CHAPTER

10

Circle Geometry

The pathways of the model airplane and the satellite shown are both circular. There are forces that are acting on these two objects to keep them in their circular orbits. What would happen to these objects if these forces were instantly removed? What direction would the airplane and the satellite move in relation to the circles?

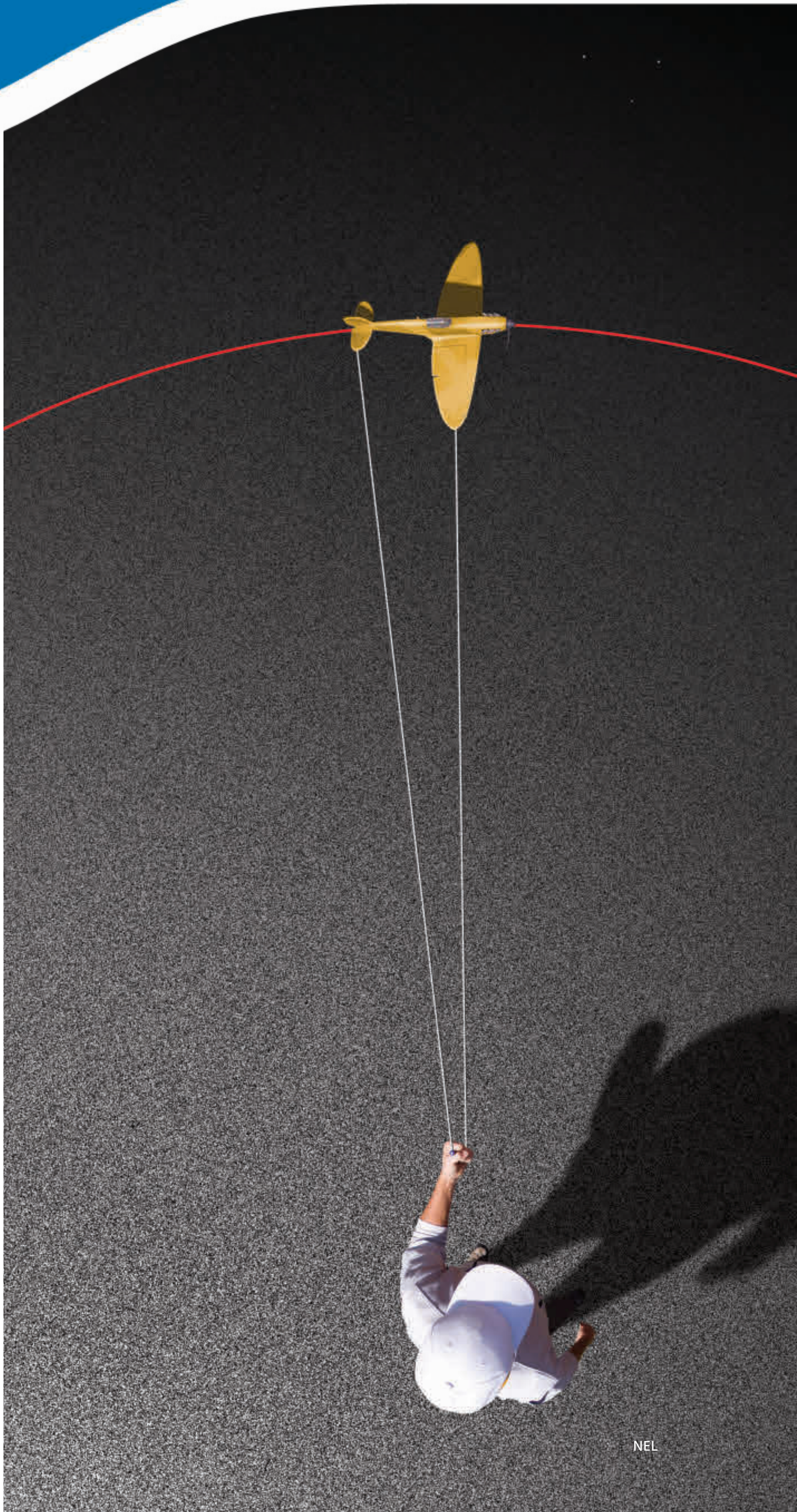
In this chapter, you will explore some properties of circles and use them to solve a variety of problems.

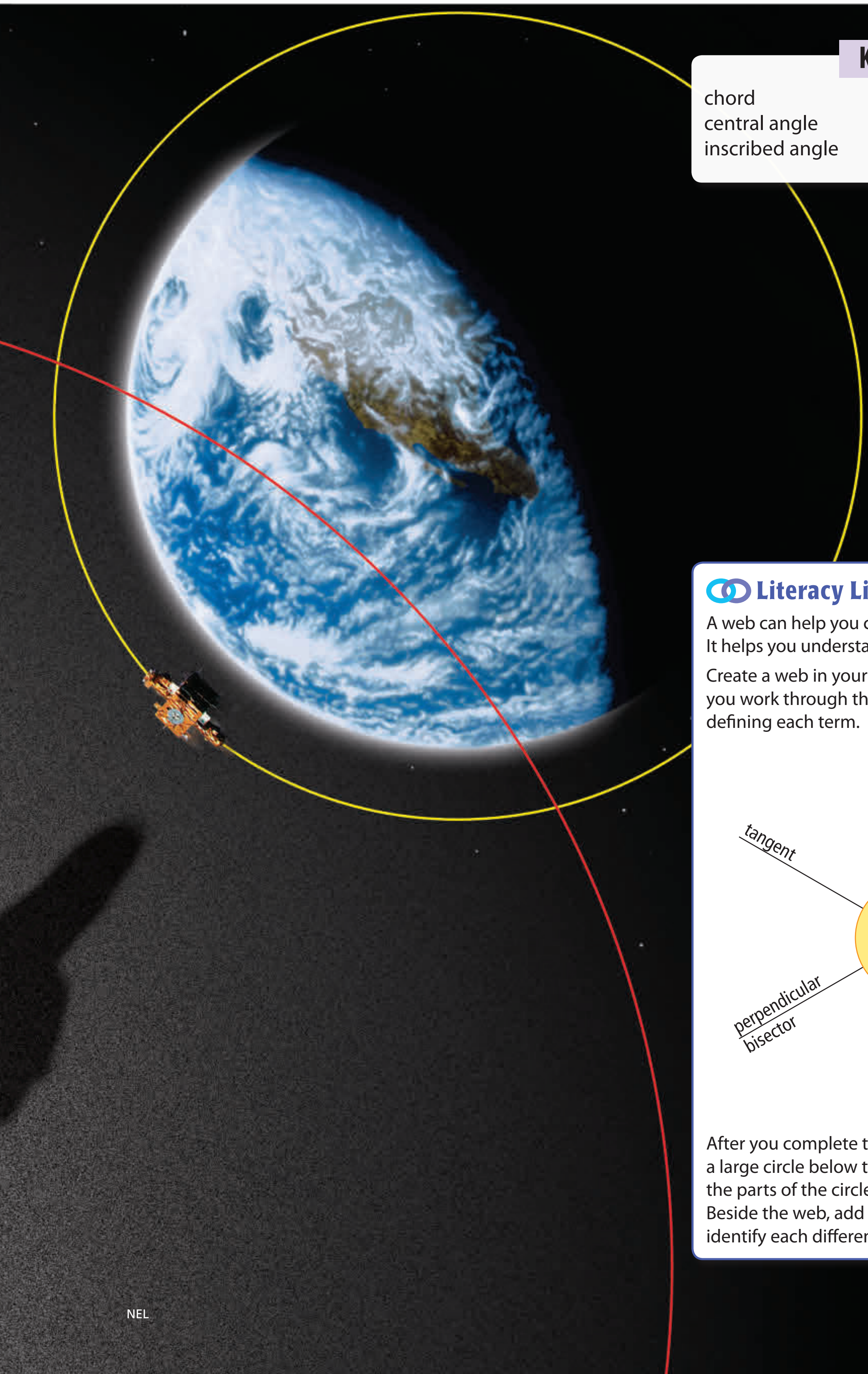
What You Will Learn

- to apply properties of circles to determine the measures of unknown angles and line segments
- to solve problems involving properties of circles

History Link

Euclid was a Greek mathematician who lived around 300 B.C. He is considered the "father of geometry." In his book *Elements* he clearly presented the fundamental principles of geometry and provided logical proofs of these principles. *Elements* was one of the first books to be published after the invention of the printing press in the fifteenth century. It is considered the oldest textbook. For more information about Euclid, go to www.mathlinks9.ca and follow the links.





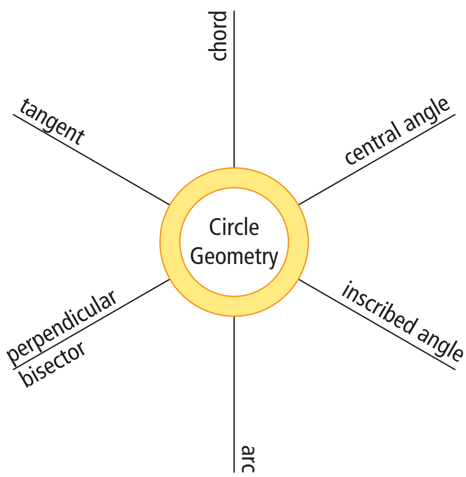
Key Words

chord
central angle
inscribed angle
arc
perpendicular bisector
tangent

Literacy Link

A web can help you create connections among ideas. It helps you understand how new ideas are related.

Create a web in your math journal or notebook. As you work through the chapter, complete the web by defining each term.



After you complete the web, use a compass to draw a large circle below the web. On your circle, draw the parts of the circle that you defined in the web. Beside the web, add a legend. Use a colour code to identify each different part of the circle.

FOLDABLES™
Study Tool

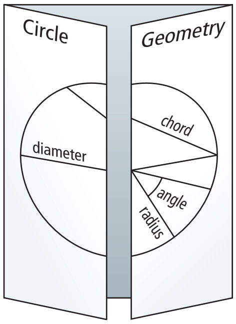
Making the Foldable

Materials

- sheet of 11 × 17 paper
- compass
- six sheets of 8.5 × 11 paper
- scissors
- stapler
- ruler

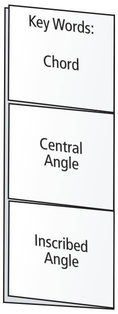
Step 1

Fold the long side of a sheet of 11 × 17 paper in half. Pinch it at the midpoint. Fold the outer edges of the paper to meet at the midpoint. Use a compass to draw a circle and label it as shown. Add the chapter title.



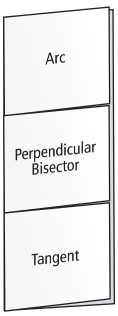
Step 2

Fold the short side of a sheet of 8.5 × 11 paper in half. Fold in three the opposite way. Make two cuts as shown through one thickness of paper, forming three tabs. Label the tabs as shown.



