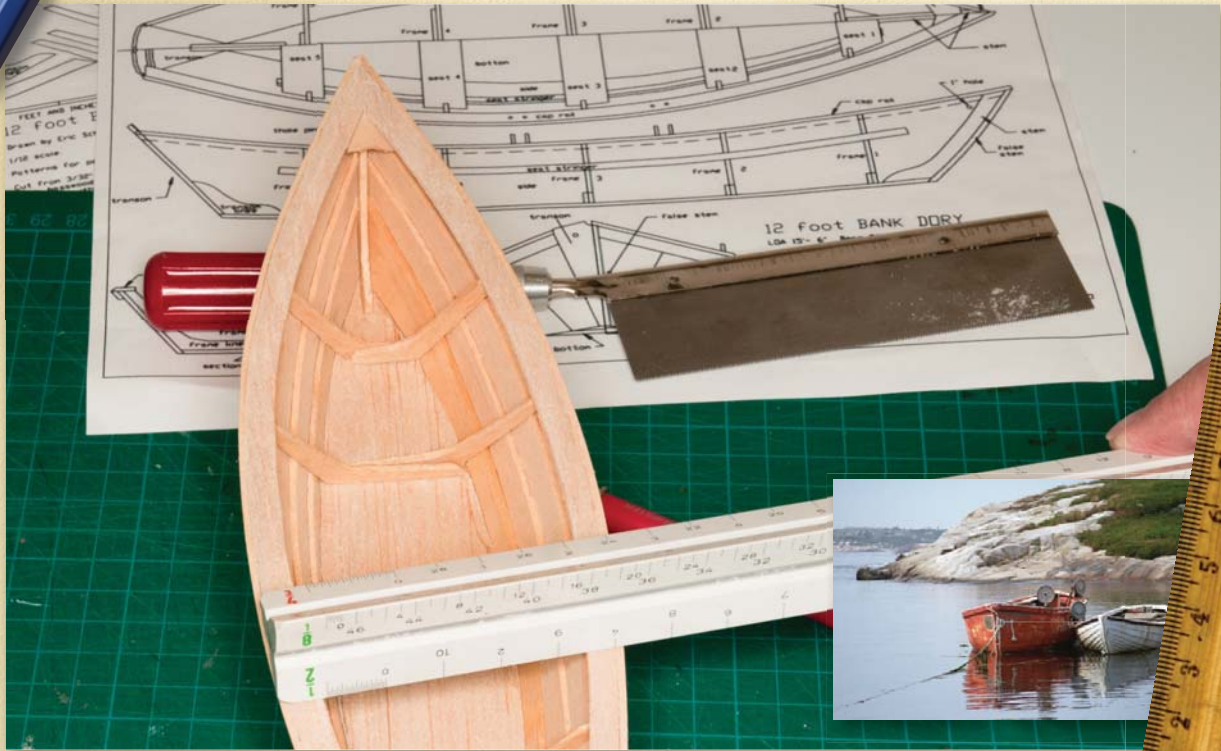


# Math at Work 11: Chapter 2

February 20, 2012, 16:15

# 2

# Drawing and Design



George builds model dories. He is meticulous with the accuracy of his measurements. His creations are exact scale replicas of Newfoundland Dories.

1. What is a scale replica?
2. Describe some scale replicas you own or have seen.
3. What measurements do you think George needs to make in order to build a model to scale?

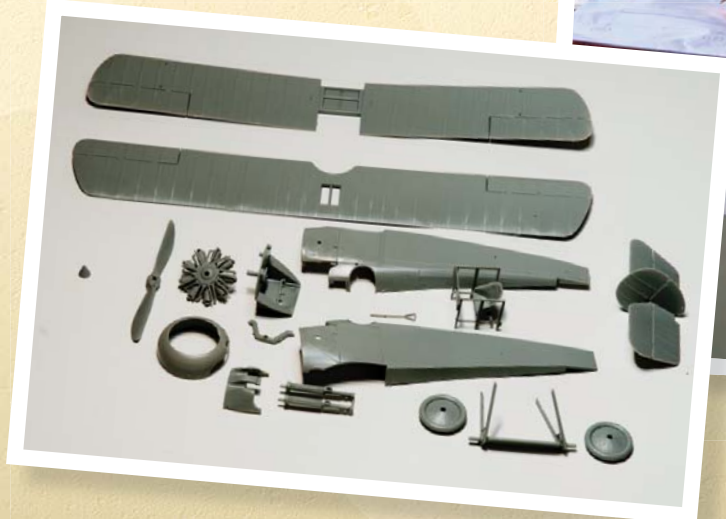
## Key Words

scale  
scale drawing  
orthographic drawing  
isometric drawing  
one-point perspective drawing  
vanishing point  
point of perspective  
exploded view diagram



## Career Link

Bryan is a framer. He builds walls, door frames, stairs, and roof trusses, among other things. Bryan uses other people's drawings to determine the size and location of the items he is responsible for building.



## Proportional Thinking

1. Solve to create equivalent fractions.

a)  $\frac{1}{4} = \frac{\square}{8}$

b)  $\frac{1}{8} = \frac{\square}{16}$

c)  $\frac{1}{4} = \frac{\square}{400}$

d)  $\frac{1}{12} = \frac{\square}{60}$

e)  $\frac{1}{4} = \frac{8}{\square}$

f)  $\frac{1}{2} = \frac{16}{\square}$

g)  $\frac{1}{10} = \frac{70}{\square}$

h)  $\frac{1}{20} = \frac{40}{\square}$

2. Solve to create equivalent ratios.

a)  $1:5 = \square:20$

b)  $1:12 = \square:72$

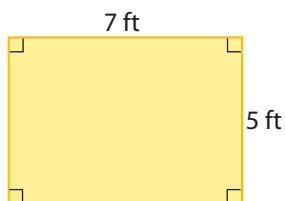
c)  $2:3 = \square:12$

d)  $9:16 = \square:48$

e)  $40:1 = 200:\square$

f)  $8:5 = 200:\square$

3.

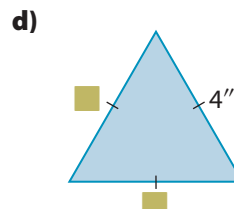
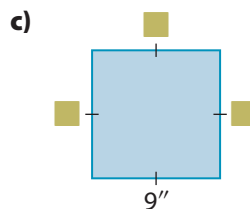
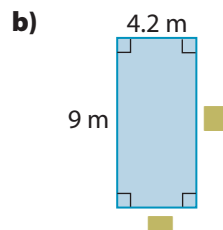
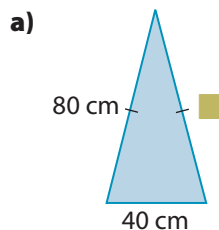


- What is the perimeter of the rectangle?
- What is the area of the rectangle?
- If the length of the rectangle is doubled, is the perimeter doubled?
- If the length of the rectangle is doubled, is the area doubled?
- Calculate the perimeter and the area of the larger rectangle.

## Working With Diagrams

These are examples of hash marks: / and //. Hash marks on a diagram identify sides that are the same length.

4. Solve for the unknown values.



5. For each right triangle, determine the missing length.

To determine the missing length of a right triangle, use the Pythagorean relationship.

$$a^2 + b^2 = c^2$$

$$4^2 + b^2 = 5^2$$

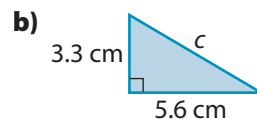
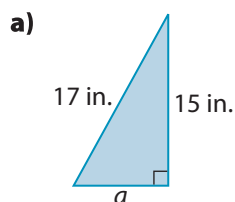
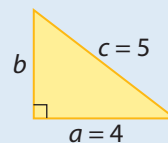
$$16 + b^2 = 25$$

$$b^2 = 9$$

$$b = \sqrt{9}$$

$$b = 3$$

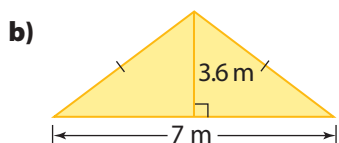
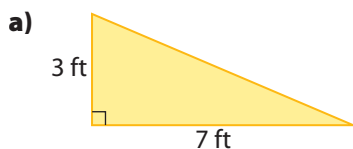
The length of  $b$  is 3 cm.



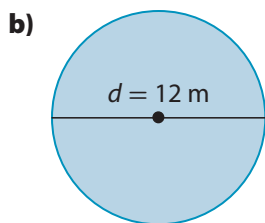
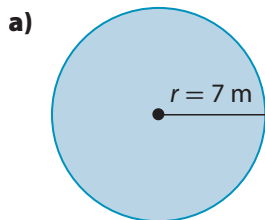


## Area

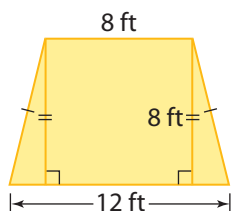
6. Determine the area of each shape.



7. Calculate each area, to the nearest square metre.

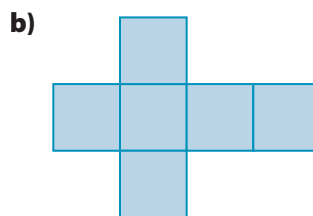
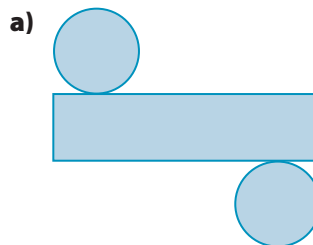


8. Calculate the area of the composite shape.

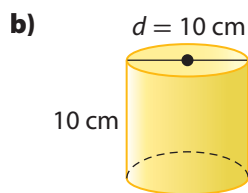
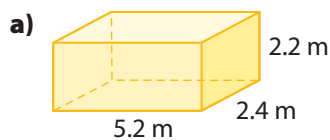


## Nets and Surface Area

9. Name the shape made by each net.



10. Estimate and then determine the surface area of each figure.



11. On grid paper, sketch a net of each of the figures in #10.

# 2.1

## Working With Scale

### Focus On ...

- using proportional reasoning to determine the dimensions of a scale drawing or model
- drawing a scale diagram of a given object
- solving problems that involve scale
- describing when a scale representation might be used
- constructing a model of a 3-D object

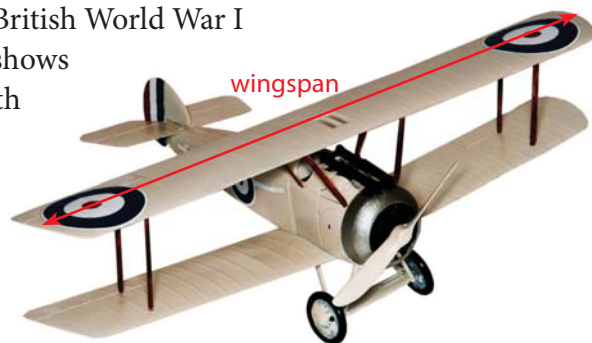
*A hobbyist building a model, a carpenter reading a drawing, and a dressmaker working from a pattern all need to have a good knowledge of scale. Also, long-distance truck drivers who must interpret the **scale** of a map.*

### scale

- the relationship between a distance on a drawing, model, or map and the actual distance
- for example, a scale of 1 cm : 1 m means that 1 cm on the diagram, model, or map represents 1 m actual size

### Explore Scale

The Sopwith Camel was a British World War I fighter plane. This picture shows a scale model of the Sopwith Camel. In a scale model, every dimension of the actual plane has been reduced by the same factor.



