NL Math 10: Chapter 5

February 24, 2011, 16:30

5 All About Angles

In 1980, Newfoundland artist Christopher Pratt designed the flag of Newfoundland and Labrador. Blue represents the sea, white represents snow and ice, red represents the struggles of the people, and gold represents the confidence that the people have in the future.

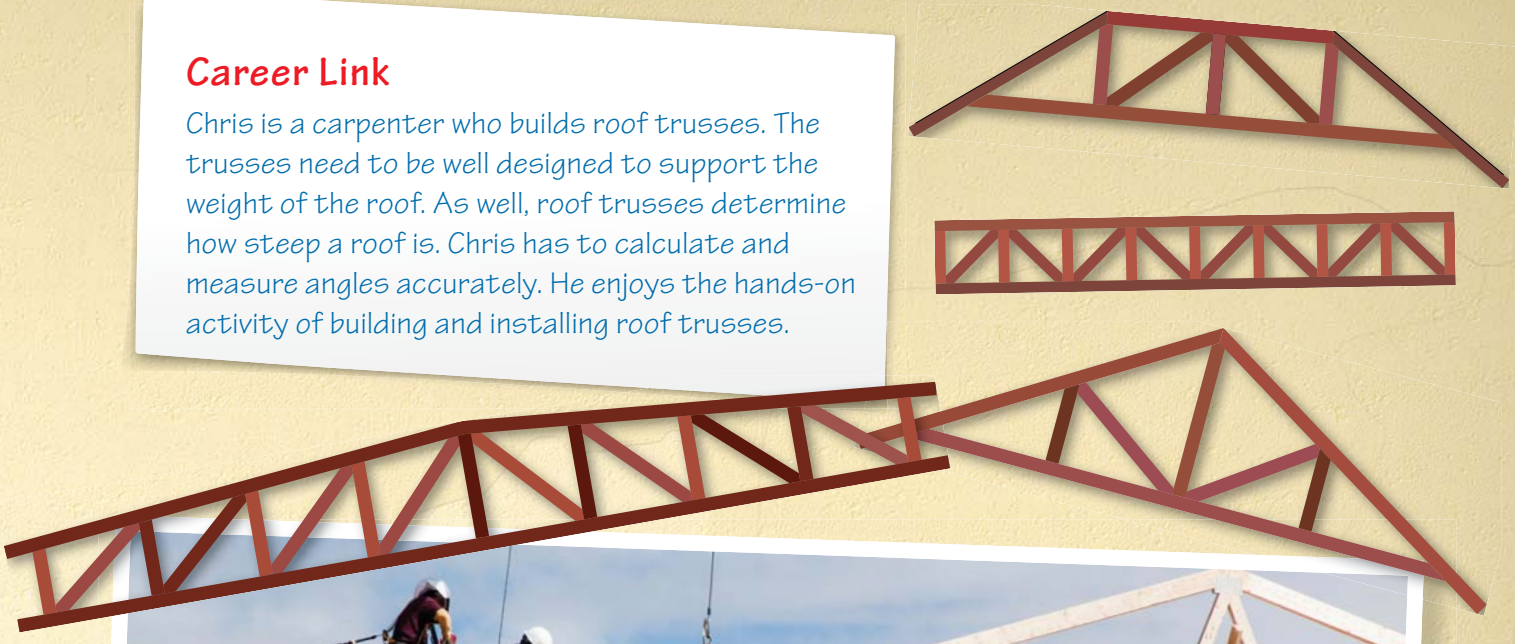
1. What kinds of angles do you notice in the design?
2. Estimate the sizes of the angles in the flag.
3. Explain why understanding how to use angles is important to an artist or designer.
4. What other jobs and activities require an understanding of angles?

Key Words

angle
obtuse angle
acute angle
straight angle
reflex angle
bisect
angle bisector
parallel lines
transversal
perpendicular lines
opposite angles
supplementary angles
complementary angles
corresponding angles
same side exterior angles
same side interior angles
alternate exterior angles
alternate interior angles

Career Link

Chris is a carpenter who builds roof trusses. The trusses need to be well designed to support the weight of the roof. As well, roof trusses determine how steep a roof is. Chris has to calculate and measure angles accurately. He enjoys the hands-on activity of building and installing roof trusses.



5 Get Ready

Evaluate Expressions

1. Calculate without using a calculator.

a) $25 + 65$

b) $32 + 58$

c) $120 + 60$

d) $30 + 60 + 90$

e) $45 + 45 + 90$

2. Calculate.

a) $90 - 45$

b) $90 - 70$

c) $90 - 35$

d) $180 - 60$

e) $180 - 125$

f) $360 - 90$

g) $360 - 180$

3. Determine each unknown value.

a) $\square + 60 = 90$

b) $45 + \square = 90$

c) $\square + 70 = 90$

d) $120 + \square = 180$

e) $130 + \square = 180$

f) $\square + 75 = 180$

g) $180 + \square = 360$

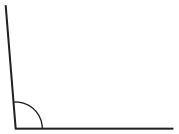
h) $210 + \square = 360$

Estimate Angles

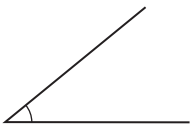
4. Match each angle with one of the following descriptions.

- a bit less than 45°
- a bit less than 90°
- a bit more than 90°
- a bit less than 180°
- a bit more than 180°
- a bit less than 360°

a)

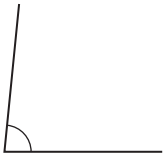


b)



c)

d)



e)

f)

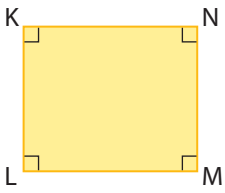
5. Estimate the angle in each sign language letter.

a)

b)

Measure Angles

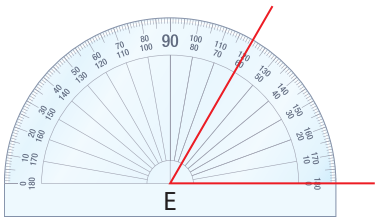
6. Name each right angle in rectangle KLMN.



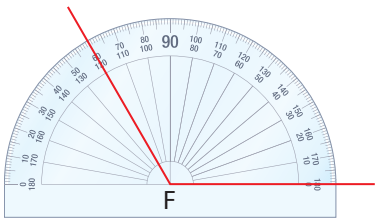
A right angle measures 90°. The symbol for a right angle is a square.

7. Read each angle shown on the protractor.

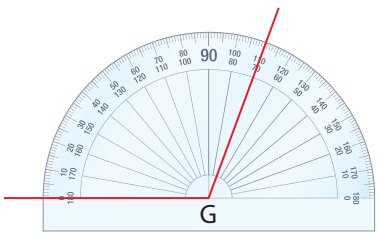
a)



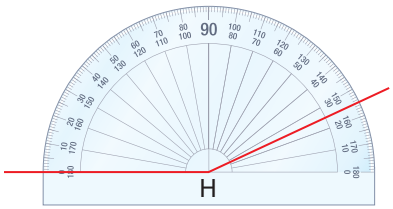
b)



c)

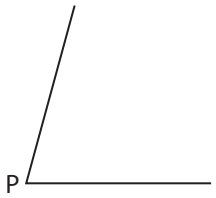


d)

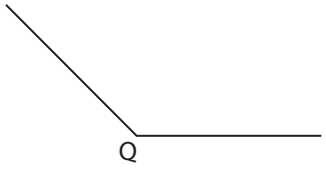


8. Use a protractor to measure each angle.

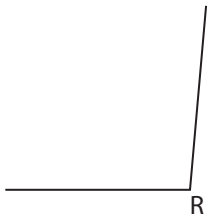
a)



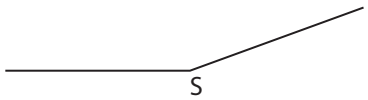
b)



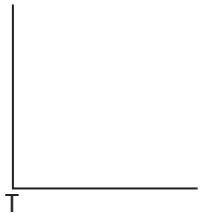
c)



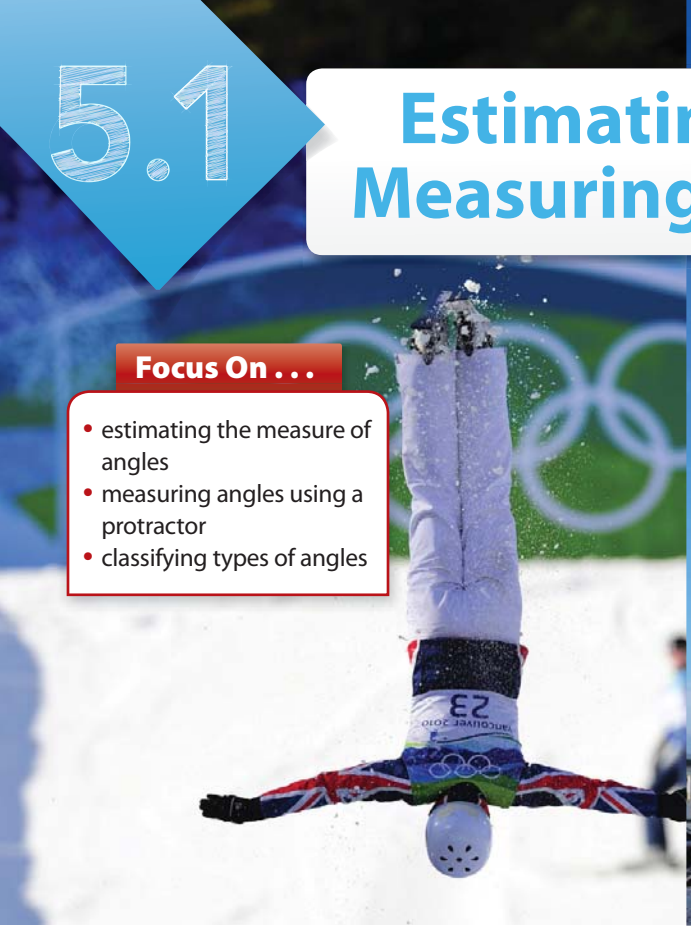
d)



e)



5.1 Estimating and Measuring Angles



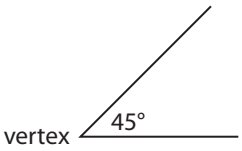
Focus On . . .

- estimating the measure of angles
- measuring angles using a protractor
- classifying types of angles

Athletes need to control the turns they make in the air. In order to land safely, they need to be able to estimate angles.

angle

- formed by two line segments that start from the same point, which is called a vertex
- measured in degrees
- the symbol for degrees is °

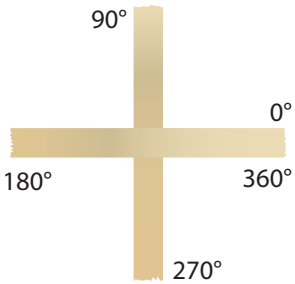


Materials

- masking tape
- Explore Angles worksheet

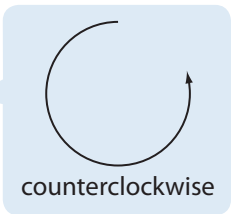
Explore Angles

1. a) Tape two pieces of masking tape on the floor as shown. Write the labels on strips of paper. Tape the labels on the floor.



b) Stand where the two lines cross each other, and face 0°. Slowly turn counterclockwise until you are facing 0° again. You have completed a "360."

Turning a full circle makes an angle of 360°.



2. Standing on the taped lines, make each of the following fractions of a turn. Record each angle measure in a table similar to the one shown. Then, justify each measure. The first one is done for you.

	Fraction of a Turn	Measure of Angle	Justify
a)	$\frac{1}{2}$	0° to 180° = 180°	$\frac{1}{2}$ of $360 = \frac{360}{2} = 180$
b)	$\frac{1}{4}$		
c)	$\frac{3}{4}$		
d)	$\frac{1}{8}$		
e)	$\frac{1}{6}$		
f)	$\frac{1}{3}$		

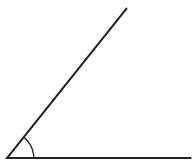
3. a) A right angle is shaped like a square corner. Think of a personal reference for 90°. Sketch your personal reference.

b) You can compare other angles to a 90° angle. Decide how to use your personal reference for 90° to estimate the measure of each of the following angles: 45°, 30°, 60°, and 180°. Sketch each angle measure using your personal reference.

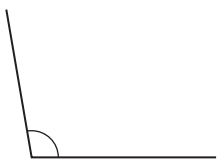
c) What other personal reference can you use for each angle in part b)?

4. Reflect How can you use your references from #3 to estimate the size of the following angles?

a)

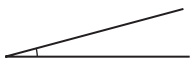


b)



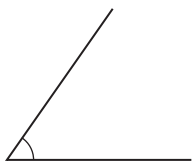
5. Extend Your Understanding Use your personal references to estimate the measure of the following angles.

a)

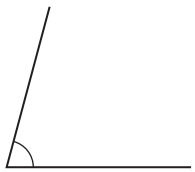


b)

c)



d)



F.Y.I.

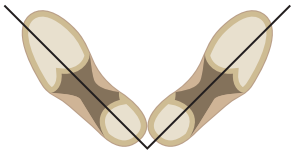
When they turn, athletes focus on a fixed point. This helps keep them balanced.

On the Job 1

Classify Angles

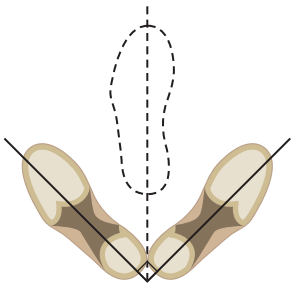
Kyla loves Scottish Highland dancing and takes lessons weekly. To help improve her footwork, Kyla estimates and classifies the following foot angles. What personal references could she use to check each foot angle?

a) For country dancing, Kyla starts in this position.



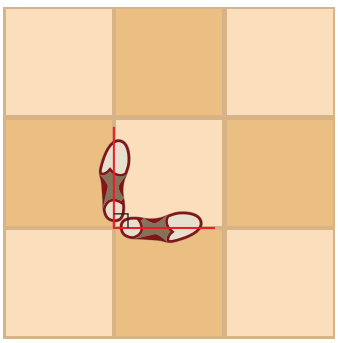
b) At other times, she starts with her feet like this.

c) Kyla often steps out at this angle.



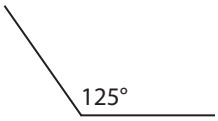
Solution

a) Kyla uses a personal reference to check this angle. The tiles on the floor where she practises are square. She places her feet along each side of a tile to check her angle. They form a right angle. This is 90°.



