1

Coordinates and Design

Look around at what you and your classmates are wearing. Is anyone wearing a piece of clothing with a repeating pattern? This design is a transformation.

Cultures all over the world use transformations on clothing, as well as on belts, blankets, baskets, and other items. Many of these designs could be drawn on a Cartesian plane.

In this chapter, you will learn how to plot points and create shapes on a Cartesian plane. You will use transformations—slides, mirror images, and turns—to make designs with these shapes.

What You Will Learn

- to use ordered pairs to plot points on a Cartesian plane
- to draw designs on a Cartesian plane
- to identify coordinates of the vertices of 2-D shapes
- to translate, reflect, and rotate points and shapes on a Cartesian plane
- to determine the horizontal and vertical distances between points

Key Words

Cartesian plane
x-axis
y-axis
origin
quadrants
coordinates
vertex
transformation
translation
reflection
rotation

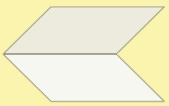
MATH LINK

A number of cultures use designs in their artwork. Many Aboriginal peoples use beads to decorate their ceremonial clothing or to create jewellery. How would you create a bead design of your own on a Cartesian plane?

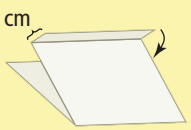
FOLDABLES™
Study Tool

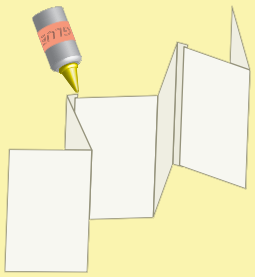
Make the following Foldable to organize what you learn in Chapter 1.

Step 1 Collect three sheets of paper. Fold each sheet of paper in half as shown.

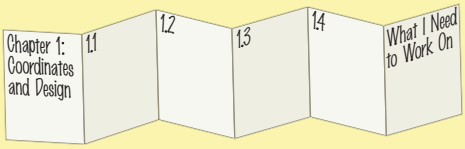


Step 2 Fold a 1-cm tab along the edge of two of the folded sheets of paper. Glue the papers together along the tabs.





Step 3 Label sections made by each fold.



Literacy Link

As you work through Chapter 1, make notes on the appropriate fold. Include information about the key words, examples, and key ideas.

1.1

The Cartesian Plane

Focus on...

After this lesson, you will be able to...

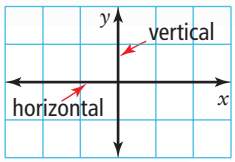
- ❑ label the axes and origin of a Cartesian plane
- ❑ identify and plot points on a Cartesian plane

Have you ever been lost? Did you look at a map for directions?

A grid can be used to show locations on a map. The seventeenth-century French mathematician René Descartes (1596–1650) developed a system for graphing points on a **Cartesian plane**. A Cartesian plane is also called a coordinate grid.

Cartesian plane

- the plane formed when a horizontal and a vertical number line cross



Materials

- grid paper
- ruler
- 11 × 17 sheet of paper
- scissors
- glue

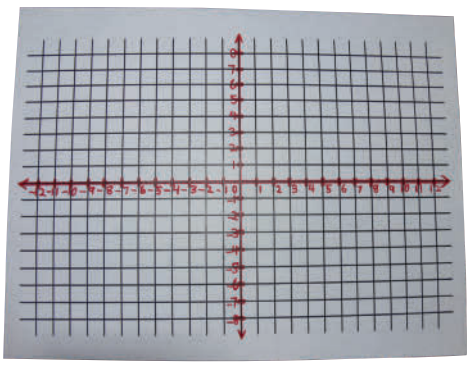
Explore the Math

How do you draw a coordinate grid?

1. a) In the middle of a sheet of grid paper, draw a horizontal number line across the whole width of the page. Place the number 0 in the middle of the number line. Label the number line as shown. This is the ***x*-axis**.

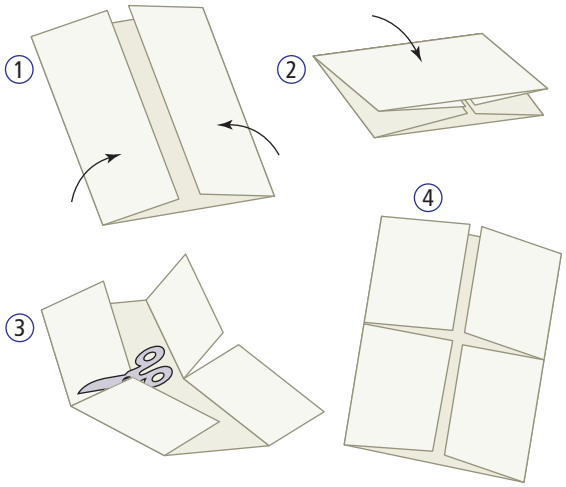
b) Draw a vertical number line through the 0 point along the whole length of the page. Label the number line as shown. This is the ***y*-axis**.

c) Label the **origin**. What ordered pair describes the origin?



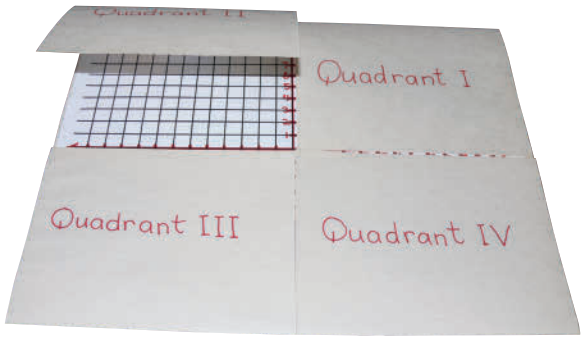
2. a) FOLDABLES™ Study Tool

Make a Foldable. Use an 11 × 17 sheet of paper to create a four-door book as shown.



b) Glue the coordinate grid you drew behind the doors. Make sure to line up the x-axis and y-axis with the cuts for the doors.

c) Label the four **quadrants** on the outside of the four doors in a counterclockwise direction.



d) Open the door of quadrant I. Mark two points in quadrant I. Label the **coordinates** of each point.

e) What do you notice about the x-coordinates of these points in quadrant I? What do you notice about the y-coordinates in quadrant I? Make a prediction about the x-coordinates and y-coordinates for any point in quadrant I.

f) Repeat parts d) and e) for quadrants II, III, and IV.

Reflect on Your Findings

3. What are the signs of the x-coordinates and y-coordinates of any point in each quadrant? On each door, write $(+, +)$, $(-, +)$, $(+, -)$, or $(-, -)$.

x-axis
- the horizontal number line on the coordinate grid

y-axis
- the vertical number line on the coordinate grid

origin
- the point where the x-axis and the y-axis cross

quadrants
- the four regions on the coordinate grid

coordinates
- the values in an ordered pair (x, y)

Literacy Link

Roman Numerals

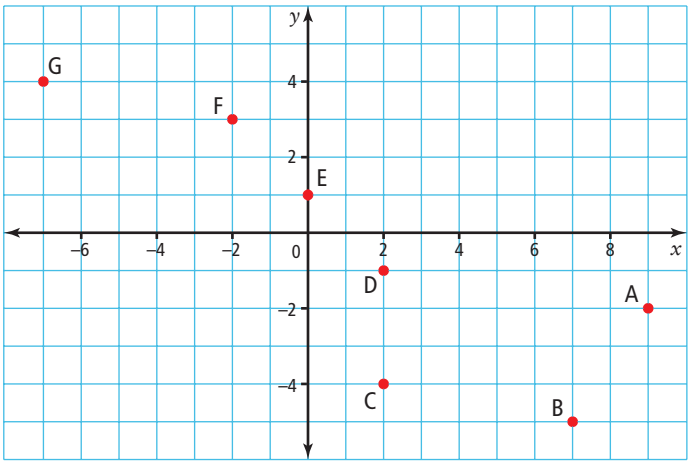
I, II, III, and IV are Roman numerals that represent 1, 2, 3, and 4.

Science Link

The points on this coordinate grid form the Big Dipper.

Example 1: Identify Points on a Coordinate Grid

State the coordinates of each point on the coordinate grid shown.



Literacy Link

Reading Coordinates

Read the *x*-coordinate first, then the *y*-coordinate.

(3, –2) is read as "the coordinate pair three, negative two" or "the ordered pair three, negative two."

Solution

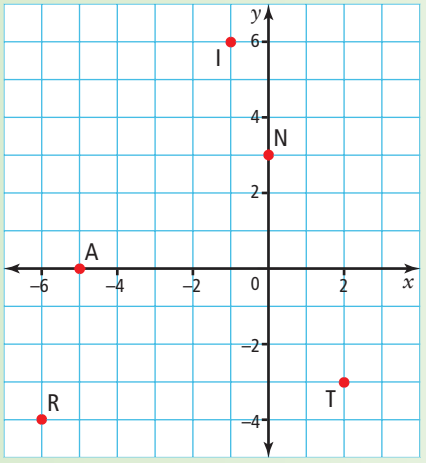
First, determine the *x*-coordinate. Then, determine the *y*-coordinate. This will give the ordered pair (*x*, *y*).

Point A is 9 units to the right of the origin. So, the *x*-coordinate is 9. Point A is 2 units down from the origin. So, the *y*-coordinate is −2. Point A has the coordinates (9, −2). This can be shown as A(9, −2).

The coordinates of the other points are B(7, −5), C(2, −4), D(2, −1), E(0, 1), F(−2, 3), G(−7, 4).

Show You Know

What are the coordinates for each point on the coordinate grid shown?



Example 2: Plot Points on a Coordinate Grid

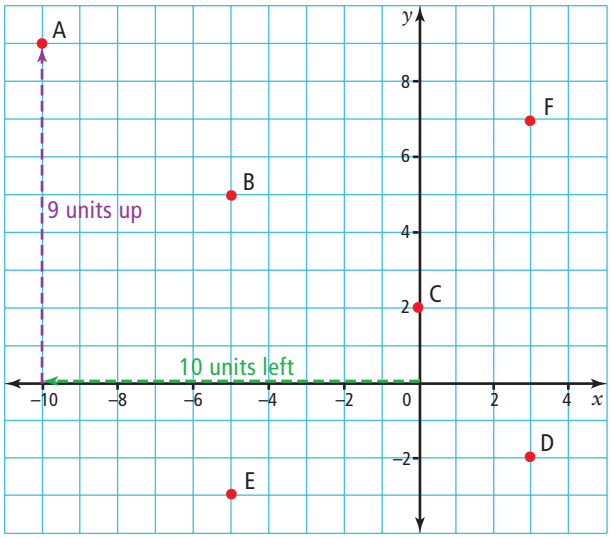
Plot the following points to form a constellation:
A(−10, 9), B(−5, 5), C(0, 2), D(3, −2), E(−5, −3), F(3, 7).

Solution

To plot points on the coordinate grid, always start at the origin.

For point A(−10, 9), count 10 units to the left.
Then, count 9 units up.
Make a dot. Label the point A.

Plot points B, C, D, E, and F.



Count *x* units right if the *x*-coordinate is positive, and left if it is negative.
Then, count *y* units up if the *y*-coordinate is positive, and down if it is negative.

If the *x*-coordinate is 0, do not count left or right.
If the *y*-coordinate is 0, do not count up or down.

Science Link

The points on this coordinate grid form the constellation called Cygnus, the swan.

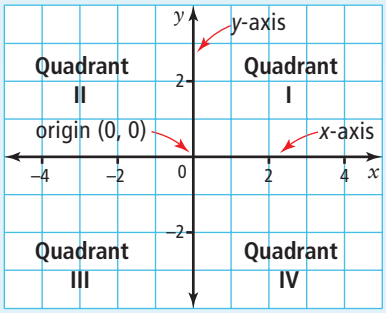
Show You Know

Plot the following points on a coordinate grid:
C(–9, 1), A(5, 10), R(0, –7).

Key Ideas

- An ordered pair (x, y) is used to locate any point on a Cartesian plane.
- All points located within the same quadrant have the same signs for their x-coordinates and the same signs for their y-coordinates.
- Points on the x-axis have the value $(x, 0)$.
- Points on the y-axis have the value $(0, y)$.

Cartesian Plane



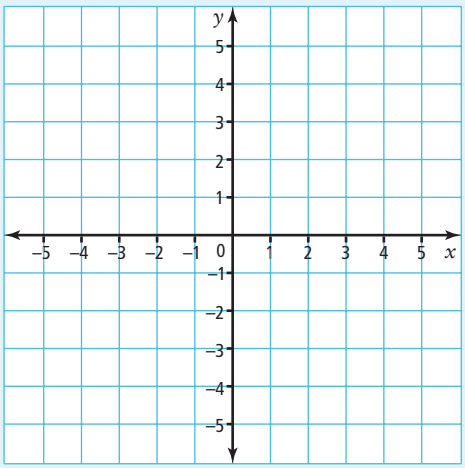
Communicate the Ideas

1. **a)** On the inside of one of the doors of your four-door book, explain how to plot points on the coordinate grid.

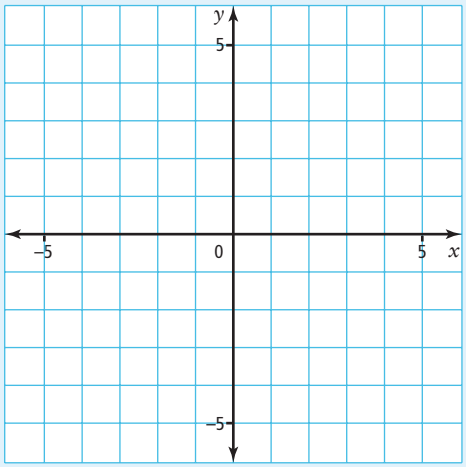
b) On another door, explain how to identify points on the coordinate grid.