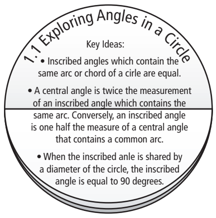
Step 3

Fold the short side of a sheet of 8.5 × 11 paper in half. Fold in three the opposite way. Make two cuts as shown through one thickness of paper, forming three tabs. Label the tabs as shown.



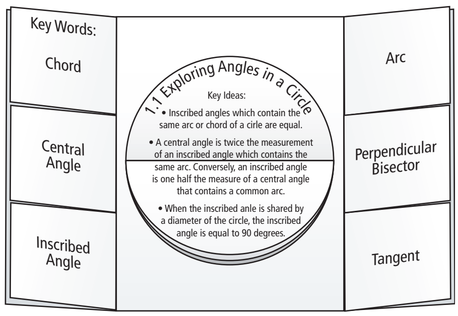
Step 4

Stack four sheets of 8.5 × 11 paper and staple at the four corners. Use a compass to draw a circle with a radius of 7.5 cm on the top sheet. Cut around the circle through all four thicknesses of paper. Fold the four circles in half. Title every second page of the booklet with a section title. The first page is shown.



Step 5

Staple the booklets from Steps 2 and 3 into the Foldable from Step 1. Staple the booklet from Step 4 along the fold line into the Foldable.



Using the Foldable

As you work through the chapter, write definitions of the Key Words beneath the tabs on the left and the right. In the centre booklet, there are two pages for each chapter section. Record the Key Ideas on the first page, and examples on the second page. There is one extra circle at the end for additional notes.

On the back of the centre panel of the Foldable, make notes under the heading What I Need to Work On. Check off each item as you deal with it.

Math Link

Geometry in Design

Architects, engineers, graphic designers, and many artists rely on their understanding of geometric principles in their work.

For Aboriginal people, the circle is a very important shape. The medicine wheel, where spiritual teachings occurred, was frequently framed in a circle using large rocks.

Work with a partner to complete the activity and questions.

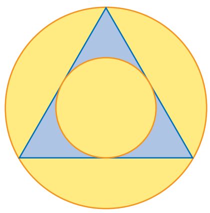
1. On tracing paper, construct a circle that has a diameter of at least 5 cm.

2. **a)** Fold the paper so that the circle is folded exactly in half. Then, reopen the tracing paper. Along the fold, draw a line segment that has both endpoints on the circle.
b) What is the mathematical term for this line segment?

3. **a)** Fold the circle in half again, making a different crease. Draw a line segment along this crease.
b) What is the mathematical term for the intersection of the two line segments created?

4. **a)** Estimate the measure of each of the four angles you created.
b) Measure the four angles with a protractor. How did your estimates compare?
c) What is the sum of the four central angles?

5. An environmental club is considering using the logo shown. What kind of triangle is used in the diagram? Explain your reasoning. How could you create this logo?



6. Experiment with drawing different regular polygons (with six or fewer sides), outside or inside a circle, so that each side or vertex of the polygon touches the circle. What difficulties did you have?

7. Brainstorm some businesses that have circles in their advertisements.

In this chapter, you will design a two-dimensional blueprint for a piece of art, a logo, or an advertisement. You will learn some geometric properties that can assist you with creating designs involving circles.

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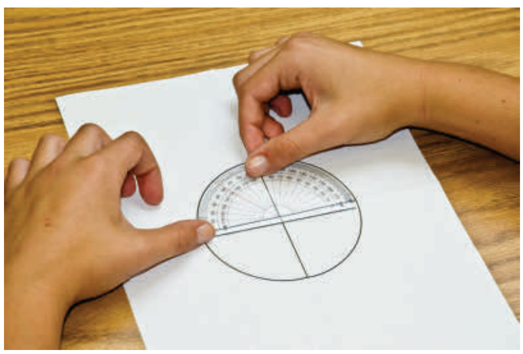


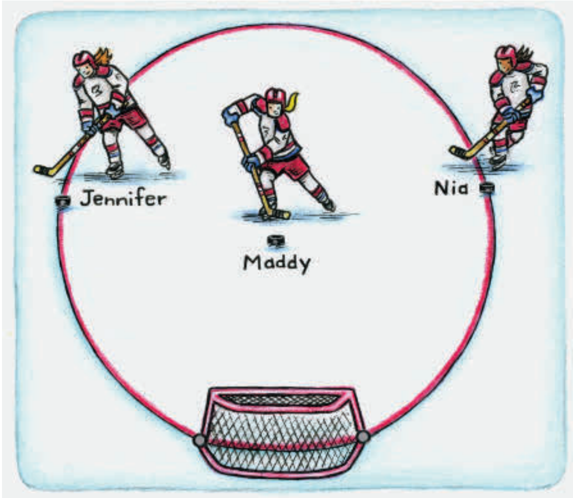
10.1 Exploring Angles in a Circle

Focus on…

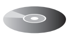
After this lesson, you will be able to…

- describe a relationship between inscribed angles in a circle
- relate the inscribed angle and central angle subtended by the same arc

Maddy, Jennifer, and Nia are each about to shoot at the empty net. If they are each equally accurate with their shot, who do you think is most likely to score?



Materials

- compass or circular geoboard with elastic bands 
- protractor
- ruler

chord

- a line segment with both endpoints on a circle

central angle

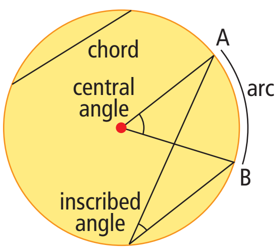
- an angle formed by two radii of a circle

inscribed angle

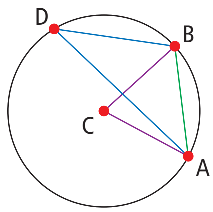
- an angle formed by two chords that share a common endpoint

arc (of a circle)

- a portion of the circumference



Explore Relationships Between Angles in a Circle



1. Construct a large circle and label its centre C. Construct a **chord** AB and a **central angle** ∠BCA. Measure ∠BCA.

2. Create the **inscribed angle** ∠BDA. What is the measure of ∠BDA?

3. How do the measures of ∠BCA and ∠BDA compare?

4. Create a second inscribed angle ∠BEA. What is the measure of ∠BEA?

5. Choose another point on the circle between D and E. Create one more inscribed angle that has its arms touching the endpoints of the **arc** AB. What is the measure of this inscribed angle?

6. Repeat steps 1 to 5 for a different sized circle, and a different sized chord AB.

Reflect and Check

7. a) What is the relationship between a central angle and an inscribed angle that stands on the same arc?

b) What is the relationship between all the inscribed angles that stand on the same arc?

8. Predict which hockey player in the opening paragraph is most likely to score on the empty net. Explain.

Link the Ideas

You can use properties related to angles in a circle to solve problems.

Inscribed Angles
The inscribed angles *subtended* by the same arc are congruent.

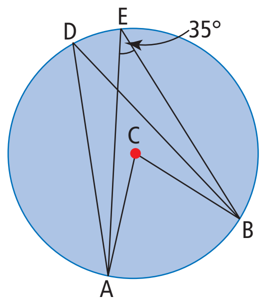
Central and Inscribed Angles
The measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc.

Example 1: Determine Angle Measures in a Circle

Point C is the centre of the circle. $\angle AEB = 35°$

a) What is the measure of $\angle ADB$? Justify your answer.

b) What is the measure of $\angle ACB$? Justify your answer.



Solution

a) The inscribed angles, $\angle ADB$ and $\angle AEB$, are equal because they are subtended by the same arc, AB.

Therefore, $\angle ADB = 35°$.

b) The central angle $\angle ACB$ is subtended by the same arc AB as the inscribed angle $\angle AEB$. A central angle is twice the measure of an inscribed angle that is subtended by the same arc.

$$\begin{aligned}\angle ACB &= 2\angle AEB \\ &= 2 \times 35° \\ &= 70°\end{aligned}$$

Therefore, $\angle ACB = 70°$.

Literacy Link

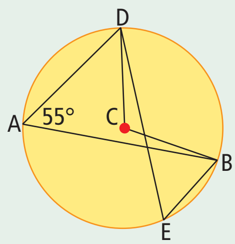
An angle that *subtends* an arc or a chord is an angle that "stands on" or is formed by the endpoints of the arc or chord.

Web Link

You may wish to explore these geometric properties on a geoboard or on a computer. Go to www.mathlinks9.ca and follow the links.

Show You Know

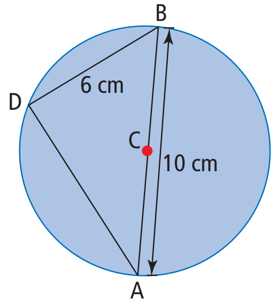
Point C is the centre of the circle. $\angle DAB = 55°$.
What are the measures of angles $\angle DEB$ and $\angle DCB$?
Justify your answers.



Example 2: Use Central and Inscribed Angles to Recognize Relationships

Point C is the centre of the circle.
diameter AB = 10 cm
chord BD = 6 cm

a) What is the measure of ∠ADB? Explain your reasoning.
b) What is the length of the chord AD? Justify your answer.



Solution

a) The diameter AB divides the circle into two semicircles. Since AB is a straight line, the central angle ∠ACB is 180°. Then, ∠ADB must be half of 180° because it is an inscribed angle that is subtended by the same arc, AB. The measure of ∠ADB is 90°.

b) Since ∠ADB = 90°, △ABD is a right triangle.
The Pythagorean relationship can be used to find the length of AD.

$$
\begin{aligned}
AD^2 + BD^2 &= AB^2 \\
AD^2 + 6^2 &= 10^2 \\
AD^2 + 36 &= 100 \\
AD^2 &= 64 \\
AD &= \sqrt{64} \\
AD &= 8
\end{aligned}
$$

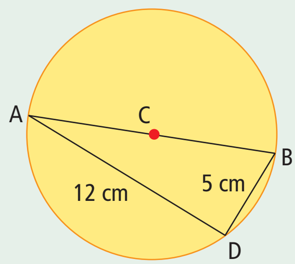
Therefore, AD = 8 cm.

Show You Know

Point C is the centre of the circle. AB is the diameter.
chord AD = 12 cm
chord BD = 5 cm

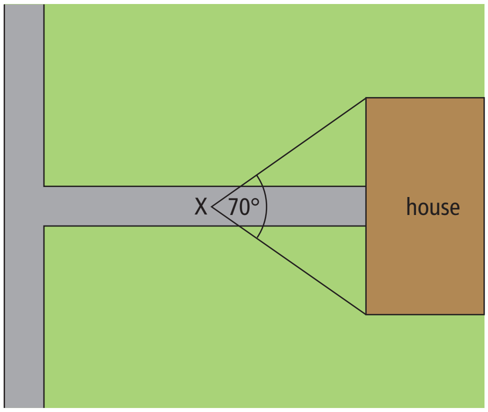
a) What is the measure of ∠ADB? Explain your reasoning.

b) What is the length of the diameter AB?