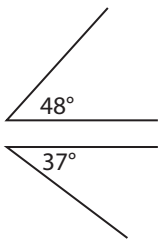
obtuse angle

- measures between 90° and 180°

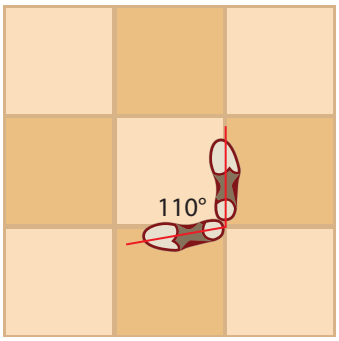


acute angle

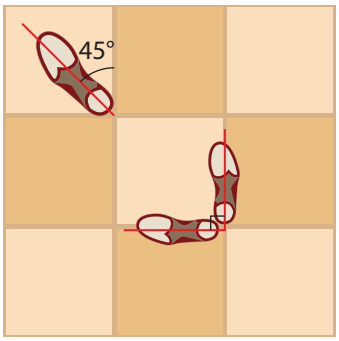
- measures less than 90°



b) Kyla compares this angle to the corner of the tile. It is larger than a right angle, but not a lot larger. Kyla thinks it is about 110°. Her dance teacher may call it an **obtuse angle**.



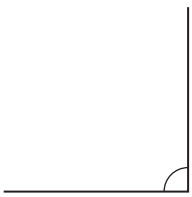
c) Kyla imagines cutting a right angle in half. She tries to step out at that angle. Half of 90 is 45. She tries to make a 45° angle with her step. Her dance teacher may call it an **acute angle**.



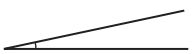
Your Turn

Use a reference to help you estimate the size of each angle. Then, classify each angle as a right angle, an obtuse angle, or an acute angle.

a)



b)



c)

Check Your Understanding

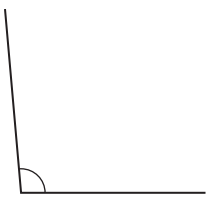
Try It

For #1 to #6, match the angle in column A with the correct fraction of a turn in column B.

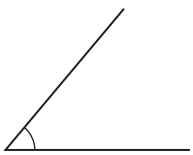
Column A	Column B
1. 45°	**a)** $\frac{1}{2}$ turn
2. 60°	**b)** $\frac{1}{3}$ turn
3. 90°	**c)** $\frac{1}{4}$ turn
4. 120°	**d)** $\frac{1}{5}$ turn
5. 180°	**e)** $\frac{1}{6}$ turn
6. 270°	**f)** $\frac{1}{8}$ turn
	g) $\frac{3}{4}$ turn

7. Classify each angle as a right angle, an acute angle, or an obtuse angle.

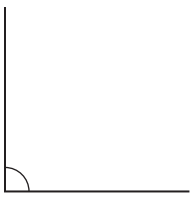
a)



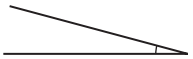
b)



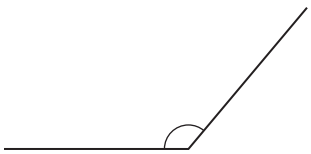
c)



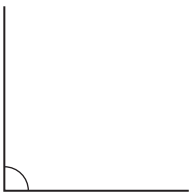
d)



e)



f)



8. Describe each type of angle. Then, sketch an example of each. Estimate the measure of each angle you drew.

a) acute angle **b)** obtuse angle **c)** right angle

Tools of the Trade

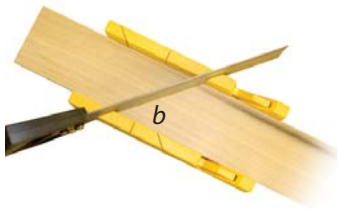
Carpenters use a mitre box to cut wood at different angles. To learn more about how to use a mitre box, go to www.mhrmathatwork10.ca and follow the links.

Apply It

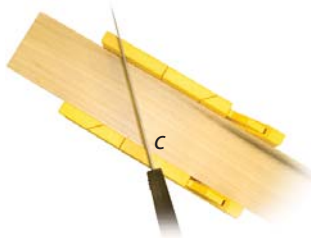
9. Stephen's mitre box can be used to cut wood at three different angles. Use a personal reference to estimate each angle measure. Then, classify each angle.

a)

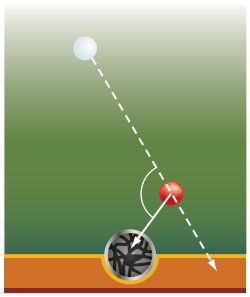
b)



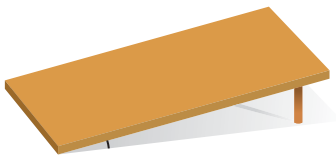
c)



10. Helen hopes to make the shot shown in a game of pool. Assume that the white cue ball moves as planned. Estimate the angle between the cue ball and the path of the red target ball into the pocket.



11. Miguel built a bike ramp.



a) Estimate the angle between the ground and the ramp.

b) Estimate the angle that the support leg makes with the ground.

c) Classify each angle as an acute angle, an obtuse angle, or a right angle.

On the Job 2

Classify Angles 180 Degrees and Larger

Kyle and Matthew are planning a video that shows how to do bike tricks safely. To go with the video, Kyle is writing an instruction booklet. The booklet will include drawings of the starting and ending position of the rider and bike for each move. Kyle plans to start with easy tricks, and then build up to more difficult ones.

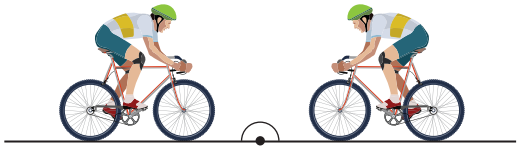
a) In a "Nollie 180," the rider and bike lift off the ground and rotate in a half circle. Kyle sketches the starting and ending positions. What size of an angle did the rider turn?

b) In a "Nollie 270," the rider and bike spin in a $\frac{3}{4}$ turn. What size of an angle did the rider turn?

Solution

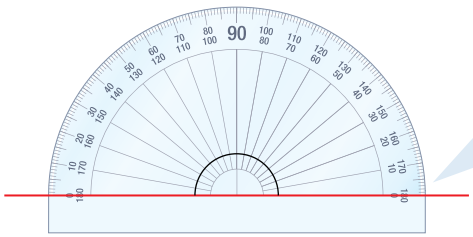
a) Kyle sketches where the bike begins and lands.

He draws a line to join the two sets of bike tires.



The bike turned 180°. The angle makes a straight line.

Kyle uses a protractor to measure 180°. This is a **straight angle**.



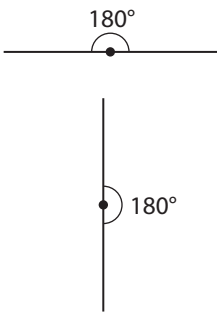
Place the centre of the protractor on top of the vertex of the angle.

The angle passes through 0 on the outside scale.

Use the outside scale to measure the angle.

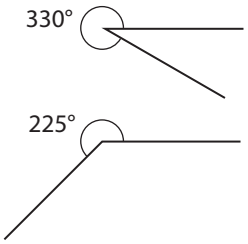
straight angle

- measures exactly 180°



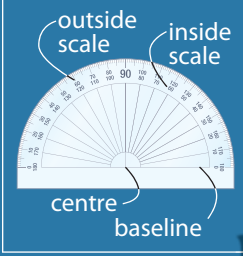
reflex angle

- measures more than 180° but less than 360°



F.Y.I.

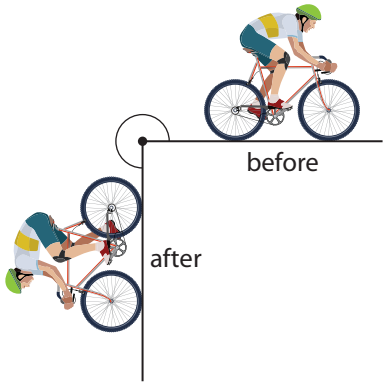
A protractor has an inside scale and an outside scale. Use the two scales to measure the number of degrees in angles. Use the inside scale to measure angles whose baseline lies on the right. Use the outside scale to measure angles whose baseline lies on the left.



Web Link

To practise estimating the size of angles, go to www.mhrmathatwork10.ca and follow the links.

b) Again, Kyle sketches where the bike begins and lands. He draws a line joining the two sets of bike tires.

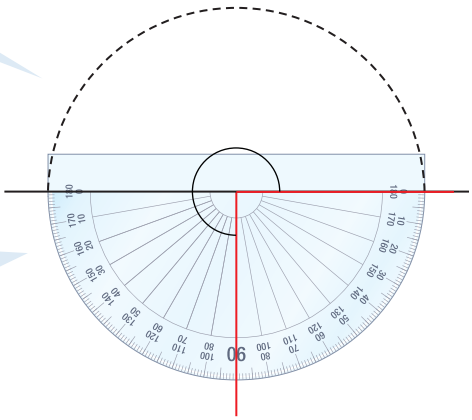


It looks like the bike turned about 90° more than 180°.

Kyle uses a protractor to measure the angle.
A **reflex angle** is larger than 180°.

This part of the angle is 180°.

Measure the size of this part of the angle.
Place the centre of the protractor on top of the vertex of the angle.
Use the inside scale to measure the angle shown.
Add 180 to your measurement.



Measure of the angle $= 180° + 90°$
$= 270°$

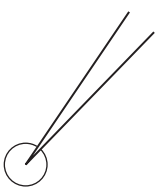
The angle measures 270°.

Your Turn

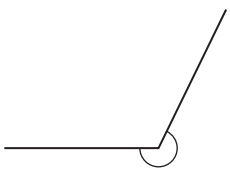
Classify each angle as a straight angle or a reflex angle. Then, measure the size of each angle

a)

b)



c)

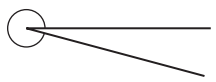


Check Your Understanding

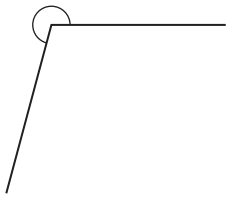
Try It

1. Classify each angle as a straight angle or a reflex angle.

a)



b)



c)

d)

2. Use a personal reference to estimate the measure of each angle in #1. Identify the reference you used for each angle.

3. Use a protractor to measure each angle in #1.

4. Describe each type of angle. Then, sketch an example of each. Estimate the measure of each angle you drew.

a) straight angle

b) reflex angle

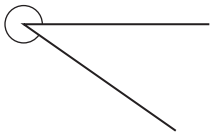
Apply It

5. Identify two straight angles and one reflex angle you see in the photos.

Figure A

Figure B

6. Ling is an aerial skier. Attempting to land a "360," she manages the turn shown.



a) Estimate the size of the angle from the starting position. Then, measure the angle using a protractor. Compare the estimate and the measurement.

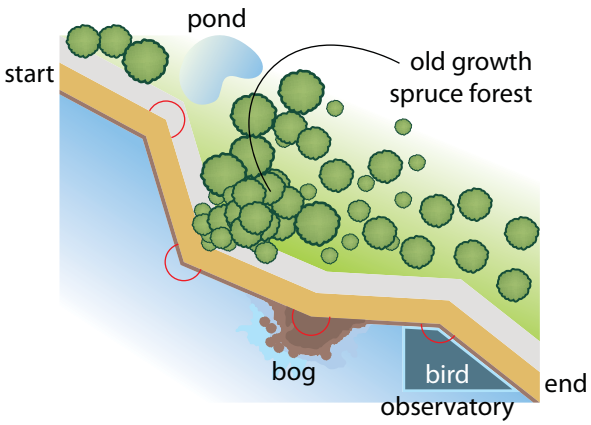
b) How far short is Ling from landing the rotation correctly?

F.Y.I.

Gros Morne National Park is a world heritage site. The Gros Morne mountain peak is part of the Long Range Mountains that run the length of Newfoundland's west coast. For more information about Gros Morne National Park, go to www.mhrmathatwork10.ca and follow the links.

7. Volunteers are building a new boardwalk at Gros Morne National Park. The sketch shows the plan for the boardwalk. Each section of the boardwalk is a straight line that changes direction and leads to points of interest.

a) Estimate each angle shown. **b)** Measure each angle.



F.Y.I.

The Cabox is the highest peak in Newfoundland. It is located near Corner Brook.

8. Mary faces north from the peak of the Cabox.

a) She turns 225° clockwise to face southwest and points toward Halifax, NS. Sketch the angle. How far does she need to keep turning clockwise to face north again?

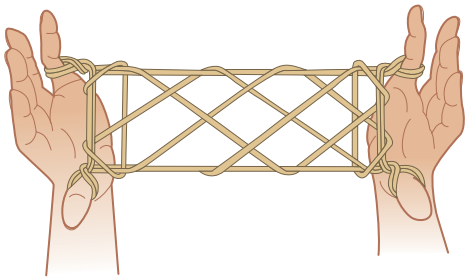
b) From her starting position, Mary turns 120° clockwise to face a little south of east. She points toward St. John's. Sketch the angle. How far does she need to keep turning clockwise to face north again?

Work With It

F.Y.I.

String figures are patterns formed when you span a looped string between your hands and twist it in different ways around your hands and wrists.

1. The string figure shown is sometimes called "tent flap."



a) Look for an example of each type of angle: acute, right, obtuse, straight, and reflex.

b) Estimate the size of the acute angles and the obtuse angles.

c) Measure each angle in part b) using a protractor. How close were your estimates?

F.Y.I.

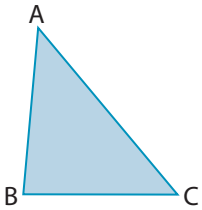
The symbol for triangle is △.

The symbol for angle is ∠.

2. a) Estimate the size of each angle in △ABC.

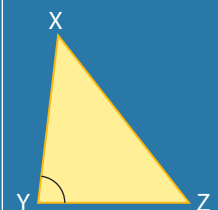
b) Use a protractor to measure each angle.

c) What is the sum of the three angles in the triangle?



F.Y.I.

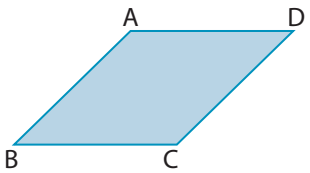
Angles are named by the "path" taken to make them. The indicated angle is named ∠XYZ since the angle is created by following the path from X to Y to Z. Note that the angle could also be named ∠ZYX.



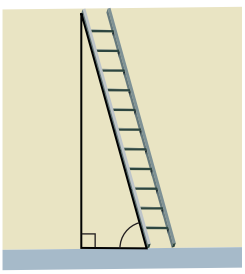
3. To make the star quilt, Jill cut fabric into diamond shapes. The diagram shows one diamond shape.

a) Estimate the size of the following angles: ∠A, ∠B, ∠C, and ∠D.

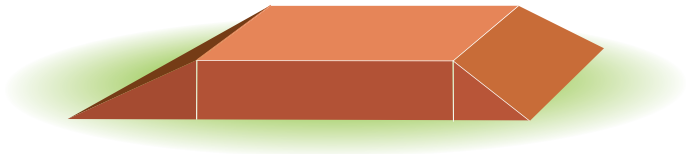
b) Measure the angles using a protractor.