### Materials

- imperial ruler

The actual wingspan of the Sopwith Camel is 28 feet.

1. Convert 28 feet to inches.
2. Measure the distance of the wingspan in the picture, in inches.
3. How many times greater is the actual wingspan than the one in the picture?
4. **Reflect**
  - a) Express the wingspan in the picture and the wingspan of the actual plane as a ratio.
  - b) State the ratio in its simplest form.
5. **Extend Your Understanding** Measure the diameter of one of the wheels in the picture. Use the ratio from step 4b) to determine the approximate diameter of the wheel on a real Sopwith Camel.

Express the ratio in the form  $\square : \square$ .



### Tools of the Trade

### Web Link

Plan a scale model of the solar system. Decide on the diameter of the sun for your scale model, and learn what sizes the planets should be. Go to [www.mcgrawhill.ca/school/learningcentres](http://www.mcgrawhill.ca/school/learningcentres) and follow the links.

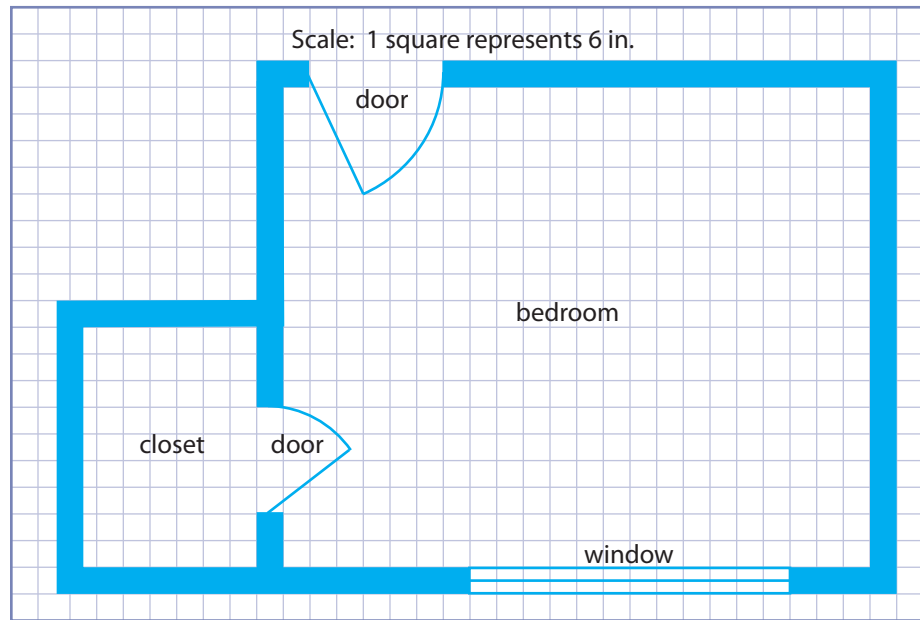
Model builders produce life-size and scale models of many vintage planes, including the Sopwith Camel and the Vickers Vimy, shown here. A replica of the Vickers Vimy made a non-stop flight from St. John's, NL, to Ireland in 2005. This trip duplicated early flights made by the original Vickers Vimy aircraft. To make replicas, model builders need precision measuring tools and a knowledge of scale. To learn more about vintage aircraft models and the people who make them, go to [www.mcgrawhill.ca/school/learningcentres](http://www.mcgrawhill.ca/school/learningcentres) and follow the links.




## On the Job 1

### Read Scale Drawings

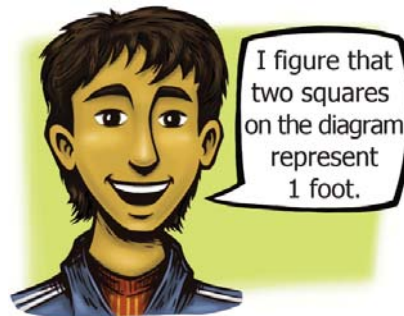
This is a **scale drawing** of Fiona's bedroom done on  $\frac{1}{4}$  inch grid paper.



- Explain the scale.
- Convert the scale of the diagram to a 1:  ratio.
- What are the dimensions of Fiona's room, in feet?
- How wide are the doors? Door dimensions are quoted in inches. Show your answer in inches.
- How deep is the closet? Show your answer in feet and inches.

### Solution

- a) One side of one square on the diagram represents a length of 6 inches in the room.



### F.Y.I.

A ratio shows the relationship between two values that have the same units. A scale can show the relationship between two values that have the same units or different units.



**Strategy**



**Look for a Pattern**

**F.Y.I.**

Ratios can also be written as fractions.

1 : 24 can also be written as  $\frac{1}{24}$ .

**Strategy**



**Look for a Pattern**

- b) The diagram is on  $\frac{1}{4}$  inch grid paper. The length of one side of each square on the diagram is  $\frac{1}{4}$  long.

$\frac{1}{4}$  represents 6".

$\frac{2}{4}$  represents 12".

$\frac{3}{4}$  represents 18".

1" represents 24".

The drawing is a 1 : 24 reduction of the room.

- c) On the diagram, the length of the room is 22 squares.

Since 2 squares represent 1 foot, the actual room is  $22 \div 2 = 11$  feet long.

The width of the room is 18 squares, so the room is 9 feet wide.

- d) One square represent 6 inches. The closet opening is 4 squares, so the closet door is  $4 \times 6 = 24$  inches wide.

On the diagram, the main door to the room is 5 squares wide.

The door to the room is  $5 \times 6 = 30$  inches wide.

- e) The depth of the closet is  $6\frac{1}{2}$  squares on the diagram.

$$6.5 \times 6 = 39$$

The closet is 39 inches deep.

Change 39 inches to feet and inches.

$$1' = 12''$$

$$2' = 24''$$

$$3' = 36''$$

$$39'' = 3' 3''$$

The closet is 3' 3" deep.

$$6 \times 6 = 36$$

$$0.5 \times 6 = 3$$

$$36 + 3 = 39$$

**Your Turn**

Refer back to the scale drawing of the bedroom.

- a) How far from the top-left corner of the room is the middle of the window? State the measurement in inches. **Hint:** Use the Pythagorean relationship.
- b) How wide is the window? State the measurement in feet.
- c) How wide is the closet? State the measurement in feet and inches.

## Check Your Understanding

### Try It

- Jackie is building a scale model of a garden shed. She will let 1 inch represent 2 feet. If the base of the shed measures 8 feet by 12 feet, what measurements will Jackie need for the model?
- Write each scale as a 1 :  $\square$  ratio.
  - 1 cm to 1 m
  - 1 inch to 1 foot
  - 1 cm to 1 km
  - $\frac{1}{4}$  in. to 1 ft
- A common scale for collectible toy cars is 1:64. Use the following measurements of the scale model to determine the actual measurements of the 1959 Volkswagen Beetle. Round your answers to the nearest centimetre.
  - length = 6.4 cm
  - width = 2.4 cm
  - height = 2.3 cm
  - wheel diameter = 5.6 mm
- The most famous aircraft designed and built in Canada was the Avro Arrow. A number of  $\frac{1}{8}$  scale models were made and tested in a wind tunnel. The length of each model was 10' 8". What was the length of the full-size Arrow?



### Web Link

Check the size of an actual Volkswagen Beetle by going to [www.mcgrawhill.ca/school/learningcentres](http://www.mcgrawhill.ca/school/learningcentres) and following the links.



### Tools of the Trade

Engineers frequently start with scale models of the items they plan to build. Scale models allow them to test how the environment will affect the item. To learn more about building models for this purpose, go to [www.mcgrawhill.ca/school/learningcentres](http://www.mcgrawhill.ca/school/learningcentres) and follow the links.

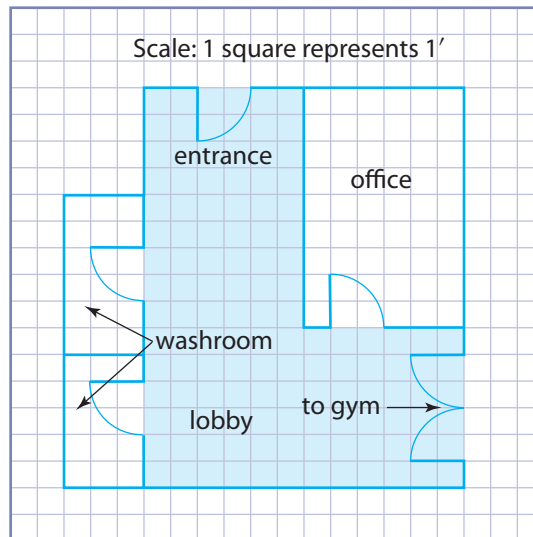
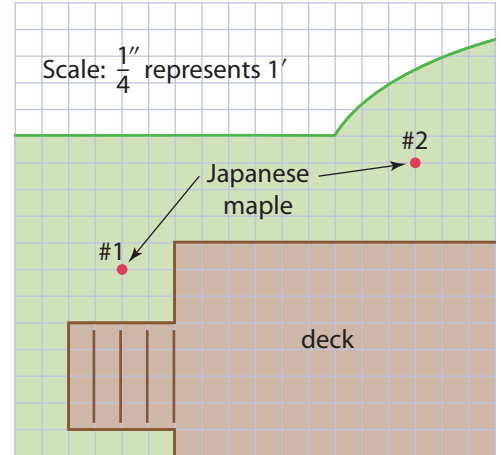


## Apply It

5. Part of a scale drawing for Josh's landscape design is shown below. The design is drawn on  $\frac{1}{4}$  inch grid paper.

*For parts a) to c), measure from the centre of the tree.*

- How far from the steps does Josh plan to plant tree #1?
  - How far from the deck will he plant tree #2?
  - Calculate the distance between the two trees, to the nearest foot. **Hint:** Use the Pythagorean relationship.
  - Using a floor with square tiles, mark the location of the two trees relative to each other.
6. Create a scale drawing of a room in your school. Choose an appropriate scale. Include the locations of all doors and windows.
7. Determine the number of 6"-by-6" floor tiles needed to cover the entrance and lobby of this recreation centre. The entrance and lobby are shaded blue.





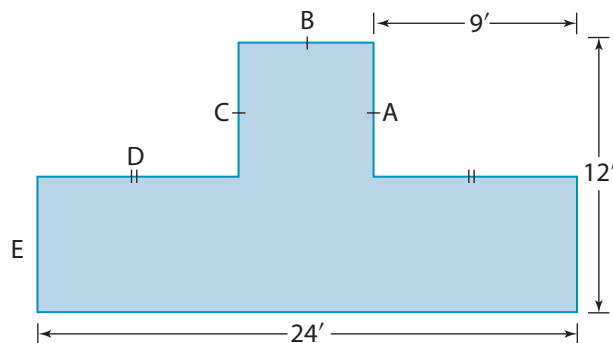
## On the Job 2

### Determine Missing Dimensions

To avoid clutter, many scale drawings include a minimum amount of information. Use the measurements given to determine any missing dimensions.



Determine the lengths of walls A to E identified on the diagram.



### Solution

The first dimension that can be determined is D.

It is the same length as the wall with the same number of hash marks. So, wall D is 9 feet long.

B is next.  $9 + 9 = 18$ . The length of the wall at the bottom of the drawing is 24 feet. The length of the wall at the top must be the same.

Therefore, B must be  $24 - 18 = 6$  feet long.