Example 3: Use Central and Inscribed Angles to Solve Problems

Jamie works for a realtor. One of his jobs is to photograph houses that are for sale. He photographed a house two months ago using a camera lens that has a 70° field of view. He has returned to the house to update the photo, but he has forgotten his original lens. Today he only has a telephoto lens with a 35° field of view.



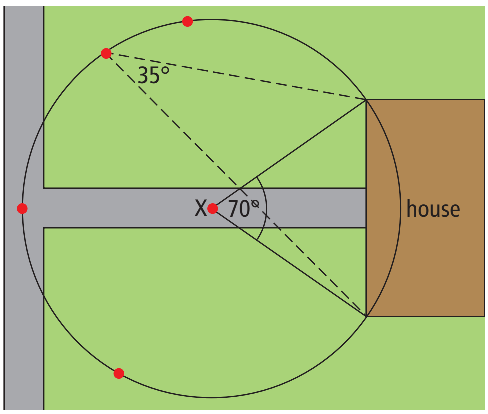
From what location(s) could Jamie photograph the house, with the telephoto lens, so that the entire house still fills the width of the picture. Explain your choices.

Solution

Draw a circle with the centre located at the vertex of the 70° angle. Use one arm of the angle as the radius of the circle. Construct any number of different inscribed angles that each contains the front of the house. Any of these points are locations from which Jamie could take the photo. The measure of each of these inscribed angles will be half the measure of the central angle.

$70° \div 2 = 35°$

Each inscribed angle will measure 35°, which corresponds to the field of view for Jamie's telephoto lens. Depending on access, and whether there are any trees or a garden in the way, any point on the major arc that is outside of the house will work.



Strategies

Draw a Diagram

Identify all Possibilities

Literacy Link

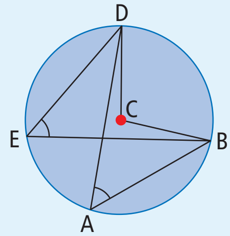
A *major arc* is more than a semicircle. A *minor arc* is less than a semicircle.

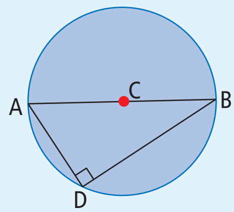
Show You Know

A flashlight has a field of view measuring 25°, and a camera has a field of view measuring 50°. How can you position the camera and flashlight so that the camera will capture the same area as the flashlight illuminates?

Key Ideas

- Inscribed angles subtended by the same arc of a circle are equal. $\angle DEB = \angle DAB$
- A central angle is twice the measure of an inscribed angle subtended by the same arc. $\angle DCB = 2\angle DAB$
- An inscribed angle is one half the measure of a central angle subtended by the same arc. $\angle DAB = \frac{1}{2}\angle DCB$
- When the inscribed angle is subtended by a diameter of the circle, the inscribed angle is equal to 90°.





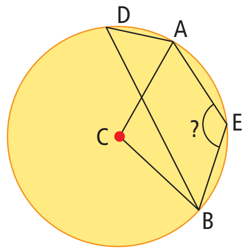
Literacy Link

The identical markings at the inscribed angles $\angle DEB$ and $\angle DAB$ indicate that the measures of these angles are equal.

Check Your Understanding

Communicate the Ideas

1. In the diagram, $\angle BDA$ measures half of $\angle BCA$. Does the rule for inscribed angles hold true for $\angle BEA$?Explain your reasoning.

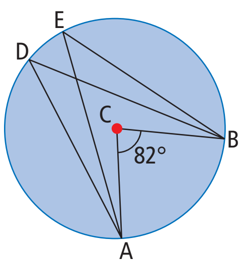


2. Manny constructed a circle using a compass. He used a straight edge to draw a diameter. Then, he constructed an inscribed angle that shared endpoints with the diameter. What is the measure of the inscribed angle he constructed? How do you know?

Practise

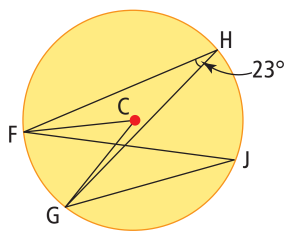
For help with #3 to #5, refer to Example 1 on page 379.

3. What are the measures of $\angle ADB$ and $\angle AEB$? Justify your answers.



4. a) What is the measure of $\angle FJG$? Explain your reasoning.

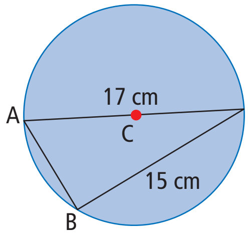
b) What is the measure of $\angle FCG$? Justify your answer.



5. Draw a circle with a central angle that measures 60°. Draw and label the measure of two inscribed angles that are subtended by the same arc as the central angle.

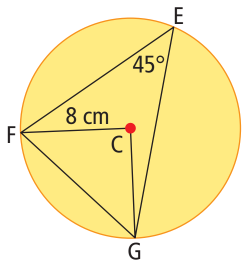
For help with #6 and #7, refer to Example 2 on page 380.

6. Point C is the centre of the circle.
diameter AD = 17 cm
chord BD = 15 cm



a) What is the measure of ∠ABD? Explain.
b) What is the length of the chord AB?

7. The circle has centre C and a radius of 8 cm. ∠FEG = 45°.



a) What is the measure of ∠FCG?
b) What is the length of the chord FG? Express your answer to the nearest tenth of a centimetre.

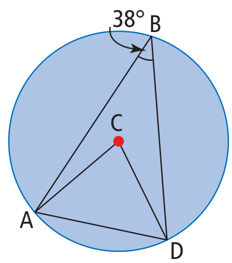
For help with #8 and #9, refer to Example 3 on page 381.

8. After a power outage, Jacob helps his mother by shining a flashlight beam at the breaker panel while she locates the tripped breakers. His flashlight projects light through an angle of 15°, while his mother's flashlight projects light through an angle of 30°. Use a diagram to show a good place for Jacob to stand so that his flashlight will illuminate the same area of the breaker panel as his mother's flashlight does.

9. For a high school drama production, three spotlights are positioned on an arc at the back of the theatre, just above the audience. Each spotlight projects light through an angle of 22° and fills the rectangular front of the stage. Use a diagram to identify an ideal location to take a photo of the performance using a camera with a lens that has a field of view of 44°.

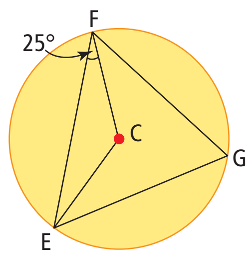
Apply

10. In the diagram, C is the centre of the circle and ∠ABD = 38°. For each of the following questions, justify your answer.



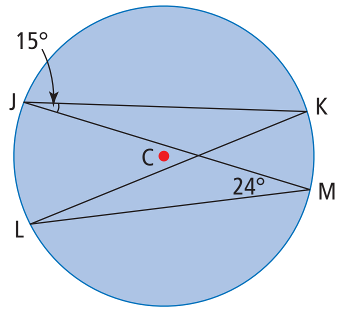
a) What is the measure of ∠ACD?
b) What type of triangle is △ACD?
c) What is the measure of ∠CAD?

11. Point C is the centre of the circle and ∠CFE = 25°. Justify each of your answers to the following questions.



a) What is the measure of ∠ECF?
b) What is the measure of ∠EGF?

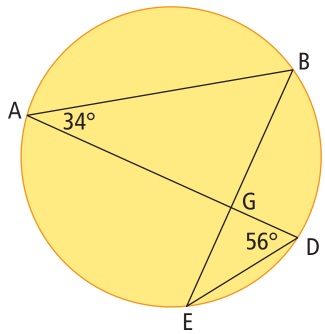
12. If $\angle KJM = 15°$, $\angle JML = 24°$, and point C is at the centre of the circle, what is the measure of each of the following angles?



a) $\angle KLM$ **b)** $\angle JKL$

c) $\angle JCL$ **d)** $\angle KCM$

13. In the diagram, $\angle BAD = 34°$ and $\angle ADE = 56°$.



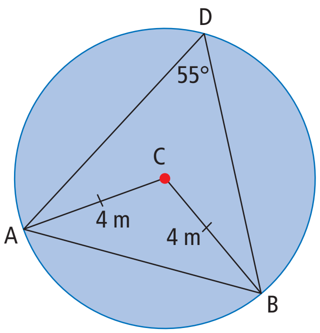
a) What is the measure of $\angle ABE$?

b) What is the measure of $\angle AGB$?

c) What type of triangle is $\triangle ABG$?

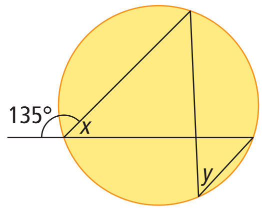
d) What is the measure of $\angle DGE$?

14. After looking at the diagram of the circle, Amanda decides to use the Pythagorean relationship to calculate the length of chord AB. Will this method work? Explain.

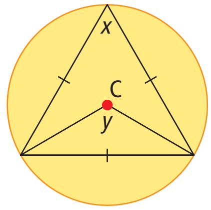


15. Find the unknown angle measures, x and y, in each diagram. Where C is labelled, it is the centre of the circle.

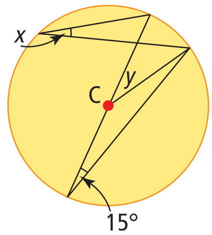
a)



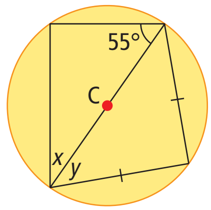
b)



c)



d)



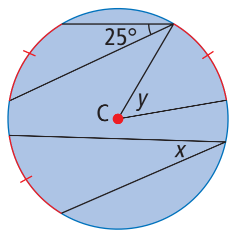
16. Design a geometry question involving a given central angle for which the answer is an inscribed angle measuring 30°. Include a diagram with your question.

17. A circle with centre C has a diameter AB. The inscribed angle $\angle ADE$ measures 14°. What are the measures of $\angle ACE$ and $\angle ABE$? Draw a diagram.