4. For ladder safety, a ladder needs to be at an angle of 75° with the ground. Is this ladder placed safely? Justify your answer.



5. Sallie drew a sketch of a skateboard ramp.



a) Measure the angle the longer ramp makes with the ground.

b) Measure the angle the shorter ramp makes with the ground.

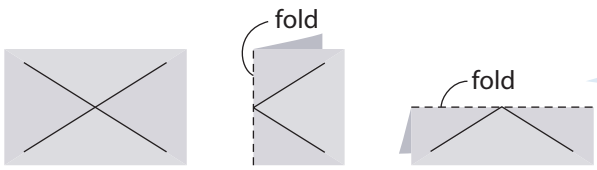
c) Identify each type of angle.

Discuss It

6. a) Using personal references, estimate the angles formed by ten items in the classroom. Find as many different types of angles as you can.

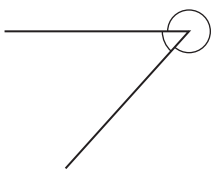
b) Which angle is the most common? Compare your results with those of a classmate.

7. William drew two lines that cross on a sheet of paper. He used paper folding to find patterns in the size of the angles. Describe what he did. What angle patterns did he find?

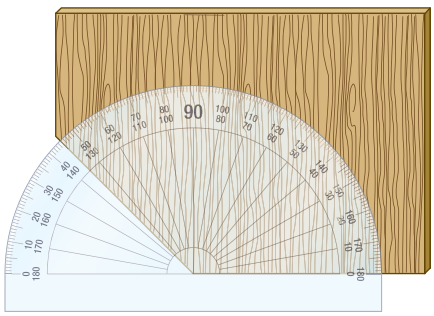


Compare the sizes of the angles on opposite sides of the fold.

8. Tyler says the diagram shows two angles. He says the larger angle is a reflex angle. The smaller angle is an acute angle. Is he correct? How do you know?



9. Peter and Andrée are measuring the angle of a saw cut. Peter says the wood is cut at an angle of 45°. Andrée says the cut is at 135°.



a) Who is correct?

b) Explain the mistake that the other person made.

10. With a partner, discuss how picture framers, artists, and tilers might use angles in their work.

5.2 Angle Constructions

Focus On . . .

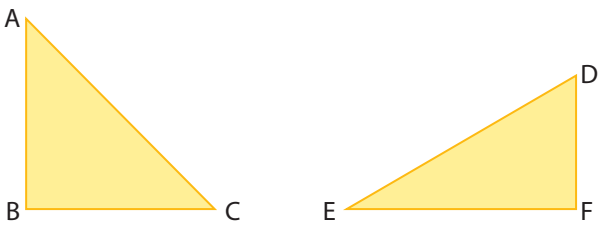
- sketching angles using a reference
- constructing angles
- bisecting angles

Drafters prepare drawings that are used to build everything from toys and spacecraft to houses and gas pipelines. Drafters use set squares to help construct commonly used angles.

Explore Set Squares

Materials

- ruler
- set squares
- protractor

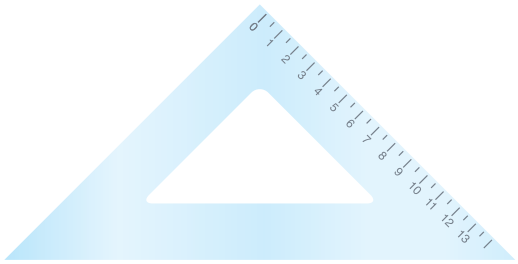


1. Draw and label the angles of the ABC set square. Draw each angle by tracing around a corner of the set square.
2. Measure each of the angles you have drawn.
3. Follow steps 1 and 2 for the DEF set square.

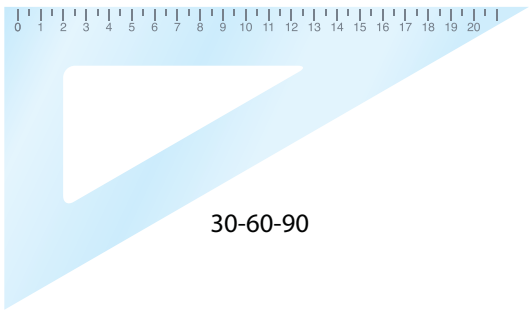
Tools of the Trade

A "set square" is a geometry tool used to construct 30°, 45°, 60°, and 90° angles without using a protractor. To learn more about using a set square, go to www.mhrmathatwork10.ca and follow the links.

4. Reflect There are two types of set squares. One is called a 90-45-45. The other is called a 30-60-90.



90-45-45



30-60-90

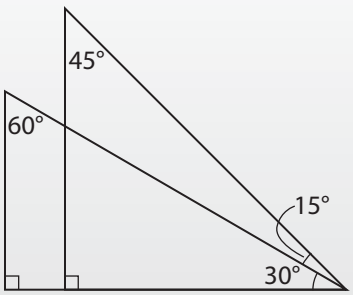
a) Which type of set square is ABC?

b) Which type of set square is DEF?

5. Extend Your Understanding You can construct larger angles using set squares. Place two set squares next to each other. Draw as many different angles as you can. Record the size of each angle you make.

Puzzler

Two set squares can be used to create a reference for 15°. How can you use the set squares to develop a reference for 55°?



On the Job 1

Draw an Angle

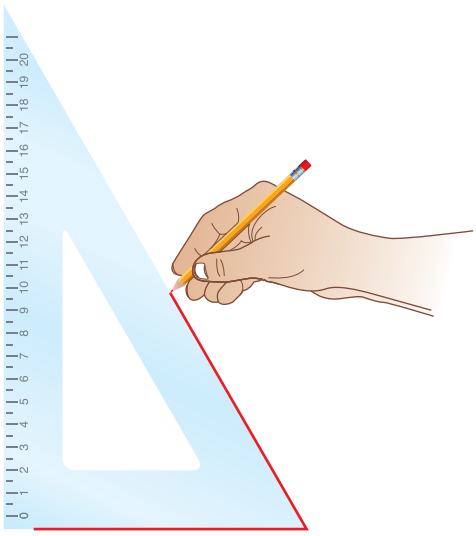
Morgan has been hired to put up an adjustable solar panel on the roof of a house. He will set it at an angle of 60° to the ground.

a) Sketch the angle that Morgan will use.

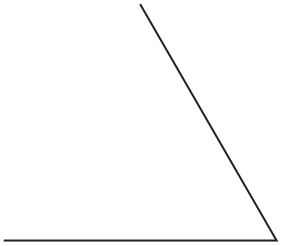
b) Construct the angle.

Solution

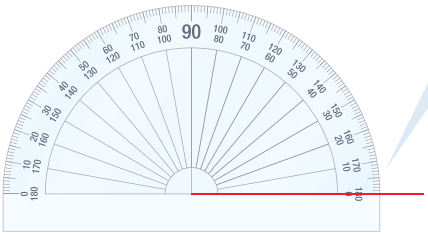
a) Use a reference.



Sketch an angle of 60°.

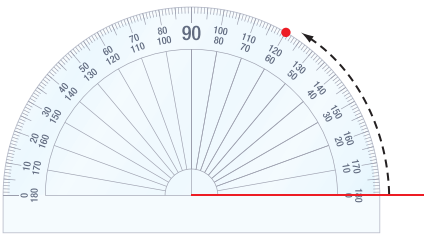


b) Use a ruler to draw a line. Place the protractor along the line as shown.

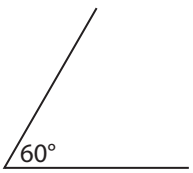


The baseline of the protractor must lie along the drawn line. The centre of the protractor must be at one end of the drawn line.

Mark a dot at 60°.



Use a ruler to draw the other arm of the angle. Label the angle size.



Web Link

You can use dynamic geometry software to construct an angle. Go to www.mhrmathatwork10.ca and follow the links.

Your Turn

Sketch each angle using a reference of your choice. Then, construct each angle.

a) 42°

b) 115°

Check Your Understanding

Try It

1. Use set squares to draw each angle.

a) 30° **b)** 45° **c)** 75°

2. Sketch an estimate of each angle. Then, use a ruler and protractor to construct each angle.

a) 46° **b)** 84° **c)** 32°

d) 178° **e)** 180° **f)** 210°

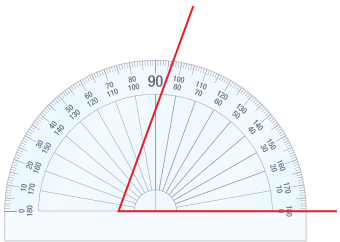
3. Use a method of your choice to construct each angle.

a) 29° **b)** 53° **c)** 69°

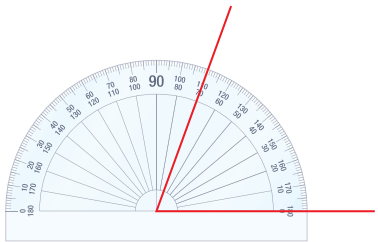
d) 117° **e)** 175° **f)** 301°

4. Katie and Beth are each using a protractor to draw a 70° angle.

Katie:



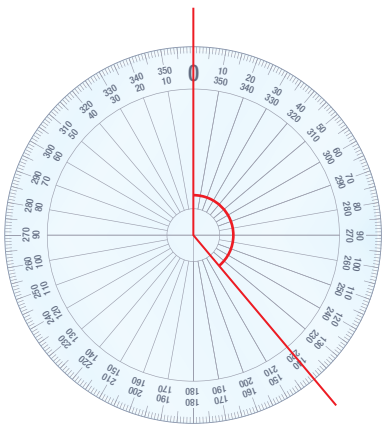
Beth:



a) Who is using the protractor incorrectly?

b) What error is she making?

5. Shaun used a protractor to draw a 220° angle.

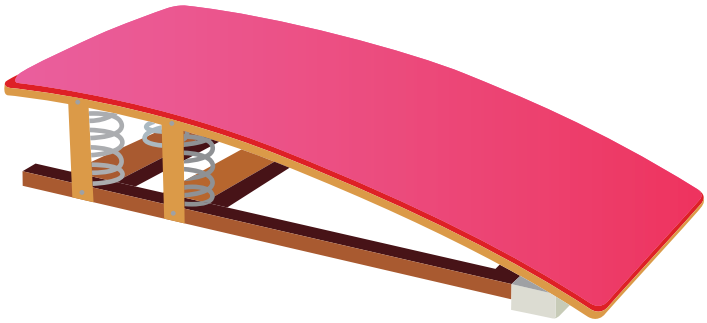


a) What error did Shaun make?

b) What is the correct method?

Apply It

6. The angle of a gymnastics springboard should be about 20° for the gymnast to get enough lift. Draw an angle of 20°.



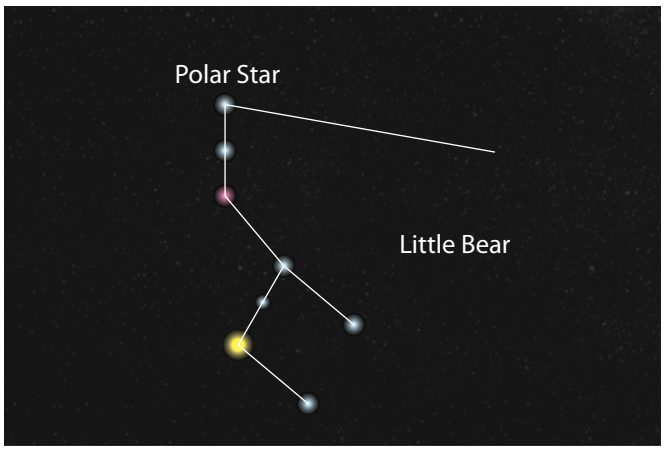
7. Justin created a logo for the Student Council.

a) Measure the three angles of the triangle.

b) Measure the lengths of the sides of the triangle.

c) Use your measurements to make a copy of the triangle.

8. This diagram shows the star system known as the Little Bear. Measure each line and angle using a ruler and a protractor. Then, use your measurements to construct a copy of the star system.



Web Link

For more information about the Mi'kmaq tale of the Great Bear, go to www.mhrmathatwork10.ca and follow the links.

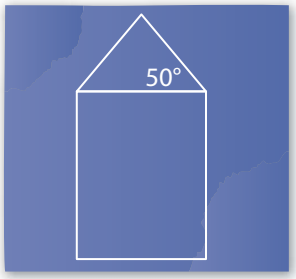
F.Y.I.

The Little Bear and Pole Star are part of the Great Bear star system. According to a Mi'kmaq tale, when the Great Bear is seen on the northern horizon in late fall and early winter evenings, it is a sign to bears that it is time to hibernate.

On the Job 2

Construct an Angle Bisector

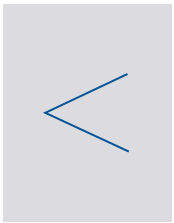
John is designing a fishing hut. The roof has an angle of 50°. He thinks that is too steep. He would like to cut the 50° angle in half. How can John **bisect** the 50° angle?



bisect
- cut in half

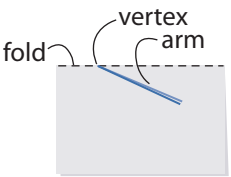
Solution

Draw the angle.

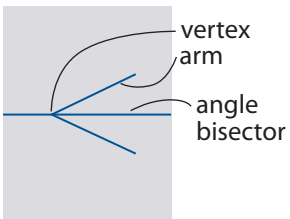


Method 1: Paper Folding

Fold through the vertex so that one arm is placed on top of the other arm.



Draw a line along the fold line. This line is the **angle bisector**.



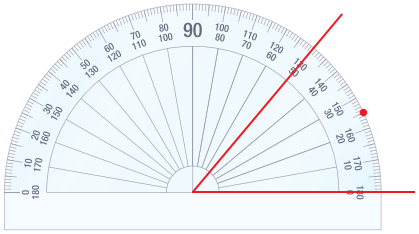
angle bisector
- line that cuts an angle into two equal pieces

Method 2: Use a Protractor

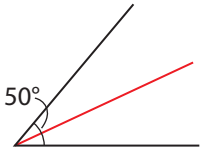
Divide the size of the angle by 2.

$50° \div 2 = 25°$

Place a protractor so that the baseline is along one arm of the 50° angle. Mark a dot at 25°.



Draw a straight line from the dot to the vertex of the angle. This line is the angle bisector.



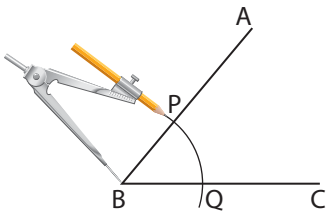
F.Y.I.

An "arc" is part of the circumference of a circle.