2.

Grid A



Grid B



a) How is Grid A the same as Grid B? How is it different?

b) Compare your answer with that of a classmate.

3. What are the similarities and differences between two points with coordinates (0, 3) and (3, 0)? Use diagrams as part of your explanation.

4. Imagine that a fly is resting on the ceiling of your classroom.

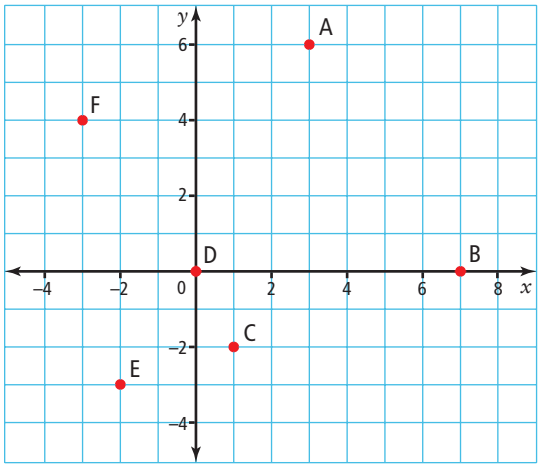
a) How might you describe the exact location of the fly?

b) How would having a coordinate grid on the ceiling make it easier for someone to locate the fly?

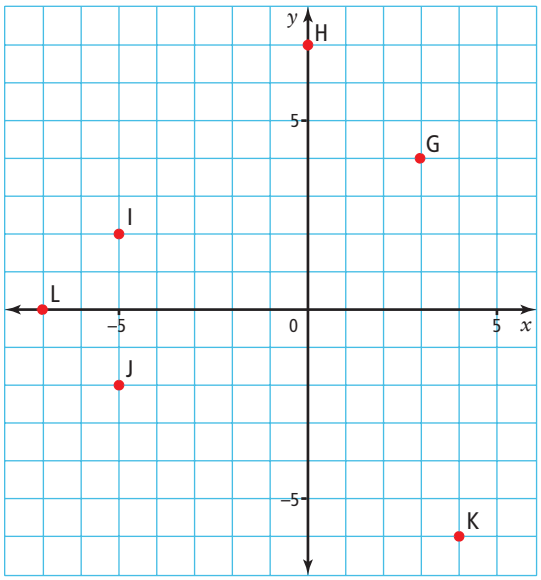
Practise

For help with #5 to #8, refer to Example 1 on page 6.

5. What are the coordinates of each point shown on the coordinate grid?



6. Identify the coordinates of each point shown on the coordinate grid.



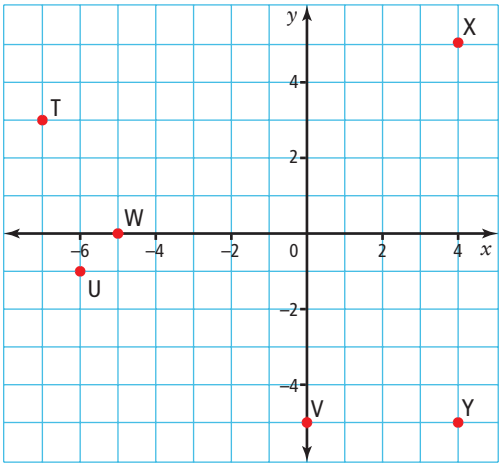
WWW Web Link

To practise plotting and identifying points on a coordinate grid, go to www.mathlinks7.ca and follow the links.

7. Which letter on the coordinate grid matches each ordered pair?

a) $(-7, 3)$ **b)** $(4, 5)$ **c)** $(-6, -1)$

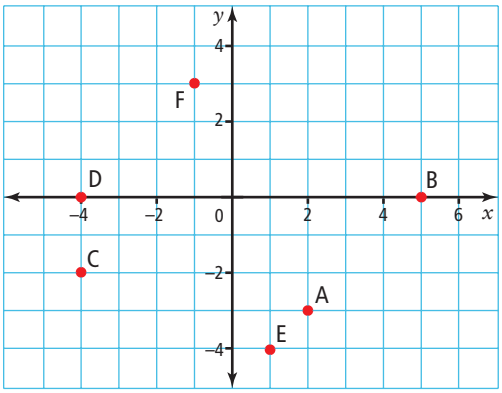
d) $(-5, 0)$ **e)** $(4, -5)$ **f)** $(0, -5)$



8. Identify the letter on the coordinate grid that matches each ordered pair.

a) $(1, -4)$ **b)** $(2, -3)$ **c)** $(-1, 3)$

d) $(5, 0)$ **e)** $(-4, -2)$ **f)** $(-4, 0)$



For help with #9 and #10, refer to Example 2 on page 7.

9. Plot these points on a coordinate grid: A(3, −6), B(0, 0), C(8, 0), D(3, 4), E(−3, 4), F(−2, −9), G(−5, 5), H(1, −3).

10. Plot each ordered pair on a coordinate grid: J(0, 0), K(−4, 2), L(3, −8), M(−7, −7), N(7, −7), P(0, −6), Q(−1, 1), R(5, 0).

Apply

11. **a)** Predict in which quadrant each of the following points will lie: A(6, −4), B(−3, 2), C(−10, −7), D(−6, 3), E(8, 7), F(0, 9), G(0, 0), H(1, −2).

b) Plot the points on a coordinate grid.

c) Were your predictions correct?

d) Which points do not lie in any quadrant? Where do these points lie?

12. Plot the following points on a coordinate grid: G(−7, 0), H(−7, −1), I(−7, −3), J(−7, −5), K(−7, −8), L(−7, −10).

a) What do you notice about the points?

b) What do the coordinates of the six points have in common?

c) Name two other points that could belong with these six points.

13. Plot the following points on a coordinate grid: A(5, 7), B(5, 3), C(9, 3), D(9, 7). Connect A to B, B to C, C to D, and D to A.

a) What shape did you create?

b) In which quadrant is the shape located?

c) What are the side lengths of the shape?

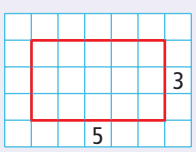
d) What are the similarities between the following pairs of points: A and B, C and D, A and D?

14. Create a rectangle by connecting points E(−3, 2), F(−3, −1), G(1, −1), and H(1, 2). What is the area of the rectangle? How do you know you are correct?

Literacy Link

Finding Area

Area = length × width
= 5 × 3
= 15 units^2

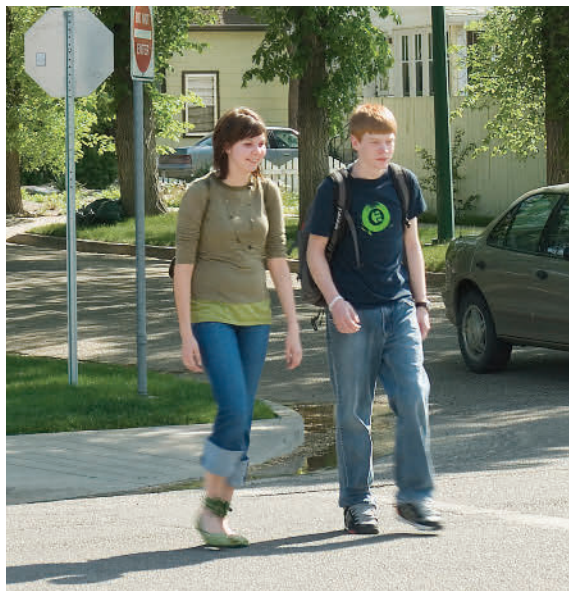


15. Amy and Joe plan to walk in a straight line from their house to their new school. Plot the route on a coordinate grid. Their house lies at H(−7, −4) and the school lies at S(4, 7). All grid lines, including the x-axis and the y-axis, are streets.

a) Draw a line from H to S. How many times do they have to cross a street?

b) There is a traffic light on the x-axis at L(−3, 0). Will they cross at the traffic light?

c) There are two crosswalks on the y-axis: one at C(0, 5) and one at W(0, 3). Will they cross at one of the crosswalks? If so, at which one, C or W?



16. Maria is creating an X pattern for a needlepoint project. She has plotted the X on a coordinate grid with these ordered pairs: J(3, 0), K(2, −1), L(1, −2), M(−3, −2), N(−1, −4), R(−1, 0), S(0, −1), T(2, −3), U(3, −4).

a) Will she make an X?

b) If not, what ordered pair should she change to fix it?

c) What is the correct ordered pair?

Extend

17. **a)** Plot each pair of points on a coordinate grid. Join each pair with a line segment:
A(−7, 4) and B(3, 4)
C(−3, 2) and D(−3, 7)
E(0, 0) and F(0, 5)
G(−5, −3) and H(7, −3)

b) What is the length of the line segment joining each pair?

18. The map shows the coordinates of Hurricane Katrina. Positive is used for north latitude and negative for west longitude.

What were the coordinates when the storm

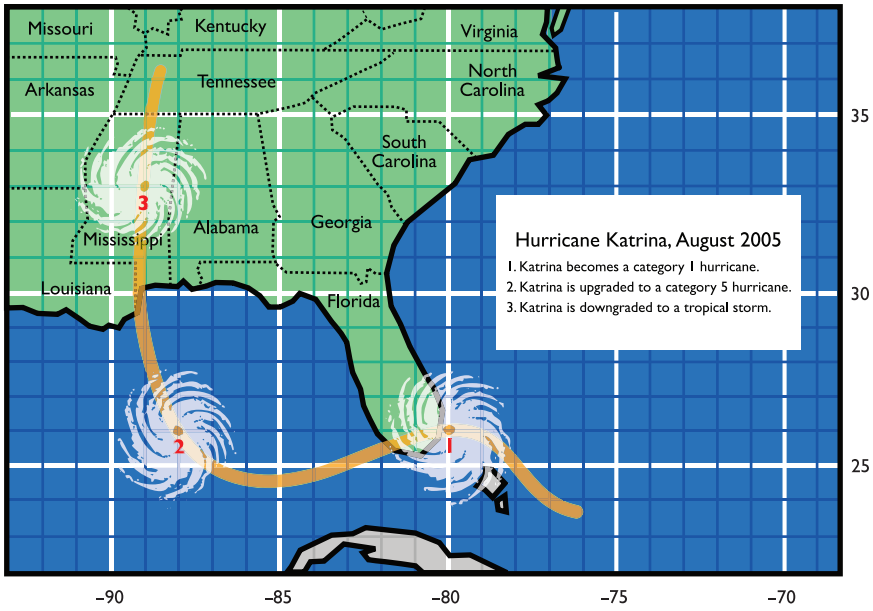
a) became a category 1 hurricane?

b) was upgraded to a category 5 hurricane?

c) was downgraded to a tropical storm?

Did You Know?

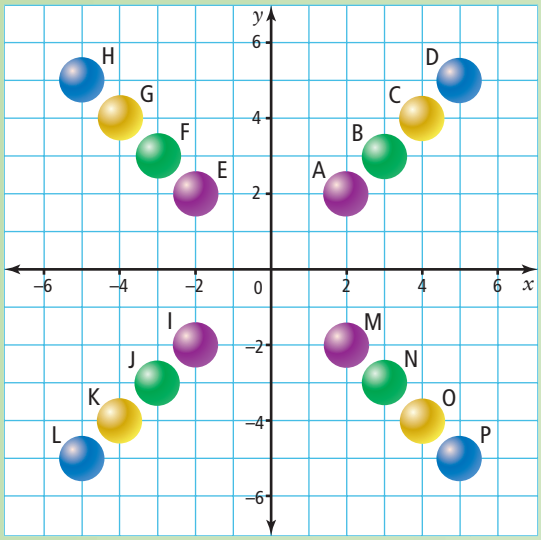
When the wind speed of a tropical storm reaches 119 km/h, it is called a hurricane.



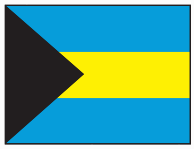
MATH LINK

a) Identify the coordinates of the centre of the beads.

b) Study the coordinates of the beads in each line. What do you notice?



1.2 Create Designs


Bahamas


Canada


Hungary


Nicaragua


Scotland


South Africa


United Arab Emirates


Vietnam

Focus on...

After this lesson, you will be able to...

- ❑ create a design and identify the coordinates used to make the design
- ❑ identify the coordinates of vertices of a 2-D shape

Many designs can be drawn on a coordinate grid. Look at the flag designs shown here. Also, think of the logo for your favourite sports team, the logo for a local bank, or the brand symbol on your shoes. Which of the designs may have been created on a coordinate grid?

Explore the Math

How do you draw a design on a coordinate grid?

Materials

- grid paper
- ruler
- coloured pencils

Literacy Link

Plural of Axis

The word axes is used to describe more than one axis.

1. Draw a coordinate grid on grid paper. Label the axes by 5s from −10 to 10.

2. Plot the following points: A(−10, −10), B(10, −10), C(10, 10), and D(−10, 10). Connect them in alphabetical order from A to D. Connect D to A.

3. On the same piece of grid paper, plot these points: E(2, 5), F(2, 2), G(5, 2), H(5, −2), I(2, −2), J(2, −5), K(−2, −5), L(−2, −2), M(−5, −2), N(−5, 2), P(−2, 2), and Q(−2, 5). Connect the points in alphabetical order from E to Q. Connect Q to E. Colour the inside of this design red.