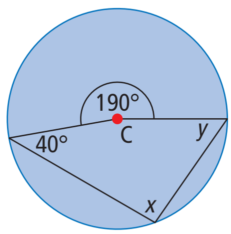
Extend

18. Find the unknown angle measures, x and y, in each diagram, given that C is the centre of the circle.

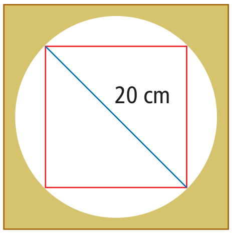
a)



b)

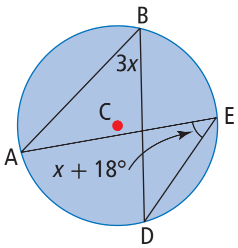


19. A hole has a diameter of 20 cm. What is the maximum side length of a square that will fit into the hole?

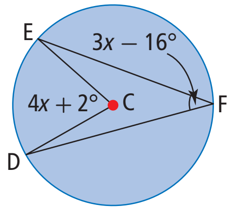


20. For each of the following diagrams, calculate the value of x.

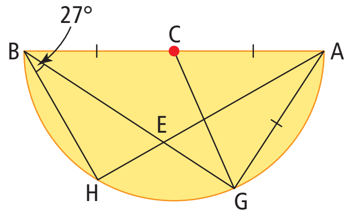
a)



b)



21. In the semicircle, $\angle HBE = 27°$. C is on the diameter and is the midpoint of AB.



Determine the measure of each angle, justifying your work mathematically.

a) $\angle BHA$

b) $\angle BEH$

c) $\angle AEG$

d) $\angle ACG$

e) $\angle BCG$

Math Link

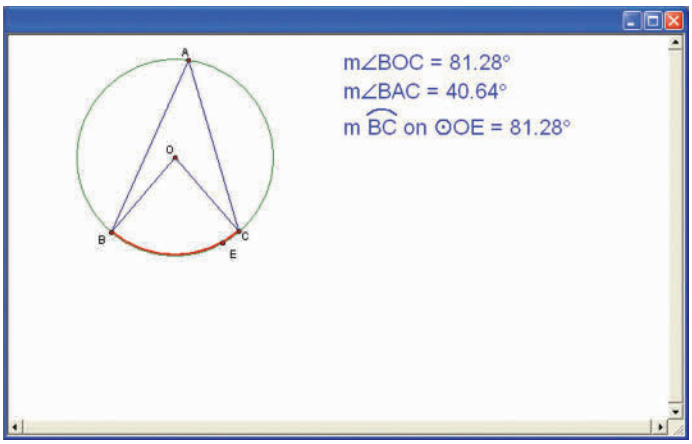
a) Design a piece of art using one circle and any number of inscribed and central angles.

b) Describe how the angles and line segments in your design are related.

Tech Link

Inscribed and Central Angles

In this activity, you will use dynamic geometry software to explore inscribed and central angles in a circle. To use this activity, go to www.mathlinks9.ca and follow the links.



Explore

1. a) What is the measure of the central angle?

b) What is the measure of the inscribed angle?

c) What is the measure of the minor arc BC?

2. Drag point A around the circle. What happens to the measure of the two angles ∠BOC and ∠BAC? Why does this happen?

3. Drag either point B or point C around the circle. Record at least four measurements of the inscribed angle and the central angle from different locations on the circle.

∠BAC	∠BOC

4. Describe any relationships between the central angle ∠BOC and the inscribed angle ∠BAC subtended by the same arc.

10.2 Exploring Chord Properties

Focus on...

After this lesson, you will be able to...

- describe the relationship among the centre of a circle, a chord, and the perpendicular bisector of the chord

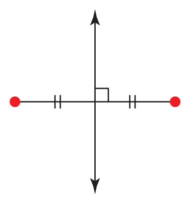
An archeologist found an edge piece of a broken Aztec medallion. If she assumes it is circular, how might she determine the circumference of the whole medallion?

Materials

- compass
- tracing paper
- ruler

Literacy Link

A *perpendicular bisector* passes through the midpoint of a line segment at 90°.



Explore Chords in a Circle

1. Construct a large circle on tracing paper and draw two different chords.
2. Construct the perpendicular bisector of each chord.
3. Label the point inside the circle where the two perpendicular bisectors intersect.
4. Share your construction method with another classmate.

What methods could you use to do this construction?

Reflect and Check

5. a) What do you notice about the point of intersection of the two perpendicular bisectors in step 3?
   b) Do you think that this will be true for any chord and any circle? How could you test your prediction?
6. How could the archeologist use perpendicular bisectors to determine the circumference of the Aztec medallion?

WWW Web Link

You may wish to explore these geometric properties on a computer. Go to www.mathlinks9.ca and follow the links.

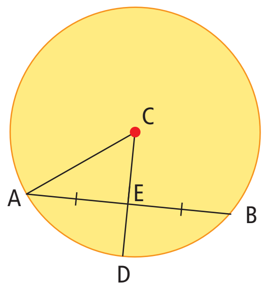
Link the Ideas

You can use properties related to chords in a circle to solve problems.

Perpendicular Bisector of a Chord
A line that passes through the centre of a circle and is perpendicular to a chord bisects the chord.

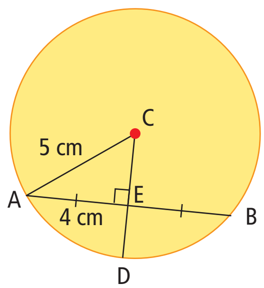
Example 1: Bisect a Chord With a Radius

Radius CD bisects chord AB. Chord AB measures 8 cm. The radius of the circle is 5 cm. What is the length of line segment CE? Justify your solution.



Solution

Since CD is a radius that bisects the chord AB, then CD is perpendicular to AB and ∠AEC = 90°.



Strategies

Organize, Analyse, Solve

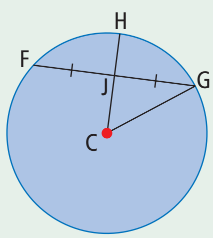
The length of AE is 4 cm because CD bisects the 8-cm chord AB. The radius AC is 5 cm. Using the Pythagorean relationship in △ACE,

$$\begin{aligned} CE^2 + AE^2 &= AC^2 \\ CE^2 + 4^2 &= 5^2 \\ CE^2 + 16 &= 25 \\ CE^2 &= 9 \\ CE &= \sqrt{9} \\ CE &= 3 \end{aligned}$$

Therefore, CE measures 3 cm. This is the shortest distance from the chord AB to the centre of the circle.

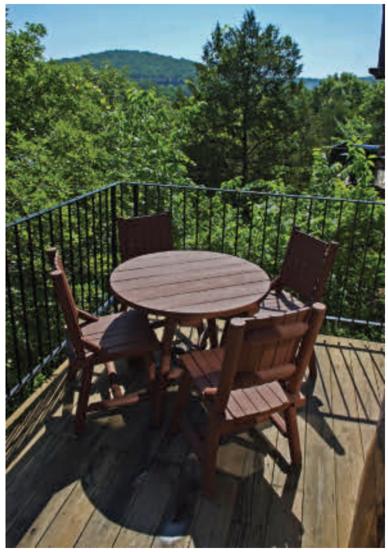
Show You Know

Radius CH bisects chord FG. Chord FG measures 12 cm. The radius of the circle measures 10 cm. What is the length of CJ?



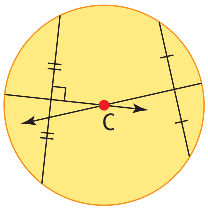
Example 2: Use Chord Properties to Solve Problems

Louise would like to drill a hole in the centre of a circular table in order to insert a sun umbrella. Use a diagram to explain how she could locate the centre.



Solution

Draw two chords. Locate the midpoint of each chord. Use a carpenter's square to draw the perpendicular bisectors of each chord. Locate the point of intersection of the two perpendicular bisectors. The point of intersection is the centre of the table.



Did You Know?

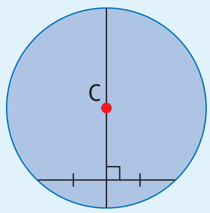
A *carpenter's square* is used in construction to draw and confirm right angles.

Show You Know

Mark would like to plant a cherry tree in the centre of a circular flower bed. Explain how he could identify the exact centre using circle properties.

Key Ideas

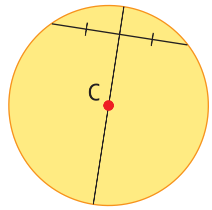
- The perpendicular bisector of a chord passes through the centre of the circle.
- The perpendicular bisectors of two distinct chords intersect at the centre of the circle.
- If a bisector of a chord in a circle passes through the centre, then the bisector is perpendicular to the chord.
- If a line passes through the centre of a circle and intersects a chord at right angles, then the line bisects the chord.
- The shortest path between the centre of a circle and a chord is a line that is perpendicular to the chord.



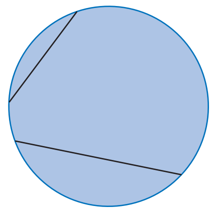
Check Your Understanding

Communicate the Ideas

1. Describe how you know that the diameter of the circle forms a right angle with the chord at their point of intersection.



2. Explain how you could locate the centre of the circle using the two chords shown.



3. Amonte was explaining the properties of perpendicular bisectors to his friend Darius.

"There are three important properties of perpendicular bisectors of chords in circles:
- The line bisector cuts the chord in two equal line segments.
- The line intersects the chord at right angles; they are perpendicular.
- The line passes through the centre so it contains the diameter.

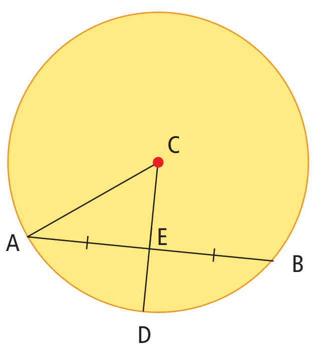
If any two of these properties are present, then the third property exists."

Is Amonte's explanation correct? What does he mean by the last statement?

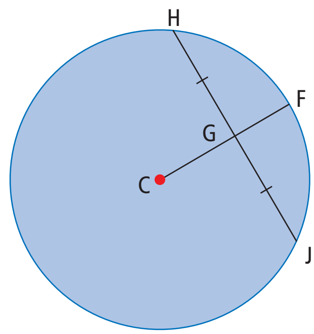
Practise

For help with #4 and #5, refer to Example 1 on page 387.

4. CD bisects chord AB. The radius of the circle is 15 cm long. Chord AB measures 24 cm. What is the length of CE? Explain your reasoning.



5. The radius CF bisects chord HJ. CG measures 4 mm. Chord HJ measures 14 mm. What is the radius of the circle, expressed to the nearest tenth of a millimetre? Justify your answer.

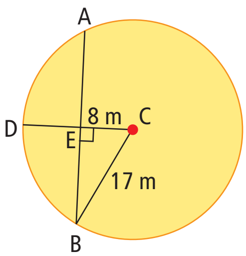


For help with #6, refer to Example 2 on page 388.

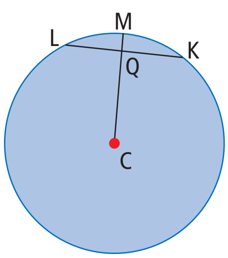
6. Hannah wants to draw a circular target on her trampoline. Explain, using diagrams, how she could locate the centre of the trampoline.

Apply

7. The radius of the circle is 17 m. The radius CD is perpendicular to the chord AB. Their point of intersection, E, is 8 m from the centre C. What is the length of the chord AB? Explain your reasoning.

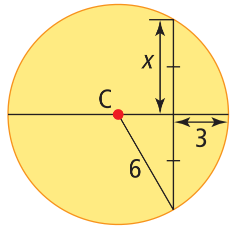


8. The radius of the circle is 11.1 cm, the radius CM is perpendicular to the chord LK, and MQ measures 3.4 cm. What is the length of the chord LK? Express your answer to the nearest tenth of a centimetre.