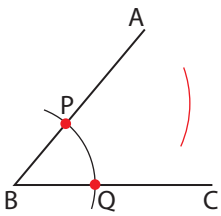
Method 3: Use a Compass

Set the compass to any length less than the shorter arm of the angle. Do not change this setting.

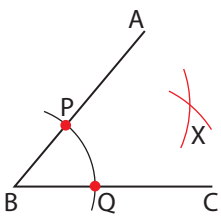
Put the point of the compass on the vertex of the angle. Draw an arc through the arms of the angle. Label P and Q.



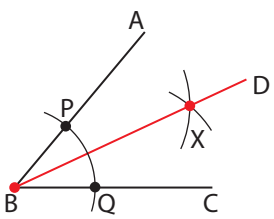
Move the point of the compass to P. Draw an arc between the arms of the angle.



Move the point of the compass to Q, and draw an arc. Label the point of intersection X.



Use a ruler to draw a straight line through X and B. This line is the angle bisector.



Your Turn

a) Draw an angle of 240°. Use a protractor to draw the angle bisector.

b) Draw an angle of 90°. Use a compass and ruler to bisect the angle.

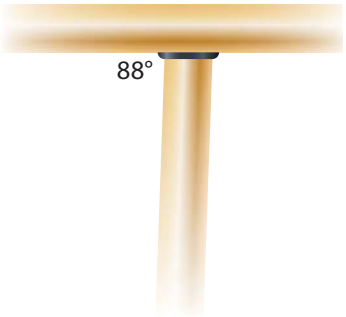
Check Your Understanding

Try It

1. Use paper folding to bisect each angle.
   a) 46°
   b) 220°
2. Use a protractor to draw the angle bisector of each angle.
   a) 124°
   b) 180°
3. Use a compass and a ruler to bisect each angle.
   a) 120°
   b) 38°

Apply It

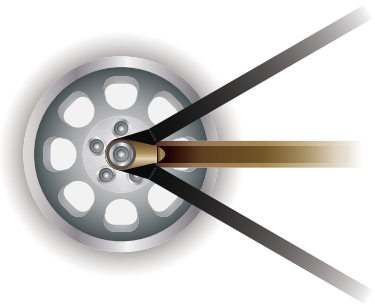
4. A steelworker needs to weld a beam that bisects the angle formed by the two beams shown. Where should the worker place the beam? Include a diagram with your answer.



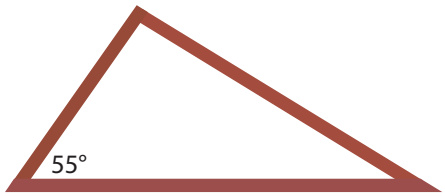
Tools of the Trade

Workers use a welding torch to join two pieces of metal. The torch heats the two pieces of metal to a temperature that produces molten metal from each piece. To learn more about welding, go to www.mhrmathatwork10.ca and follow the links.

5. The design for a new motorcycle model includes three rods that form a support. The metal rod in the middle should be exactly halfway between the outer rods. Is the middle rod placed correctly? Show your work.

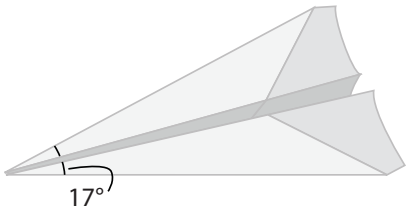


6. a) The design for a roof is shown. How can you bisect the roof angle? Show your work.



b) Check that the angles you have formed have the same measure.

7. Jake is drawing sketches for making a paper airplane. He needs to draw the angle bisectors for the angles indicated. Make a copy of one of the angles, and then bisect it.



Work With It

1. Use set squares to draw each angle.

a) 30° **b)** 45° **c)** 120°

2. Use a protractor to construct each angle.

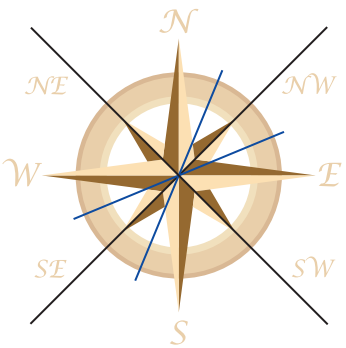
a) 23° **b)** 59° **c)** 180° **d)** 337°

3. Draw an 80° angle. Use a compass or protractor to construct the angle bisector.

4. **MINI LAB** Work with a partner or small group. Use masking tape to copy this compass rose on the floor of your classroom. Do the following steps.

Materials

- masking tape
- ruler
- large protractor
- marker
- Compass Rose worksheet



STEP 1

Stand facing E. Turn 180° clockwise. What direction are you facing?

STEP 2

a) Bisect the 180° angle and turn to face the bisected angle. What direction are you facing?

b) What is the size of each angle created after the original angle was bisected?

STEP 3

c) Identify four other angles in the compass rose. Provide their measurements.

d) What is the size of the bisected angle for each of your angles from part c)?

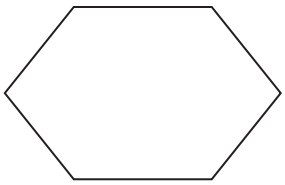
STEP 4

Create two new questions using the angles on the compass rose. Trade them with your partner or with someone in your group. Answer each other's questions.

Tools of the Trade

Boaters, trappers, and many other people who work or play in the outdoors use a compass to help them identify direction. A compass rose is a figure on a map or nautical chart that shows the directions. To learn more about the symbols on a compass rose and how to use them, go to www.mhrmathatwork10.ca and follow the links.

5. A town is adding a sandbox to the local playground. Make a copy of the design for the sandbox. Label the sizes of all angles.



6. A certain car door is designed to swing open a maximum of 64°. When pushed partly closed, it will stay open at half of that angle. At what angle will it stay open? Include a diagram with your answer.

Discuss It

7. **a)** Draw a reflex angle of 200°.

b) Use a compass and a ruler to bisect the angle.

c) Read the speech balloon again. Why is this step necessary?

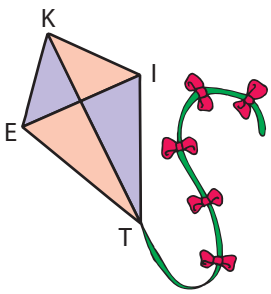
d) What is another way you could bisect a reflex angle?

When you draw the X, make sure you set the compass to a length greater than the length you used to draw the arc.

8. Describe a pro and a con of each of the three methods you have used to bisect an angle.

9. **a)** Sketch a diagram of the kite. Identify the angles that are bisected.

b) What are two different examples of angle bisectors in the real world? Sketch each example. Identify the angles that are bisected.



5.3 Lines and Angles

Focus On . . .

- identifying perpendicular, parallel, and transversal lines
- identifying patterns of angles formed by parallel lines
- identifying patterns of angles when two lines cross

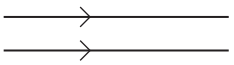
Materials

- Street Map of St. John's
- ruler
- protractor

The map shows a section of St. John's. On the map, Duckworth St. and Gower St. are parallel. Cochrane St. cuts across them. The streets form eight angles. Which angles look the same size to you?

parallel lines

- lines that do not cross each other
- are marked by matching arrowheads
- two parallel lines are always the same distance apart



Explore Angles in Parallel Lines and Perpendicular Lines

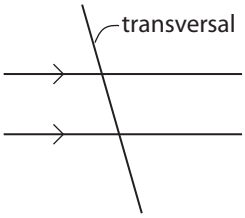
Use the street map of St. John's to help answer the questions.

1. a) Identify two other streets that are **parallel**.

b) Will these streets ever cross each other? Explain.

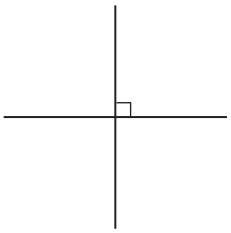
transversal

- a line that crosses two or more parallel lines



perpendicular lines

- lines that cross each other at right angles
- often marked with one right angle symbol



F.Y.I.

The symbol for parallel is ||.

The symbol for perpendicular is ⊥.

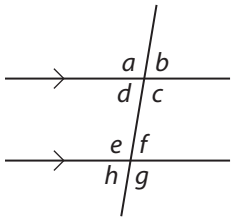
Web Link

Explore parallel lines using dynamic geometry software. Go to www.mhrmathatwork10.ca and follow the links.

2. a) Draw two parallel lines to represent Duckworth St. and Gower St. Then, draw a **transversal** to represent Cochrane St. crossing the two streets.

You can draw parallel lines using opposite sides of a ruler.

b) Measure each acute angle and obtuse angle.



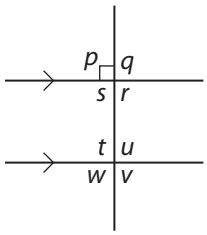
c) Choose a different pair of streets that are parallel and a street that crosses them.

d) Use a protractor to measure the angles. What patterns do you notice in the angle measures?

3. a) Identify a street that crosses Duckworth St. and Gower St. at a right angle. This street should be **perpendicular** to Duckworth St. and Gower St.

b) Draw two parallel lines to represent Duckworth St. and Gower St. Then, draw a line perpendicular to the parallel lines.

c) Use a protractor to measure the angles. Describe the patterns you notice in the angle measures.



4. Reflect Compare your results with those of your classmates.

a) Describe how the angles formed by parallel lines and a transversal are related.

b) How are the angles formed by parallel lines and a perpendicular line related?

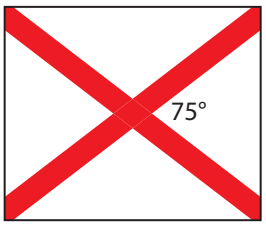
5. Extend Your Understanding Jessie is driving a bus along Gower St. toward downtown and wants to go to Duckworth St. The bus cannot make turns at angles less than 90°. Plan a driving route for Jessie. How many ways can you find for her to do this? Show the route and label the street names. Can you find another route? If so, describe it.

On the Job 1

Work With Angles Formed by Intersecting Lines

Marine signal flags are used by ships at sea. Each signal spells out a short message or has a special meaning. Each flag is a rectangle. For each flag, identify how the lines meet or cross. Determine the measures of the angles formed by the lines.

a)

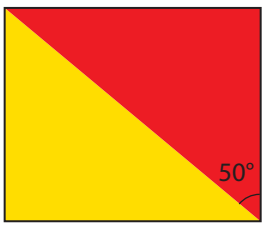


I require assistance.

b)

You should stop. I have something important to communicate.

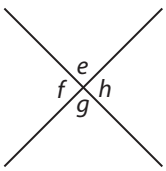
c)



Man overboard.

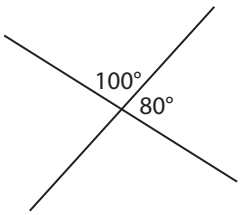
opposite angles

- a pair of equal angles formed by two lines that cross
- they form an X pattern
- on the diagram, $e = g$ and $f = h$



supplementary angles

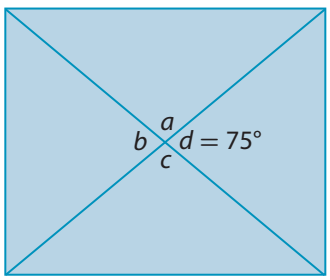
- two angles that add to 180°
- they form a straight angle



Solution

a) The lines cross.

Sketch a diagram.



The diagonals form four angles.

b and d are **opposite angles**.
They are equal.
$\angle b$ is 75°.

a and d form a straight angle.
They are **supplementary angles**.

$$a + d = 180°$$
$$a + 75° = 180°$$
$$a + 75° - 75° = 180° - 75°$$
$$a = 105°$$

Estimate: $180° - 80° = 100°$

Check: $105° + 75° = 180°$

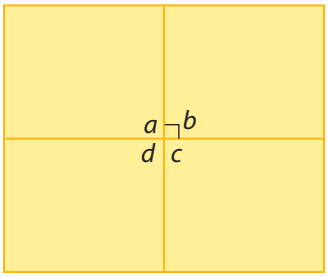
$\angle a$ is 105°.

a and c are opposite angles. They are equal.
$\angle c$ is 105°.

The three angles have measures of 105°, 75°, and 105°.

b) The lines are perpendicular. They cross at a right angle.

Sketch a diagram.



The diagonals form four angles.
$\angle b$ is 90°.

Using the X pattern, b and d are equal.
$\angle d$ is 90°.

a and d are supplementary angles.

$$a + d = 180°$$
$$a + 90° = 180°$$
$$a + 90° - 90° = 180° - 90°$$
$$a = 90°$$

Check: $90° + 90° = 180°$

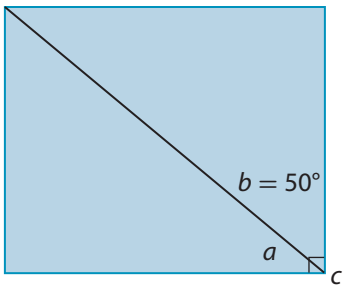
$\angle a$ is 90°.

Using the X pattern, $a = c$.
$\angle c$ is 90°.

All four angles each measure 90°.

c) The line is a transversal that meets parallel lines.

Sketch a diagram.



The diagonal forms two angles.

c is a right angle, so it measures 90°.

a and b are **complementary angles**.

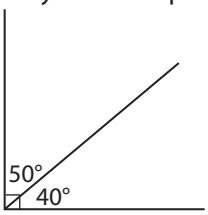
$$a + b = 90°$$
$$a + 50° = 90°$$
$$a + 50° - 50° = 90° - 50°$$
$$a = 40°$$

Check: $40° + 50° = 90°$

$\angle a$ is 40°.

complementary angles

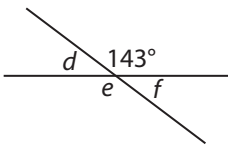
- two angles that add to 90°
- they form a right angle
- they form an L pattern



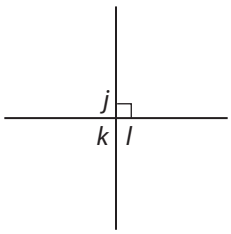
Your Turn

Identify how each of the sets of lines meet or cross. Then, determine the size of each unknown angle.

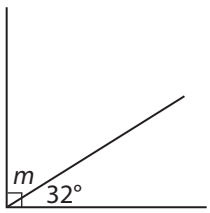
a)



b)



c)



Web Link

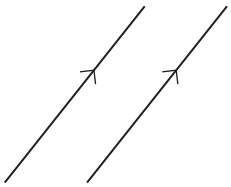
To explore complementary angles and supplementary angles, go to www.mhrmathatwork10.ca and follow the links.