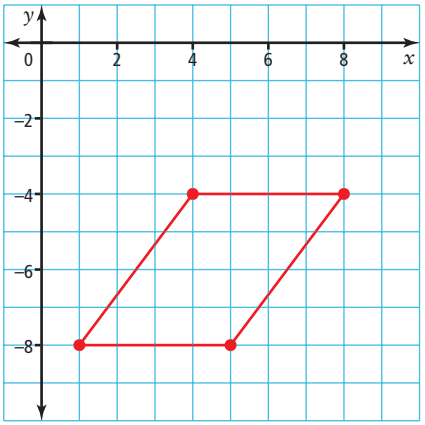
Reflect on Your Findings

4. **a)** What does the flag you created look like?

   b) Do you think you would have drawn the same design if you had connected the points in a different order? Explain.

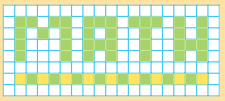
Example 1: Identify the Coordinates of Vertices

Identify the coordinates of the vertices of the shape.



Art Link

For needlepoint, cross-stitch, and rughooking, the design is drawn on a grid first.

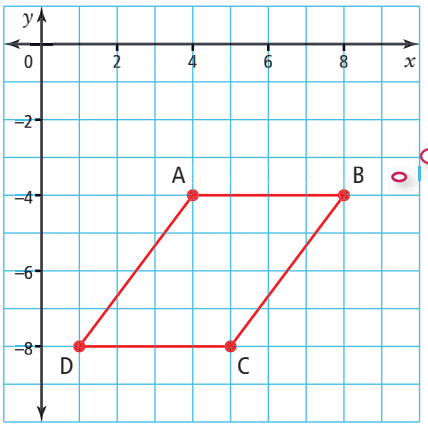


Solution

Label each **vertex** of the shape.

vertex

- a point where two sides of a figure meet
- plural is *vertices*

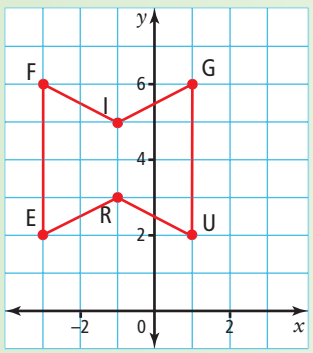


Label the vertices with capital letters.

Identify each vertex using ordered pairs:
A(4, −4), B(8, −4), C(5, −8), D(1, −8).

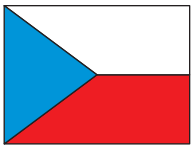
Show You Know

What are the coordinates of the vertices of the figure shown?



Example 2: Draw a Design

Draw the flag of the Czech Republic on a coordinate grid.



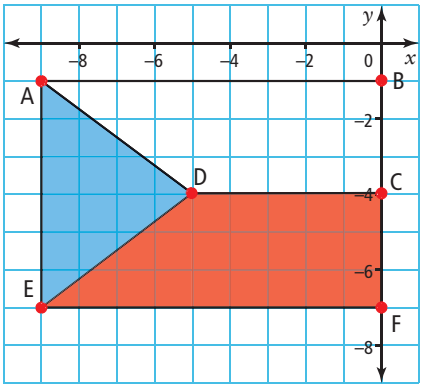
Solution

Draw a coordinate grid. Place all of the vertices of the design where you think they belong on the coordinate grid.

Place the vertices where the grid lines cross.

Label the vertices with capital letters. Name the vertices using ordered pairs:

A(−9, −1), B(0, −1), C(0, −4), D(−5, −4), E(−9, −7), F(0, −7).



Connect A to B, B to C, C to D, and D to A. Colour the inside of this shape white.

Connect C to F, F to E, and E to D. Colour the inside of this shape red.

Connect A to E. Colour the inside of this triangle blue.

Show You Know

Draw a square on a coordinate grid. Write the coordinates for the vertices of your square. Then, write instructions for how to connect the vertices.

Key Ideas

- To draw a design on a Cartesian plane,
  - plot the vertices of your design
  - identify the vertices and name their coordinates
  - connect the vertices to make your design
  - colour your design, if it has colour

Communicate the Ideas

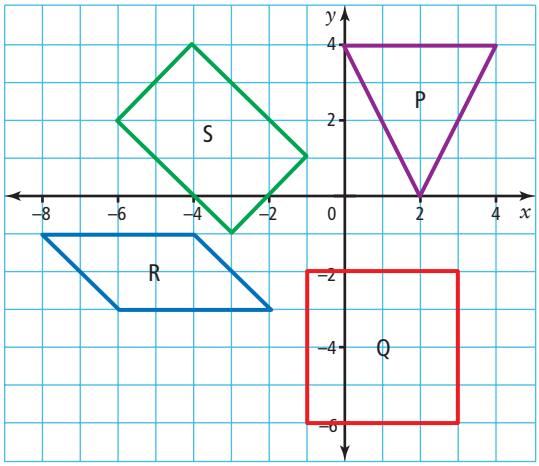
1. How do you create a design on a coordinate grid? Explain each step.

2. Why is it easier for someone to follow instructions to draw a design if the design is drawn on a coordinate grid?

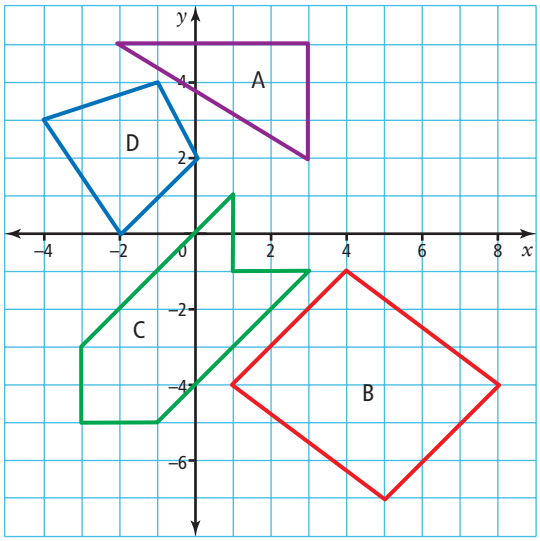
Practise

For help with #3 and #4, refer to Example 1 on page 13.

3. Identify the coordinates of the vertices of figures P, Q, R, and S.



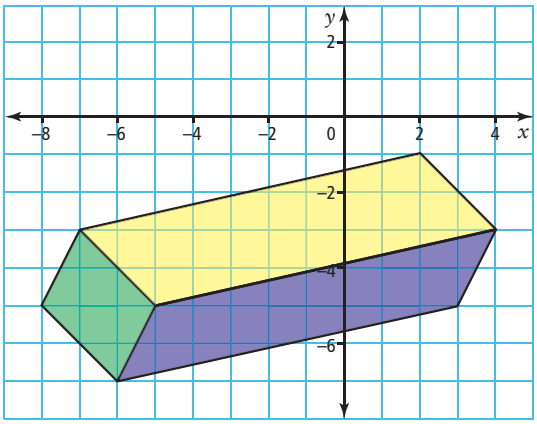
4. What are the coordinates of the vertices of figures A, B, C, and D?



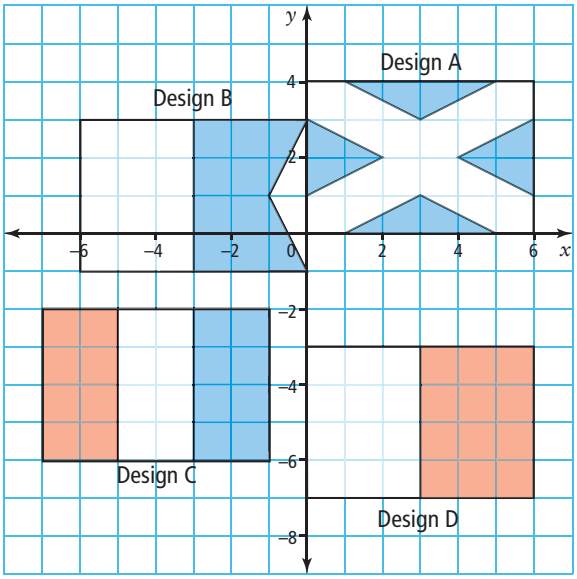
For help with #5 and #6, refer to Example 2 on page 14.

5. **a)** For the following design, name the coordinates of the vertices.

b) Describe the steps you would follow to copy the design.

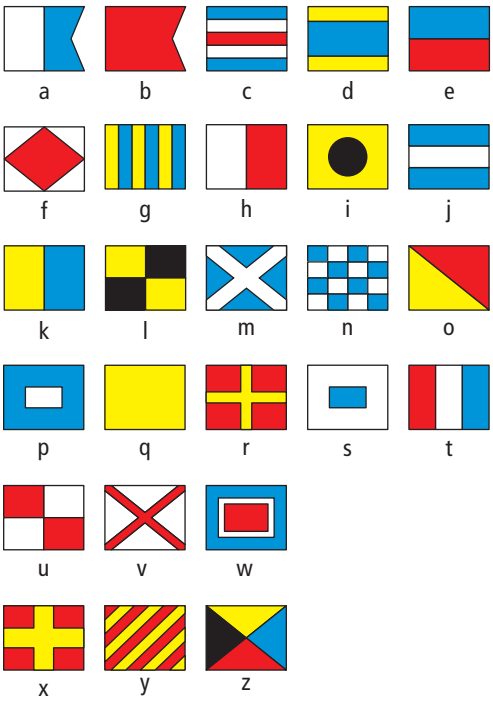


6. For each of the following designs, what are the coordinates of the vertices? What steps would you follow to copy each design?



Apply

7. Maritime signal flags like the ones shown below are used to communicate at sea. What word do the flags shown in #6 spell? Hint: Read the flags in counterclockwise order beginning in quadrant I.



8. **a)** Using the Maritime flags, design a word on the coordinate grid. Write instructions for drawing your design. Note: Your word should have four letters. Write the letters in counterclockwise order, beginning in quadrant I.

b) Exchange your instructions with a classmate. Draw your classmate's flags, using his or her instructions. What is the word?

9. On a coordinate grid, draw the number 4. Write instructions for drawing the number. Compare your instructions to a classmate's.

10. **a)** Draw and label a coordinate grid by 5s from −10 to 10. Draw a design by plotting the following points:

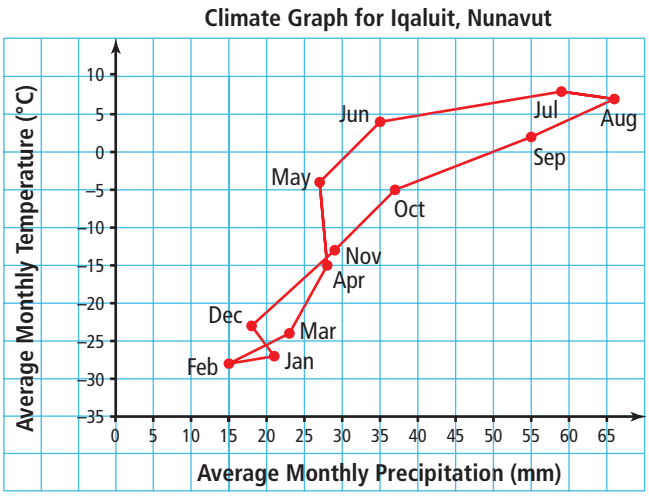
A(0, −4), B(−4, −5), C(−2, −3), D(−6, 2), E(−5, 2), F(−6, 4), G(−4, 3), H(−4, 4), I(−1, 0), J(−2, 5), K(−1, 4), L(0, 6), M(1, 4), N(2, 5), P(1, 0), Q(4, 4), R(4, 3), S(6, 4), T(5, 2), U(6, 2), V(2, −3), W(4, −5).

Using a red pencil, connect the dots in alphabetical order from A to W. Then, connect W to A. Plot point X at (0, −7) and connect X to A.

b) What symbol have you drawn?

Extend

11. The climate graph shows the climate for Iqaluit, Nunavut. The x-coordinate gives the average monthly precipitation (rain and snow). The y-coordinate gives the average monthly temperature.



What is the average precipitation and temperature for each of the following months? Write each answer as an ordered pair.

a) January **b)** April

c) July **d)** October

12. A rectangle has a perimeter of 36 units.

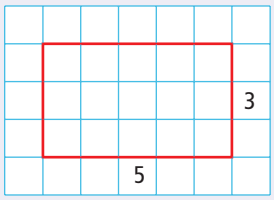
a) What could the length and width of the rectangle be? Give all possible whole number answers.

b) List each length and width as an ordered pair (l, w).

c) Plot the points on a coordinate grid.

d) In which quadrant are the points located? Why are the points in this quadrant?

Literacy Link



Perimeter

Perimeter $= 2l + 2w$
$= 2(5) + 2(3)$
$= 10 + 6$
$= 16$ units

13. **a)** Plot and connect the following pairs of points:
(0, 10) and (1, 0)
(0, 9) and (2, 0)
(0, 8) and (3, 0)
(0, 7) and (4, 0)

b) Write the coordinates of the points that will complete the design. Plot and connect these points.

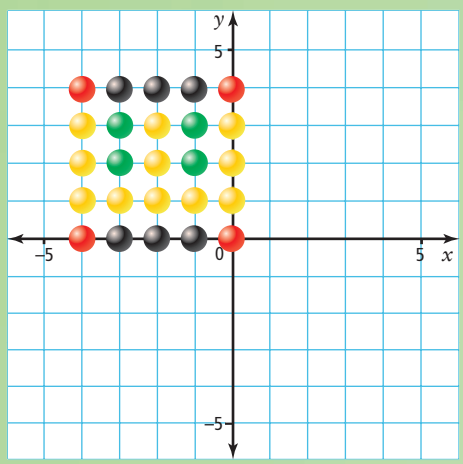
14. **a)** Draw the shape in #13 again, but this time in quadrants II, III, and IV. Use the same coordinate grid you used for #13.

b) How are the ordered pairs of the shape in quadrant III different from the ones of the shape in quadrant I?

c) Describe the design.