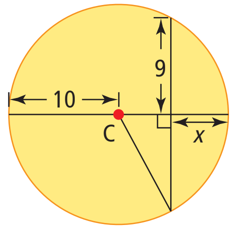


9. Calculate the unknown length, x. Give each answer to the nearest tenth.

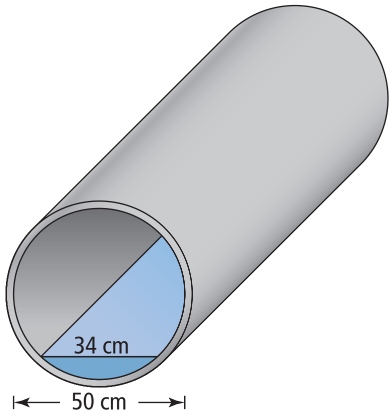
a)



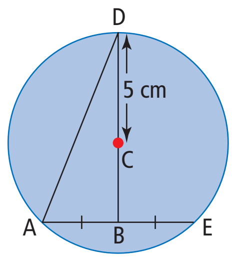
b)



10. The circular cross section of a water pipe contains some water in the bottom. The horizontal distance across the surface of the water is 34 cm. The inner diameter of the pipe is 50 cm. What is the maximum depth of the water? Express your answer to the nearest centimetre.



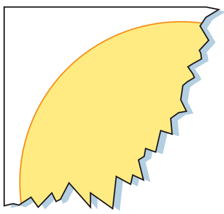
11. If you know that the radius CD = 5 cm, and BC = 3 cm, what is the area of △ABD?



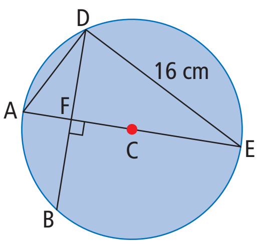
12. A circle has a diameter of 50 mm. A chord is 14 mm long. What is the shortest distance from the centre of the circle, C, to the chord? Include a diagram with your solution.

13. How could you locate the centre of a regular octagonal table using chord properties? Include a diagram in your explanation.

14. Your classmate used a compass to draw a circle with a radius of 8 cm. He felt the circle was inaccurate and tore it into small pieces. How could you use the following piece to check his accuracy?



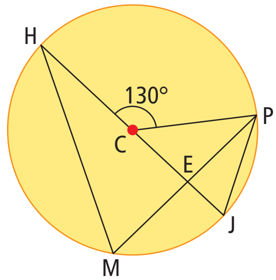
15. In this circle, the diameter AE = 20 cm, the chord DE = 16 cm, AF = 7.2 cm, and ∠BFE = 90°.



Determine the following measures and justify your answers. Express lengths to the nearest tenth of a centimetre.

a) ∠ADE **b)** AD
c) DF **d)** BD

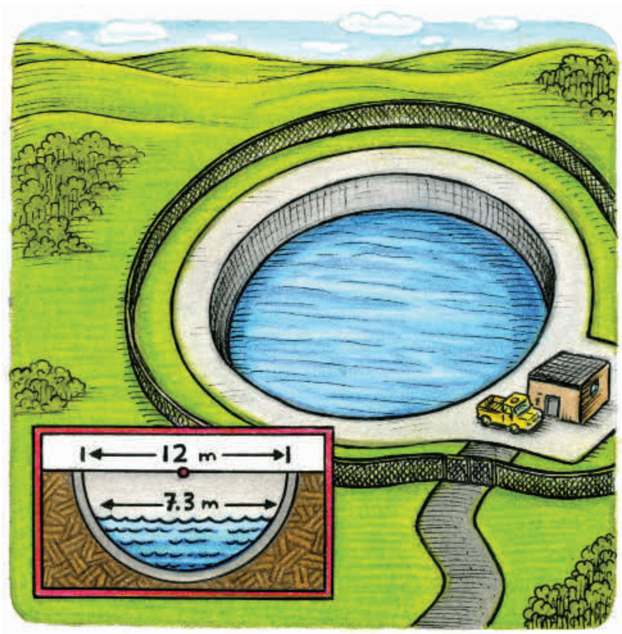
16. Point E is the midpoint of the chord MP. HJ is a diameter of the circle. C is the centre of the circle, and ∠HCP = 130°.



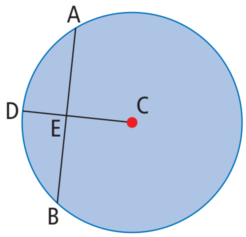
Determine the following angle measures. Justify your answers.

a) ∠HMP **b)** ∠HEM
c) ∠MHJ **d)** ∠MPJ
e) ∠PCE **f)** ∠CPE

17. A helicopter pilot surveys the water level in an aqueduct in a remote section of the country. From the air, the pilot measures the horizontal width of the water to be 7.3 m. The aqueduct is a hemisphere and has an inner diameter of 12 m. What is the depth of the water? Express your answer to the nearest tenth of a metre.

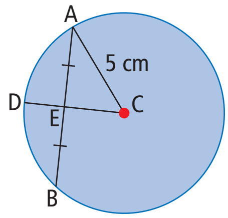


18. Gavyn was asked to find the length of the chord AB. He was told that the radius of the circle is 13 cm, radius CD is perpendicular to chord AB, and chord AB is 5 cm from the centre C.



Determine the mistakes that Gavyn made and find the correct length of AB.

Gavyn's Solution



Draw the radius AC, which is the hypotenuse of right triangle $\triangle AEC$.

By the Pythagorean relationship,

$$EC^2 + AC^2 = AE^2$$
$$13^2 + 5^2 = AE^2$$
$$169 + 25 = AE^2$$
$$194 = AE^2$$
$$AE = \sqrt{194}$$
$$AE \approx 13.9$$

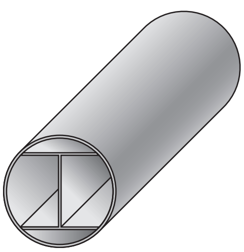
Since CD is a radius and it is perpendicular to AB, then CD bisects the chord AB.

$AB \approx 2 \times 13.9$

$AB \approx 27.8$

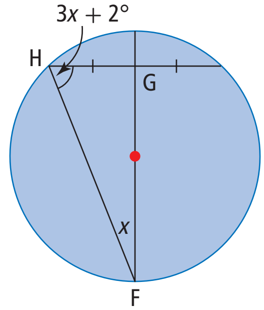
Therefore, AB is approximately 27.8 m.

19. Some plastic tubing is moulded with an I-beam on the inside to provide extra strength. The length of each of two parallel chords is 10 mm, and the perpendicular distance between these two chords is 12 mm. What is the diameter of the circular tubing? Express your answer to the nearest tenth of a millimetre.



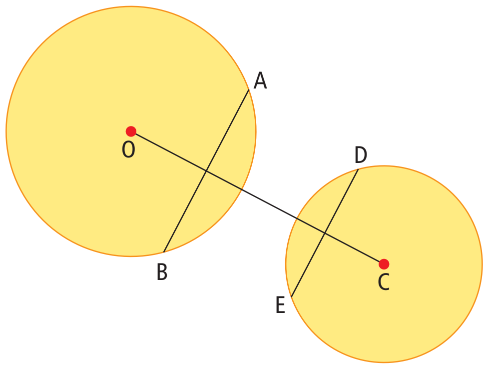
Extend

20. a) How do you know that $\triangle FGH$ is a right triangle?



b) Solve for x algebraically and determine the measures of both acute angles in $\triangle FGH$.

21. Line segment OC is a bisector of chords AB and DE. If O is the centre of the circle on the left and C is the centre of the circle on the right, explain how you know that AB is parallel to DE.



Math Link

The North American Plains Indians and Tibetan Buddhists create mandalas. A mandala is a piece of art framed within a circle. The design draws the viewer's eyes to the centre of the circle. Mandalas have spiritual significance for their creators. The photo shows a Buddhist monk using coloured sand to create a mandala.

a) Refer to the portion of the sand mandala shown in the picture. Design a mandala with a similar pattern but your own design. For example, you could create a mandala to celebrate the work of a famous mathematician. Your design should show only part of the mandala.

b) If you want to display your mandala, you will need to know how much room the entire design will take up. What is a reasonable estimate for the circumference of your mandala? Explain your reasoning.

c) How do you think the monks ensure symmetry in their mandalas? How could you use your knowledge of circle properties to help you?

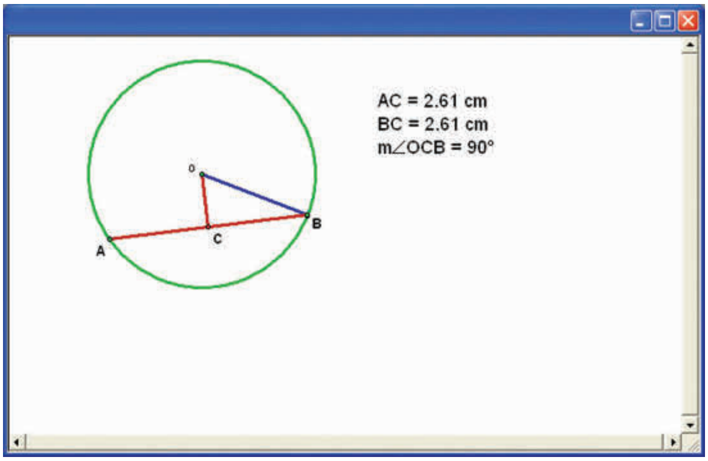
WWW Web Link

For more information about sand mandalas, go to www.mathlinks9.ca and follow the links.

Tech Link

Perpendicular Lines to a Chord

In this activity, you will use dynamic geometry software to explore perpendicular lines from the centre of a circle to a chord. To use this activity, go to www.mathlinks9.ca and follow the links.



Explore

1. a) What is the measure of ∠OCB?
   b) What is the measure of line segment AC?
   c) What is the measure of line segment BC?

2. Drag point A to another location on the circle.
   a) Describe what happens to the measure of ∠OCB when you drag point A to a different location on the circle.
   b) What happens to the measures of the line segments AC and BC? Explain.

3. Drag point B around the circle.
   a) What effect does this have on the measure of ∠OCB?
   b) What effect does this have on the lengths of line segment AC and line segment BC?

4. What conclusions can you make about ∠OCB, the angle formed by the segment from the centre of the circle to the midpoint of the chord?

5. What conclusions can you make about the relationship between line segment AC and line segment BC?

10.3 Tangents to a Circle

Focus on...

After this lesson, you will be able to...

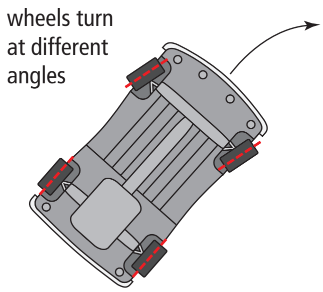
- relate tangent lines to the radius of the circle.

When a car turns, the wheels are at different angles in relation to the car. Each wheel is turning through its own circle. What is the relationship between the four circles where the tires turn?