Check Your Understanding

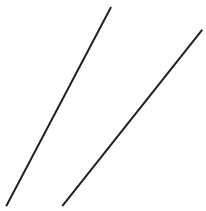
Try It

1. Identify each pair of lines as parallel, perpendicular, or neither. Justify your answer.

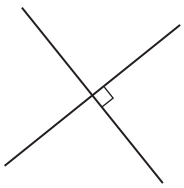
a)



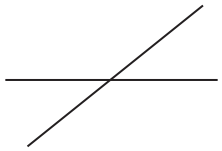
b)



c)

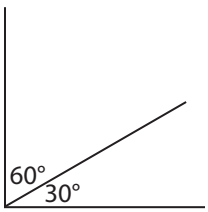


d)

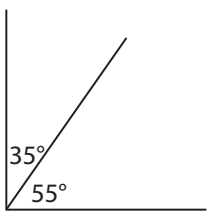


2. Which pairs of angles are complementary? How do you know?

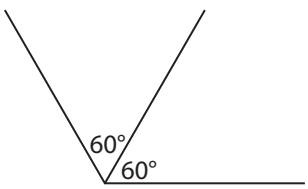
a)



b)

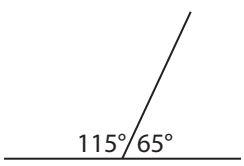


c)



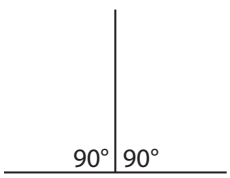
3. Which pairs of angles are supplementary? Explain how you know.

a)



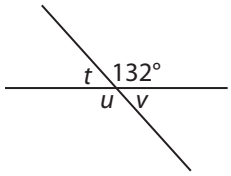
b)

c)

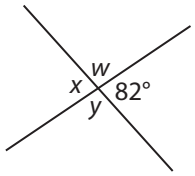


4. What is the measure of each unknown angle?

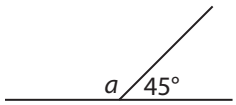
a)



b)



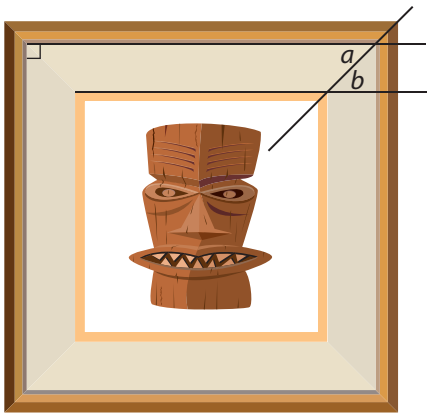
5. a) What is the measure of a?



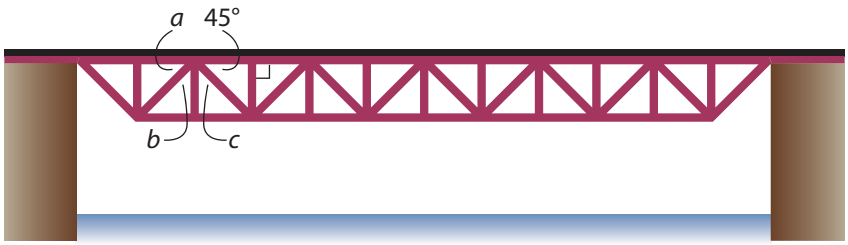
b) What kind of angle is a? How do you know?

Apply It

6. Emily uses a cutter to cut angles for picture frame mats. The mat shown is a square. Determine the measures of a and b. Show your work.



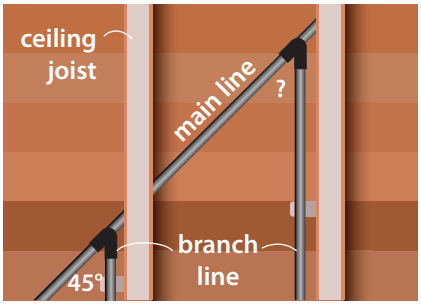
7. A bridge has the framework shown. All the vertical supports are parallel and the two horizontal lines are parallel. Determine the measure of each angle. Show your work.



On the Job 2

Work With Angles Formed by Parallel Lines

Brent is installing a central vacuum system. He checks the plan to find out how the lines run. The main line runs from the power unit along the joists in the basement ceiling. The joists are parallel to each other. The main line connects to branch lines that go to the valves upstairs. It is important to know the angles at which the lines meet, so Brent can buy the correct elbow fittings.



He needs an elbow joining the main line to a branch line. What size of elbow does he need?

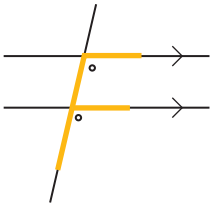
45° elbow 60° elbow 90° elbow

F.Y.I.

Elbows connect tubes at different angles. For example, a 90° elbow is shaped like a right angle. It provides a joint between two pipes at right angles to each other.

corresponding angles

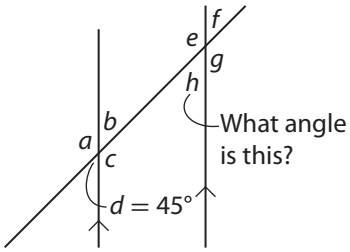
- a pair of angles on the same side of a transversal crossing the parallel lines
- they are equal
- they form an F pattern



Solution

Sketch a diagram. Name the angles to make them easier to discuss. Brent wants to know the measure of $\angle h$.

Determine the measures of all the unknown angles.



Method 1: Use Corresponding Angles

d and h are **corresponding angles**. They are equal.

$\angle h$ is 45°.

Method 2: Use Same Side Exterior Angles

d and c are supplementary.

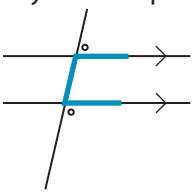
$c = 180° - 45°$

$c = 135°$

$\angle c$ is 135°.

same side exterior angles

- a pair of angles on the same side of a transversal and outside the parallel lines
- they add to 180°
- they form a C pattern



d and g are **same side exterior angles**. They are also supplementary. So, they add to 180°.

d and $g = 180°$

$45° + g = 180°$

$g = 180° - 45°$

$g = 135°$

$\angle g$ is 135°.

g and h are supplementary too. They add to 180°.

$g + h = 180°$

$135° + h = 180°$

$h = 180° - 135°$

$h = 45°$

$\angle h$ is 45°.

Method 3: Use Same Side Interior Angles

c and h are **same side interior angles**. They are supplementary too. So, they add to 180°.

$c + h = 180°$

$135° + h = 180°$

$135° - 135° + h = 180° - 135°$

$h = 45°$

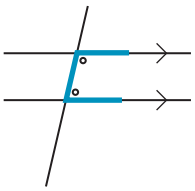
Check: $135° + 45° = 180°$

$\angle h$ is 45°.

Brent needs a 45° elbow.

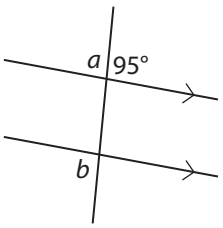
same side interior angles

- a pair of angles on the same side of a transversal and inside the parallel lines
- they add to 180°
- they form a C pattern

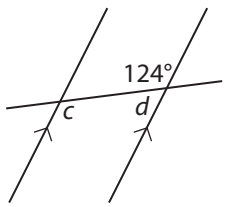


Your Turn

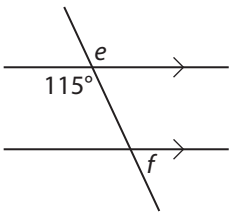
a) Determine the measures of a and b.



b) Determine the measures of c and d.



c) Determine the measures of e and f.



Check Your Understanding

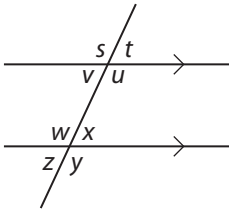
Try It

1. Use the diagram to identify

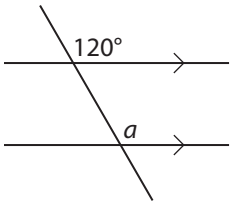
a) two pairs of same side exterior angles

b) two pairs of same side interior angles

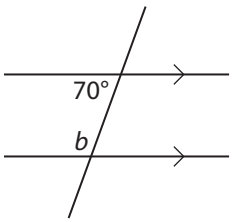
c) four pairs of corresponding angles



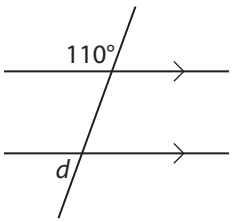
2. a) Use corresponding angles to help determine a.



b) Use same side interior angles to help determine b.

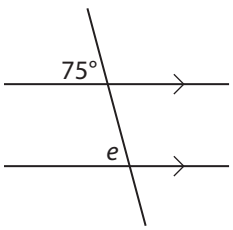


c) Use same side exterior angles to help determine d.

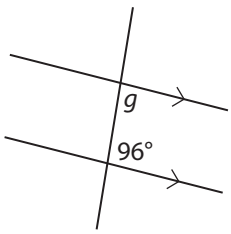


3. Describe the relationship of each marked angle to the given angle. Then, determine the measure of the marked angle.

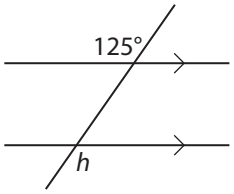
a)



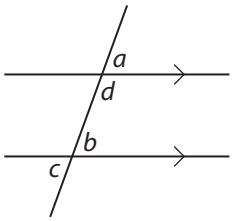
b)



4. What is the size of h? Explain your reasoning.

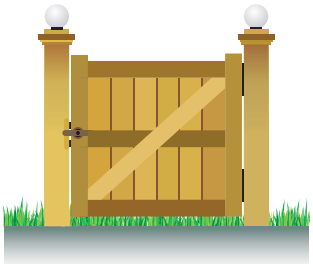


5. Angle a measures 80°. Use angle patterns to determine the measures of b, c, and d.



Apply It

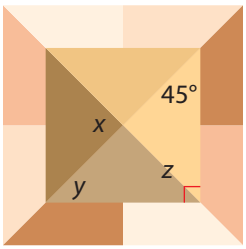
6. The gate shown has three parallel boards and a diagonal board. Make a sketch of the gate. Show as many pairs of each kind of angle as possible.



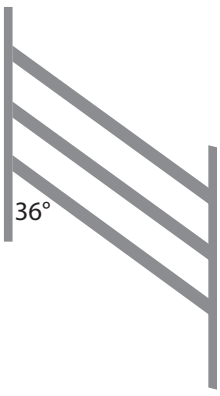
a) corresponding angles

b) same side interior angles

7. Khali is designing a square floor tile. He wants the angles to fit together. At what angles are the pieces placed in this tile? Identify the three angles. Give reasons for your answers.



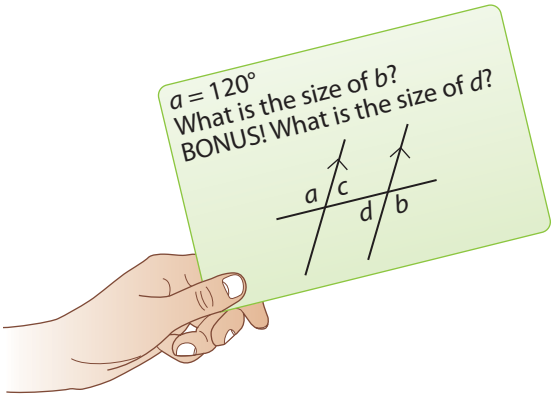
8. Megan is installing a railing beside a staircase. Three parallel rails are supported on two posts that are parallel. Megan wants to attach each rail to the post at the same angle. The size of one angle is shown. Sketch your own diagram. Show the measures of as many angles as you can.



On the Job 3

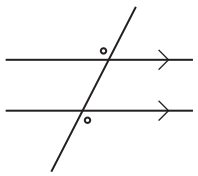
Work With Alternate Angles

Lyle and Amy have a part-time job developing brainteasers. Lyle created the following card. Solve this brainteaser.



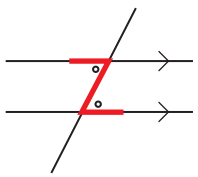
alternate exterior angles

- a pair of angles on opposite sides of a transversal and outside the parallel lines
- they are equal



alternate interior angles

- a pair of angles on opposite sides of a transversal and inside the parallel lines
- they are equal
- they form a Z pattern



Solution

a and b are **alternate exterior angles**.

The angles are equal.

$\angle a$ and $\angle b$ are both 120°.

Bonus!

***Method 1: Work from* a**

$$a + c = 180°$$
$$a = 120°$$
$$c = 180° - 120°$$
$$c = 60°$$

$\angle c$ is 60°.

c and d are **alternate interior angles**. Alternate interior angles form a Z pattern. Alternate interior angles are equal.

$\angle d$ is 60°.

***Method 2: Work from* b**

$b + d = 180°$

$b = 120°$

$d = 180° - 120°$

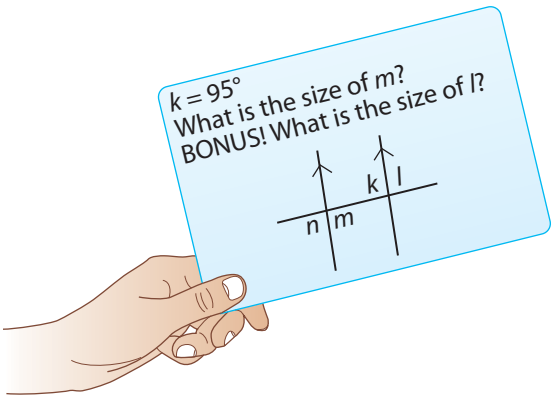
$d = 60°$

$\angle d$ is 60°.

What other methods can you use to check the size of these angles?

Your Turn

Amy liked the idea but wanted a different set of lines. Solve Amy's brainteaser.



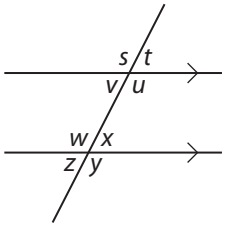
Check Your Understanding

Try It

1. Use the diagram to identify

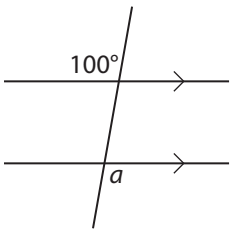
a) two pairs of alternate exterior angles

b) two pairs of alternate interior angles

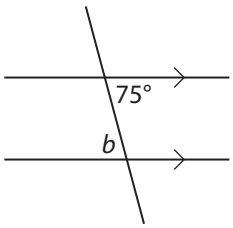


2. Describe the relationship of each marked angle to the given angle. Then, determine the measure of the marked angle.

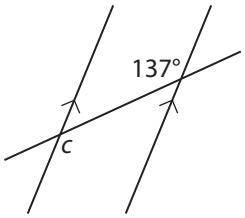
a)



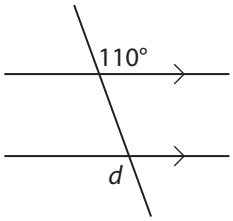
b)



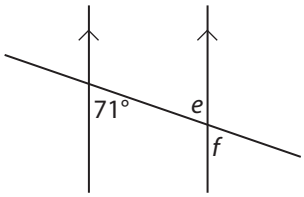
c)



d)

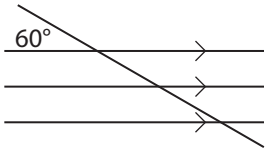


3. Determine the size of e and f. Be prepared to explain your reasoning.

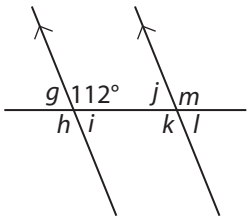


4. **a)** How many angles in the diagram measure 60°?

b) How many angles measure 120°?



