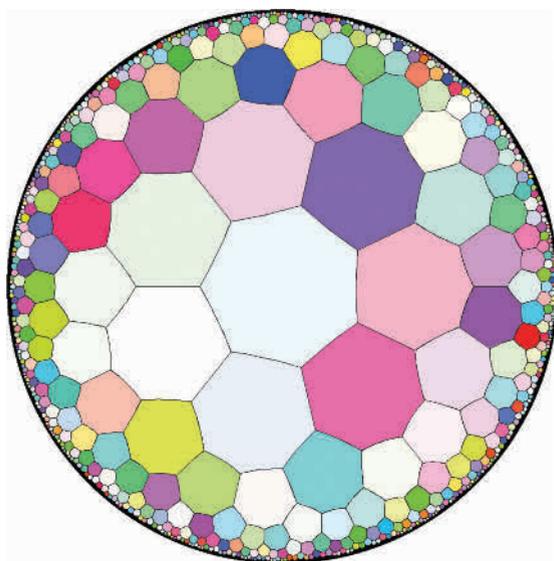
Math Link

For your design project, include a polygon.
• Use a polygon that is similar to one shown here.
• Use an appropriate scale factor and draw a scale diagram of the polygon to fit on your design project. Identify the scale factor used on your design.

4.4 Similar Polygons   **159**
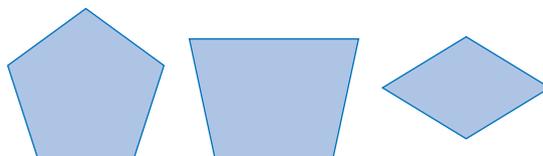
Key Words

For #1 to #4, unscramble the letters for each term. Use the clues to help you.

1. L G Y N O P O

A � is a closed figure with sides that are line segments.

2. I R I S L M A

▭ figures have equal corresponding angles and proportional corresponding side lengths.

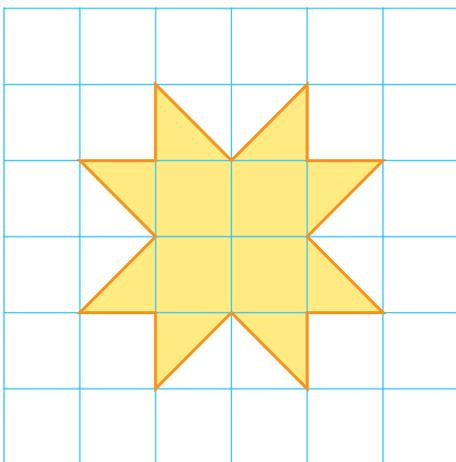**3.** C A S E L O T C A F R

The ▭ ▭ is the constant amount by which any dimension of a shape is enlarged or reduced.

4. R R O O P P T I N O

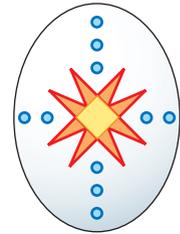A ▭ is a statement that says two ratios are equal.

4.1 Enlargements and Reductions, pages 130–138

5. Use grid paper to draw the design using each scale factor.

a) scale factor of 2
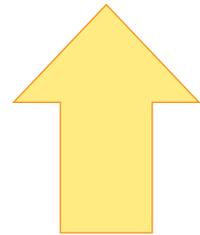
b) scale factor of 0.5

6. Draw an image of the egg design that is three times as large as the original.
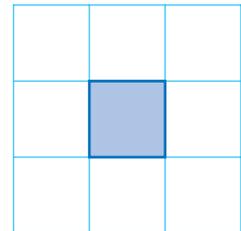
> **Art Link**
> Pysanky is the traditional Eastern European art that uses beeswax and dyes to create designs on eggs.

7. Draw a reduction of the arrow that is half as large as the original.

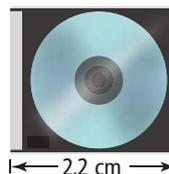**8.** Draw an image of the square to illustrate each of the following.

a) a scale factor equal to 1

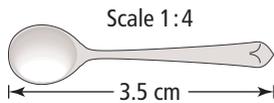b) a scale factor greater than 1

c) a scale factor less than 1

4.2 Scale Diagrams, pages 139–145

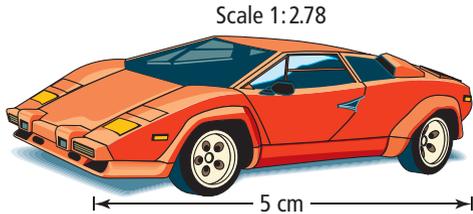**9.** An actual CD jewel case measures 14.3 cm. Determine the scale factor used to create the image.