MATH LINK

a) Copy this bead design onto a coordinate grid.



b) Repeat the same design in quadrants I, III, and IV. The design in each quadrant should have one edge along the x-axis and one edge along the y-axis.

1.3 Transformations

Focus on...

After this lesson, you will be able to...

- ❑ use a translation, a reflection, and a rotation
- ❑ describe the image resulting from a transformation

transformation

- moves a geometric figure
- examples are translations, reflections, and rotations

How does this image show **transformations**?

Explore the Math

How do you describe a translation?

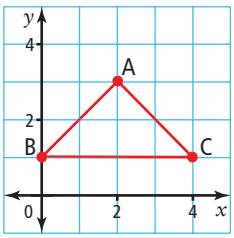
Materials

- grid paper
- ruler
- scissors

translation

- a slide along a straight line

1. On grid paper, draw and cut out the triangle shown. Place it on a coordinate grid. Draw around the outline of the cutout and label it ABC.



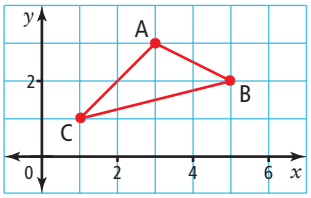
2. Slide the cutout 6 units to the right and 2 units up. Draw around the outline of the cutout in its new position. Label the new triangle A′B′C′.

Reflect on Your Findings

3. What are the coordinates of △A′B′C′?

How do you describe a reflection?

4. On grid paper, copy and cut out the triangle shown. Place it on a coordinate grid. Draw around the outline of the cutout and label it ABC.



5. Flip the cutout over the x-axis. Draw around the outline of the cutout. Label the new triangle A′B′C′.

Reflect on Your Findings

6. **a)** Compare the distance of A and A′ from the line of reflection.

b) Compare the distance of B and B′ from the line of reflection.

c) Predict the distance of C and C′ from the line of reflection.

reflection

- a mirror image
- a mirror line is called a line of reflection

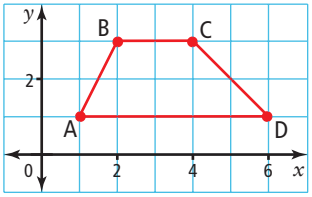
Literacy Link

Reading Prime

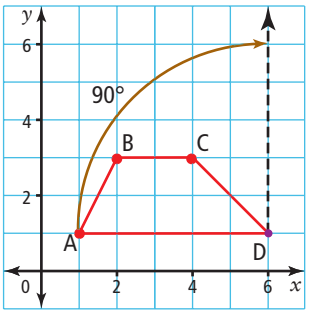
A′ is read "A prime." It is used to label the point that matches point A after a transformation.

How do you describe a rotation?

7. On grid paper, draw and cut out the quadrilateral shown. Place it on a coordinate grid. Draw around the outline of the shape and label it ABCD. Mark the centre of rotation as D.



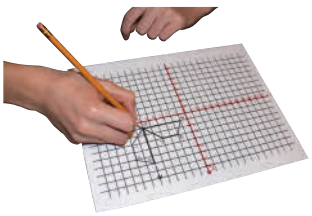
8. Find the line connecting A to the centre of rotation at D. Draw a broken line from D at a 90° angle clockwise.



rotation

- a turn about a fixed point called the centre of rotation

9. Place the cutout over the original figure. Put your pencil tip on the cutout at point D. Turn the cutout 90° clockwise until point A is on the broken line. Draw around the outline of the cutout. Label the new quadrilateral A′B′C′D′.



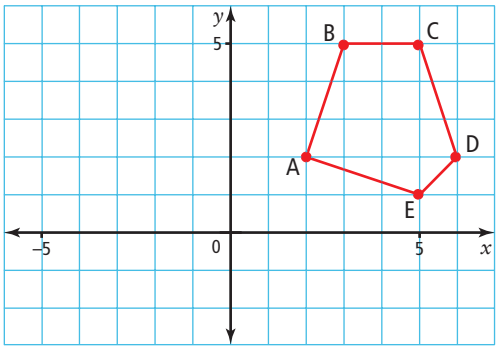
Reflect on Your Findings

10. **a)** Compare quadrilateral ABCD with its rotation image A′B′C′D′. How are the figures the same? How are they different?

b) Look at the lines that join A to D, and A′ to D′. What do you notice about the lengths of these lines?

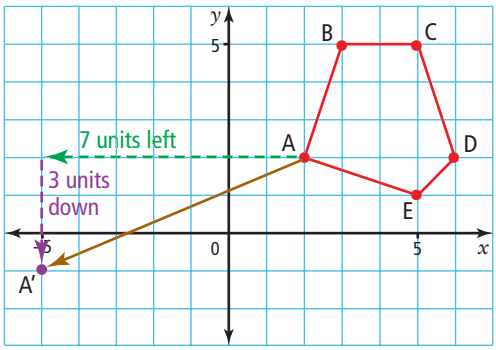
Example 1: Draw a Translation

Translate this figure 7 units to the left and 3 units down.

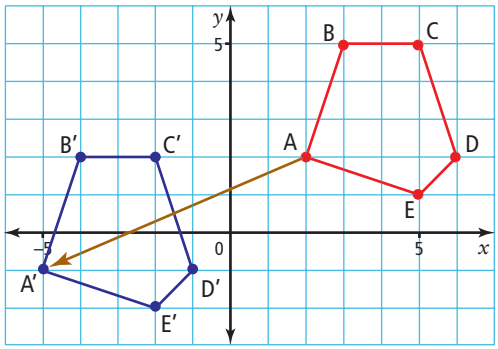


Solution

Start at point A. Count 7 units left and 3 units down. Plot the point and call it A′.

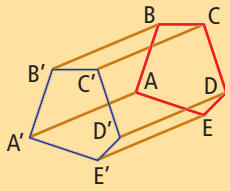


Plot points B′, C′, D′, and E′, counting 7 units left and 3 units down. Connect the points to form A′B′C′D′E′.



Art Link

You can use translations to make 3-D drawings.



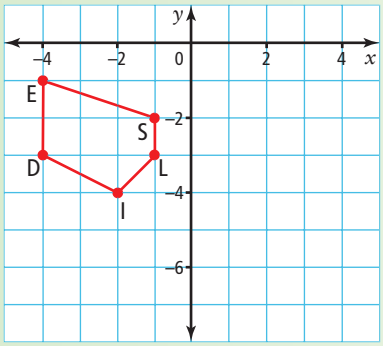
Literacy Link

Reading the Translation Arrow

The translation arrow → shows the distance and direction a figure has moved.

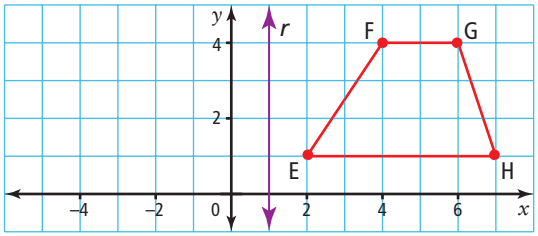
Show You Know

Copy this figure on a coordinate grid. Translate the figure 5 units right and 3 units down.



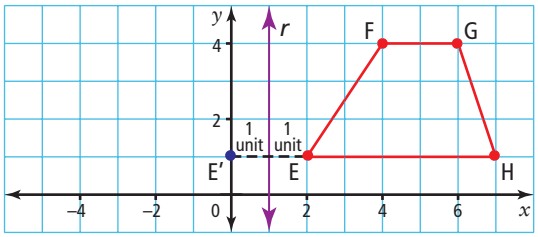
Example 2: Draw a Reflection

Reflect the figure in line of reflection r.



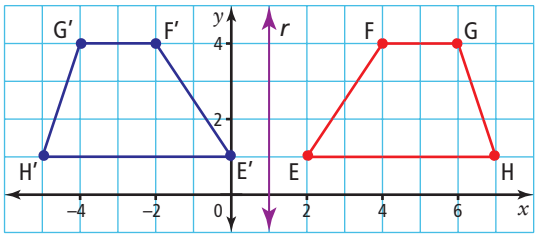
Solution

Count how many spaces each point is from line of reflection r. Point E is 1 unit away from the line of reflection. Point E′ will be 1 unit away on the other side of the line of reflection. Plot point E′.



Distance should be measured perpendicular (90°) to the line of reflection.

Plot points F′, G′, and H′. Connect the points to form E′F′G′H′.

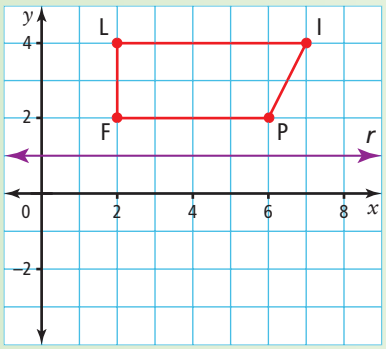


Did You Know?

When an object is reflected in a mirror, the line of reflection is the mirror.

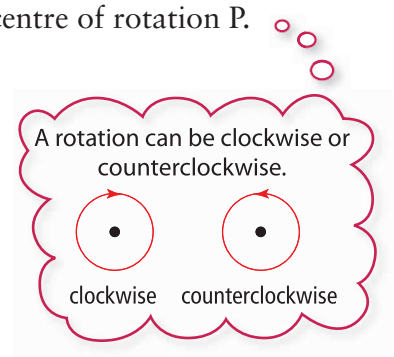
Show You Know

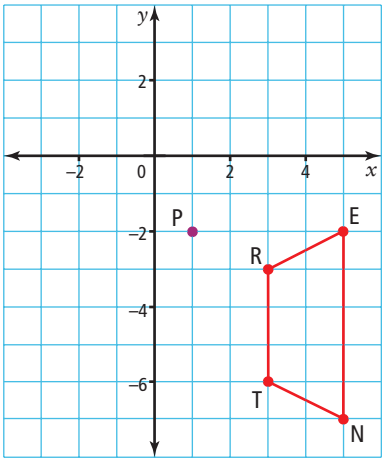
Copy this figure on a coordinate grid. Reflect the figure in line of reflection r.



Example 3: Draw a Rotation

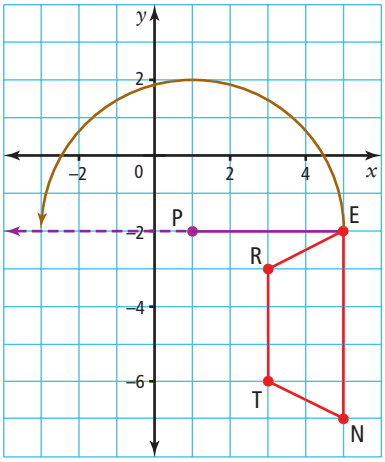
Rotate the trapezoid 180° counterclockwise about centre of rotation P.



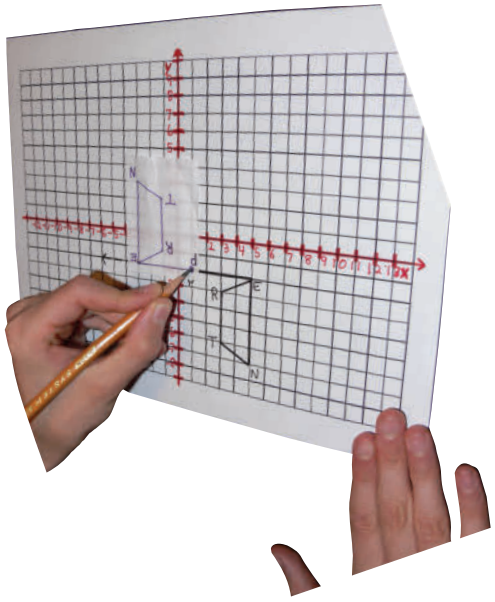


Solution

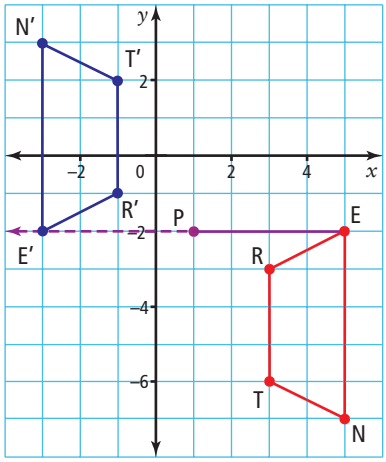
Draw a line connecting E to centre of rotation P. Draw a broken line from P at a 180° angle of rotation counterclockwise. Copy the trapezoid and point P onto tracing paper.



Place the tracing over the original figure. Place your pencil tip on point P. Turn the tracing 180° counterclockwise until E is on the broken line. Mark the points R′, E′, N′, and T′.

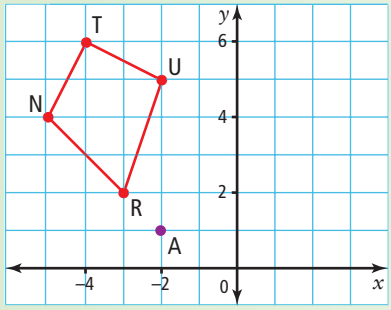


Remove the tracing paper and connect the points. Label trapezoid R′E′N′T′.



Show You Know

Copy this figure on a coordinate grid. Rotate the figure 270° counterclockwise about centre of rotation A.