5. What is the measure of each unknown angle? Explain your reasoning.

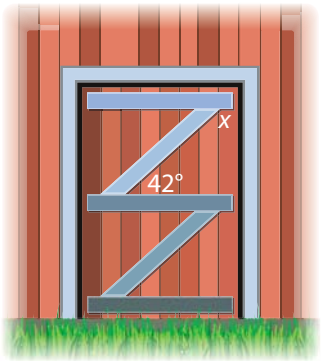


Apply It

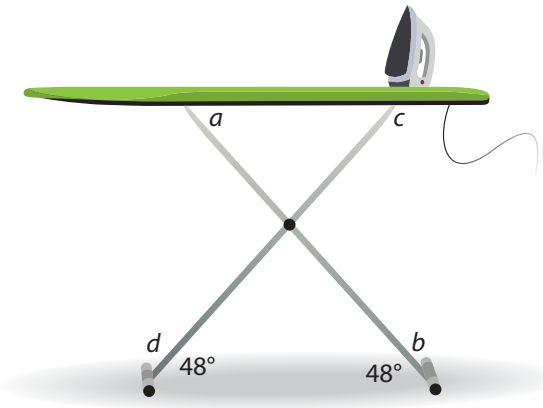
6. There are three parallel boards in a barn door. The diagonal boards make the door stronger.

a) What is the measure of x? Show your work.

b) Show another way to determine x. Justify your method.



7. The top of an ironing board is parallel to the floor. What are the measures of the angles marked? Justify your answers.



Work With It

F.Y.I.

Memorial windows are often placed in churches to remember loved ones.

1. Artists use lines and angles to create art. Newfoundland artist Christopher Pratt created this piece called *Memorial Window*. It shows the ocean from a glass window.

a) Sketch a diagram of the art piece. On your diagram, identify as many perpendicular and parallel lines as you can.

b) How are perpendicular lines and parallel lines different?

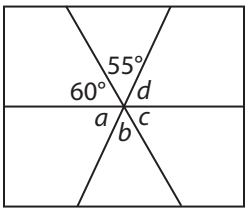
c) Identify pairs of equal angles. How did you identify them?

d) Highlight a pair of complementary angles and supplementary angles. How did you identify them?

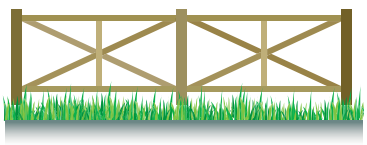
F.Y.I.

A photo mosaic is a large image made from several smaller images.

2. Annie is creating a photo mosaic. She needs to decide at which angles to cut the images. Determine the unknown angle measures in Annie's design.



3. Michael is designing a new gate. The horizontal pieces are parallel. So are the vertical pieces.

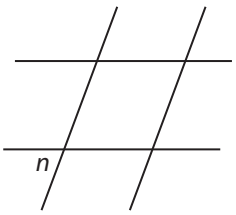


a) Sketch your own diagram. List all the types of angles you see. Label them on your diagram.

b) Show the measures of as many angles as you can.

c) Show two pairs of angles that have equal measures. Explain how you know they are equal.

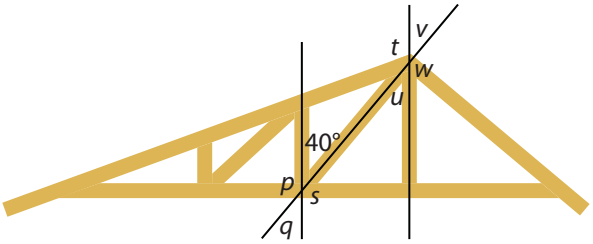
4. The number symbol, #, is used on computer keyboards and phone pads.



a) Sketch a diagram. Identify all the angles that are equal to n.

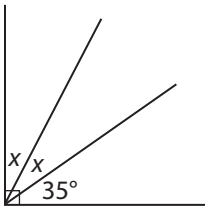
b) On another diagram, identify all the angles that are supplementary to n.

5. Robert is making the roof truss shown. He wants to know the measures of the angles. The measure of one angle is shown. Determine the measures of all the other labelled angles. Justify your answer.



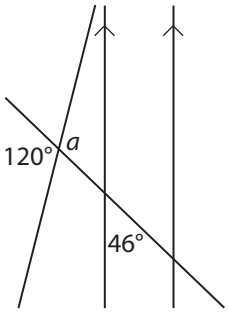
Discuss It

6. a) Determine the measure of x.
Hint: The x angles are equal.

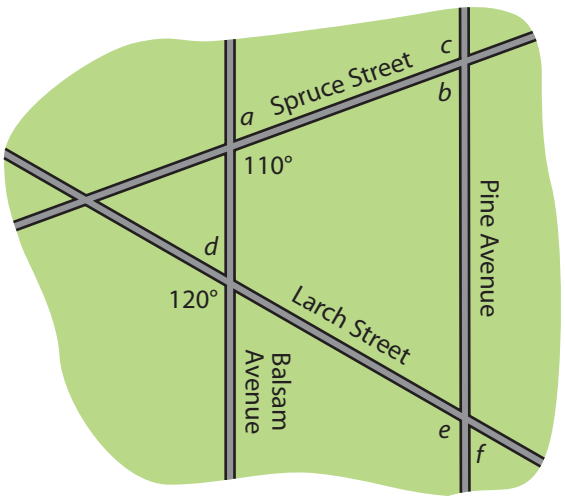


b) Are any two of the angles shown complementary? supplementary? Explain why or why not to a classmate.

7. Ellie and Paul are solving for the unknown angles in the diagram. Paul says that a is equal to 120°. Ellie says that a is equal to 46°. Who is correct? Explain your reasoning.



8. a) On the street map, Balsam Avenue and Pine Avenue are parallel. Determine the measures of a, b, c, d, e, and f.

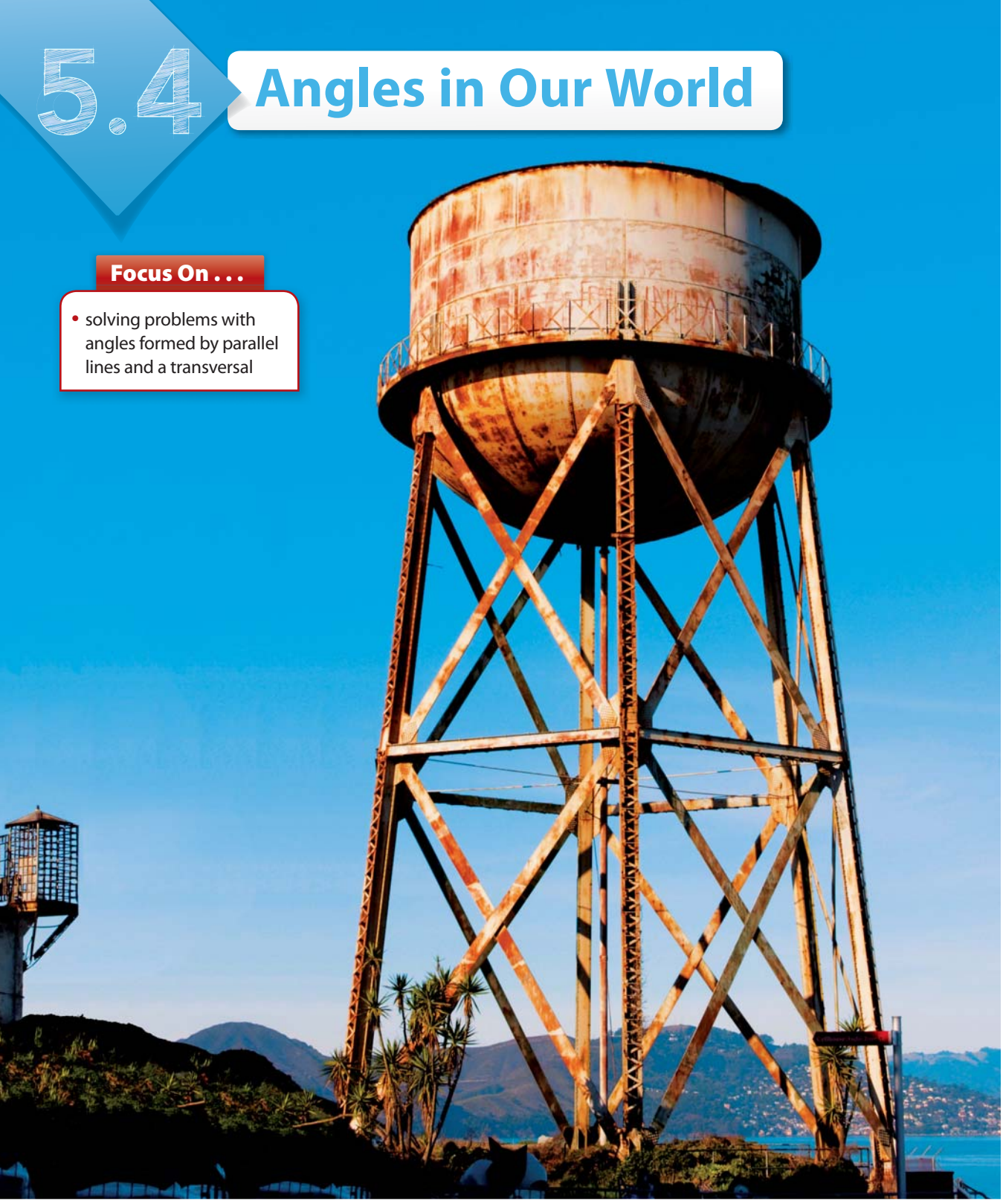


b) Compare your answers with those of a classmate. What different methods did you use? Did you get the same answers? Discuss why or why not.

5.4 Angles in Our World

Focus On . . .

- solving problems with angles formed by parallel lines and a transversal



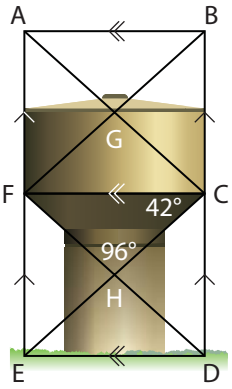
The design of a water storage tower includes many angles and parallel line segments. Identify pairs of equal angles. How do you know they are equal?

Explore Relationships Between Angles

Materials

- protractor
- Water Storage Tower worksheet

Eric is designing a water storage tower. He plans to make his design similar to the one shown. He needs to determine some measures that were used in designing the water tower, such as angle measures used for the support structure.



1. a) What size is ∠EHD?

b) How do you know?

2. a) What size is ∠HCD?

b) How do you know?

3. a) Identify the size of as many other angles as you can using angle properties.

b) How do you know the size of each of these angles?

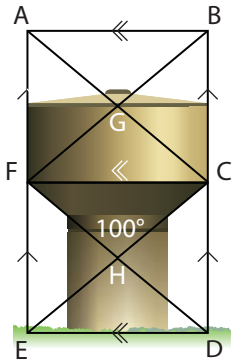
4. Reflect

a) Which angle sizes were you unable to identify?

b) Why were you unable to identify their sizes?

5. Extend Your Understanding Eric decided to change the design.

a) Suppose he redrew it so that ∠FHC was 100°. How would this affect the other angles?

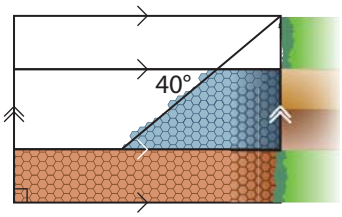


b) Suppose he changed it so that line segments AE and BD were not parallel. How would this affect the angles?

On the Job 1

Determine Angle Measures

Melanie is finishing a mosaic tile floor for the entrance of a house. She needs to cut three tile pieces to fill in the white spaces. The tiles she is cutting are square. What are the unknown angle measures at each corner of each missing tile piece?

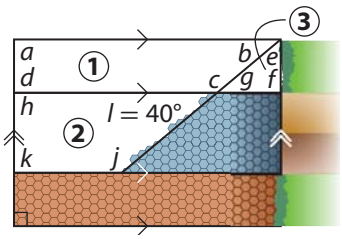


Tools of the Trade

Using a tile cutter allows you to make precise cuts on ceramic tiles. Cutting ceramic tile is done using a carbide or tungsten-steel wheel. For more information on ceramic tile cutters, go to www.mhrmathatwork10.ca and follow the links.

Solution

Draw a diagram. Use a number to identify each tile that needs cutting. Use a letter to identify every corner of each tile. Identify any angles that are the same size.



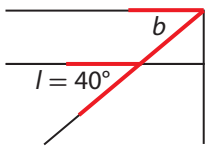
Angles a, d, f, k, and h are all the same size. This is true because

- each angle occurs where perpendicular lines meet
- perpendicular lines meet at right angles

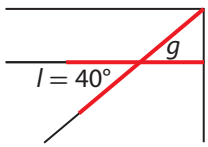
So, a, d, f, k, and h are each 90°.

l, b, and g are all the same size. This is true because

- l and b are corresponding angles that follow the F pattern

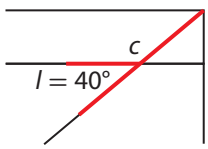


- l and g are opposite angles that follow the X pattern



Since l is 40°, b and g are each 40°.

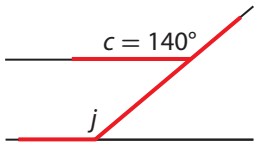
c and l are supplementary angles.
These two angles form a straight line.



$$c + l = 180°$$
$$c + 40° = 180°$$
$$c + 40° - 40° = 180° - 40°$$
$$c = 140°$$

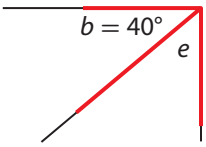
$\angle c$ is 140°.

c and j are the same size. This is true because c and j are corresponding angles that follow the F pattern.



So, j is 140°.

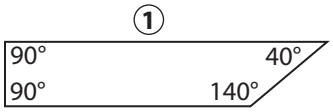
e and b are complementary angles. These two angles form a right angle.