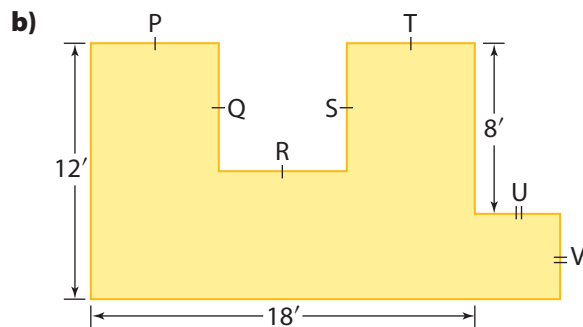
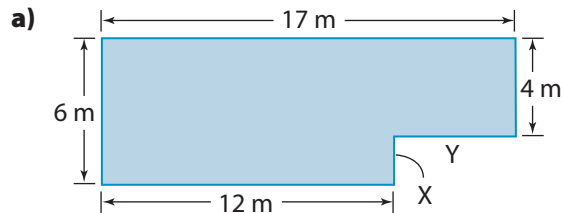
The hash marks on the diagram indicate that A and C are the same length as B, so they are 6 feet long as well.

Since A is 6 feet and the overall dimension on the left of the diagram is 12 feet, E must also be 6 feet long.

*C is also 6' long. You could use the length of C to calculate the length of E.*

### Your Turn

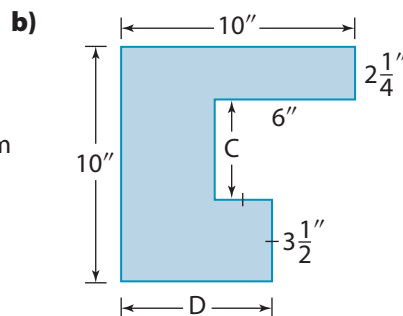
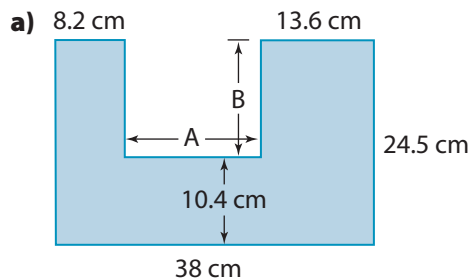
Determine the missing dimensions.



## Check Your Understanding

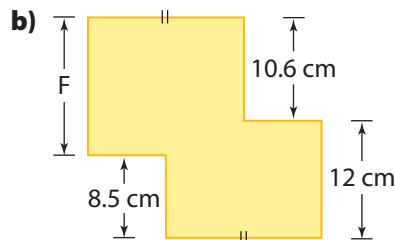
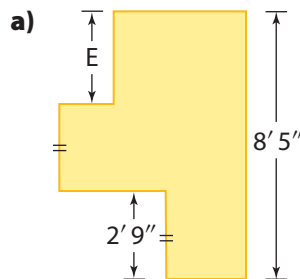
### Try It

1. Determine the missing dimensions.



2. Calculate the perimeters of the figures in #1.

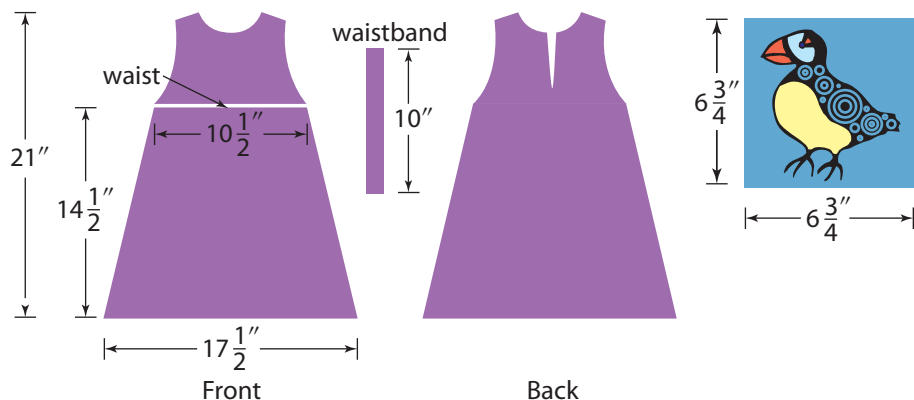
3. Determine the missing dimensions.



4. Can you calculate the perimeters of the figures in #3? Explain.

### Apply It

5. Krista wants to make a simple dress for her niece. She downloads a dress pattern from the Internet. Krista wants to put a decal above the waistband on the front. Is there enough room for the decal shown? Show your work.



### Strategy



#### Develop a Strategy

$$\begin{array}{r} 8' 5'' \\ - 2' 9'' \\ \hline \end{array}$$

What strategy might you use to solve this subtraction question? Discuss your strategy with a peer. Try different strategies. Do they produce the same answer?



#### Tools of the Trade

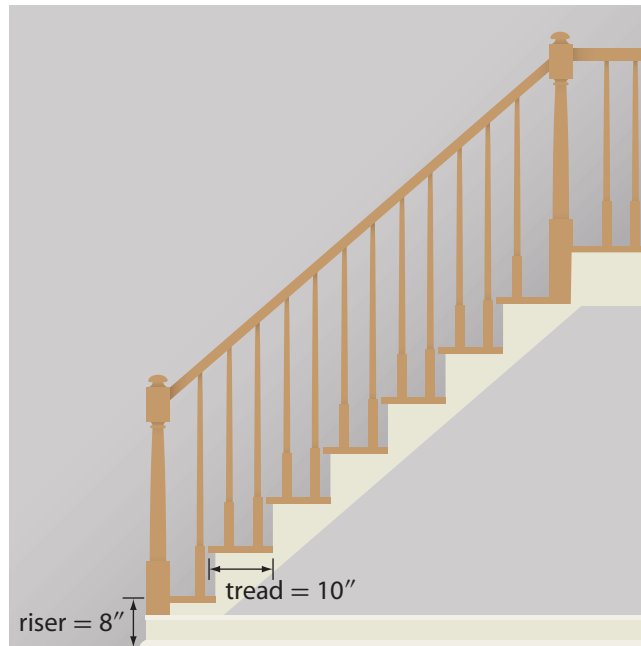
The patterns used by seamstresses and tailors are scale drawings. To learn how to adjust a pattern for another size, go to [www.mcgrawhill.ca/school/learningcentres](http://www.mcgrawhill.ca/school/learningcentres) and follow the links.



**F.Y.I.**

The riser is the vertical distance from one step to another. The tread is the width of a step that you walk on.

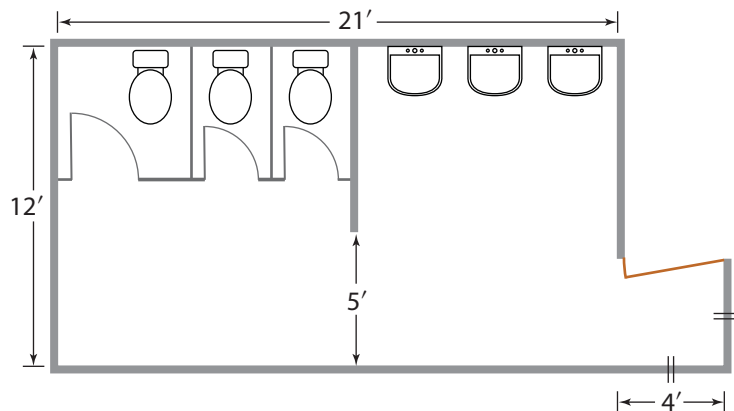
6. Determine the overall height and the overall length of the stairs from the floor to the landing.



7. A hospital's maintenance schedule calls for the walls of all washrooms to be painted. Below is a scale drawing of one washroom.

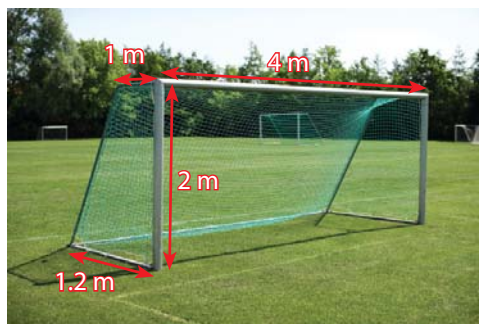
- All of the walls are made from concrete block that is 6 inches thick.
- The wall separating the sinks from the toilets goes from the floor to the ceiling.
- The ceiling is 9' 6" high.
- The door is 7' high.

Determine the total area to be painted.

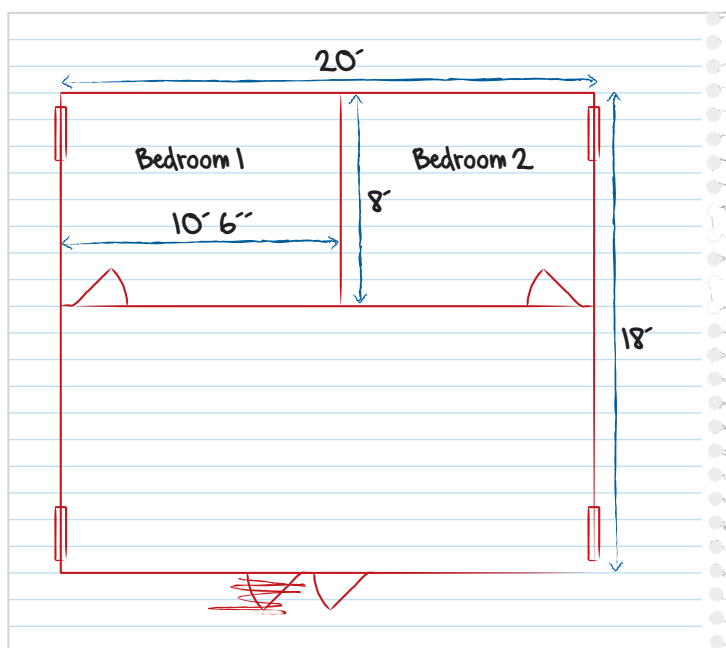


## Work With It

- Use a scale of 1 square to 10 cm to create a scale drawing of the front of the soccer goal shown.
  - Use the same scale to draw the side of the goal.



- Bryce plans to renovate his family's cabin. To help plan, he made a quick sketch of the layout. The diagram is not to scale.



- The wall between the bedrooms is made of log and is 8 inches thick. Determine the missing dimension of bedroom 1 and of bedroom 2.
- Draw a scale diagram using the scale 1 square: 1 ft. The bedroom doors are 30 inches wide and 6 inches from the corner of the room. The main door is 36 inches wide and is centred on the wall. Each window is 36 inches wide and starts 2 feet from the closest corner.

### F.Y.I.

One square foot has  
144 square inches.  
 $1 \text{ ft}^2 = 144 \text{ in.}^2$

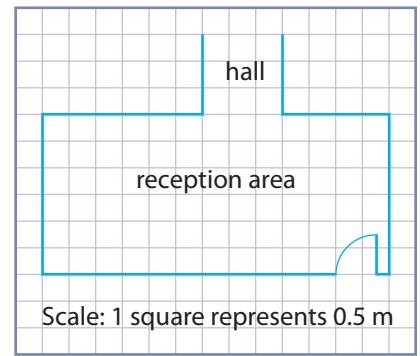


### Tools of the Trade

Artists and builders use scale drawings to get an accurate picture of a project they plan to build. To learn more about careers that use scale drawings, go to [www.mcgrawhill.ca/school/learningcentres](http://www.mcgrawhill.ca/school/learningcentres) and follow the links.