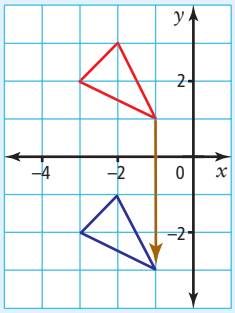


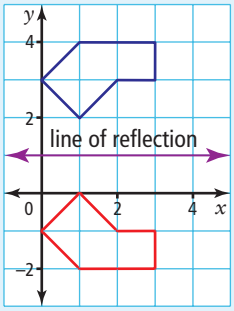
Did You Know?

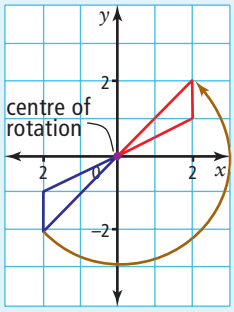
The first Ferris wheel was introduced at the World's Fair in Chicago in 1893. A moving Ferris wheel is an example of a rotation. The centre of the wheel is the centre of rotation.

Key Ideas

- Transformations include translations, reflections, and rotations.
- A translation is a slide along a straight line.
- A reflection is a mirror image in a line of reflection.
- A point and its reflection are the same distance from the line of reflection.
- A rotation is a turn about a centre of rotation.
- The rotation can be clockwise or counterclockwise.





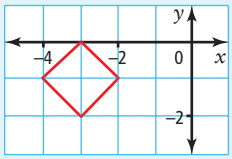


Communicate the Ideas

1. Give an example of each of the following in real life. Share your ideas with a friend.

a) translation **b)** reflection **c)** rotation

2. Think of a translation, a reflection, and a rotation for this square. Explain how you would make each of these transformations.

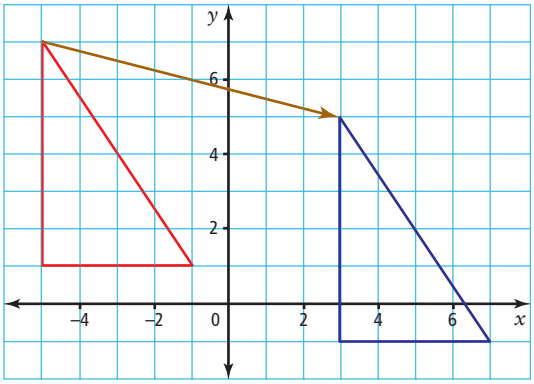


Practise

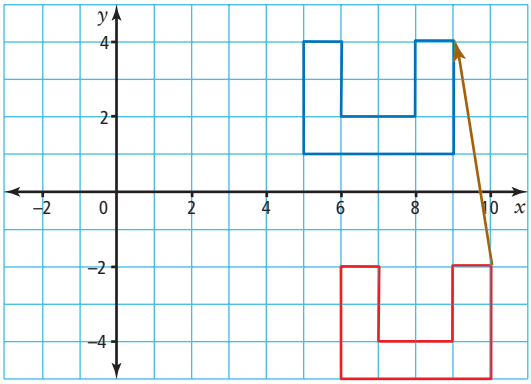
For help with #3 to #6, refer to Example 1 on page 20.

3. What is the translation shown in each diagram?

a)

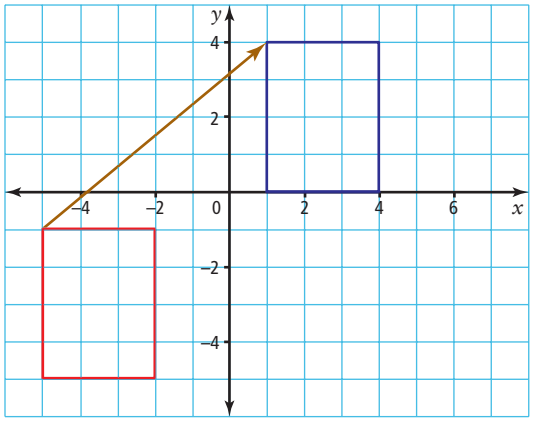


b)

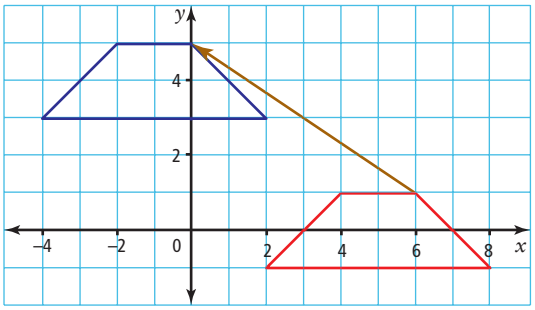


4. Identify the translation in each diagram.

a)



b)

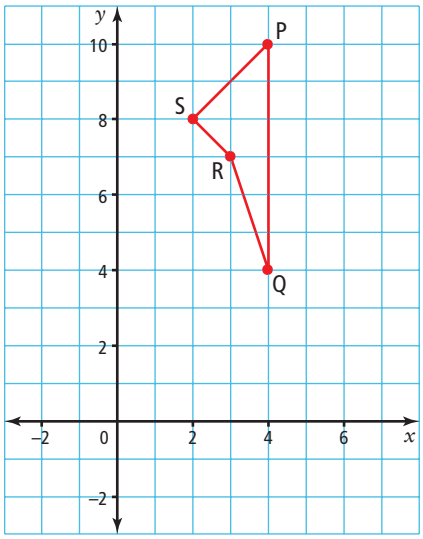


5. Copy figure PQRS onto a coordinate grid.

a) Translate figure PQRS 3 units right and 6 units down.

b) What are the coordinates of the translation image?

c) Draw the translation arrow.

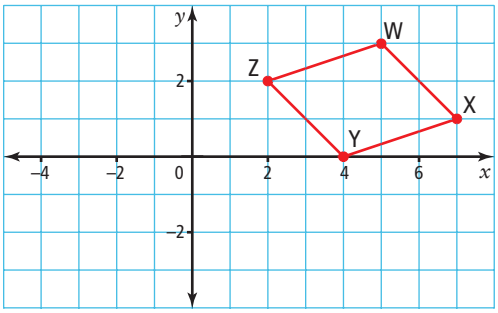


6. Copy parallelogram WXYZ onto a coordinate grid.

a) Translate WXYZ 6 units left and 3 units down.

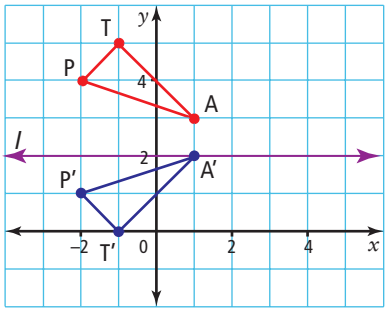
b) Identify the coordinates of the translation image.

c) Draw the translation arrow.

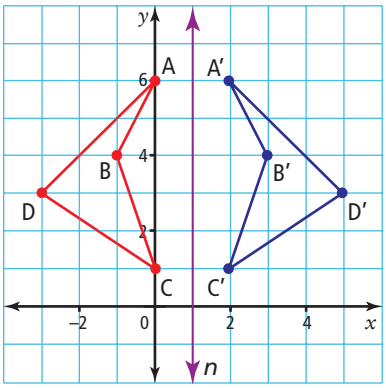


For help with #7 to #12, refer to Example 2 on page 21.

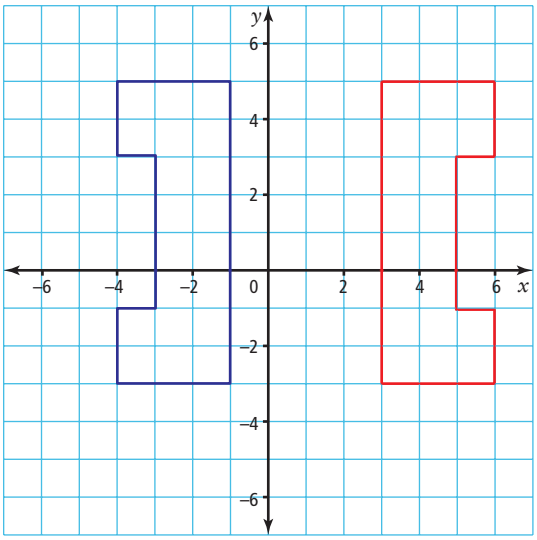
7. Is $\triangle T'A'P'$ a reflection image of $\triangle TAP$ in the line of reflection, l? How do you know?



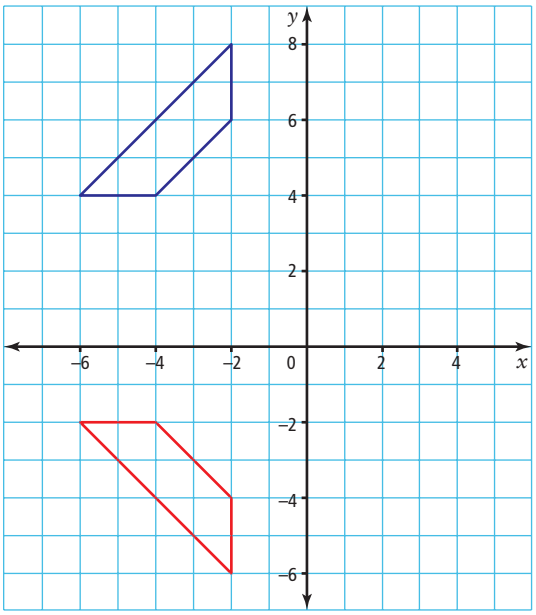
8. Is figure A′B′C′D′ a reflection image of figure ABCD in the line of reflection, n? How do you know?



9. Copy the figure and its reflection image on a coordinate grid. Describe the line of reflection. Show it on your diagram.



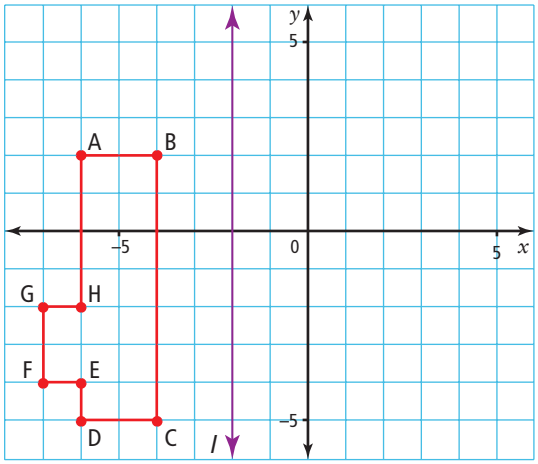
10. Copy the trapezoid and its reflection image on a coordinate grid. Describe the line of reflection. Show it on your grid.



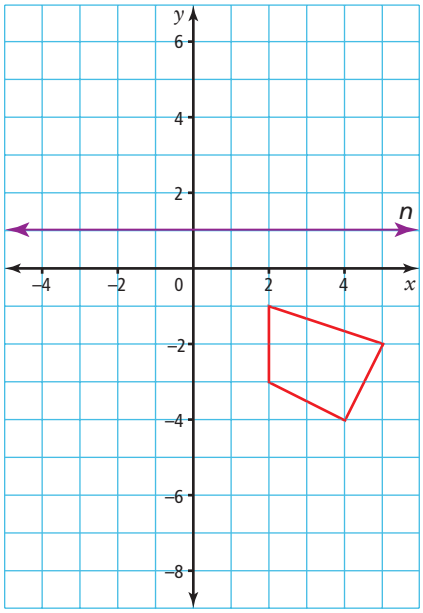
11. Copy the figure and line of reflection l on a coordinate grid.

a) Draw the reflection image.

b) What are the coordinates of A′B′C′D′E′F′G′H′?



12. Copy the quadrilateral and line of reflection n on a coordinate grid. Draw the reflection image.

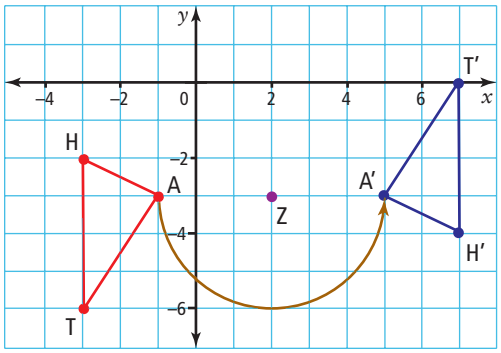


For help with #13 to #18, refer to Example 3 on pages 22–23.

13. The diagram shows △HAT, its rotation image, and centre of rotation Z.

a) What are the coordinates of △HAT and △H′A′T′?

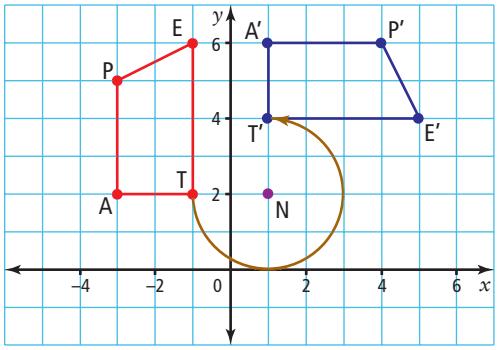
b) What are the direction and angle of rotation?



14. The diagram shows figure TAPE, its rotation image, and centre of rotation N.

a) What are the coordinates of TAPE and T′A′P′E′?