2.2 cm

10. Determine the actual length of each object from its scale diagram.

a) spoon

Scale 1:4
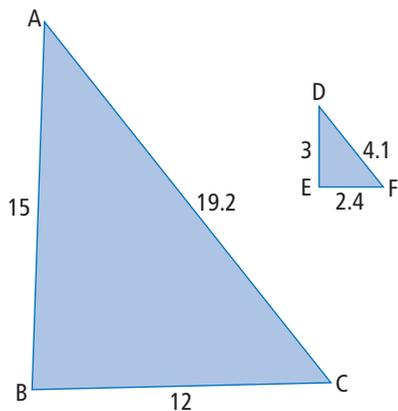

|← 3.5 cm →|

b) toy car

Scale 1:2.78

|← 5 cm →|

11. The scale for an image of a tower is 1 cm represents 12.5 m. If the actual tower measures 108.75 m in height, what is its height on the drawing?

12. A highway is 600 km in length. If the length of the highway on a map is 6 cm, what is the scale factor? Hint: 1 km = 100 000 cm.

4.3 Similar Triangles, pages 146–153

13. Are these triangles similar? Explain.

14. △UVW is similar to △UYZ. Determine the length x.
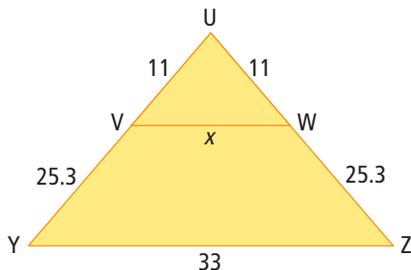
15. Given that △GHI is similar to △KLM, what is the length of side IG?
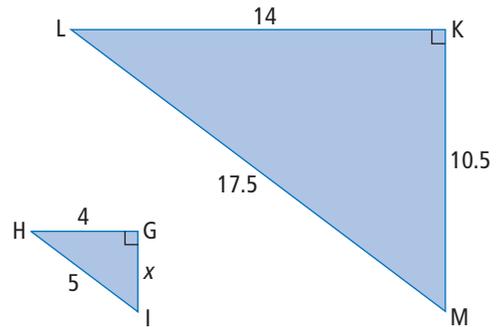
4.4 Similar Polygons, pages 154–159

16. Determine if the two polygons are similar.
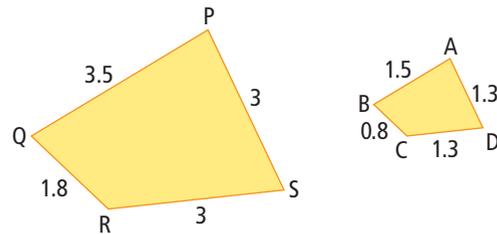
17. The sides of one quadrilateral measure 3 cm, 9 cm, 12 cm, and x. The corresponding sides of a similar quadrilateral measure 2.25 cm, 6.75 cm, 9 cm, and 13.5 cm. What is the value of x?

18. The pentagons DEFGH and JKLMN are similar. Determine the missing side lengths, to the nearest tenth.

Chapter 4 Practice Test

For #1 to #4, choose the best answer.

1. What is the value of x if $\frac{1}{x} = \frac{8}{32}$?

  A 2   **B** 3   **C** 4   **D** 7

2. $\triangle$GHI $\sim$ $\triangle$KLM. Determine the missing length.