3. A dentist is hiring a local renovation company to update the reception area of the office.

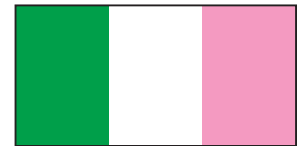
- How many squares make up 1 metre on the drawing?
- Count the squares on the drawing to determine the dimensions of the reception area.
- Determine the area of the floor of the reception area.
- The dentist has a lot of riding toys to keep young clients happy while they are waiting. To protect the walls, the dentist wants to run carpet along the bottom 10 cm of each wall. How many square metres of carpet will it take to cover the bottom of the walls in the reception area?



### Discuss It

4. Discuss with a group of peers the strategies used to answer #3d). Which strategy do you feel the most comfortable with?

5. The Newfoundland Tricolour can be of any size. The only condition is that the ratio of the flag's height to its width must be 1 : 2. Each colour takes up one third of the area of the flag.



- Explain the meaning of "a height to width ratio of 1 : 2."
- Create a scale model of the flag with a height of 15 centimetres.
- Many tourist shops sell miniature flags. On grid paper, or by using a computer, create a scale model of a flag that is 6 inches wide.

6. Make a sketch of an L-shaped room. Include some, but not all, of the dimensions. Give your sketch to a partner to solve. Have your partner explain the strategy used for determining the missing dimensions.

7. A town is considering expanding its curling facility. Explain why an architect bidding on the contract might want to develop scale drawings or a scale model of the project.



### F.Y.I.

The Newfoundland Tricolour is a popular but unofficial provincial flag. The green colour or pale is always on the hoist side, which is the side closest to the flagpole.

### Strategy



Draw or Model



# 2.2


## Representing Views of 3-D Objects

### Focus On ...

- drawing top, front, and side views of a given 3-D object
- drawing a 3-D object, given the top, front, and side views
- creating an isometric drawing of a given 3-D object
- determining whether the given views of a 3-D object represent the object

*Construction crews often work from scale drawings. Some drawings are 2-D views of what the final project will look like. Other drawings give the impression of being 3-D, even though they are drawn on paper.*

### Materials

- linking cubes
- grid paper 
- ruler

### Explore Different Views of a 3-D Object

1. Use linking cubes to build a model of a set of three steps. The tread of each step is 40 cm. The riser of each step is 20 cm. The steps are 1 m wide. Let one side of each cube represent 20 cm.



### Strategy



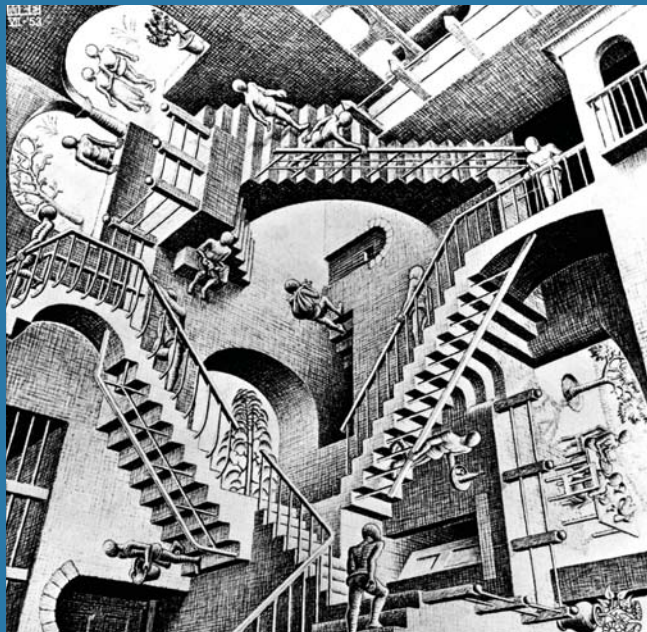
### Draw or Model

2.
  - a) Set the model on a desk. At eye level, look directly at the front of the steps. What shape do the outside edges of the model make?
  - b) Sketch what you see.
  - c) Set the model on the floor. Look directly at the top of the steps. What shape do the outside edges of the model make?
  - d) Sketch what you see.
  - e) Set the model on a desk. Look directly at the right side of the model at eye level. What shape do the outside edges of the model make?
  - f) Sketch what you see.
3. **Reflect** Where would you have to position yourself to have the model of the steps look like a 3-D object?
4. **Extend Your Understanding** Make a sketch of just one cube so that your 2-D drawing gives the impression of being 3-D.

In this Explore, you do not need to draw to scale.

### F.Y.I.

The artist M.C. Escher created a number of works of art that play with perspective.



*Relativity, M.C. Escher*

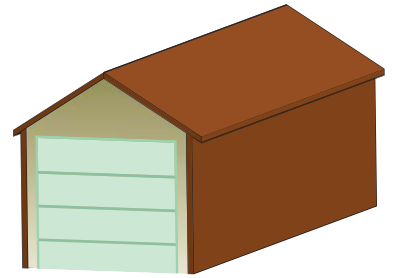
## orthographic drawing

- a 2-D view of a 3-D object
- often includes a front view, a top view, and a side view of the object

## On the Job 1

### Work With Orthographic Drawings

Nicole's company is bidding on a job to design and build a garage. Part of Nicole's presentation will be a set of **orthographic drawings** of what the garage will look like after it is built. The dimensions of the garage will be approximately 20 feet long, 12 feet wide, and 12 feet high.



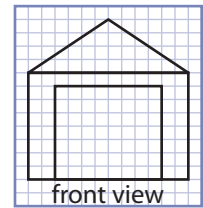
Create orthographic drawings of the garage using a scale of 1 square to 1 foot.

### Solution

#### Method 1: Create Orthographic Drawings by Hand

Front view:

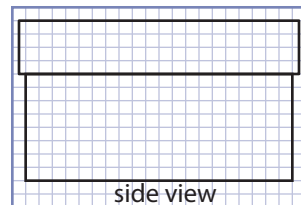
From the front, the garage appears as a rectangle with a triangular top. You can see the garage door from this view.



Side view:

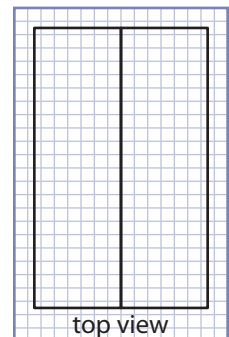
From the side, the garage appears as a longer rectangle. The roof also appears as a rectangle.

In the side view of an orthographic drawing, often only the right side view is given.



Top view:

From the top, only the roof is visible. Since the roof overhangs the garage a bit, the top view measures approximately 21 feet by 13 feet. The ridge at the top of the roof is also visible. It is shown by the line down the centre.



## Method 2: Create Orthographic Drawings Using GSP

Open *The Geometer's Sketchpad*®. Choose **Open** from the **File** menu. Navigate to the directory that stores *The Geometer's Sketchpad*® on your computer. This will likely be called **Sketchpad**, and be located in the **Program Files** directory. Choose **Samples**, then **Sketches**, and then **Geometry**. Select and open the file **Dot Paper.gsp**.

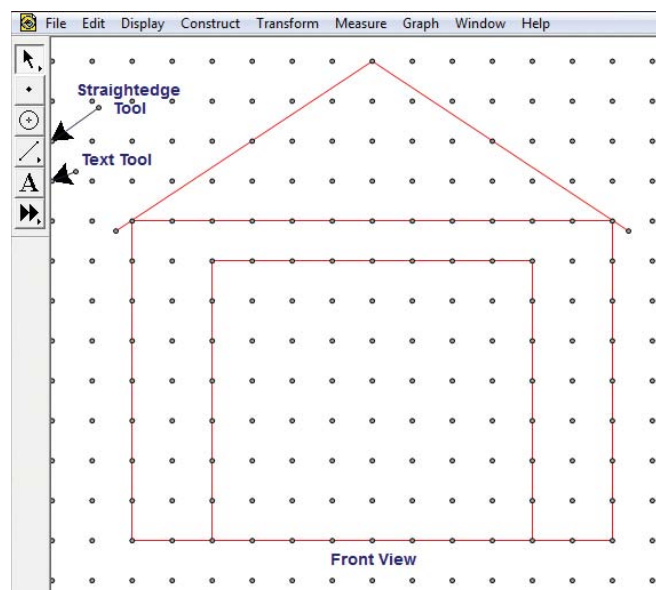
Press the button **Square dot paper**.

You can use the red slider to adjust the spacing of the dots in the pattern.

Select the **Straightedge Tool**. Use the square dot pattern to help you draw the front view of the garage. Use the **Text Tool** to label the drawing.

In the same way, draw the side view and top view of the garage.

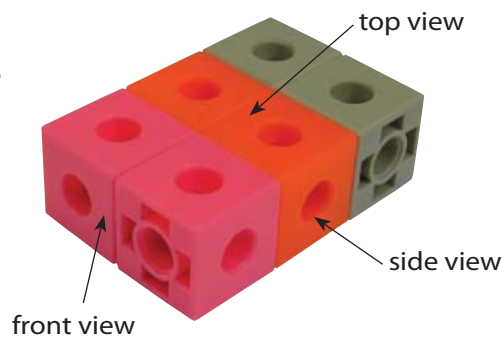
Save your file with a different name.



### Your Turn

Draw and label the front view, side view, and top view of the rectangular prism shown. Use a scale of 1 square to 1 cube. Draw the prism in two ways:

- on grid paper by hand
- using technology

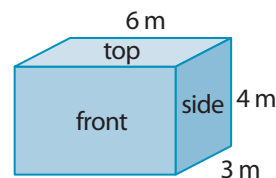




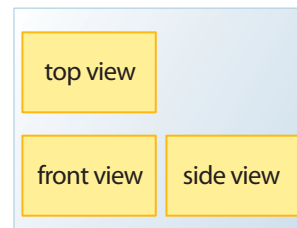
## Check Your Understanding

### Try It

1. State the dimensions for the front view, side view, and top view of the rectangular prism shown.



2. On one sheet of grid paper, draw the three orthographic views of the prism in #1. Use a scale of 1 square to 1 metre. Arrange the drawings on the page as shown.



3. Redraw the views in #2 using a scale of 1 square to 50 cm.
4. Use technology to create scale drawings of the three views of the rectangular prism in #1.
5. Sketch the three orthographic views of the set of steps you built in Explore Different Views of a 3-D Object on page 72. Use a scale of 1 square to 10 cm.

### Apply It

6. On paper or using technology, draw a set of orthographic drawings of the cereal box shown. The dimensions of the box are approximately 12" by 8" by 3".



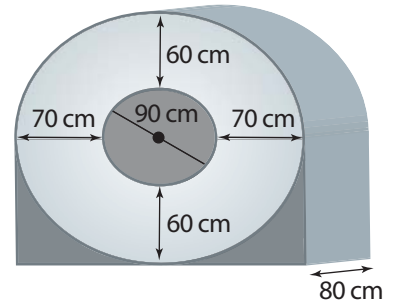
7. Draw the three views of a soup can that is 12 cm high and has a diameter of 8 cm.
8. A steam room in a fitness centre measures 10 feet by 6 feet. It has a U-shaped seating area around three sides of the room. The top view of the seating area is shown. Sketch the front view and the side view. The height of the seating area is 24 inches.