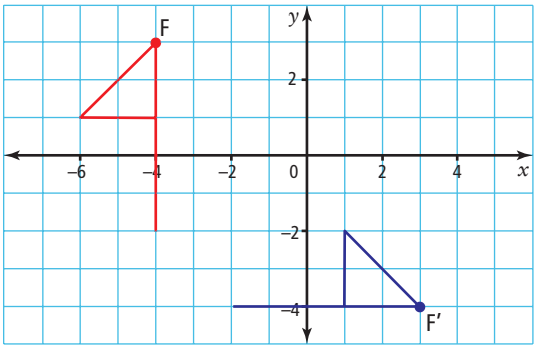
b) What are the direction and angle of rotation?



15. The diagram shows a figure and its rotation image.

a) What are the coordinates of the centre of rotation?

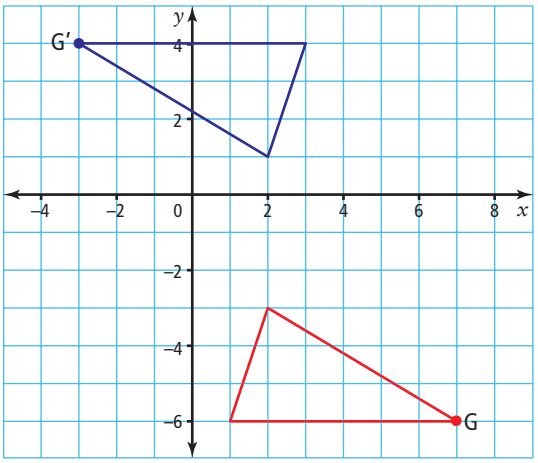
b) What are the direction and angle of rotation? Give more than one answer, if possible.



16. The diagram shows a triangle and its rotation image.

a) What are the coordinates of the centre of rotation?

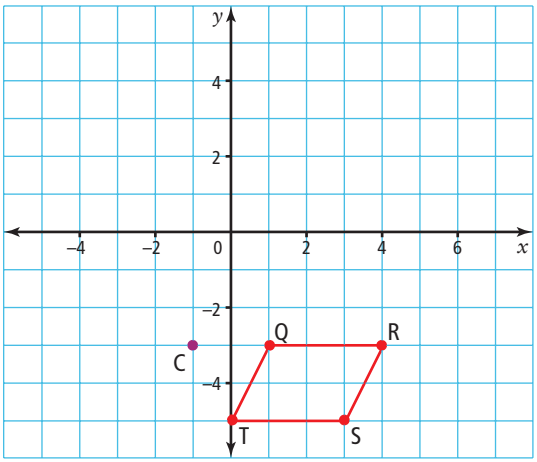
b) What are the direction and angle of rotation? Give more than one answer, if possible.



17. Copy parallelogram QRST and centre of rotation C onto a coordinate grid.

a) Rotate the parallelogram about C, 270° clockwise.

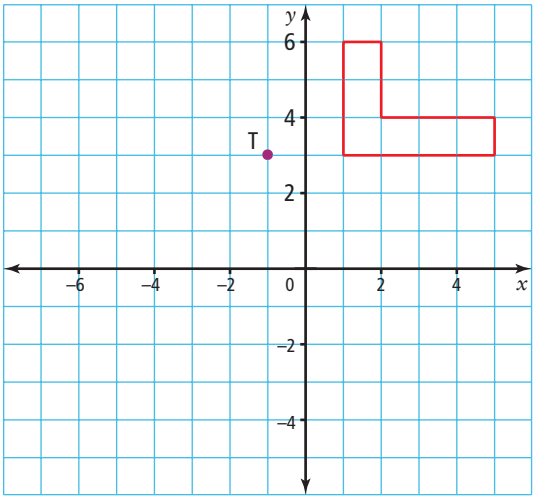
b) What are the coordinates of Q′R′S′T′?



18. Copy the figure and centre of rotation T onto a coordinate grid.

a) Rotate the figure about T, 360° counterclockwise.

b) What do you notice about the figure and its rotation image?

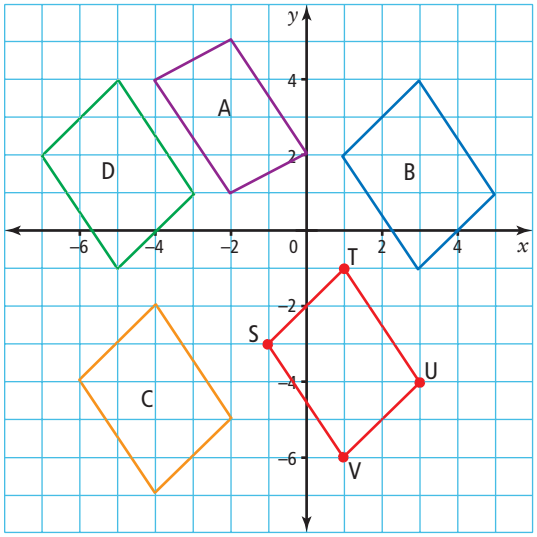


Apply

19. a) Look at figures A, B, C, and D. Which are translation images of parallelogram STUV?

b) Copy STUV and each translation image on a coordinate grid. Draw the translation arrow to each image.

c) Describe each translation in words.



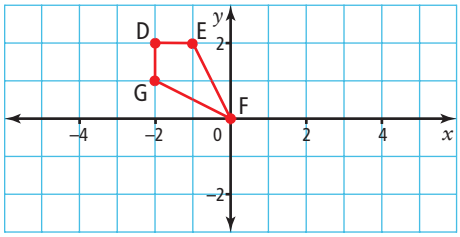
20. The plans for a new schoolyard have been drawn on a coordinate grid. The climbing equipment has been placed at coordinates M(−4, 4), N(−3, 5), R(−2, 4), P(−2, 2), and Q(−4, 2). The architect wants to move it into quadrant IV. Points M and Q will lie on the *y*-axis and point N will lie on the *x*-axis.

a) Plot MNRPQ on a coordinate grid.

b) Move MNRPQ to its new position.

c) What translation would do this?

Use parallelogram DEFG for #21 to #23.



21. Copy parallelogram DEFG onto a coordinate grid.

a) Reflect DEFG over the x-axis.

b) On the same coordinate grid, reflect these two figures over the y-axis.

c) How are the figures the same? How are they different?

22. Copy parallelogram DEFG onto a different coordinate grid.

a) Rotate DEFG 90° clockwise about the centre of rotation at (0, 0).

b) On the same coordinate grid, rotate DEFG 90° counterclockwise about (0, 0).

c) On the same coordinate grid, rotate DEFG 180° clockwise about (0, 0).

23. How are the designs created in #21 and #22 the same? How are they different?

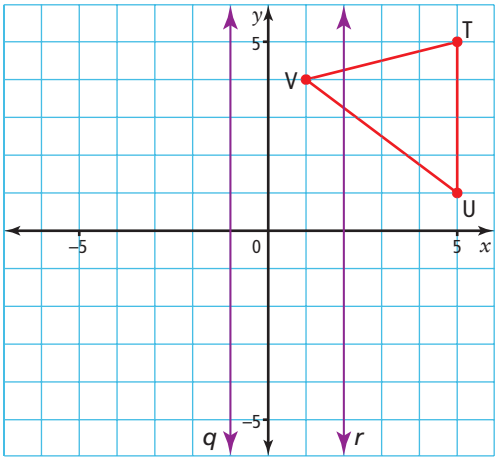
Extend

24. Copy triangle TUV onto a coordinate grid.

a) Reflect TUV in line of reflection q.

b) Reflect TUV in line of reflection r.

c) How is the reflection in a) different from the reflection in b)?



25. a) How can a reflection and a translation make images that look the same?

b) Give an example. Label all the vertices.

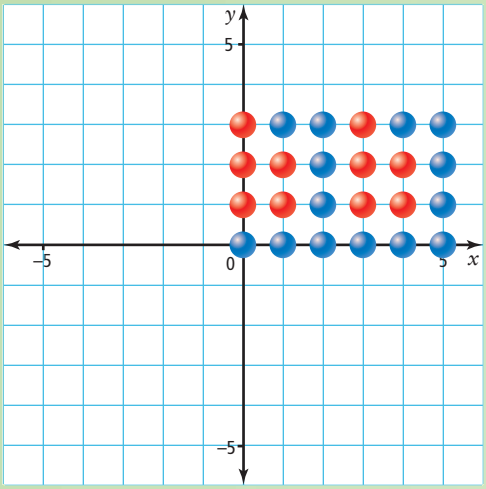
c) Are the coordinates the same for the reflection image and the translation image?

MATH LINK

a) What type of transformation(s) do you see in this bead design?

b) Reflect or rotate the entire design to make a different pattern.
- If you use a reflection, one side of the image should touch one side of the original design.
- If you use a rotation, one vertex of the image should touch one vertex of the original design.

c) Describe the transformation you used.

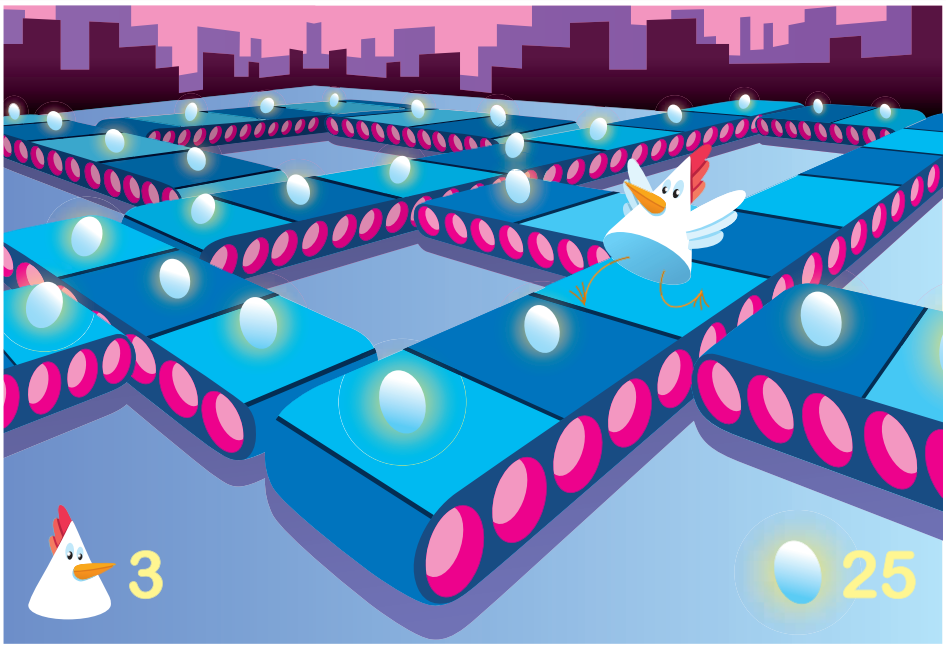


1.4 Horizontal and Vertical Distances

Focus on...

After this lesson, you will be able to...

- describe the movement of a point on a Cartesian plane, using the terms horizontal and vertical
- determine the horizontal and vertical distance between two points
- describe how vertices of a 2-D shape change position when they are transformed one or more times



In some computer and video games, you play on a grid that is very much like a coordinate grid. You can travel only in a horizontal or vertical direction.

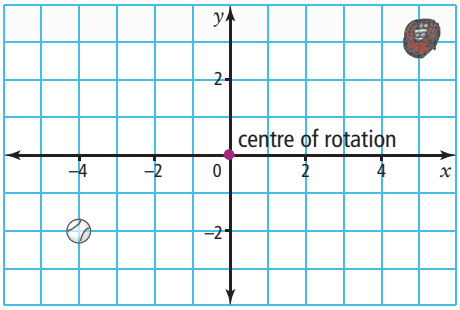
Explore the Math

Materials

- grid paper
- ruler

How do you describe horizontal and vertical movement?

1. You can describe movement by playing a game on a grid. The object of this game is to get the baseball in the glove. You can move the ball using only rotations, reflections, and translations.



a) Rotate the ball 180° clockwise around the centre of rotation. What is the ball's new position?

b) Translate the ball from the position after rotation to the glove. How would you describe this translation?

c) How many transformations did it take to put the ball in the glove?

2. Describe the total horizontal and vertical distance the ball travelled from start to finish.

Reflect on Your Findings

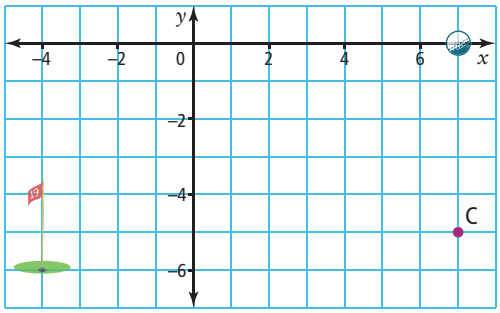
3. a) Give another example of a set of transformations that would move the baseball into the glove. Use at least two different kinds of transformations.

b) What single transformation would move the ball into the glove?

Example 1: Describe Horizontal and Vertical Movement

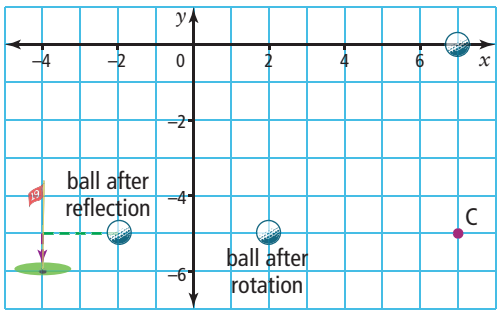
a) Use translations, reflections, and rotations to get the golf ball into the hole.

b) Describe the total horizontal and vertical distance the ball travelled from start to finish.