  A 4

  B 8

  C 10

  D 14

3. On a scale diagram, what does 1 in the scale 1 : 5 represent?

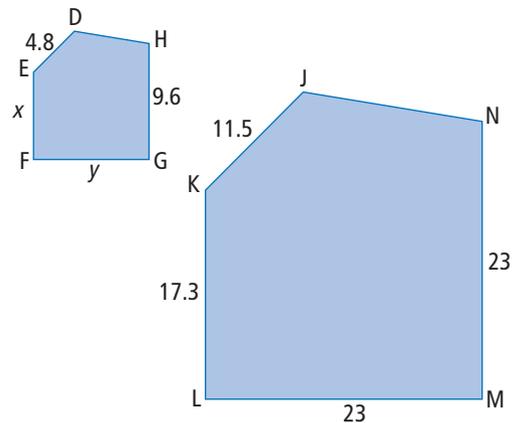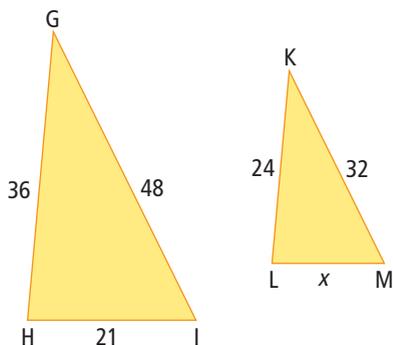 **A** how many times larger the object is

  B one unit of the actual size

  C one unit of the diagram size

  D the total size of the scale diagram
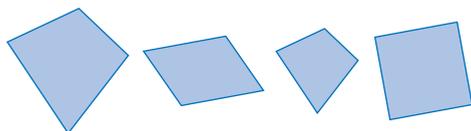
4. Which pair of quadrilaterals appears to be similar?

    Figure 1   Figure 2   Figure 3   Figure 4

  A Figure 1 and Figure 2

  B Figure 1 and Figure 3

  C Figure 1 and Figure 4

  D Figure 2 and Figure 3

Complete the statements in #5 and #6.

**5.** An umbrella is 75 cm in length. Using a scale of 1 : 5, the length of an image of the umbrella is ■.

6. The constant amount by which the dimensions of an object are enlarged or reduced is called the ■ ■.

Short Answer

7. Draw a reduction that is half the size of this figure.

8. If the actual pencil has a length of 18.8 cm, determine the scale factor used to create this image. Give your answer to the nearest tenth.

  4 cm
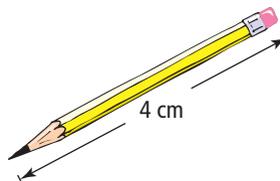
9. The flagpole in front of city hall is 5.5 m tall. If the height of a model of the flagpole is 6.5 cm, what is the scale factor of the model? Express your answer to the nearest hundredth.

10. An actual western spruce budworm larva can grow to 32 mm in length. Using a scale of 1 : 1.43, what would be the length of an image of the larva? Express your answer to the nearest tenth.

> **Did You Know?**
>
> Western spruce budworm larvae feed mostly on the foliage, flowers, and developing cones of fir and spruce trees. These insects cause serious damage to Douglas firs in the interior of British Columbia.

11. Is the image proportional to the original shape? Explain how you know. If it is proportional, state the scale factor.

original   image
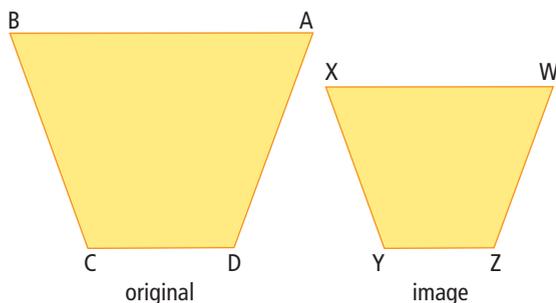
Extended Response

12. At noon one day, a 20-m vertical pole casts a shadow that is 28 m long. A nearby building casts a shadow 35 m in length. Sketch the situation. How tall is the building?
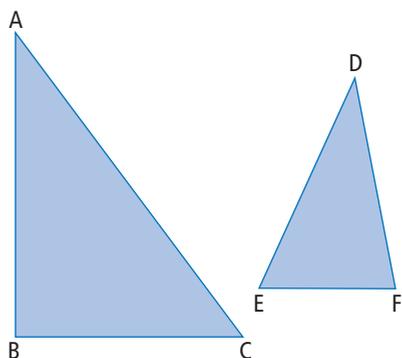
13. Determine if △ABC and △DEF are similar. Show all your work.

14. Bees made the hexagonal-shaped cells in the honeycomb shown here. Draw a hexagon similar to one of these cells. Explain why the two hexagons are similar.