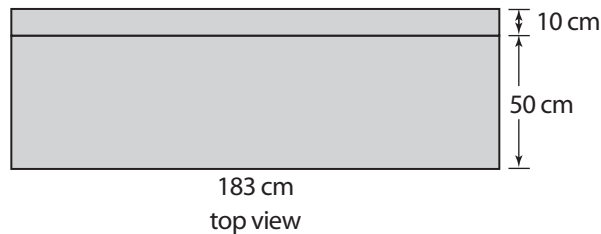
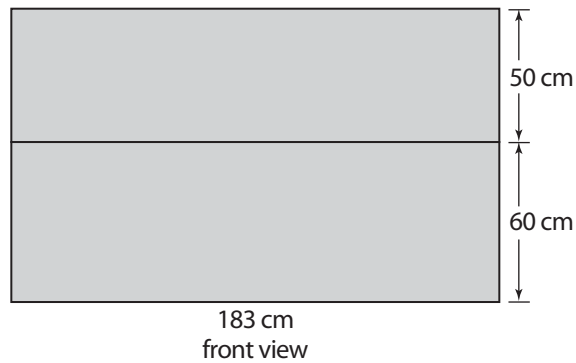
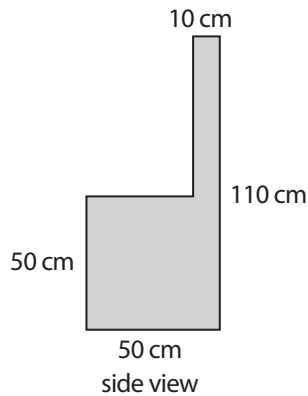
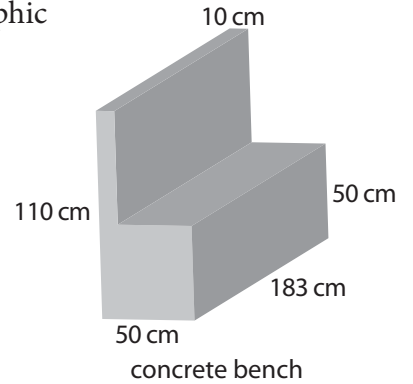
**F.Y.I.**

A CT scanner is used to examine patients. The scanner takes a series of X-ray views from many different angles to produce cross-sectional images of the bones and soft tissues inside the body. Patients lie on a bed that slides inside the centre hole of the scanner.

9. A medical CT scanner is shown. Sketch the front, side, and top views of the scanner.



10. a) Does each of the three orthographic drawings represent the concrete bench shown?

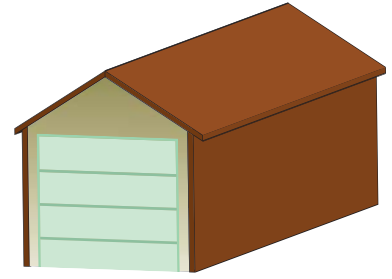


- b) If not, draw and label the correct view(s).

## On the Job 2

### Work With Isometric Drawings

Sometimes people get a better sense of what an object will look like if a drawing of it appears 3-D. Photographs have this type of 3-D effect. The drawing of the garage shown is not a photograph, but it gives the illusion of being 3-D. You can create this effect using an **isometric drawing**.



### isometric drawing

- a view of a 3-D object in which
  - all horizontal edges of the object are drawn at a  $30^\circ$  angle
  - all vertical edges of the object are drawn vertically
  - all lines are drawn to scale

How would Nicole create an isometric drawing of the garage?

### Solution

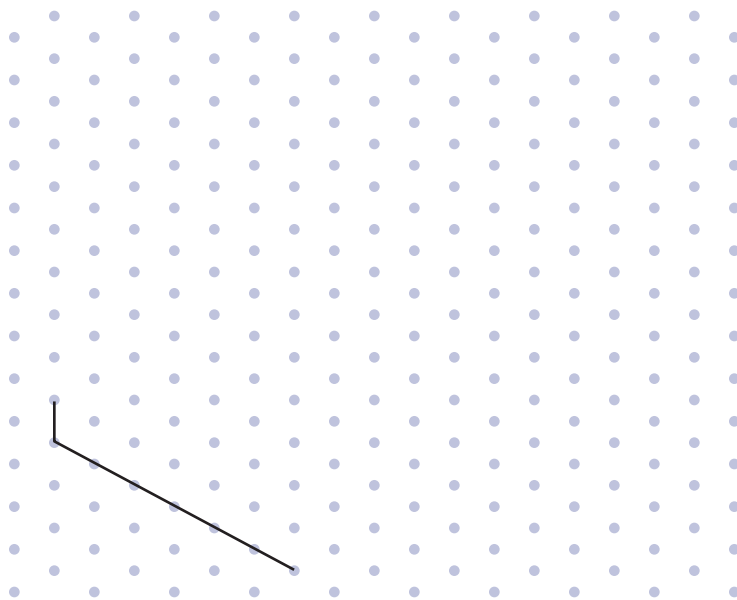
Nicole uses isometric dot paper. Her drawing does not include the slight overhang of the roof.

**Step 1** Draw a vertical line to connect two dots. This line is the beginning of the left side of the garage.

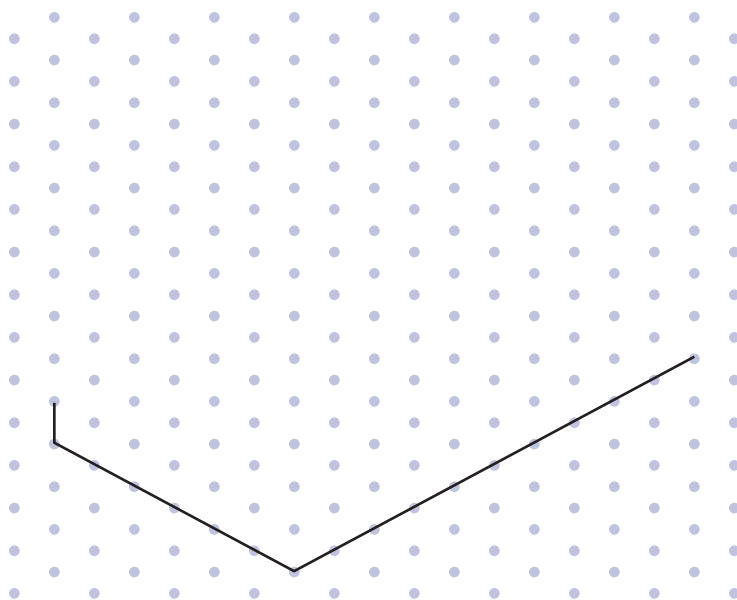
For the scale, this distance between adjacent dots represents 2 feet.



**Step 2** Connect the bottom of this line to a dot that is at a  $30^\circ$  angle from it. This distance represents 2 feet along the base of the garage. Extend this line to represent the 12-foot width of the garage.

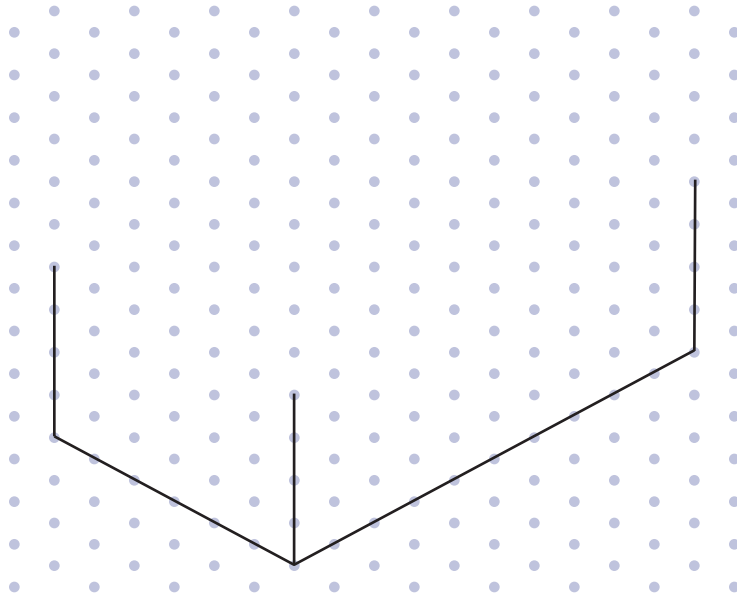


**Step 3** From the endpoint of this line, draw a line to represent the 20-foot length of the garage.

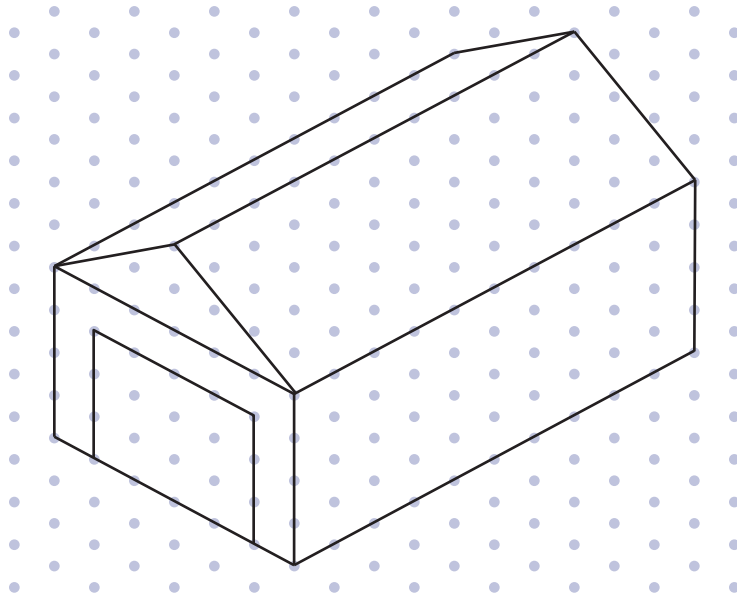




**Step 4** The height of the garage, below the roof, is 8 feet. Draw three lines to represent the height of the garage.



**Step 5** Keeping in mind that the distance between adjacent dots represents 2 feet, complete the diagram of the garage.



### Web Link

To use an online isometric drawing tool, go to [www.mcgrawhill.ca/school/learningcentres](http://www.mcgrawhill.ca/school/learningcentres) and follow the links.

### Your Turn

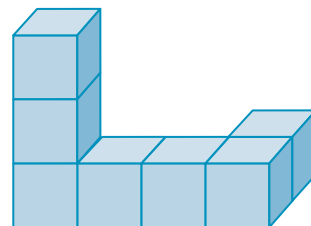
Create an isometric drawing of the cube shown.



## Check Your Understanding

### Try It

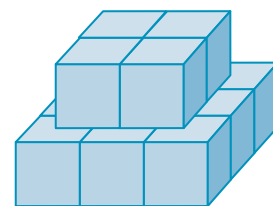
- Use linking cubes to build a rectangular prism that is 1 cube by 2 cubes by 3 cubes.
  - Use isometric dot paper to create an isometric drawing of the prism.
- Create the following shape using linking cubes. Then, create an isometric perspective drawing of it.



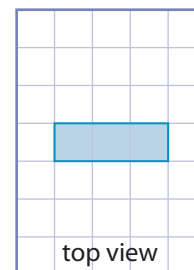
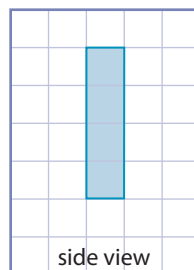
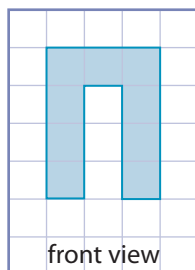
- Create an isometric drawing of the model of the steps that you built in Explore Different Views of a 3-D Object at the beginning of this section.