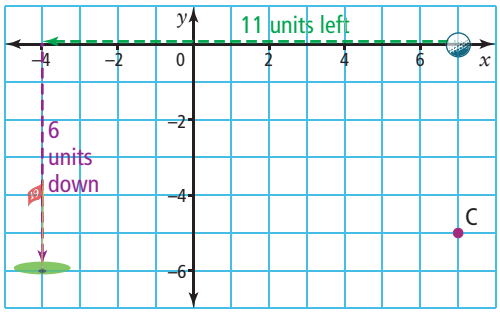


Solution

a) A 90° counterclockwise rotation about centre of rotation C will move the ball from (7, 0) to (2, −5). A reflection in the y-axis will move the ball from (2, −5) to (−2, −5). A translation of 2 units horizontally left and 1 unit vertically down will put the ball into the hole.



b) The ball travelled 11 units horizontally left and 6 units vertically down.



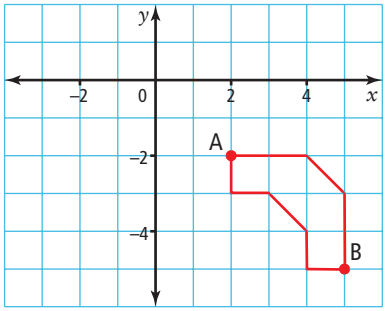
WWW Web Link

To play the golf game online, go to www.mathlinks7.ca and follow the links.

Show You Know

There is more than one solution to this game. What is another set of transformations you could use to put the golf ball into the hole? Use a rotation, a reflection, and a translation.

Example 2: Describe the Movement of the Vertices of a Shape



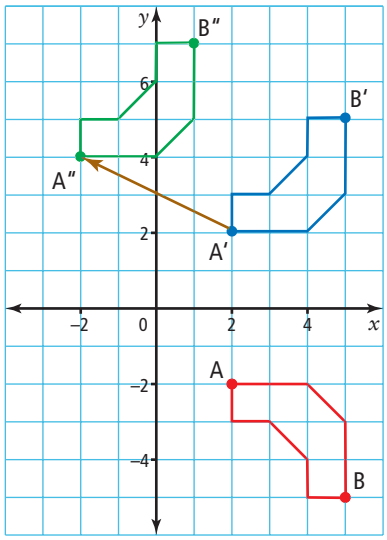
a) Reflect this shape in the x-axis. Then, translate it 4 units horizontally left and 2 units vertically up.

b) What are the coordinates of vertex A″ and vertex B″?

c) Describe the movement of vertex A to vertex A″ and vertex B to vertex B″.

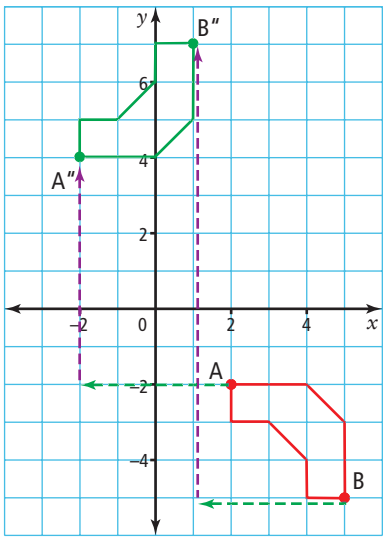
Solution

a)



b) A″(−2, 4), B″(1, 7)

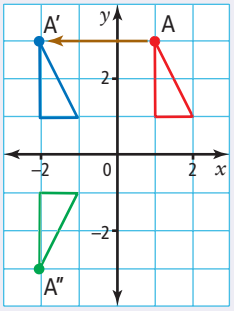
c) Vertex A moved 4 units horizontally left and 6 units vertically up. Vertex B moved 4 units horizontally left and 12 units vertically up.



Literacy Link

Reading Double Prime

A″ is read as "A double prime." It is used to label the point that matches point A after a second transformation.

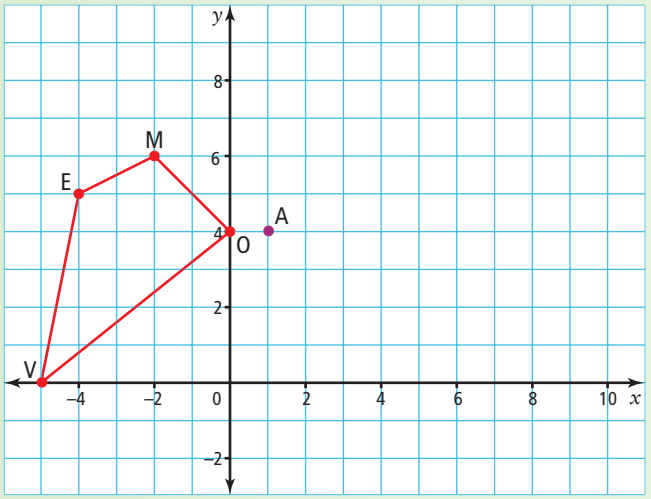


Show You Know

a) Copy the figure onto a coordinate grid. Rotate this figure 180° clockwise about centre of rotation A. Then, translate it 3 units right and 2 units down.

b) What are the coordinates of vertex V″?

c) Describe the horizontal and vertical movements of vertex V to vertex V″.



Key Ideas

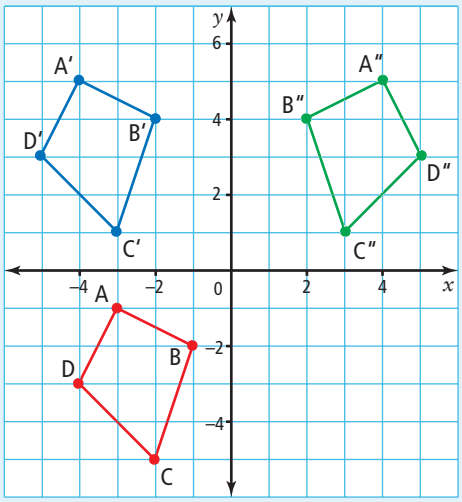
- To describe the change in position of a point, count the horizontal and vertical movements of the point.
- To describe the change in position of a shape, count the horizontal and vertical movements of its vertices.

Communicate the Ideas

1. a) What is the change in position of A to A′? A′ to A″?

b) What is the change in position of B to B′? B′ to B″?

c) Why is the change in position of A to A″ different from the change in position of B to B″?



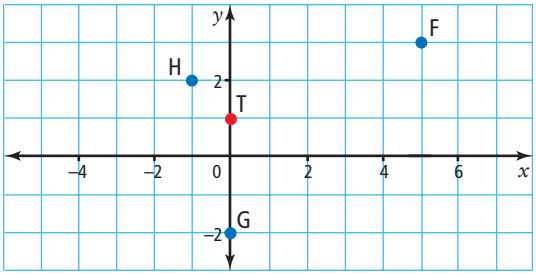
2. Point E is 3 units horizontally left from point E′. How is this different from point E being 3 units vertically down from point E′? Discuss your answer with a partner.

Practise

For help with #3 and #4, refer to Example 1 on page 31.

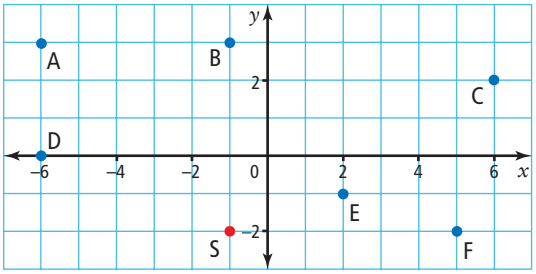
3. What are the horizontal and vertical movements of point T to each of the following points?

a) F **b)** G **c)** H



4. Describe the horizontal and vertical movements of point S to each of the following points.

a) A **b)** B **c)** C
d) D **e)** E **f)** F

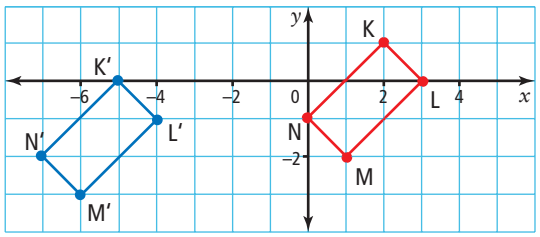


For help with #5 and #6, refer to Example 2 on page 32.

5. Rectangle KLMN has been translated.

a) What are the coordinates of K′L′M′N′?

b) What are the horizontal and vertical movements of KLMN to K′L′M′N′?



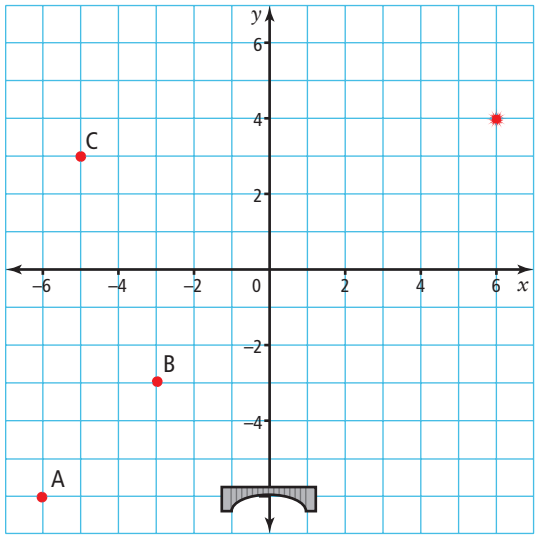
6. Draw a quadrilateral with vertices at A(3, 8), S(4, 9), R(8, 7), and T(5, 6). Rotate ASRT 90° counterclockwise around the centre of rotation at (3, 4). Reflect A′S′R′T′ over the x-axis.

a) What are the coordinates of A″S″R″T″?

b) What are the horizontal and vertical movements of ASRT to A″S″R″T″?

Apply

7. You are a dispatcher at the local police station. You must send a car to a traffic accident scene at (6, 4). The y-axis represents a river. The police cars are at points A, B, and C. Cars can travel only along grid lines and must go over the bridge to cross the river.



a) Which car is closest to the accident?

b) Which car would you send to the accident? Why?

c) Write directions for the car you chose to get to the accident. Use the words horizontal and vertical.

8. Marissa and Nigel are racing their cars in the annual model car rally.

The table shows the results of one race as ordered pairs.

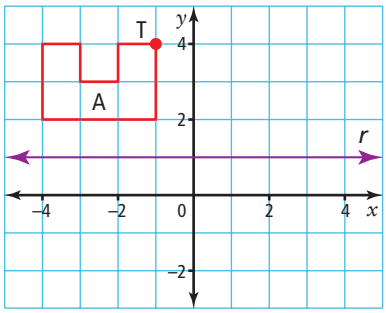
Competitor	Start	Finish
Marissa	(3, 0)	(−6, 0)
Nigel	(3, 1)	(−7, 1)

a) On a coordinate grid, plot the start and finish points of each person's car.

b) Describe the distance each car travelled.

c) Who won? Explain.

9. Copy figure A onto a coordinate grid. Reflect figure A in the y-axis to form figure A′. Now, reflect figure A′ in line of reflection r to make figure A″.



a) Describe the horizontal and vertical change in position of T to T″.

b) Is it possible to get from A to A″ in one transformation? If so, describe the transformation. If not, why not?

10. a) Create a simple cartoon character in one quadrant of a coordinate grid. Use only straight lines.

b) Move the character to the other quadrants using a different transformation each time. Use all three types of transformations.

c) Create a comic strip, using each quadrant as a frame in your comic strip.

Extend

11. There are three types of transformations studied in this chapter: translation, reflection, and rotation. If you had to choose only two to work with, which two would you choose? Explain why.

12. Which point is closest to point X(7, 6): A(−1, 2), B(−4, 6), or C(6, −2)?

13. Point A is rotated 90° clockwise about (0, 0). Then, it is reflected in the x-axis. Its new location is (4, 6).

a) What are the coordinates of point A before the rotation and reflection?

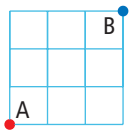
b) Describe the change in position from point A to point A′.

c) What is one translation that would have the same result as the rotation and reflection?

14. You are allowed to move only right or up.

a) What is the shortest distance from A to B?

b) What is the total number of different paths from A to B?



1 Chapter Review

Key Words

For #1 to #9, match each description on the left with one of the terms on the right.

1. A pair of numbers in the form of (x, y)

2. Another name for a Cartesian plane

3. A translation, a reflection, or a rotation

4. A slide along a straight line

5. Looking in a mirror

6. Doing a "360" on a skateboard

A coordinate grid

B ordered pair

C origin

D vertex

E x-axis

F y-axis

G transformation

H translation

I reflection

J rotation

7. The horizontal axis of a coordinate grid

8. The vertical axis of a coordinate grid

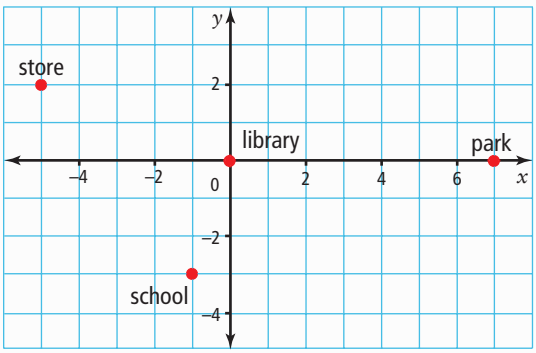
9. The name for point (0, 0)

1.1 The Cartesian Plane, pages 4–11

10. What are the ordered pairs for each location?

a) library **b)** school

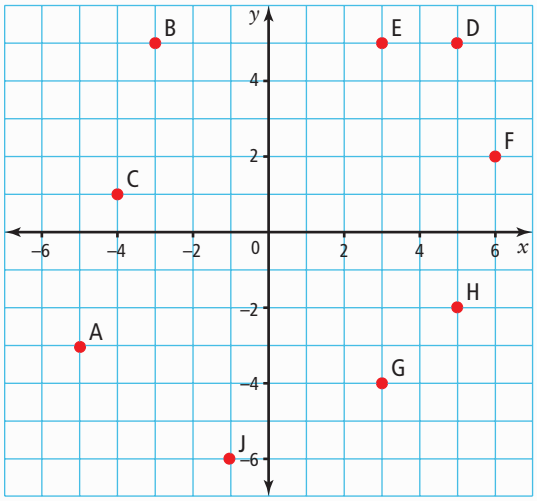
c) park **d)** store



11. What are the ordered pairs in

a) quadrant I? **b)** quadrant II?

c) quadrant III? **d)** quadrant IV?