15. These polygons are similar. Determine the missing lengths x and y. Show your work.
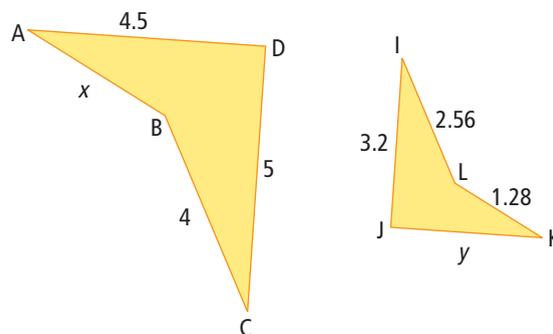
Math Link: Wrap It Up!

Finalize your design project.

a) Decide on the layout. Include the following elements:
- an enlarged or reduced image of your design
- a similar triangle for the logo
- a similar polygon that features the title of your design project
- a scale diagram of your design

b) Make a presentation that includes:
- your design and the scale you used
- a description or actual sample of the completed design project
- what you learned about scale diagrams and similarity

Challenges

Shadow, Shadow

When was the last time you made a shadow puppet? What was the largest shadow puppet you have ever made? How did you do it?

You be the puppeteer for a children's show. Work with a partner to develop shadow puppets that can be used to explore some of the mathematics you learned in this chapter.

1. Create a bird shadow like the one shown.

   a) What is the scale factor between the hands and the corresponding shadow? Explain how you determined the scale factor.

   b) Use a different set of dimensions to calculate the scale factor.

   c) What do you notice about the two scale factors?

   d) What is the mathematical relationship between the hands and the corresponding shadow?

2. a) Create a shadow puppet of your own design. Do not move the light source. Instead, move your hands, changing the distance between them and the wall.

   b) How does moving your hands affect the scale factor of the shadow? Record your observations and justify your response mathematically.

3. a) Cast the shadow puppet on the wall. Keep your hands in the same location but this time move the light source closer to and farther away from your hands.

   b) How does moving the light source affect the scale factor of the shadow? Record your observations and justify your response mathematically.
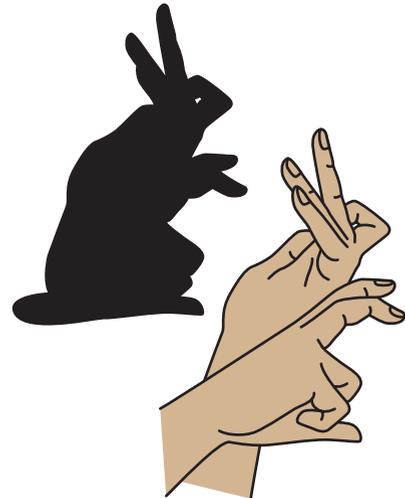
Graphic Designer

You be a graphic designer. Design a logo for a community, an organization, a school, a product, or a service. Your logo must

• include a feature that sets the community, organization, school, product, or service apart. For example, you might represent
  – a community award using a symbol such as a trophy
  – a culture using a symbol such as a totem pole
  – an organization using a symbol such as an animal with qualities that reflect those of the organization
  – a service using symbols such as a computer, a calculator, or a hammer and nails
  – an industry using symbols such as an oil well or a grain elevator
  – geography using symbols such as mountains, trees, or plains

• include polygons or composite shapes made of polygons

• be scalable, meaning that it can be enlarged or reduced

1. **a)** Design the logo.

   b) Explain why you chose the particular elements of your logo.

2. **a)** Decide on the dimensions of an enlargement of the logo for a billboard or a banner.

   b) Determine the scale factor.

   c) Mathematically justify the scale factor for three measurements in your logo.

3. **a)** Decide on the dimensions of a reduction of the logo for a business card or a web site.

   b) Determine the scale factor.

   c) Mathematically justify the scale factor for three measurements in your logo.

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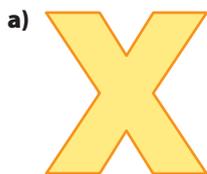
Digital rights not available.

Digital rights not available.

Canadian Blood Services
Société canadienne du sang

Chapters 1–4 Review

Chapter 1 Symmetry and Surface Area

1. Sketch each shape showing its line(s) of symmetry. Describe the lines of symmetry and the type of symmetry each shape has.

a)

b)

2. Describe two ways you could complete the drawing if the dashed line represents a line of symmetry. Complete the drawing.
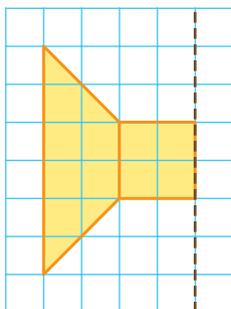
3. Create a design within a circle that shows both line and rotation symmetry.

a) How many lines of symmetry are in your design? Describe them.

b) What is the order of rotation in your design?

c) Give the angle of rotation in both degrees and fractions of a revolution.