### Apply It

- Create any 3-D model you wish using linking cubes.
  - On isometric paper, sketch an isometric drawing of your model.
- Draw the front, top, and side views of the object shown.



- Create an isometric drawing of the 3-D object shown in the orthographic drawings. The scale is 1 square represents 3 ft.



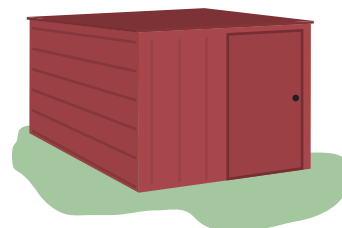
#### Tools of the Trade

Isometric drawing technology is used for a variety of purposes. For example, software for designing piping systems enables drafters to create isometric drawings of piping systems for industries such as

- oil and gas
- power generation
- chemical
- water/waste water
- food and beverage
- pulp and paper
- life sciences

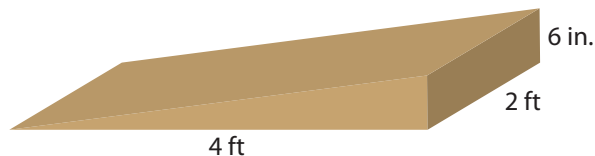
## Work With It

1. Pete wants to build a shed that is 2 metres wide and 3 metres deep. He wants the front of the shed to be 2 metres high and the back to be 1.8 metres high. The rectangular roof will be on a slope so that snow can slide off.

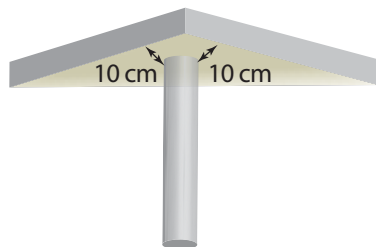


- a) Create a set of three orthographic drawings showing the front view, top view, and right-side view of Pete's shed.
- b) Create an isometric drawing of the shed. Use a scale in which the distance between vertical dots represents 20 cm.

2. A skateboard ramp is in the shape of a triangular prism.



- a) Create orthographic drawings of the front, top, and side views. Use a scale of your choice.
- b) Create an isometric drawing of the ramp. Use a scale of your choice.
3. The legs of a table have a diameter of 10 cm. The legs are attached 10 cm from the edges of the tabletop, as shown in the diagram. The table is 150 cm long, 90 cm wide, and 75 cm high.



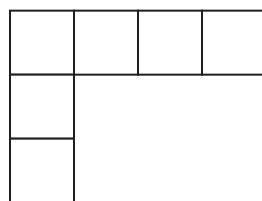
- a) Draw the front view and side view of the whole table.
- b) Describe what the top view would look like.

### F.Y.I.

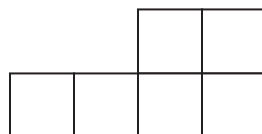
1 ft = 12 in.

$\frac{1}{2}$  ft = 6 in.

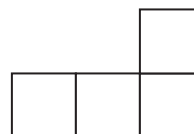
4. a) Build a model of the 3-D figure that has the following orthographic views.



top view



front view



right side view

- b) Create an isometric drawing of the object.

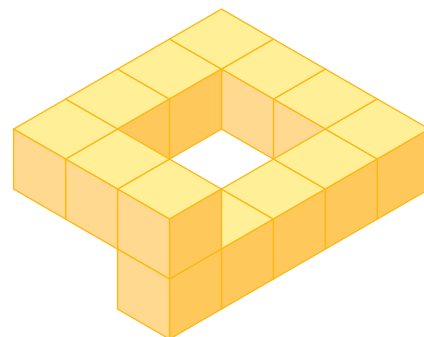
### Discuss It

5. Suppose you began an isometric diagram by drawing a circle on isometric dot paper. Would it look like a sphere when you completed the drawing? Explain. Include an isometric drawing with your explanation.

6. Explain why the front view and the side view of this cylinder look the same.



7. Explain why the isometric drawing shown cannot represent a real object.



8. Research and describe at least three jobs that involve working with scale diagrams.



# 2.3

## Representing Perspectives of 3-D Objects



### Focus On . . .

- drawing a one-point perspective view of a 3-D object
- identifying the point of perspective of a given one-point perspective drawing
- drawing the components of an exploded view diagram
- sketching an exploded view diagram of a 3-D object
- sketching a 2-D representation of a 3-D object, given its exploded view diagram

*Artists, architects, and interior designers draw 3-D objects in various ways. The way you draw a 3-D object depends on how far away you are from the object and what angle you are looking at it from. How things appear depends on your perspective.*

### Explore Perspective

#### Part 1: The Effect of Perspective on Parallel Lines

1. Go outside and find a pair of parallel lines, such as the two edges of a sidewalk. Look down near your feet and notice the distance between the parallel lines.



**2. Reflect** Slowly lift your eyes up and look out along the parallel lines. What do you notice about how the parallel lines appear as they get farther away from you?

**3. Extend Your Understanding**

- a) Sketch what you see in step 2.
- b) What appears to happen to the parallel lines if they go on forever?

**Part 2: The Effect of Perspective on Size**

- 4. a) Stand next to a large object, such as a car or a truck.
- b) Stretch your arms out so that you see the entire object between your outstretched hands.
- c) Estimate the distance between your hands.

**5. Reflect**

- a) Locate a similar car or truck that is farther away from you.
- b) Repeat steps 4b) and c). What do you notice?
- c) Repeat the steps for a similar object that is even farther away. What do you notice?

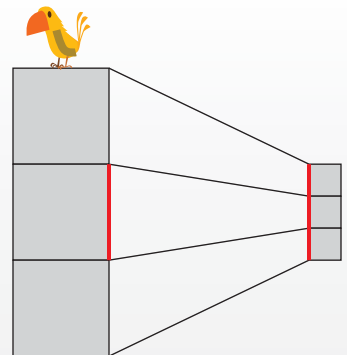
**6. Extend Your Understanding**

- a) Sketch the object from step 4a) on the bottom half of a piece of paper.
- b) Sketch the two objects from step 5 as if they were lined up behind the first item.

**Puzzler**

The picture shows a bird on a wall.

- a) Without measuring, predict whether the two red lines are the same length.
- b) Measure the two lines. Was your prediction correct?
- c) If your prediction was correct, explain your reasoning for making that prediction. If not, explain why you think you predicted incorrectly.



## On the Job 1

### Work With One-Point Perspective

- a) Janet is an artist who is working on a landscape scene. The scene includes a row of shrubs. Janet makes a **one-point perspective drawing** of the shrubs.
- b) Next, Janet works on a cityscape. She makes a one-point perspective drawing of a light rail transit train.

### Solution

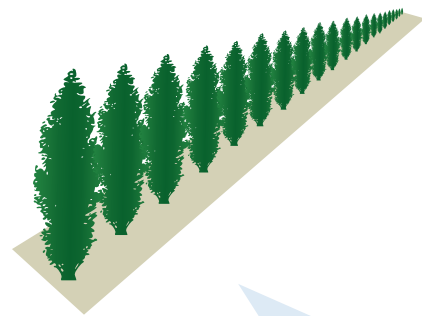
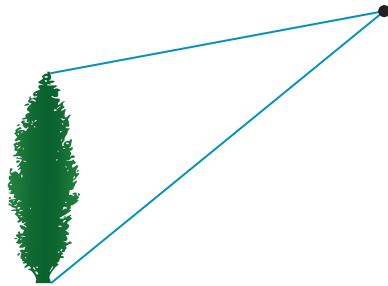
- a) On the left side of her paper, Janet sketches a shrub.

On the right side of her paper, Janet sketches a vanishing point.



Janet draws a line from the top of the shrub and from the bottom of the shrub to the vanishing point.

The lines show Janet the correct height for drawing the remaining shrubs so it looks like a row of shrubs going off into the distance.



The **point of perspective** of this one-point perspective drawing is slightly above and to the right of the original shrub. How do you know?

### one-point perspective drawing

- a drawing in which it looks like objects in the background join at a **vanishing point** in the distance

### vanishing point

- a point toward which parallel lines appear to join in the distance

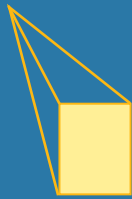
### point of perspective

- the position from which an object is being viewed

## F.Y.I.

When the point of perspective is

- above an object—the object appears to go upward into the distance
- below an object—the object appears to go downward into the distance
- to the right of an object—the object appears to go to the right into the distance
- to the left of an object—the object appears to go to the left into the distance



Point of perspective:  
up and to the left



Point of perspective:  
down and to the right

- b) On the right side of her paper, Janet sketches the face of a train.

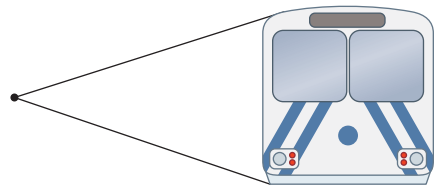


On the left side of her paper, Janet sketches a vanishing point.

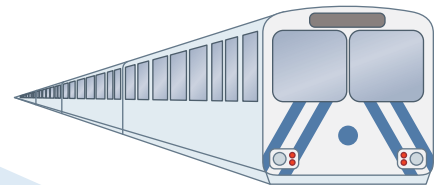
vanishing point



Janet draws a line from the top and from the bottom of the face of the train.



The lines show Janet how to draw the rest of the train so it looks like it is going off into the distance.



What is the point of perspective of the drawing?

### Your Turn

- On grid paper, draw a 3-by-3 square in the bottom-left quarter.
- Choose a vanishing point in the top-right quarter of the page.
- Draw lines from the top left, top right, and bottom right corners of the square to the vanishing point.
- Does the square begin to look like a rectangular prism? Explain.

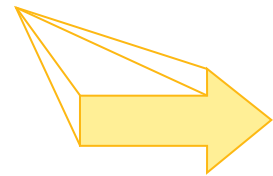
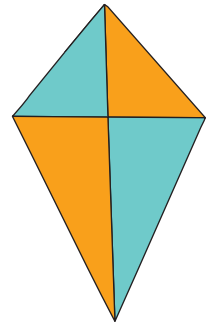


## Check Your Understanding

### Try It

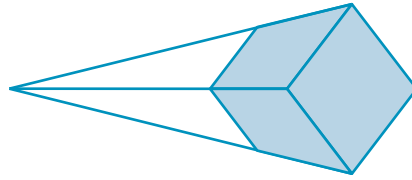
- Build a model of a rectangular prism with linking cubes.
  - On graph paper, sketch a scale diagram of one rectangular face of the prism in the bottom-left quarter of the page.
  - Locate a vanishing point on the right side of the page near the middle. Create a one-point perspective drawing of the rectangle.
  - Does the drawing appear to represent your model of a rectangular prism?
- In the middle of a piece of paper, copy the shape shown.
  - Mark a vanishing point on the left side of the paper. Create a one-point perspective drawing of the shape.
  - What is the point of perspective of the drawing? How do you know?
- The drawing shows a one-point perspective drawing.