Materials

- Turning Circle diagram
- protractor
- ruler

Explore Circles and Their Tangents



1. Find the midpoint of each line segment that represents a tire.

2. Draw a perpendicular line from each midpoint toward the inside of the turning circle.

Reflect and Check

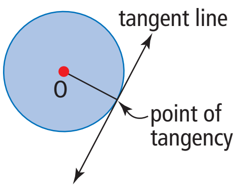
3. What do you notice about the intersection of these perpendicular lines?

4. a) Each wheel of a car travels through a different circular path. What do these circles have in common?

b) Based on your observations, what is the measure of the angle between a **tangent** to a circle and the radius at the point of tangency?

tangent (to a circle)

- a line that touches a circle at exactly one point
- the point where the line touches the circle is called the point of tangency



WWW Web Link

You may wish to explore these geometric properties on a computer. Go to www.mathlinks9.ca and follow the links.

Link the Ideas

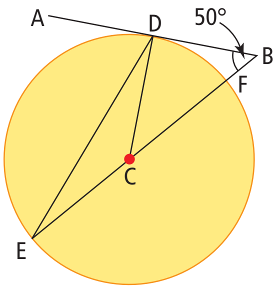
You can use properties of tangents to a circle to solve problems.

Tangent to a Circle
A tangent to a circle is perpendicular to the radius at the point of tangency.

Tangent Chord Relationship
A chord drawn perpendicular to a tangent at the point of tangency contains the centre of the circle, and is a diameter.

Example 1: Determine Angle Measures in a Circle With a Tangent Line

In the diagram shown, AB is tangent to the circle at point D, BE contains the diameter FE, and $\angle ABE = 50°$.



a) What is the measure of $\angle BDC$? Justify your answer.
b) What is the measure of central angle $\angle DCE$? Explain your reasoning.
c) What type of triangle is $\triangle CDE$? Justify your answer.
d) What is the measure of $\angle DEC$? Explain your reasoning.

Solution

a) Since AB is tangent to the circle at point D, then radius CD is perpendicular to line segment AB. Therefore, $\angle BDC = 90°$.

b) The sum of the angles in a triangle is 180°.
In $\triangle BCD$, $\angle DCB = 180° - 90° - 50°$
$\angle DCB = 40°$

Since $\angle DCE$ and $\angle DCB$ form a straight line, they are supplementary.

$$\begin{aligned} \angle DCE + \angle DCB &= 180° \\ \angle DCE + 40° &= 180° \\ \angle DCE &= 180° - 40° \\ \angle DCE &= 140° \end{aligned}$$

Literacy Link
Supplementary angles add to 180°.

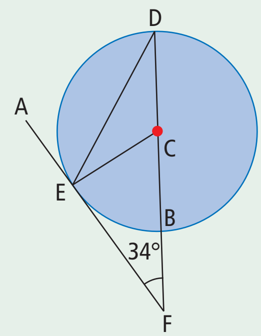
c) Triangle CDE is an isosceles triangle because CD and CE are radii of the circle and radii are equal in length.

d) ***Method 1: Use Angles in a Triangle***
The sum of the angles in a triangle is 180°. ∠DCE = 140°. Since △CDE is an isosceles triangle, then the angles opposite the equal sides are equal. ∠DEC = $\frac{1}{2}$ × 40° or 20°.
∠DEC = 20°.

Method 2: Use Inscribed Angles
∠DEF is the same as ∠DEC because the points F and C lie on the same line. This is an inscribed angle subtended by the same arc as the central angle, ∠DCF. Since an inscribed angle is one half the measure of a central angle subtended by the same arc, then ∠DEC = $\frac{1}{2}$ × 40° or 20°.
∠DEC = 20°

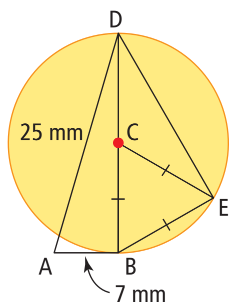
Show You Know

Line segment AF is tangent to the circle at point E. Line segment DF contains the diameter DB, and ∠CFE = 34°. What are the measures of angles ∠CEF, ∠ECF, and ∠EDF? Explain your reasoning.



Example 2: Use the Tangent Chord Relationship

In the diagram, AB is tangent to the circle at point B. BD is a diameter of the circle. AB = 7 mm, AD = 25 mm, and △BCE is an equilateral triangle.



a) What is the length of diameter BD? Justify your answer.
b) What is the length of chord BE? Explain your reasoning.
c) What is the measure of the inscribed angle ∠BED?
d) What is the length of chord DE? Justify your answer and express your answer to the nearest millimetre.

Solution

a) Diameter BD is perpendicular to tangent AB because B is the point of tangency on the circle. Therefore, $\angle ABD = 90°$ and $\triangle ABD$ is a right triangle.

Use the Pythagorean relationship in $\triangle ABD$.

$$\begin{aligned} AB^2 + BD^2 &= AD^2 \\ 7^2 + BD^2 &= 25^2 \\ 49 + BD^2 &= 625 \\ BD^2 &= 576 \\ BD &= \sqrt{576} \\ BD &= 24 \end{aligned}$$

The length of diameter BD is 24 mm.

b) BC and CE are radii of the circle. Since $\triangle BCE$ is an equilateral triangle, side BE is equal the length of the radius, or one half of the diameter.

$\frac{1}{2}(24) = 12$

The length of chord BE is 12 mm

c) The inscribed angle $\angle BED$ is subtended by a diameter, so it is a right angle. $\angle BED = 90°$.

The inscribed angle $\angle BED = 90°$.

d) Use the Pythagorean relationship in $\triangle BDE$.

$$\begin{aligned} BE^2 + DE^2 &= BD^2 \\ 12^2 + DE^2 &= 24^2 \\ 144 + DE^2 &= 576 \\ DE^2 &= 576 - 144 \\ DE^2 &= 432 \\ DE &= \sqrt{432} \\ DE &\approx 21 \end{aligned}$$

The length of chord DE is 21 mm, to the nearest millimetre.

Strategies

Organize, Analyse, Solve

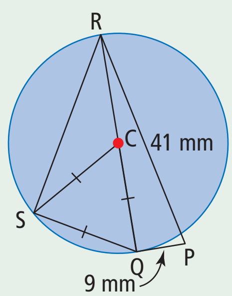
Show You Know

In the diagram shown, PQ is tangent to the circle at point Q. QR is a diameter of the circle. Line segment PQ = 9 mm, PR = 41 mm, and $\triangle QCS$ is an equilateral triangle.

a) What is the length of diameter QR? Justify your answer.

b) What is the length of chord QS? Explain your reasoning.

c) What is the length of chord RS? Justify your answer and express your answer to the nearest millimetre.



Science Link

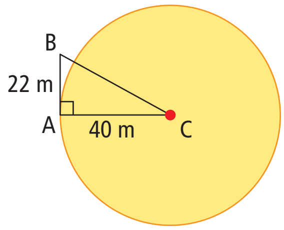
An object that is moving in a circular path will move in a straight line tangent to that circle if the force pulling the object toward the centre is suddenly removed. This force is known as *centripetal force*.

Example 3: Solve Problems With Tangents to Circles

A speed skater is practising on a circular track with a radius of 40 m. He falls and slides off the track in a line tangent to the circle. If he slides 22 m, how far is he from the centre of the rink? Express your answer to the nearest tenth of a metre. Include a diagram in your explanation.

Solution

In the diagram, the speed skater fell at point A and slid to point B.



Since the line segment AB is tangent to the circle, then it will be perpendicular to radius AC. The Pythagorean relationship can be used to calculate the distance BC, which represents how far the speed skater is from the centre of the rink.

$$\begin{aligned}BC^2 &= AB^2 + AC^2\\ BC^2 &= 22^2 + 40^2\\ BC^2 &= 484 + 1600\\ BC^2 &= 2084\\ BC &= \sqrt{2084}\\ BC &\approx 45.7\end{aligned}$$

After sliding 22 m, the speed skater is approximately 45.7 m from the centre of the rink.

Sports Link

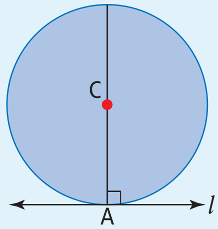
Jeremy Wotherspoon from Red Deer, Alberta, is one of Canada's best speed skaters. He has set several records at the 500-m distance.

Show You Know

Callan is attempting to land his model airplane when the wire breaks just before touchdown. If the length of the control wire is 10 m and the plane stops at a location 74 m from Callan, how far does the plane travel after the wire breaks. Express your answer to the nearest tenth of a metre.

Key Ideas

- A line that touches a circle at exactly one point is tangent to the circle.
- Point A is known as the point of tangency.
- A line l that is tangent to a circle at point A is perpendicular to the radius AC.
- A chord drawn perpendicular to a tangent line at the point of tangency contains the centre of the circle, and is a diameter.

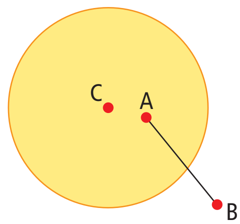


Check Your Understanding

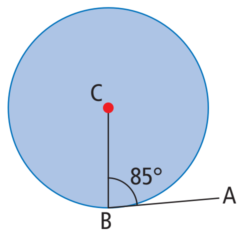
Communicate the Ideas

1. Raven and Elliott are discussing the diagram shown.

Elliott claims that line segment AB is a tangent to the circle because it touches the circle in one place. Raven disagrees. Who is correct, and why?



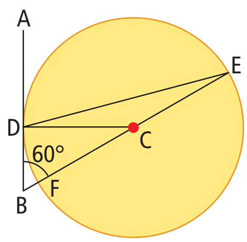
2. If BC is a radius of the circle, is AB tangent to the circle? Explain how you know.



Practise

For help with #3 and #4, refer to Example 1 on pages 395–396.

3. In the diagram, AB is tangent to the circle at point D, BE contains the diameter EF, and $\angle ABE = 60°$.



Explain your reasoning when answering each of the following questions.

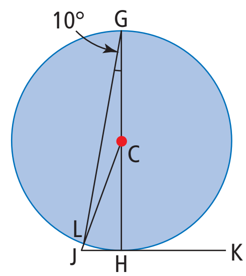
a) What is the measure of $\angle BDC$?

b) What is the measure of central angle $\angle DCE$?

c) What type of triangle is $\triangle CDE$?

d) What is the measure of $\angle DEC$?

4. Line segment JK is tangent to the circle at point H. GH is a diameter and $\angle CGL = 10°$.



Justify your answers to the following questions.

a) What type of triangle is $\triangle CGL$?

b) What is the measure of $\angle GCL$?

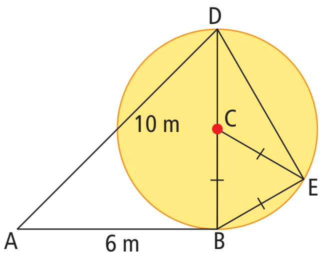
c) What is the measure of $\angle JCH$?

d) What is the measure of $\angle JHG$?

e) What is the measure of $\angle CJK$?