4. Draw a diagram of a square cake and a round cake. Select any dimensions you like as long as the side of the square cake is the same length as the diameter of the round cake.

a) Find the surface area of each cake. Use all sides except the bottom.

b) Cut each cake into four equal pieces. If the pieces of cake are separated from each other, by what percent does the surface area of each cake increase? Again, do not consider the bottom.

5. Reproduce the triangle on a coordinate grid.

a) Complete a diagram that has rotation symmetry of order 4 about the origin.
- Label the vertices on the original triangle.
- Show the coordinates of their images after each rotation.
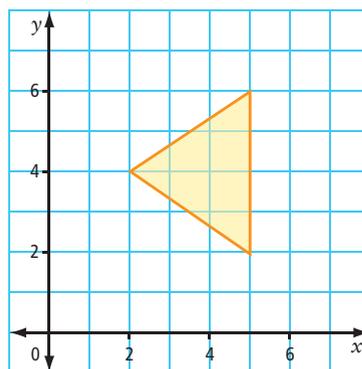
b) Start again, this time using line symmetry to make a new diagram. Use the y-axis and then the x-axis as lines of symmetry.

6. Four cubes, each with side lengths of 25 cm, are joined as shown.
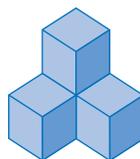
a) Find the surface area of the solid that is formed.

b) If the four cubes are rearranged as shown, how does the surface area change?
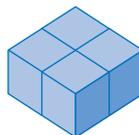
Chapter 2 Rational Numbers

7. Write the following rational numbers in ascending order.

$$0.\overline{6} \quad -0.9 \quad -\frac{4}{5} \quad 2.7 \quad -2\frac{3}{4} \quad -\frac{2}{3}$$

8. Identify a fraction between -6.3 and -6.4.

9. Estimate, then calculate.
- **a)** $-2.52 + 1.84$
- **b)** $-2.4 \times (-1.5)$
- **c)** $-4.37 \div (-0.95)$
- **d)** $0.76 + (-1.83)$
- **e)** $8.48 - 10.51$
- **f)** $-5.3(4.2)$
- **g)** $-2.31 - (-5.72)$
- **h)** $-5.5 \div (-5.5)$

10. Estimate, then calculate.
- **a)** $1\frac{1}{10} - \left(-1\frac{1}{10}\right)$
- **b)** $3\frac{3}{5} \div \left(-3\frac{3}{8}\right)$
- **c)** $-1\frac{1}{2} - \frac{1}{12}$
- **d)** $-\frac{1}{6} + \left(-\frac{1}{8}\right)$
- **e)** $\frac{1}{10} \times \left(-\frac{3}{7}\right)$
- **f)** $\frac{2}{3} \div \frac{4}{5}$
- **g)** $-4\frac{1}{2} + 2\frac{5}{9}$
- **h)** $-2\frac{1}{2}\left(-2\frac{1}{2}\right)$

11. Estimate and then calculate the side length of each square from its area. If necessary, round your answer to the nearest hundredth of a unit.
- **a)** 2.56 cm²
- **b)** 0.01 km²
- **c)** 0.048 mm²
- **d)** 1.02 km²

12. Mary is sewing a square quilt. If the area of her quilt is 2.89 m², what is its perimeter?


Chapter 3 Powers and Exponents

13. Write $4^2 \times (4^3)^5$ as a single power.

14. Evaluate the expression $(-6)^0 + 2^3 \div (7 - 5)^2$.

15. Write $\dfrac{(-4)^2(-4)^{10}}{(-4)^3}$ as a single power and then evaluate.

16. Write $(3 \times 7)^4$ as repeated multiplication without any exponents and as a product of two powers.

17. A population of 50 bacteria doubles in number every hour. The formula $N = 50(2)^t$ determines the number of bacteria, N, that are present after t hours. How many bacteria will there be after each number of hours?
- **a)** 5 h
- **b)** 9 h

Chapter 4 Scale Factors and Similarity

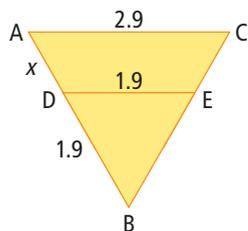**18.** Make an enlargement of the figure using a scale factor of 3.
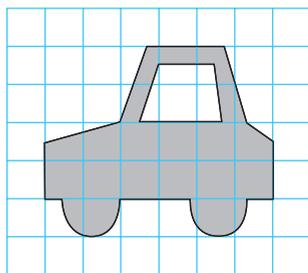
19. Determine the missing values.