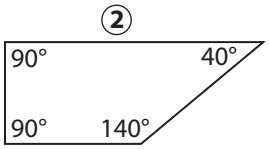


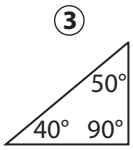
$$e + b = 90°$$
$$e + 40° = 90°$$
$$e + 40° - 40° = 90° - 40°$$
$$e = 50°$$

$\angle e$ is 50°.

Here are the angles at the corners of the three tiles.



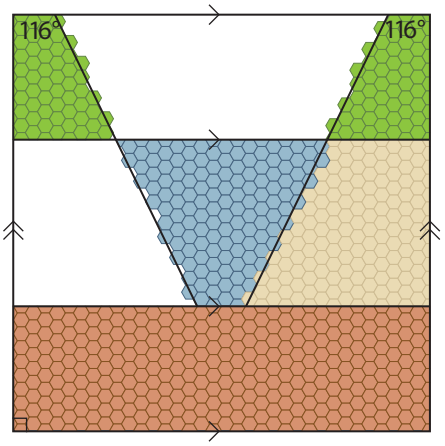




What other methods could you use to solve for these angles?

Your Turn

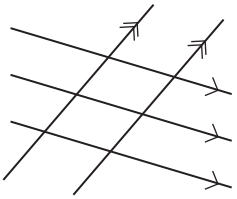
Melanie needs to cut two more tile pieces to complete the tile floor. What are the angle measures at each corner of each missing tile piece?



Check Your Understanding

Try It

1. The diagram shows one pair of parallel lines that cross a set of three parallel lines. Make two sketches of the diagram.



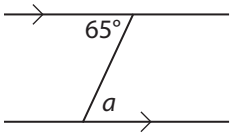
a) On one sketch, mark two pairs of angles in the Z pattern. Use two different colours.

b) On the other sketch, mark three pairs of angles in the X pattern. Use three different colours.

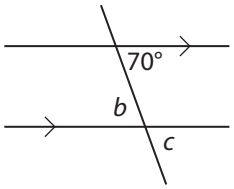
2. Cody constructed a K shape using straight lines. Do you think that there are any special angle relationships in this letter? Explain your answer.

3. Determine the sizes of the angles shown.

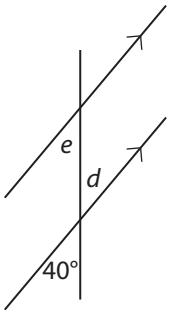
a)



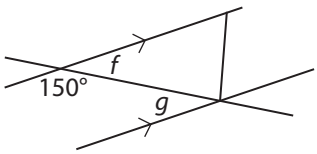
b)



c)

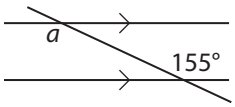


d)

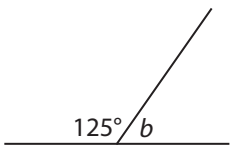


4. Determine the marked angles. Give a reason for your answer.

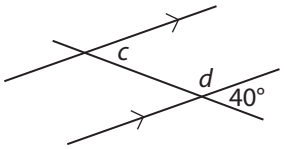
a)



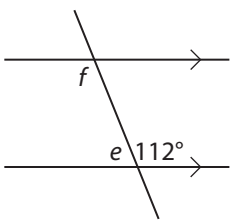
b)



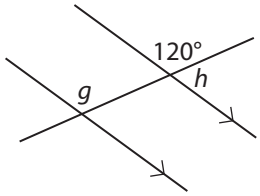
c)



d)

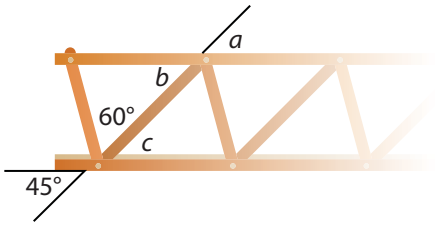


e)

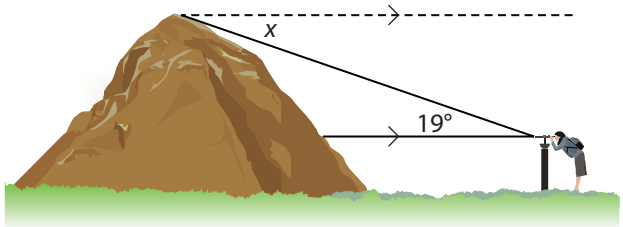


Apply It

5. Damian is making a railing and wants to know at what angles to cut the support pieces. The top rail and bottom rail are parallel. Use angle patterns to determine the measures of a, b, and c. Justify your work.



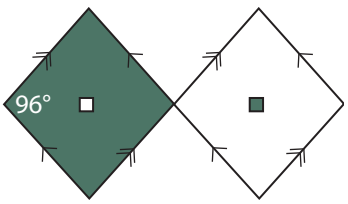
6. Emma is looking up at the top of a mountain through a telescope in Gros Morne National Park. She estimates that the angle from the telescope to the top of the mountain is 19°. What is the measure of the angle from the top of the mountain down to the telescope?



7. The diagram shows a detail of the traditional Newfoundland and Labrador diamond pattern used for mittens.

a) Determine the size of as many angles as you can without using a protractor.

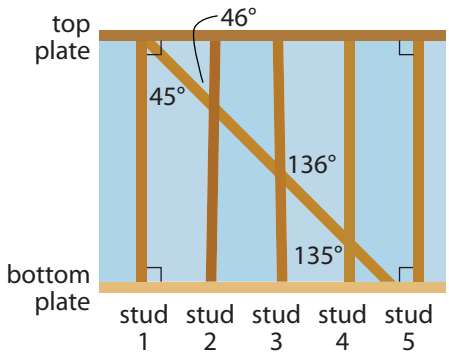
b) Use a protractor to check the accuracy of your answers. What was the reason for any errors that you made?



On the Job 2

Test for Perpendicular or Parallel Lines

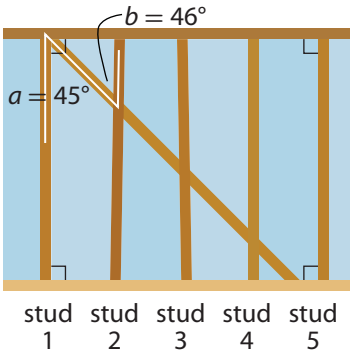
Kent is framing a wall. Studs 1 and 5 are perpendicular to the top and bottom plates. Kent measures between the wall studs at the top and bottom to ensure that Studs 2, 3, 4, and 5 are parallel to Stud 1. He double-checks by measuring the angles formed by the diagonal board. Are all of the studs parallel to Stud 1?



Solution

Check Stud 2.

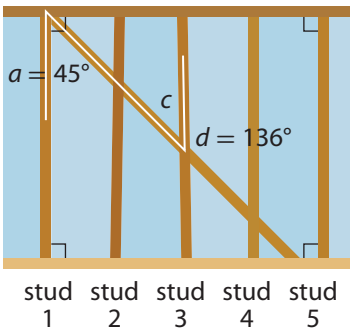
The diagonal forms a Z pattern with Studs 1 and 2. If Studs 1 and 2 are parallel, b should equal a.



$\angle b$ is 46°. Stud 2 is not parallel to Stud 1.

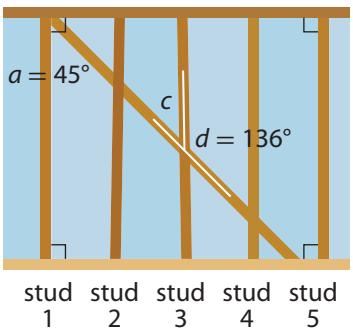
Check Stud 3.

The diagonal forms a Z pattern with Studs 1 and 3. If Studs 1 and 3 are parallel, c should be equal to a.



c and d are supplementary angles. They form a 180° angle.

$$d + c = 180°$$
$$136° + c = 180°$$
$$c = 180° - 136°$$
$$c = 44°$$



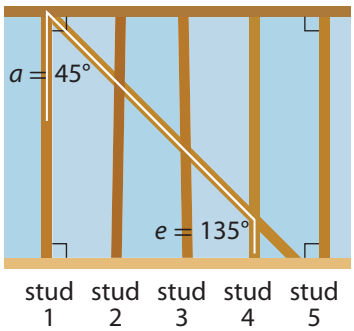
$\angle c$ is 44°. Stud 3 is not parallel to Stud 1 because c is not equal to a.

Check Stud 4.

The diagonal forms a C pattern with Studs 1 and 4. If Studs 1 and 4 are parallel, e and a should add up to 180°.

$$e + a = 180°$$
$$e + 45° = 180°$$
$$e = 180° - 45°$$
$$e = 135°$$

$\angle e$ is 135°. Stud 4 is parallel to Stud 1.

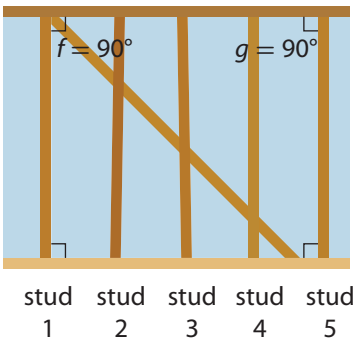


Check Stud 5.

Studs 1 and 5 are both perpendicular to the top and bottom plates. So, f and g are both 90°.

The top plate forms a C pattern with Studs 1 and 5. If Studs 1 and 5 are parallel, f and g should add up to 180°.

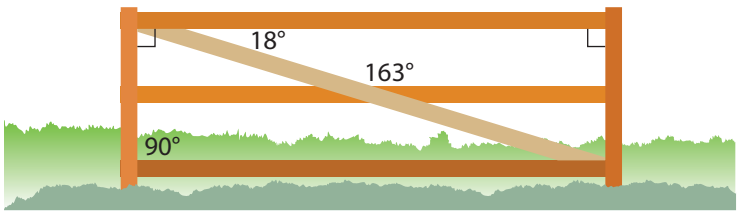
$\angle f$ and $\angle g$ add up to 180°.
Stud 5 is parallel to Stud 1.



What other methods can you use to check that these studs are parallel?

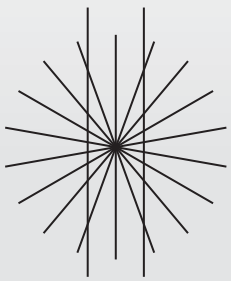
Your Turn

Connor has built a section of fence. The top and end pieces of wood meet at right angles, as shown. Connor measures some angles to check that the three horizontal pieces are parallel. Are they parallel? Give reasons for your answers.



Puzzler

Are the two side-by-side lines parallel? First guess, and then check using what you have learned about parallel lines.

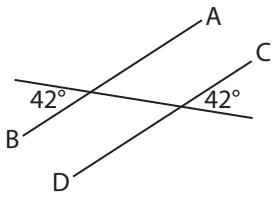


Check Your Understanding

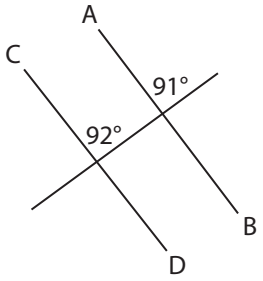
Try It

1. In each diagram, are AB and CD parallel? Explain using angle patterns or mathematical terms.

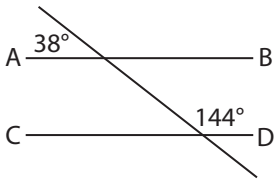
a)



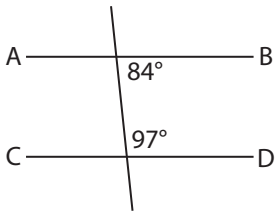
b)



c)

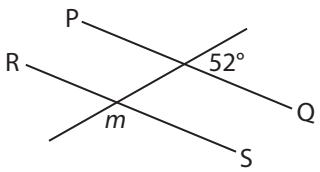


d)

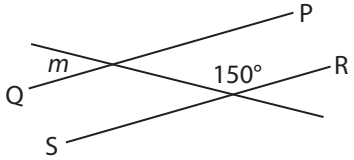


2. What must m measure for PQ and RS to be parallel?

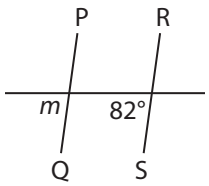
a)



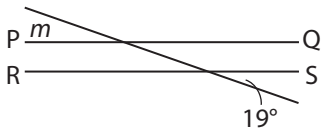
b)



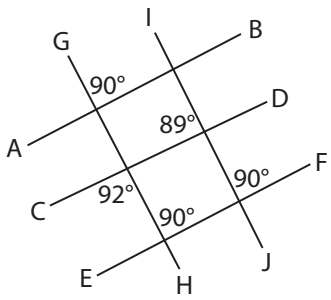
c)



d)

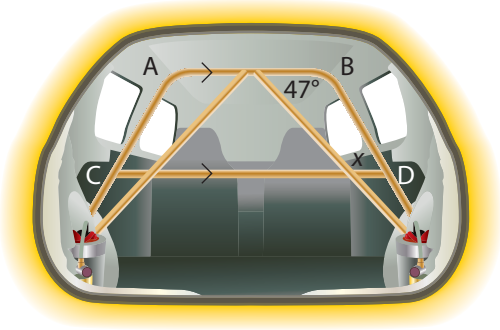


3. Which lines are parallel? Which lines are perpendicular? Give reasons for your answers.



Apply It

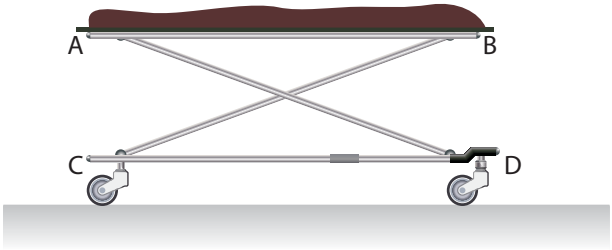
4. Race cars have built-in roll cages to protect drivers. The back of one type of roll cage includes bars in the formation shown. What does x need to measure for bars AB and CD to be parallel?



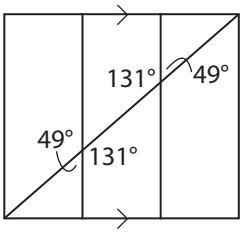
5. You can think of a gurney used by paramedics as being made up of two parallel lines and two transversals.

a) Identify the two parallel lines and two transversals.

b) The height of a gurney can be raised or lowered. If the height is changed, are the lines still parallel?

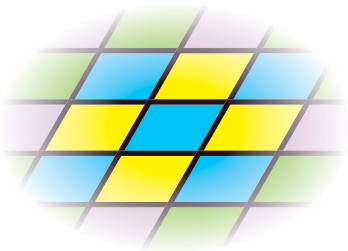


6. A Tudor-style house has "half timbering," which is exposed framing. Are the four vertical lines in this section of half timbering parallel? Give reasons for your answers.