For help with #5 and #6, refer to Example 2 on pages 396–397.

5. In the diagram, AB is tangent to the circle at point B. BD is a diameter of the circle, AB = 6 m, AD = 10 m, and △BCE is an equilateral triangle.



Justify your answers to the following questions.

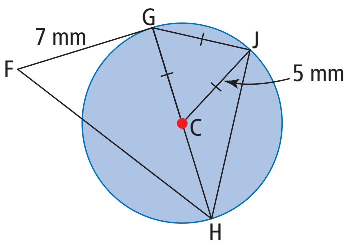
a) What is the length of the diameter BD?

b) What is the length of chord BE?

c) What is the measure of the inscribed angle ∠BED?

d) What is the length of chord DE, to the nearest metre?

6. In the diagram, FG is tangent to the circle at point G. GH is a diameter, CJ = 5 mm, FG = 7 mm, and △CGJ is an equilateral triangle.



a) What is the length of the diameter? Justify your answer.

b) Is △GHJ a right triangle? Justify your answer.

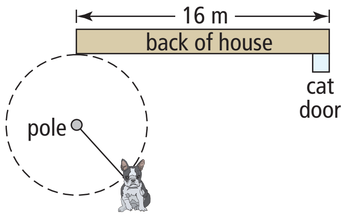
c) What is the length of chord HJ? Explain your reasoning. Express your answer to the nearest tenth of a millimetre.

d) What is the measure of angle ∠FGH? Justify your answer.

e) What is the length of FH? Explain your reasoning. Express your answer to the nearest tenth of a millimetre.

For help with #7, refer to Example 3 on page 398.

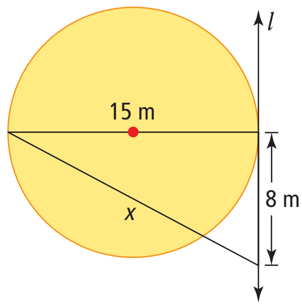
7. A dog is tied on a leash to the clothesline pole in the backyard. The leash is 5 m long and the pole is a perpendicular distance of 5 m from the edge of the house. What is the distance from the pole to the cat door? How close to the cat door can the dog get? Express your answers to the nearest tenth of a metre.



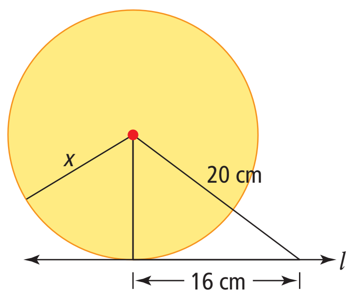
Apply

8. Find the length of x in each diagram. Line l is tangent to the circle. Express your answer to the nearest tenth, where necessary.

a)

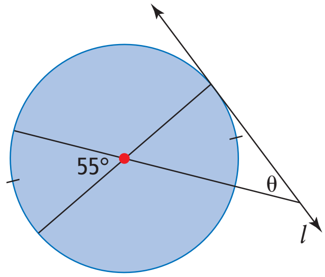


b)

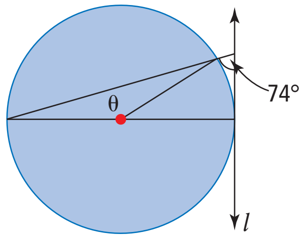


9. Find the measure of the angle θ in each diagram. Line l is tangent to the circle.

a)



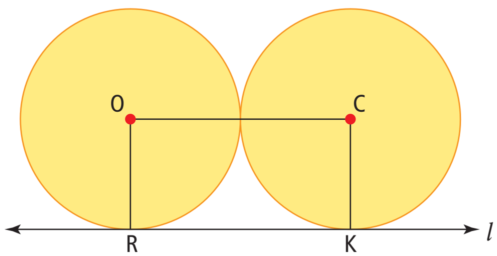
b)



Literacy Link

The Greek letter θ is *theta*. It is often used to indicate the measure of an unknown angle.

10. Both circles are identical in size. They are tangent to each other and to line l.

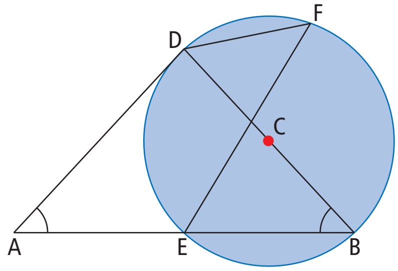


a) What type of quadrilateral is ROCK? Explain your reasoning.

b) If the radius of each circle is 5 cm, what is the perimeter of ROCK?

11. Line segment AB is tangent to a circle at point A. The diameter AD of the circle is 7.3 cm. If the length of AB is 4.2 cm, determine the length of BD. Include a diagram in your solution. Express your answer to the nearest tenth of a centimetre.

12. In the diagram, △ABD is an isosceles triangle. AD is a tangent to the circle at point D, and BD is a diameter of the circle.



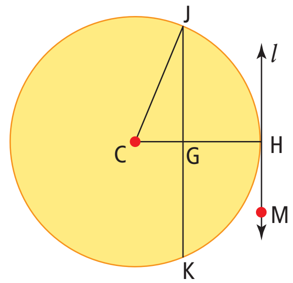
Justify your answers for each question.

a) What is the measure of ∠ADB?

b) What is the measure of ∠DBE?

c) What is the measure of ∠DFE?

13. Answer each question, given the following information.



- The line l is tangent to the circle at point H.
- The line l is parallel to the chord JK.
- The radius of the circle measures 9.1 cm.
- The chord JK measures 17 cm.

Explain your reasoning for each answer.

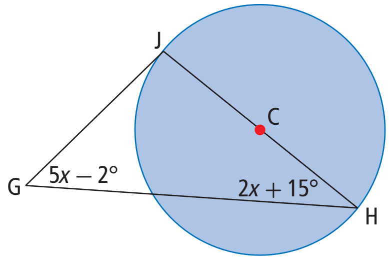
a) What is the measure of ∠CHM?

b) What is the measure of ∠CGJ?

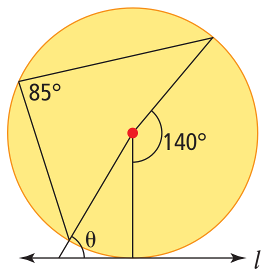
c) What is the length of JG?

d) What is the length of CG? Express your answer to the nearest tenth of a centimetre.

14. If JG is a tangent to the circle, what is the value of x and and the measure of $\angle$JGH?



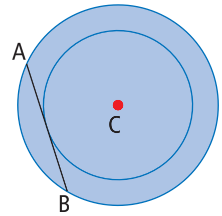
15. Line l is tangent to the circle as shown. Use properties of inscribed and central angles to find the value of angle θ. Explain your reasoning.



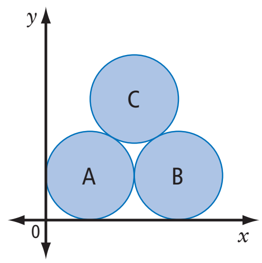
16. The aerial picture represents farmland. The circular green areas represent fields that are watered using a centre-pivot watering system. Design a question and solution using the relationship between tangents and radii of circles.

Digital rights not available.

17. Two concentric circles have their centres at point C. The radius of the smaller circle is 8 cm. The length of chord AB is 26 cm and is tangent to the smaller circle. What is the circumference of the larger circle? Express your answer to the nearest centimetre.

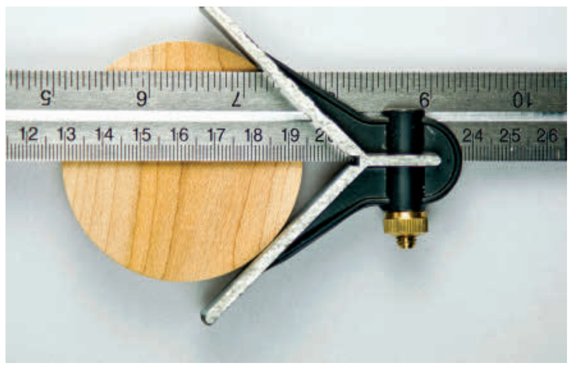


18. Three congruent circles are tangent to one another as shown. Circle A is tangent to both the x-axis and the y-axis. Circle B is tangent to the x-axis. The centre of circle A has coordinates (2, 2). What are the coordinates of the centres of circles B and C?

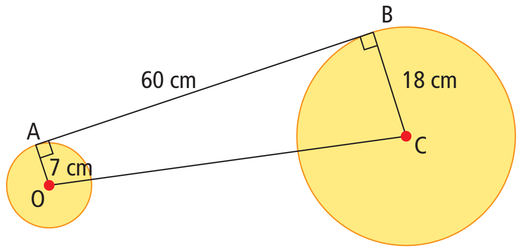


Extend

19. A steel centre square is used in woodwork to locate the centre of a wooden cylinder. Sketch the picture in your notebook and identify the edge(s) that most closely resemble a tangent to a circle. How do you think the centre square is used to locate the centre of the cylinder?

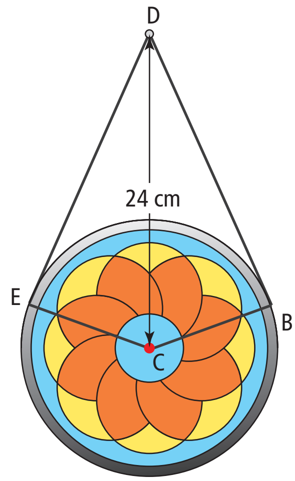


20. Two poles with radii of 18 cm and 7 cm are connected by a single metal band joining their centres and points of their outer edges. This is shown below. Determine the length of the metal band that is needed, if AB is tangent to both poles.



21. A rubber ball with a diameter of 6 cm is found on a frozen pond with only 2 cm sticking above the ice surface at its highest point. What is the circumference of the circle where the ball touches the ice surface? Express your answer to the nearest tenth of a centimetre.

22. A length of chain is attached to a suncatcher with a diameter of 20 cm. The chain is attached at points E and B such that the segments BD and ED are tangent to the circle. What is the total length of chain needed to hang the suncatcher on a nail at point D? Show your reasoning.



Math Link

a) Design a piece of art or a company logo using at least one circle. Incorporate at least one tangent. Remember that two circles can be tangent to each other.

b) Determine the measures of any chords, radii, diameters, or tangent lines in your design.