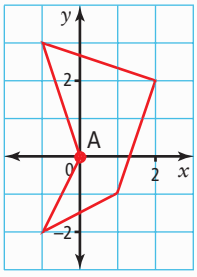
12. Draw and label the axes of a coordinate grid by 2s. Plot the following points: A(−5, 2), B(−4, 0), C(−2, −1), D(0, −3), E(1, −4), F(3, −6).

a) Which point seems out of place?

b) What do the coordinates of the other five points have in common?

1.2 Create Designs, pages 12–17

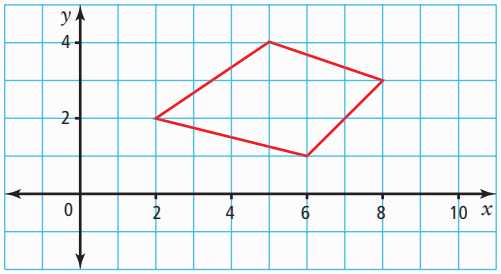
13. Label the vertices of the design. Start at A and continue in a clockwise direction. What are the ordered pairs?



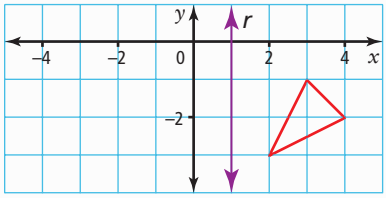
14. Create the letter H on a coordinate grid. Start at (2, −2). The letter must be 5 units high and 4 units wide. The points must lie in all four quadrants.

1.3 Transformations, pages 18–29

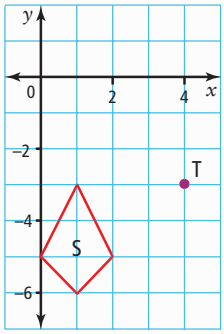
15. Copy the figure on a coordinate grid. Translate the figure 3 units left and 2 units down. What are the coordinates of the translation image?



16. Copy the figure on a coordinate grid. Reflect this image in line of reflection *r*.



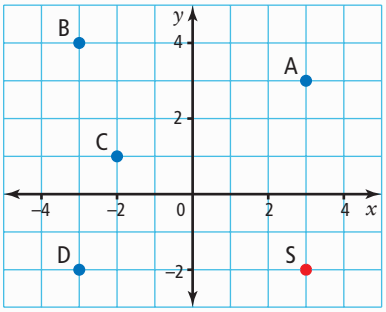
17. Copy figure S onto a coordinate grid. Skydivers form this figure in the air. The divers rotate the figure 90° counterclockwise about centre of rotation T. What are the coordinates of the vertices of the rotation image?



1.4 Horizontal and Vertical Distances, pages 30–35

18. What are the horizontal and vertical movements of point S to each of the following points?

a) A **b)** B **c)** C **d)** D

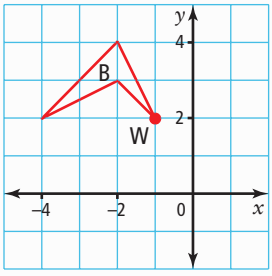


19. Draw the square S(−5, −2), T(−3, −2), E(−3, −4), P(−5, −4). Translate the square 8 units to the right and 3 units down. Draw S′T′E′P′. What is the horizontal and vertical change in position

a) from S to S′? **b)** from T to T′?

20. Draw triangle T(−2, 5), R(−2, 3), I(−4, 3). Rotate △TRI 180° counterclockwise about a centre of rotation at (−2, −1). Reflect △T′R′I′ in the *x*-axis to make △T″R″I″. What is the horizontal and vertical change in position from T to T″? from R to R″? from I to I″?

21. Reflect figure B over the *x*-axis to make figure B′. Now reflect figure B′ over the *y*-axis to make figure B″.



a) Describe the horizontal and vertical change in position of W to W″.

b) Is it possible to get from B to B″ in one transformation? If so, describe the transformation. If not, why not?

1 Practice Test

For #1 to #5, select the best answer.

1. What are the signs of the coordinates in quadrant I?

A (−, −) **B** (−, +)
C (+, +) **D** (+, −)

2. Which statement describes the point (0, 3)?

A It is in quadrant I.
B It is in quadrant II.
C It lies along the x-axis.
D It lies along the y-axis.

3. △XYZ is reflected in a line of reflection. A line connecting X to X′ will be _____ to the line of reflection.

A vertical
B perpendicular
C horizontal
D parallel

4. A fan turning is an example of a transformation. What type?

A reflection
B rotation
C translation
D slide

5. The following points are plotted on a coordinate grid: (4, −2), (4, 0), (4, 3), (4, 5), (4, 6). The points form a line that goes through the _____.

A origin
B centre of rotation
C y-axis
D x-axis

Short Answer

6. Maata is drawing a design to decorate her Inuit boots, called kamiks. She begins by plotting a line on a coordinate grid: (−6, −2), (−3, 1), (0, 4), (2, 8), (6, 10). She has made an error. Which point seems out of place?

7. **a)** Draw the following triangles on a coordinate grid:
△A: (−3, −4), (−3, −8), (−5, −8)
△B: (3, −2), (5, −2), (5, 2)
△C: (3, −4), (3, −8), (5, −8)

b) What transformation would move △A to △B?

c) What transformation would move △A to △C?

8. A square is 6 units in length. The square lies in all four quadrants and one vertex is at (−4, 4). What are the coordinates of the other three vertices?

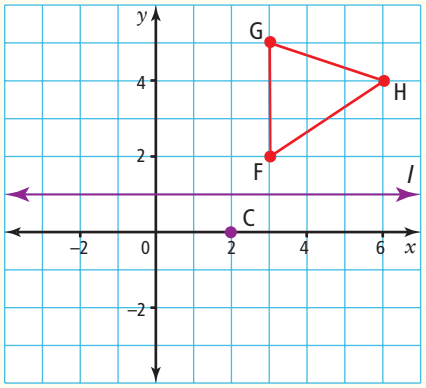
9. A(4, −2) goes through the following transformations. What are the coordinates of A′ after each transformation?

a) a reflection in the x-axis

b) a reflection in the y-axis

c) a translation of 4 units left and 9 units up

10. Copy △FGH onto a coordinate grid. △FGH is reflected in line of reflection l to make △F′G′H′. It is then rotated 90° clockwise about centre of rotation C to make △F″G″H″.



a) Draw △F′G′H′. What are the coordinates of the vertices?

b) Draw △F″G″H″. What are the coordinates of the vertices?

c) Describe the horizontal and vertical distance from vertex F to F″.

d) Describe the horizontal and vertical distance from vertex H to H″.

11. Point B(−2, 1) goes through the following transformations. What are the coordinates of B′ after each transformation?

a) a 90° clockwise rotation about (0, 0)

b) a 90° counterclockwise rotation about (3, −1)

c) a 270° clockwise rotation about (−3, 4)

d) a 180° counterclockwise rotation about (0, 0)

12. Give an example of a translation, a reflection, and a rotation in the real world.

13. Point A is at (0, 2). Point A′ is at (1, −4).

a) What are the horizontal and vertical movements from A to A′?

b) Describe one or more transformations that would move A to A′.

Extended Response

14. You reflect a design and then you reflect it again. You see that this is the same as translating the design once. Explain, and give an example.

WRAP IT UP!

On a coordinate grid, create a bead design.
Follow these guidelines:

- The design lies in one quadrant of a coordinate grid.
- The edges of the design lie along both axes.
- It includes at least one transformation.
- It has no more than 30 beads.

Then, follow these steps:

- Reflect the design over one of the axes.
- Now reflect the two designs over the other axis.
- Write a description of your design that explains the transformations you used.
- If possible, re-create your design using real beads.

See the Math Links in sections 1.2 and 1.3 for examples of bead designs.

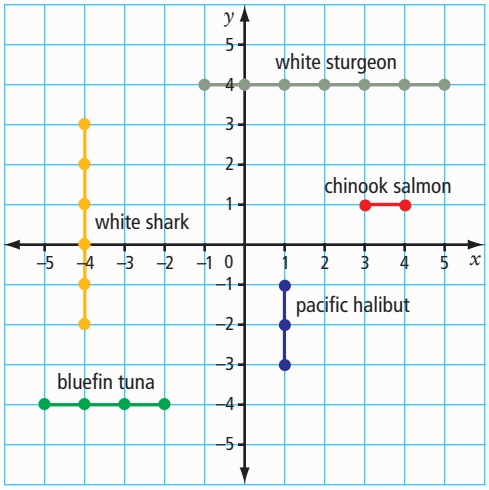
Math Games

Going Fishing

Materials

- 2 Going Fishing game boards
- coin (one per pair of students)

1. The diagram shows one way of representing the lengths of five different fish on the game board. How many units long is each fish?

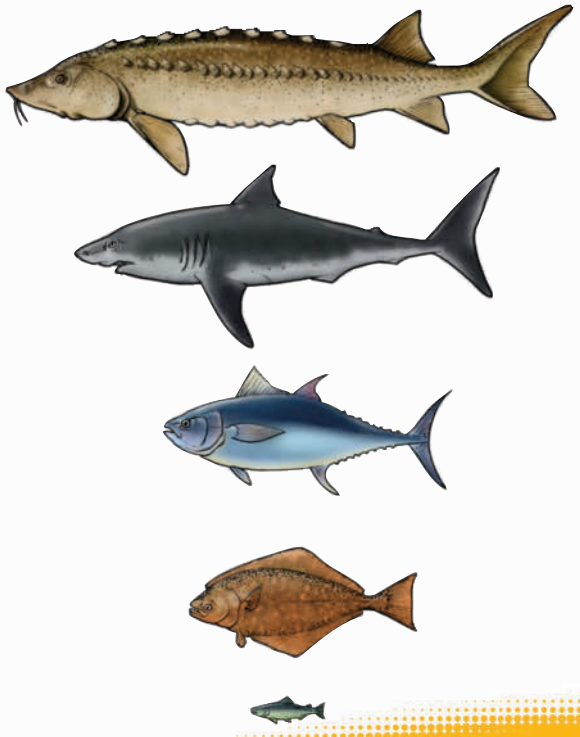


2. Draw the five fish on one copy of the game board using the following rules. Keep the locations of the fish secret.

- Make the length of each fish the same as shown in the diagram.
- Draw each fish horizontally or vertically on a grid line.
- Draw the ends of each fish where grid lines cross.
- Fish should not touch or overlap.
- Mark a point wherever grid lines cross on a fish.

3. Play the game with a partner using the following rules. The aim is to catch all of your partner's fish by finding all the points marked on them. The winner is the first player to catch all of the other player's fish.

- Flip a coin to decide who will start.
- The first player states the ordered pair for a point on the partner's game board. This player should also keep a record of this point on another copy of the board.
- The partner states whether or not the point locates a fish on his or her game board. Record the point as a hit or miss.
- If you locate a fish on your partner's board, take another turn.
- If you do not locate a fish on your partner's board, let your partner take a turn.



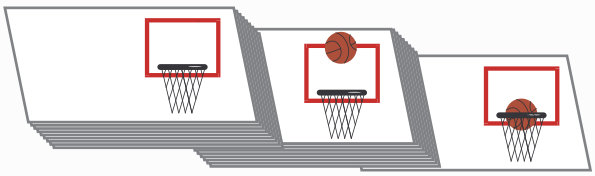
Challenge in Real Life

Make an Animation

You be the animator!
Create an animation flip pad that shows the following transformations, in any order, of an image moving over a coordinate plane. You may choose to create an animation that shows all of the transformations or create separate animations for each transformation.

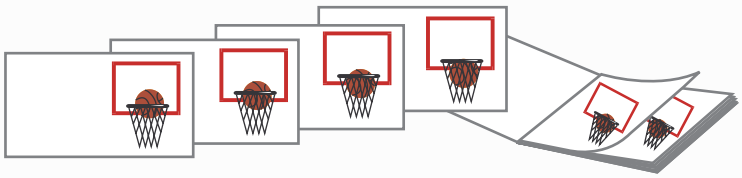
- translation
- reflection
- rotation

The idea of your animation is to show what the motion of transformations could look like in animated form.

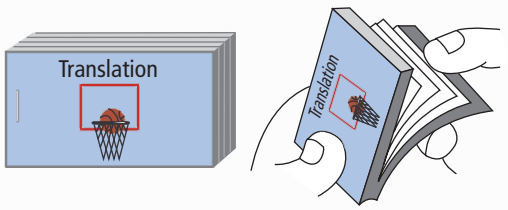


a) Draw each step in your animation on a separate piece of paper.

The more sheets of paper you use with smaller changes in movement, the more effective your animation will be.



Make a title page. Staple all the pages of your animation together in order.



b) What transformations did you use to create your animation? Explain how you used them.