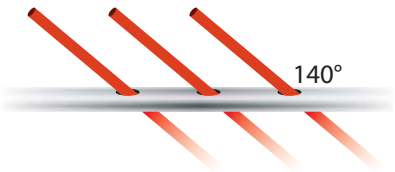
Work With It

1. Kali has drawn up a design for a square stained-glass window. The horizontal lines are parallel. The slanted lines are parallel. What is the minimum number of angles she needs to measure with a protractor to be able to determine the rest of the angles without a protractor? Explain your answer.



2. **a)** An outdoor TV antenna can be used to get HD TV. The antenna has parallel lines. For three parallel lines and the transversal, identify the measures of as many angles as you can.

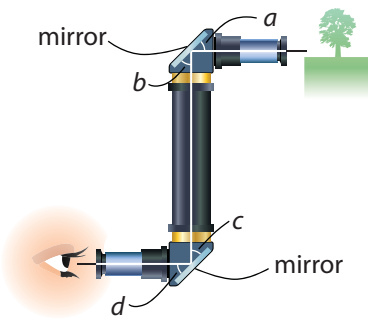
b) How many different sizes of angles are there besides straight angles?



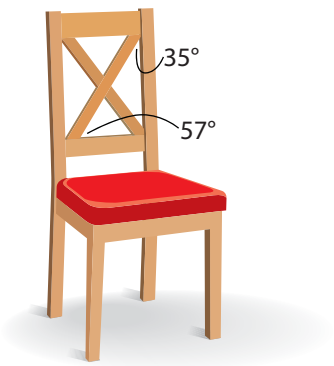
3. A periscope includes two parallel mirrors that face each other.

a) Where is the transversal between the two mirrors in the diagram?

b) Angle a is 45°. What are the measures of b, c, and d?



4. Lauren has partial plans for a wooden chair that she wants to make. She needs to know if the vertical pieces of wood on the backrest are supposed to be parallel. Determine whether they are parallel, and explain how you know.



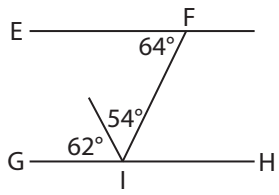
Discuss It

5. You have learned about the X, C, Z, and F patterns. What other letters do you think can be made with parallel lines and a transversal? Explain each answer.

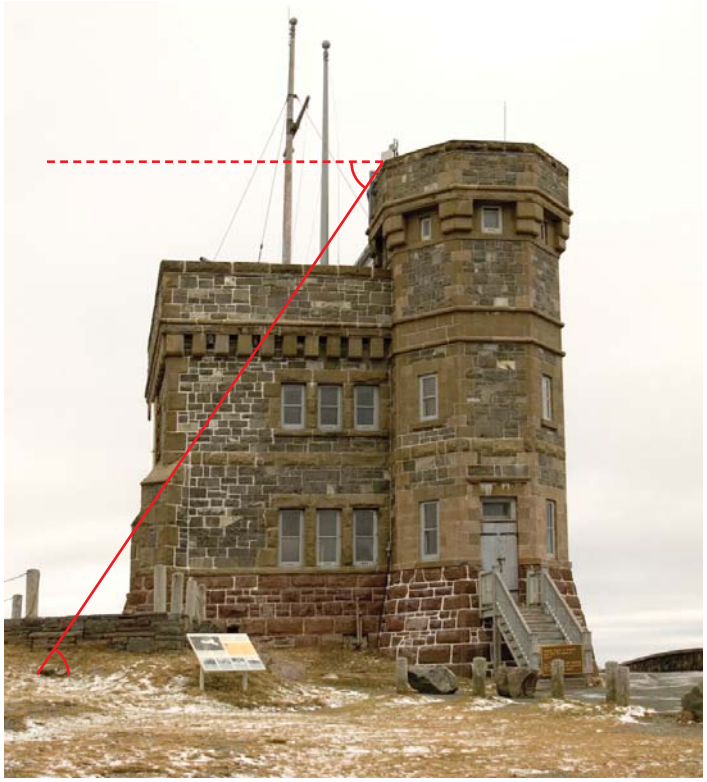
6. George is trying to solve the following puzzle. Line AB is parallel to line CD. Line EF is perpendicular to line CD. Line EF forms an 80° angle with line AB. George says that it is not possible to draw this diagram. Is he correct? Explain.

7. Explain how two parallel lines and a transversal can result in all equal angles.

8. Kristen says that EF is not parallel to GH because ∠EFI should be 62°. Is she correct? Explain.



9. Will says that the angle looking up at the top of Cabot Tower is the same as the angle looking down at the ground from the top of the tower. Do you agree with Will? Explain your answer.



F.Y.I.

Cabot Tower was built in 1897 to mark the 400th anniversary of John Cabot's discovery of Newfoundland, and Queen Victoria's Diamond Jubilee. It is found on Signal Hill near St. John's. In 1901, Guglielmo Marconi received the first trans-Atlantic wireless message in Morse code near the tower. For more information on Cabot Tower, go to www.mhrmathatwork10.ca and follow the links.

5 Skill Check

What You Need to Know

Section	After this section, I know how to . . .
5.1	■ estimate the measure of angles ■ measure angles using a protractor ■ classify types of angles
5.2	■ construct angles with a protractor ■ construct angles with set squares ■ bisect an angle
5.3	■ identify perpendicular, parallel, and transversal lines ■ identify patterns of angles formed by parallel lines ■ identify patterns of angles when two lines cross
5.4	■ solve problems involving angles formed by parallel lines

If you are unsure about any of these questions, review the appropriate section or sections of this chapter.

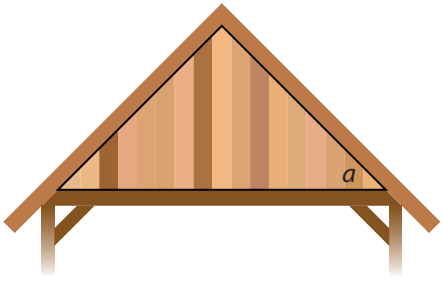
5.1 Estimating and Measuring Angles, pages 222–233

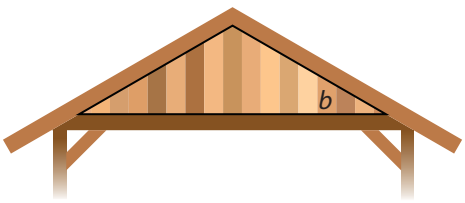
1. Keith is building a carport. He knows that the steeper the roof, the more it will cost to build. A roof that is less steep is safer to walk on and needs no special equipment to build.

a) Which roof is steeper?

b) Estimate the angle that each roof slants.

c) Measure the angle that each roof slants. How close were your estimates?





2. a) Sketch and label an example of an acute, a right, an obtuse, a straight, and a reflex angle.

b) Estimate the measure of each angle you drew.

c) Measure each angle.

5.2 Angle Constructions, pages 234–245

3. Sketch each angle.

a) 15° **b)** 40° **c)** 90° **d)** 180°

4. Use a protractor to construct each angle. Then, bisect each angle.

a) 20° **b)** 46° **c)** 102° **d)** 337°

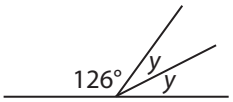
5.3 Lines and Angles, pages 246–261

5. Draw parallel lines and a transversal to show each type of angle.

a) same side interior angles **b)** corresponding angles

c) opposite angles **d)** alternate exterior angles

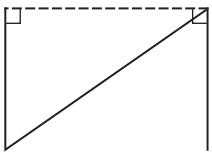
6. What are the measures of the angles labelled y? Justify your answer.



7. An angle measures 72°. How would you determine the measure of its complementary angle?

5.4 Angles in Our World, pages 262–273

8. The support structure for the Long Studio on Fogo Island has a structure similar to the diagram shown. Explain how you can determine each of the angles in the diagram using what you have learned in this chapter.



F.Y.I.

The Long Studio is one of a series of artist studios being built on Fogo Island. Artists will come from around the world to live and work in the area for several months at a time. For more information about the Long Studio, go to www.mhrmathatwork10.ca and follow the links.

5 Test Yourself

For #1 to #5, select the best answer.

1. Two angles have measures of 24° and 72°. Which expression best describes these angles?

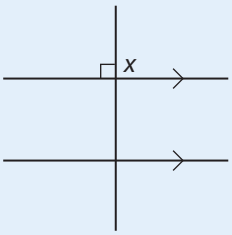
A complementary

B supplementary

C complementary and supplementary

D neither complementary nor supplementary

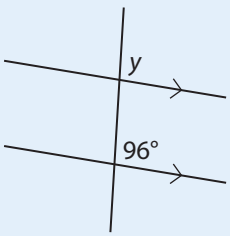
2. In the diagram shown, how many other angles have the same measure as x?



A 2 **B** 3

C 6 **D** 7

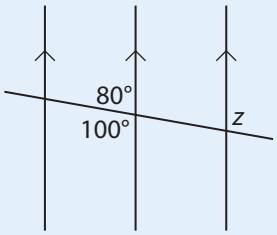
3. In the diagram shown, what is the measure of y?



A 46° **B** 84°

C 96° **D** 100°

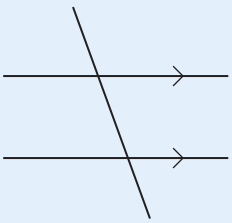
4. In the diagram shown, what is the measure of z?



A 35° **B** 80°

C 100° **D** 145°

5. For the diagram shown, which statement is correct?



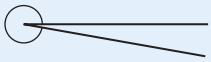
A Four pairs of angles are alternate interior or alternate exterior angles.

B Three pairs of angles are opposite angles.

C Three pairs of angles are same side interior or same side exterior angles.

D Two pairs of angles are corresponding angles.

6. Describe or sketch two strategies you can use to measure the angle shown.



7. Sketch each angle. Then, use a protractor to construct each angle.

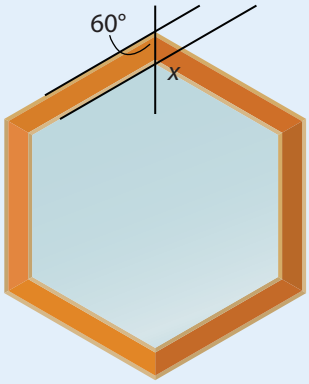
a) 30°

b) 263°

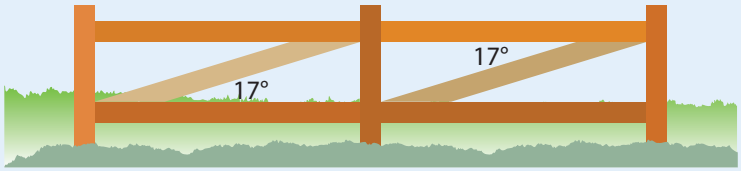
8. Draw a 65° angle. Use a compass or a protractor to bisect the angle.

9. A pair of complementary angles is the same size. What is the measure of each angle?

10. Alicia uses a mitre saw to cut angles for a picture frame. All pairs of inside and outside edges of the frame are parallel. Identify the measure of x.

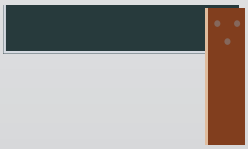


11. A fence is made from wood. Your friend challenges you to determine if the horizontal pieces of fencing are parallel to each other. How would you do this?



Tools of the Trade

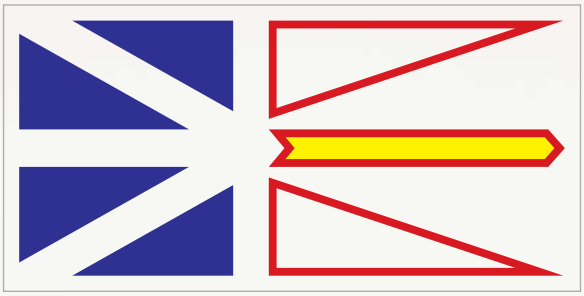
Carpenters use a carpenter's square to draw and confirm right angles. It has a long arm and a short arm that meet at 90°. Carpenters often use the term "square" to talk about right angles. Why do you think they use this term? To learn more about the tools carpenters use, go to www.mhrmathatwork10.ca and follow the links.



5 Chapter Project

Design a Flag

Before designing your own flag, work with the flag of Newfoundland and Labrador. This flag contains a number of different angles.



1. a) Some triangles in the flag have the same angles and side lengths. How many different triangles are used in the flag?

b) For each type of triangle, determine the sizes of the angles.

c) What sizes are the angles in the arrow?

2. a) On grid paper, sketch the flag.

b) Label the sketch with the sizes of the angles.

c) Label the side lengths of all the shapes in the flag.

3. Design your own flag using colour. Include at least four of the features in the list.

- ✓ parallel lines
- ✓ a transversal
- ✓ perpendicular lines
- ✓ an angle bisector
- ✓ alternate exterior angles
- ✓ alternate interior angles
- ✓ corresponding angles
- ✓ opposite angles
- ✓ same side exterior angles
- ✓ same side interior angles

a) On grid paper, draw two rectangles that are each 12 cm by 20 cm.

b) Use one rectangle for a sketch. Label the sizes of the angles. Highlight the features you chose.

c) Use the other rectangle for the final copy.

4. Explain how your flag design shows what you have learned about angles.

5 GAMES AND PUZZLES

Within 10

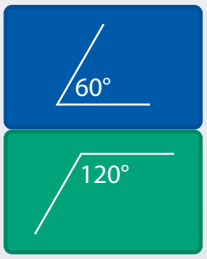
Materials
- small squares of paper
- ruler
- protractor
- Within 10 game cards

1. Play this game with a partner. Start at zero and count angles by tens up to 180°. Write each angle on a small square of paper. Use these as game cards.
2. Place the cards face down on a table. Turn over one card. Both of you use estimation to help draw the angle. Use a pencil and ruler. Then, classify the angle.
3. Trade drawings. Use a protractor to measure your partner's angle. If the angle is within 10° of the angle on the card, your partner earns one point.
4. Discard that card, and record your scores. Then, pick another card. When all the cards are gone, add up the points to see who wins.

By the Letter

Materials
- By the Letter game cards
- coin

The angle cards for this game are labelled with angle measures. You can put the angle cards together to make the letters C, F, X, or Z. An example is shown.



1. Play this game with a partner. Flip a coin to decide who will be Player 1 and Player 2.
2. Player 1 shuffles and deals all of the cards. Player 2 puts down one card.
3. Player 1 puts down one card to complete the letter C, F, X, or Z with the correct angle measures.
   - If Player 1 is successful, Player 1 gets one point.
   - If Player 1 is unsuccessful, Player 2 puts down a card to complete the letter.
   - If Player 2 is successful, Player 2 gets one point.
   - If Player 2 is unsuccessful, the cards are discarded. No one gets a point.
4. Repeat step 2 taking turns to put down the first card.
5. When both players run out of cards, add up the points to see who wins.
6. With your partner, discuss strategies you can use to help you win the game.