  - Identify the shape.
  - What is the point of perspective of the drawing?
  - Using the same shape, create a one-point perspective drawing from a different point of perspective.
  - What is the point of perspective of your drawing?
- Draw a triangle in the middle of a sheet of paper.
  - Create a vanishing point on the paper.
  - Sketch a one-point perspective drawing.
  - Which 3-D figure does the drawing appear to represent?
  - What is the point of perspective of the drawing?

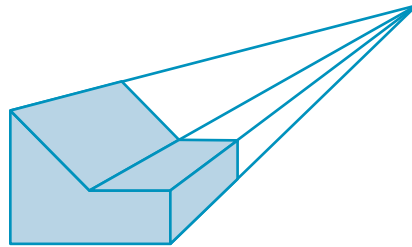


5. Identify the point of perspective for each drawing.

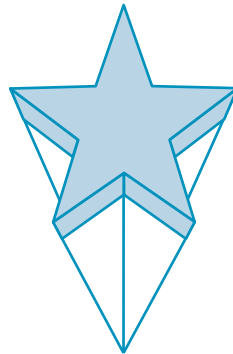
a)



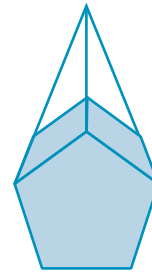
b)



c)



d)



### Apply It

6.
  - a) On grid paper, draw a 4-by-4 square in the bottom-right quarter of the page. Place a vanishing point in the upper-left quarter. Create a one-point perspective drawing of a rectangular prism.
  - b) On another sheet of grid paper, draw a 4-by-4 square in the bottom half of the page near the middle. Place a vanishing point above the square in the top half of the page. Create a one-point perspective drawing of a rectangular prism.
  - c) Compare the two drawings. Does changing the location of the vanishing point give you a different impression of the rectangular prism? Explain.
7.
  - a) On the left side of a piece of paper, sketch any object.
  - b) On the right side of the paper, place a vanishing point.
  - c) Draw lines from the top of the object and from the bottom of the object to the vanishing point.
  - d) Using the lines as a guide, draw a vanishing row of the object you drew.

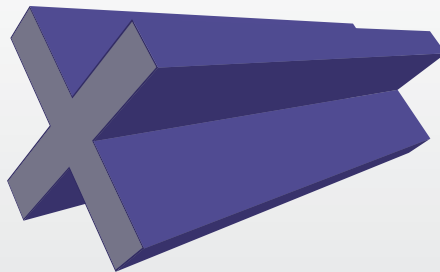
- 8. a)** On grid paper, near the centre of the page, write your first name in block letters. For example, a J might appear as shown.



- b)** Locate a vanishing point an inch or so above the midpoint of your name.
- c)** Create a one-point perspective drawing of your name.
- d)** What is the point of perspective of your drawing?
- 9.** Create four one-point perspective drawings of a 3-D object of your choice. Use a different point of perspective for each drawing.
- 10. a)** Create a drawing or painting of an outdoor scene. Include at least three objects drawn from a one-point perspective.
- b)** Show your artwork to a classmate. Discuss the 3-D effect in each other's artwork.
- 11. a)** Draw a four-sided 2-D shape.
- b)** Create an isometric drawing of the shape.
- c)** Create a one-point perspective drawing of the shape.
- d)** What are the similarities and differences between your drawings in part b) and part c)?

### Puzzler

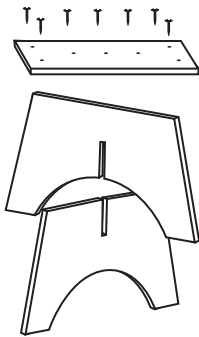
The picture shows a representation of a 3-D figure. Identify the total number of sides of the figure.



## On the Job 2

### exploded view diagram

- a drawing that shows the components of an object with the parts slightly separated
- often used to show the sequence of steps for assembling an object
- usually shown as a set of isometric drawings



### Strategy

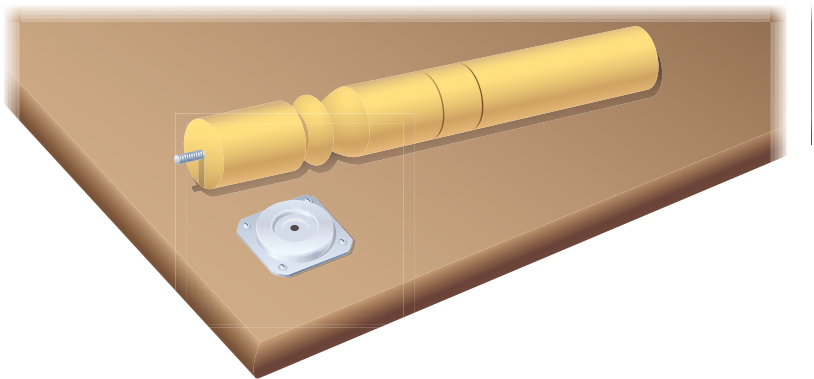


**Make a  
Systematic  
List**

### Work With Exploded View Diagrams

George writes instruction sheets for how to assemble “build-it-yourself” furniture. His instructions include a list of parts, their quantity, and the steps needed to build the piece of furniture. He also includes an **exploded view diagram**. George’s completed instructions are always done using drawing software, but he often starts by making sketches by hand.

Currently, George is working on the assembly instructions for putting the legs on a coffee table.



- List the steps for assembling the table.
- Create an exploded view diagram of the assembly.

### Solution

- Follow these steps:

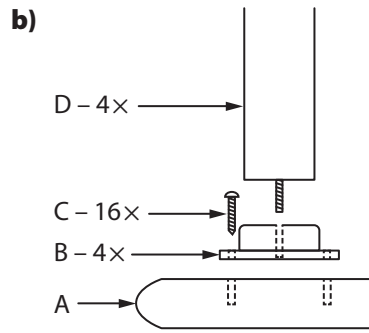
**Step A** Place the tabletop face down on the floor.

**Step B** Line up the holes in the plate with the predrilled holes in the underside of the tabletop.

**Step C** Insert four screws to attach the plate to the underside of the tabletop.

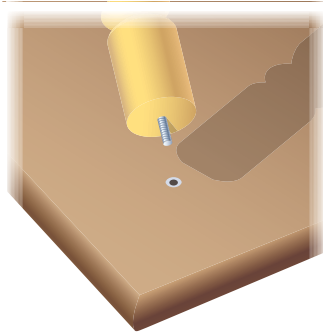
**Step D** Screw the leg of the table into the threaded hole in the plate.

**Step E** Repeat steps B, C, and D for the other three legs.



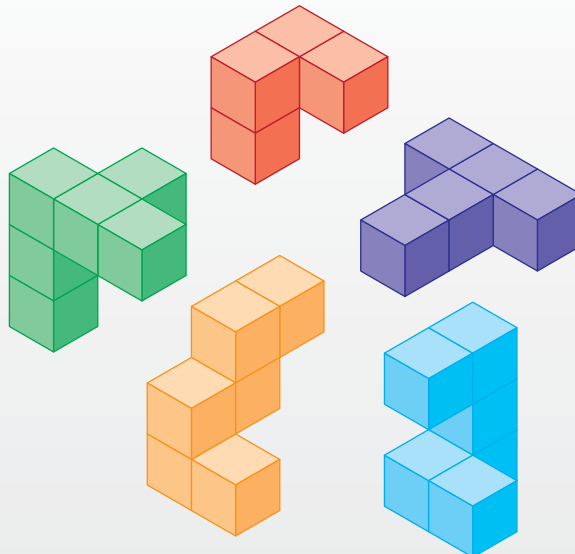
### Your Turn

The end table that matches the coffee table does not use plates. Each of the four legs screws directly into a predrilled hole in the underside of the table. Sketch an exploded view diagram that shows the assembly of one leg into the end table.



### Puzzler

The diagram shows an exploded view of the pieces of a 3-D puzzle. Create a colour isometric drawing of the completed puzzle.

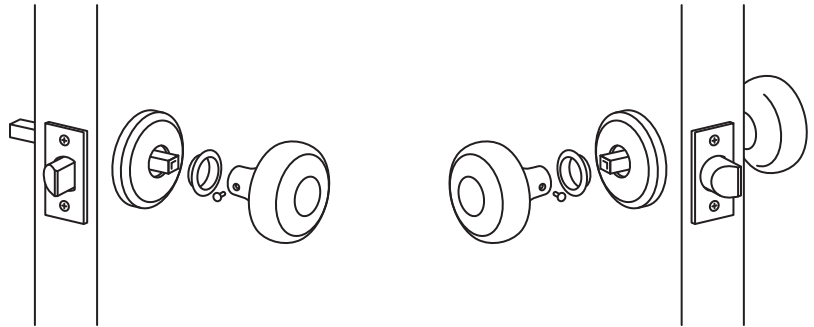




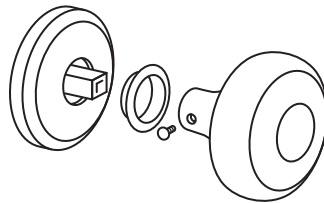
## Check Your Understanding

### Try It

1. a) Identify the item shown in the exploded view diagrams.

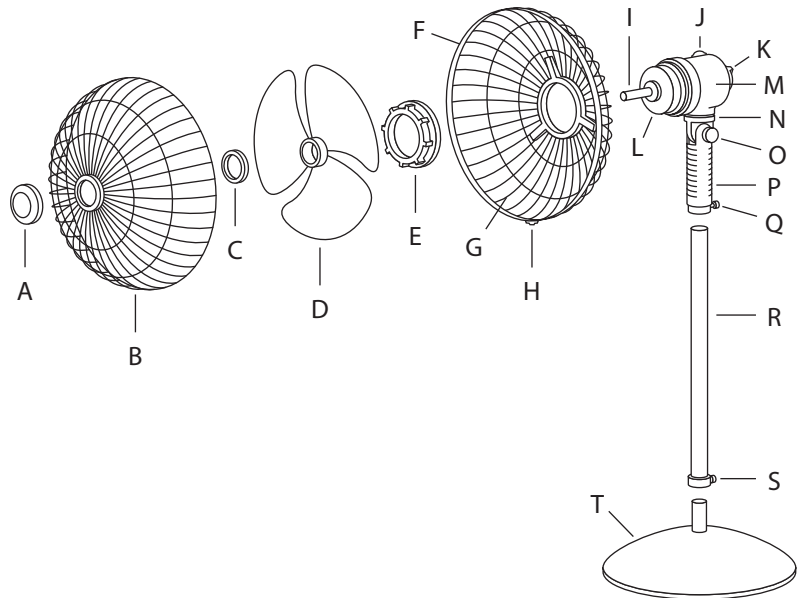


b) How many parts are shown in the diagram below?



2. a) Identify the item shown in the exploded view diagram.

b) What names would you give to parts B, D, M, and T?



### F.Y.I.

Fast food chains often use exploded view diagrams of food items, such as sandwiches. They use these drawings to train employees and to ensure that the food items are made exactly the same way in every location.

3. a) Using linking cubes, build a rectangular prism that is 3 by 1 by 1.  
b) Separate the cubes. Line them up in the order you assembled them in part a).  
c) On isometric dot paper, sketch an exploded view diagram of the rectangular prism. Make it a scale diagram. Label your diagram with the scale you used.