Tech Link

Tangents to a Circle

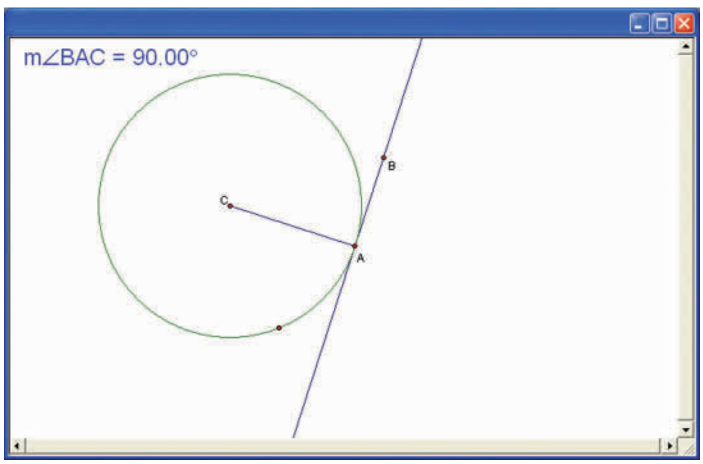
In this activity, you will use dynamic geometry software to explore tangent lines to a circle. To use this activity, go to www.mathlinks9.ca and follow the links.

Explore

1. What is the measure of ∠BAC?

2. a) Describe what happens to the measure of ∠BAC as you drag point A to different locations on the circle.

b) What conclusion can you make?



Chapter 10 Review

Key Words

Unscramble the letters for each puzzle in #1 to #4. Use the clues to help you solve the puzzles.

1. I R U S D A
the distance from the centre to any point on the circle

2. I S C B D E I N R E A G L N
an angle formed by two chords that share a common endpoint

3. R O C H D
a line segment that has both endpoints on the same circle

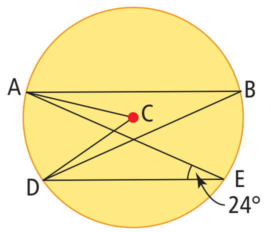
4. C R P P D E E N L A U R I S I T C R O B E
a line or line segment that passes through the midpoint of a line segment at 90°

10.1 Exploring Angles in a Circle, pages 378–385

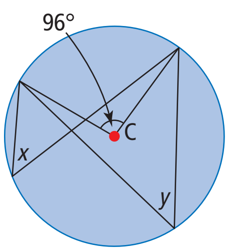
5. Determine the measure of each angle.

a) ∠ABD

b) ∠ACD



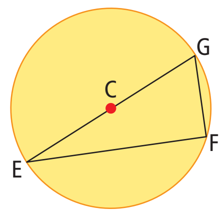
6. What are the measures of unknown angles x and y?



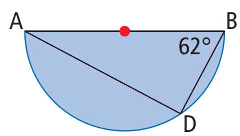
7. Parmjeet explains that if an inscribed angle in a circle has a measure of 13.5°, the central angle subtended by the same arc will also measure 13.5°. Do you agree with her thinking? Why or why not?

8. What is the measure of each central angle on the dartboard?

9. What is the measure of ∠EFG?

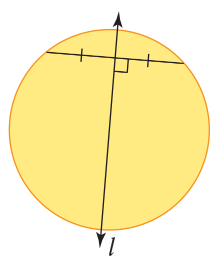


10. What is the measure of ∠BAD in the semicircle?



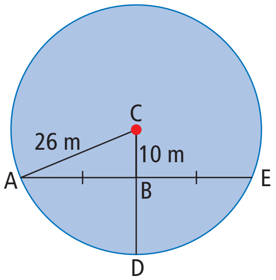
10.2 Exploring Chord Properties, pages 386–393

11. Explain how you know that the line l must pass through the centre of the circle.

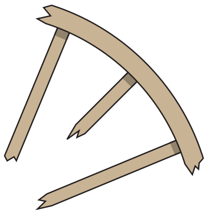


12. Andrea tries to find the centre of a wooden table top by placing a string across the top and finding its midpoint. She misses the centre by 1 cm. What went wrong? How might she have found the centre more accurately?

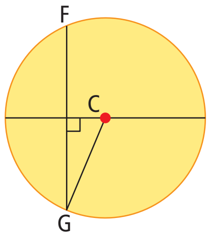
13. What is the length of chord AE? Explain how you determined your answer.



14. Archaeologists have found a broken piece of a wagon wheel as shown. Show how they can determine the circumference of the entire wheel from this broken piece.

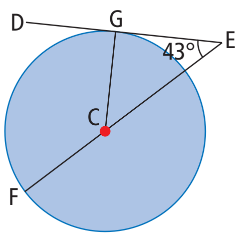


15. If chord FG has a length of 18 cm and the diameter of the circle is 22 cm, what is the shortest distance between FG and the centre of the circle? Express your answer to the nearest tenth of a centimetre.

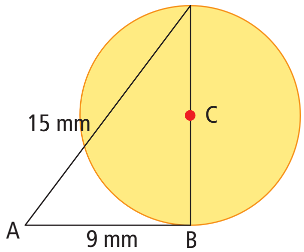


10.3 Tangents to a Circle, pages 394–403

16. What is the measure of $\angle FCG$ if DE is tangent to the circle?



17. If AB is tangent to the circle at B, what is the length of the radius?



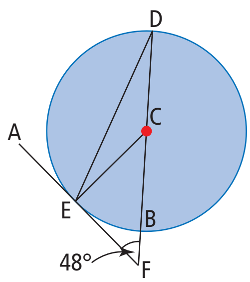
18. Jasmine was flying a remote-control airplane in a circle with a radius of 50 m. The signal was lost by the airplane which then flew along a tangent from the circle until it crashed 140 m from Jasmine's location. How far did the airplane travel horizontally along the tangent? Include a diagram in your answer. Calculate the distance to the nearest metre.

19. Line segment AF is tangent to the circle at E, and $\angle AFD = 48°$. Find the measure of each angle. Justify your answers.

a) $\angle CEF$ **b)** $\angle ECF$

c) $\angle ECD$ **d)** $\angle DEC$

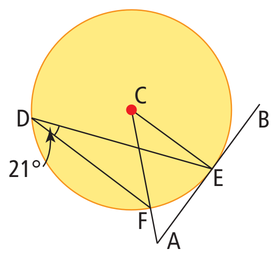
e) $\angle AED$ **f)** $\angle EDB$



20. Line segment AB is tangent to the circle at E. Determine the measure of the requested angles.

a) $\angle ACE$

b) $\angle CAB$



Chapter 10 Practice Test

For #1 and #2, choose the best answer.

1. Which statement is true?

A A central angle is smaller than an inscribed angle subtended by the same arc.

B Two inscribed angles are never equal in size.

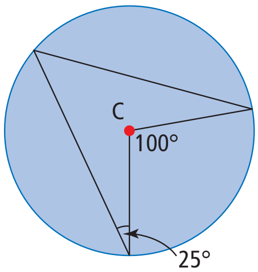
C An inscribed angle subtended by a diameter of the circle is always 90°.

D An inscribed angle in a semicircle can be larger than 90°.

2. What is the measure of the inscribed angle?

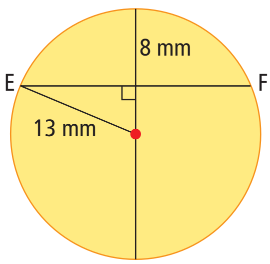
A 25° **B** 50°

C 100° **D** 200°

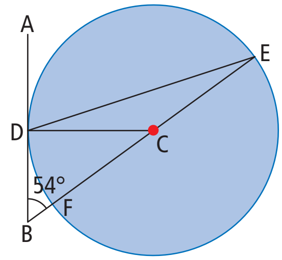


Complete the statements in #3 and #4.

3. The length of EF is ■.

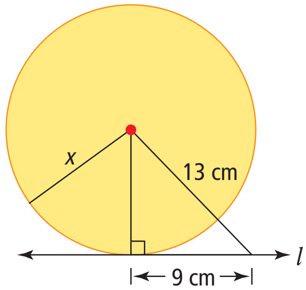


4. If AB is tangent to the circle, the measure of ∠BCD is ■.

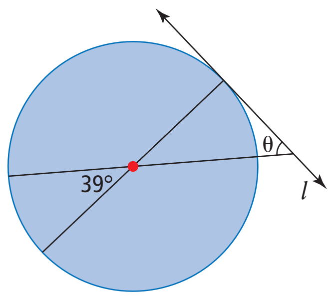


Short Answer

5. What is the length of radius x? Express your answer to the nearest tenth of a centimetre.

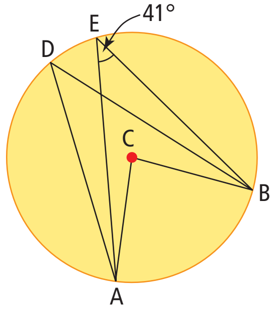


6. Find the measure of angle θ. Line l is tangent to the circle.

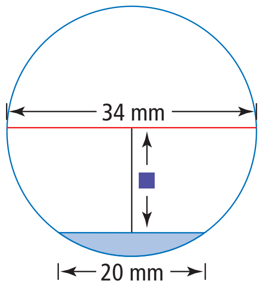


Extended Response

7. What are the measures of ∠ADB and ∠ACB? Explain your reasoning.



8. The diagram represents the water level in a pipe. The surface of the water from one side of the pipe to the other measures 20 mm, and the inner diameter of the pipe is 34 mm. What is the shortest distance from the centre of the pipe to the water level, rounded to the nearest millimetre? Explain your reasoning.



9. The Namdaemun gate, a two-storey, pagoda-style wooden building on a stone base, was Korea's premier national treasure. This 600-year-old structure in Seoul was destroyed in a fire in 2008.

To rebuild the gate, square beams were cut from logs. What is the largest dimension of square that can be cut from a log with a diameter of 40 cm?

Math Link: Wrap It Up!

Design a piece of art or a logo using at least two circles.

- Incorporate each circle property that you have studied into your design, and label where you use these properties.
- You may wish to use some or all of the designs that you created in the Math Link revisits throughout the chapter.

Challenges

Dream Catcher

The legend of the Dream Catcher exists in varying forms among Aboriginal Peoples. In the design, the Dream Catcher is formed into a loop. Its centre is woven in a web-like pattern.

It is said that the night air contains good dreams and bad dreams. According to the legend, the good dreams go through the web into the sleeper. The bad dreams become hopelessly entangled in the web and perish at the first light of dawn.

The number of points connected to the ring is often eight, in honour of the spider. The webbing is made of sinew. The web can be adorned with natural objects such as stones, beads, shells, bark, and feathers. A bloodstone is often hung in the centre.

You be the artist. In this challenge, you are going to draw a Dream Catcher and explore how its construction relates to circle geometry.

1. a) Draw a circle with a minimum radius of 8 cm. Place eight equally spaced markings on the circle.

b) What are two different ways to determine the placement of the markings?

Materials
- compass
- ruler
- protractor

2. Draw the first row of webbing by joining each pair of consecutive markings with a straight line.

   a) What are two different measures of the central angle made between the first and seventh markings and the circle centre? Show your thinking.

   b) What is the measure of the inscribed angle subtended by the same arc as the central angle you found in 2a)?

3. Draw the second row of webbing. How do the central and inscribed angles of the layers compare? Show your thinking.

4. Continue drawing the rows of webbing until an opening of approximately 5 cm in diameter is in the centre. How many rows did you need?

5. Compare your drawing to an actual Dream Catcher or pictures of Dream Catchers. How does your design differ from the actual constructed ones?