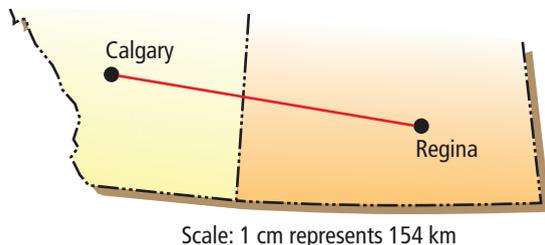**a)** $\dfrac{1}{3.5} = \dfrac{\blacksquare}{42}$

b) $\dfrac{1}{\blacksquare} = \dfrac{2.7}{49.95}$

c) $\dfrac{1}{0.09} = \dfrac{4.6}{\blacksquare}$
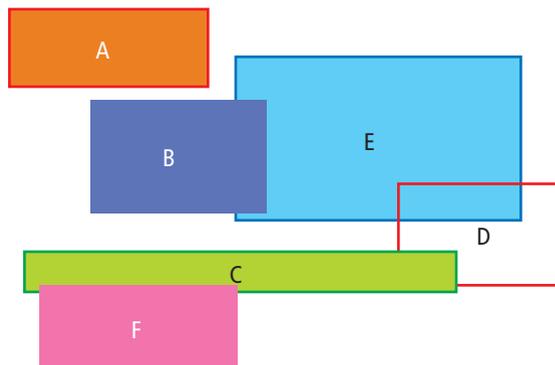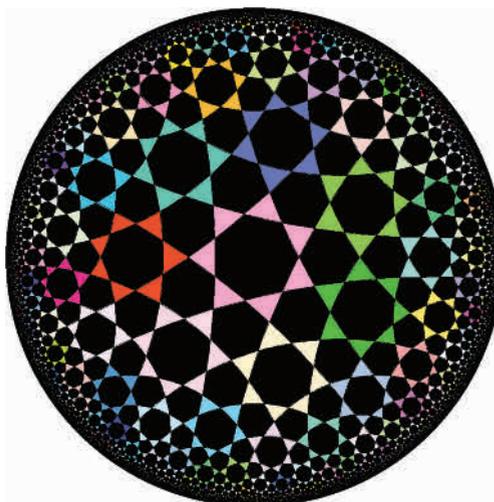
20. Determine the missing length.

21. Use the scale to calculate the actual flying distance from Calgary to Regina. Express your answer to the nearest kilometre.

Scale: 1 cm represents 154 km

22. Are any of the rectangles similar? Justify your answer.


23. a) Identify the different types of polygons shown in the tessellation.

b) Identify any similar polygons. Describe the pattern verbally.

c) Create your own tessellation that features similar polygons.

How Many Times Can You Fold a Piece of Paper?

Bruce claims that no one can fold a piece of paper in half more than seven or eight times, no matter how large the sheet or how thin the paper. Check it out. Is Bruce correct?
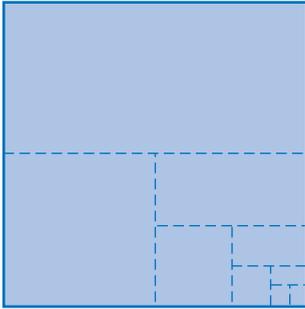
1. Use three different thicknesses of paper.
   a) For each type of paper, estimate the thickness of a single sheet.
   b) Devise a strategy to show how to determine the thickness of a sheet of paper. Support your work mathematically.

2. Use three different sizes of paper to explore the number of times in a row that a piece of paper can be folded in half.
   a) For each piece of paper, predict how many times you will be able to fold it in half.
   b) Fold each piece of paper in half as many times in a row as possible. Record your results. Compare your results with those of your classmates.

3. a) Write expressions for the thickness of the stack after each fold for a piece of paper of thickness, t.
   b) Write expressions for the area of the top of the stack after each fold for a piece of paper of area, a.
   c) Compare the patterns in the expressions you wrote. Use the patterns to help explain why it becomes difficult to fold a piece of paper after only a few folds.