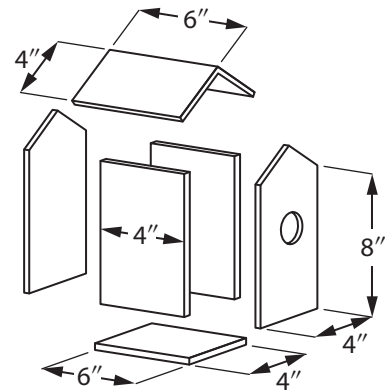
### Apply It

4. a) List the ingredients that someone would use to make a peanut butter and jam sandwich.



- b) Starting with the bottom piece of bread, list the ingredients in the order that the person would build the sandwich.
  - c) Create and label an exploded view diagram of the sandwich.
5. Choose an item in your classroom or school, such as a chair, that is made up of a number of parts.
    - a) List the parts and the quantities of each part that make up the item.
    - b) Write the sequence of steps for assembling the item.
    - c) Draw an exploded view diagram of the assembly of the item.

6. The exploded view diagram shows a birdhouse. On grid paper, sketch the front view and the side view of the birdhouse.



## Work With It

- On grid paper, sketch a rectangle near the centre of the page. This rectangle represents the front of a bus.
  - Locate a vanishing point near the bottom-left corner of the page. Create a one-point perspective drawing so it looks like a bus coming toward you.
  - As the viewer of the bus, what is your point of perspective?
  - Create a similar diagram, this time locating the vanishing point near the top-left corner of the page.
  - As the viewer of the bus, what is your point of perspective now?
- Create and label an exploded view diagram of a double cheeseburger. Include the condiments of your choice.



- A high school runs an annual plant sale in the spring. They plan to advertise with flyers. Design the heading for the flyer by making a one-point perspective drawing of the words PLANT SALE.

## Discuss It

- Create a product logo of your choice that appears 3-D.
  - Explain how you created the effect.
- Describe a specific situation in which you might choose to create an exploded view diagram.
- How does the location of the vanishing point affect the point of perspective for a one-point perspective diagram?

### What You Need to Know

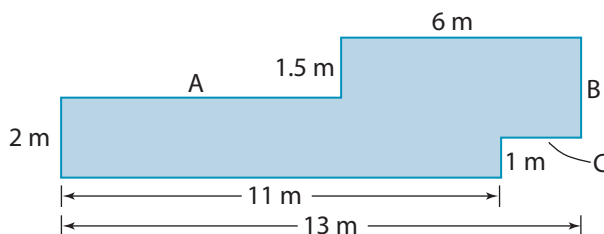
#### Section After this section, I know how to ...

- 2.1**
- use proportional reasoning to determine the dimensions of a scale drawing or model
  - draw a scale diagram of a given object
  - solve problems that involve scale
  - construct a model of a 3-D object
- 2.2**
- draw top, front, and side views of a given 3-D object
  - draw a 3-D object, given the top, front, and side views
  - create an isometric drawing of a given 3-D object
  - determine whether the given views of a 3-D object represent the object
- 2.3**
- draw a one-point perspective view of a 3-D object
  - identify the point of perspective of a given one-point perspective drawing
  - draw the components of an exploded view diagram
  - sketch an exploded view diagram of a 3-D object
  - sketch a 2-D representation of a 3-D object, given its exploded view diagram

If you are unsure about any of these questions, review the appropriate section or sections of this chapter.

#### 2.1 Working With Scale, pages 60–71

1. Determine the missing dimensions.



2. Danny's toy tractor is a 1 : 16 scale model of the actual tractor.
- a) The wheelbase of the toy tractor is 16 cm. How long is the wheelbase of the real tractor?
  - b) The front tread range of the real tractor is 60 in. What is the front tread range of the toy tractor?

#### F.Y.I.

The wheelbase of a vehicle is the distance between the front axle and the rear axle. The tread range is the distance between the centre of the tread of one of the tires to the other.



## 2.2 Representing Views of 3-D Objects, pages 72–83

3. The object shown is made from linking cubes. On paper or using technology, create a set of three orthographic drawings of the object.



4. The picture shows a set of orthographic drawings of a 3-D object.



front view



top view

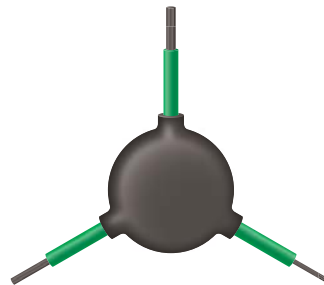


right side view

- a) Use linking cubes to build a model of the object.  
b) On isometric dot paper or using technology, create an isometric drawing of the object.

## 2.3 Representing Perspectives of 3-D Objects, pages 84–95

5. On paper or using technology, draw your initials in large block letters. Create a one-point perspective drawing of your initials.
6. The picture shows a set of Allen keys. The keys fit into the green arms, which fit into the centre piece. On paper or using technology, draw an exploded view diagram of the Allen key.



### Tools of the Trade

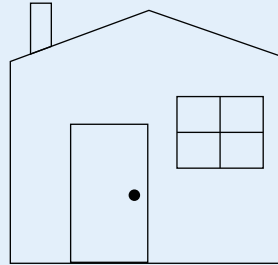
An Allen key, sometimes called an Allen wrench, is a tool used to loosen or tighten screws and bolts with hexagonal slots. Some models of “build-it-yourself” furniture require an Allen key.



For #1 to #5, select the best answer.

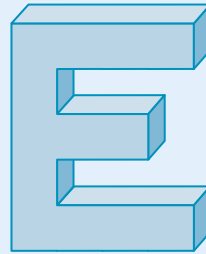
1. What kind of drawing is this diagram?

- A exploded view drawing
- B isometric drawing
- C one-point perspective drawing
- D orthographic drawing



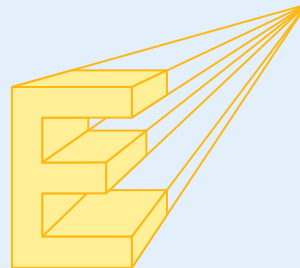
2. What kind of drawing is this diagram?

- A exploded view drawing
- B isometric drawing
- C one-point perspective drawing
- D orthographic drawing



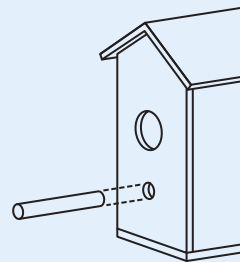
3. What kind of drawing is this diagram?

- A exploded view drawing
- B isometric drawing
- C one-point perspective drawing
- D orthographic drawing



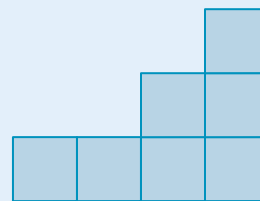
4. What kind of drawing is this diagram?

- A exploded view drawing
- B isometric drawing
- C one-point perspective drawing
- D orthographic drawing

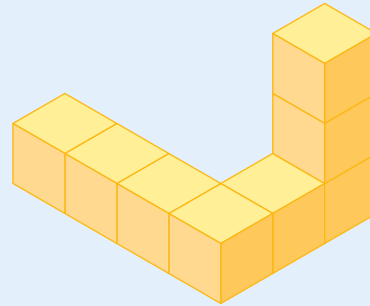


5. What kind of drawing is this diagram?

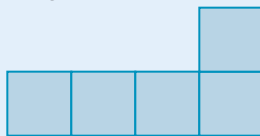
- A exploded view drawing
- B isometric drawing
- C one-point perspective drawing
- D orthographic drawing



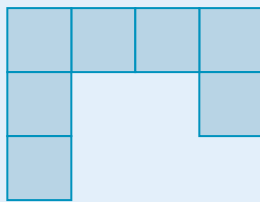
6. Draw the front view, top view, and side view of the object shown here.



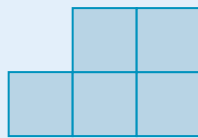
7. Create an isometric drawing of the object shown in the orthographic diagrams.



front



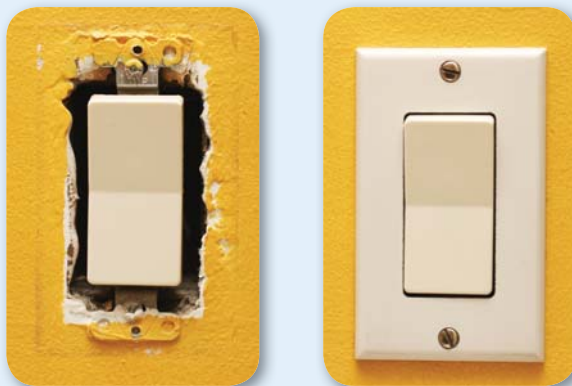
top



right side

8. a) Create a one-point perspective drawing of a triangular prism.  
 b) Identify the point of perspective of your drawing.

9.



- a) Write the sequence of steps for installing the light switch cover.  
 b) Sketch an exploded view diagram for installing the light switch cover.



### Design Your Own Model

You are now ready to design your own model.


1. Choose something you would like to design. It can be as simple or as complex as you like. For example:
  - ✓ You want to build a model or a piece of scenery for a train set, a miniature dollhouse, or a stage performance.
  - ✓ You wish to design and create an article of clothing for a collectible doll or for a sports mascot.
  - ✓ You plan to make money selling crafts during the summer. What miniature would remind tourists of where you live? For example, in Newfoundland and Labrador, they might buy a miniature flag, a small replica of Signal Hill, or a fridge magnet in the shape of the province.
2.
  - a) Choose a scale for your design.
  - b) Draw your design using two of the following:
    - a set of orthographic drawings
    - a one-point perspective drawing
    - an isometric drawing
    - an exploded view drawing
3.
  - a) Include a list of the materials that you will need in order to build the model.
  - b) Estimate the cost of these materials.
4. Build all or part of your design.

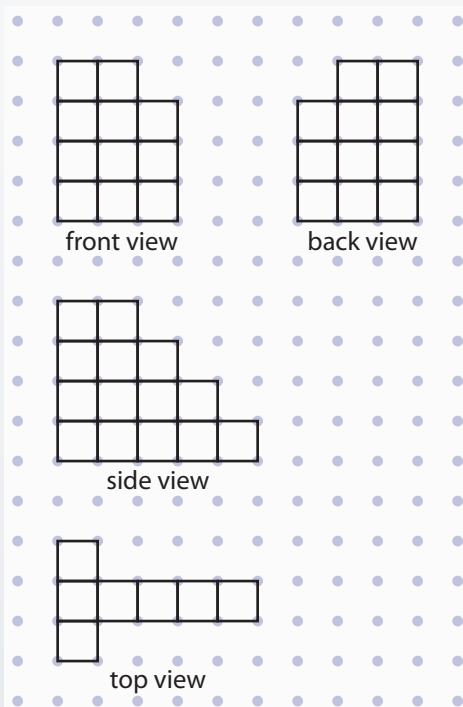


**Model It**

1. **a)** How many linking cubes would you need to build the 3-D sculpture shown in the four diagrams below?
- b)** Use linking cubes to build the 3-D sculpture represented in the four diagrams.

**Materials**

- linking cubes
- grid paper 
- ruler



2. **a)** Use linking cubes to build an original 3-D object. (Keep it hidden from most of the class.)
- b)** Draw the front, back, side, and top views of the sculpture.
- c)** Trade views with a partner. Draw the views of each other's objects.
- d)** Assess the orthographic views drawn by your partner.
  - Where do you agree with the views?
  - Where do you disagree?
  - Work with your partner to develop a correct set of orthographic views for your 3-D sculpture.