Chapter 2 Surface Area and Volume

2.1 Units of Area and Volume

《《 **KEY IDEAS** 》》

- Area is measured in square units. Volume is measured in cubic units.

Proportional reasoning can be used to

- solve problems involving area or volume units within SI

- solve problems involving area or volume units within the imperial system

- solve problems requiring the conversion of area or volume within and between the SI and imperial systems using linear dimensions

Convert $0.62\ m^2$ to square centimetres.

$1\ m = 100\ cm$

$1\ m^2 = (100\ cm)(100\ cm)$

$\qquad = 10\ 000\ cm^2$

$0.62\ m^2 = (0.62)(10\ 000\ cm^2)$

$\qquad\quad = 6200\ cm^2$

$0.62\ m^2$ is equal to $6200\ cm^2$.

Calculate the volume of a rectangular prism with dimensions 1 ft by 3 ft by 5 ft in cubic metres.

$1\ ft = 0.3048\ m$

$3\ ft = 3(0.3048\ m)$

$\qquad = 0.9144\ m$

$5\ ft = 5(0.3048\ m)$

$\qquad = 1.524\ m$

$V = (0.3048\ m)(0.9144\ m)(1.524\ m)$

$V = 0.424\ 752\ 698\ 9\ m^3$

The volume of the prism is approximately $0.42\ m^3$.

Example

For a friend's birthday, Sam plans to wrap the door of her locker with wrapping paper and fill the locker with balloons. The dimensions of the locker are shown.

a) How much wrapping paper will he need to cover the door?

b) How much space is there inside the locker for the balloons?

1.05 m

40 cm
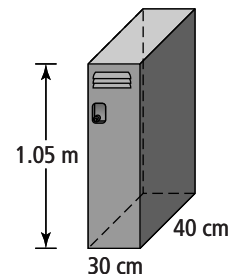
30 cm

Solution

a) The amount of wrapping paper required is equal to the area of the door. Sam decides to work in metres and converts 30 cm to 0.3 m.

Calculate the area of the door.

$A = lw$

$A = (1.05)(0.30)$

$A = 0.315$

The area of the door is $0.315\ m^2$.

Sam learns that the wrapping paper is available only in square centimetres. Sam decides to convert to square centimetres.

$$\left(\frac{0.315 \text{ m}^2}{1}\right)\left(\frac{10\,000 \text{ cm}^2}{1 \text{ m}^2}\right) = 3150 \text{ cm}^2$$

Recall that 10 000 cm² = 1 m².

Sam needs 0.315 m² or 3150 cm² of wrapping paper.

b) Sam plans to fill the locker with balloons. She calculates the volume of the locker.
Sam decides to work in metres and converts 30 cm to 0.3 m and 40 cm to 0.4 m.

$V = lwh$

$V = (1.05)(0.3)(0.4)$

$V = 0.126$

The volume of the locker is 0.126 m³.

Sam decides to convert to cubic centimetres by solving a proportion.

$$\left(\frac{x}{0.126 \text{ m}^3}\right) = \left(\frac{1\,000\,000 \text{ cm}^3}{1 \text{ m}^3}\right)$$

Recall that 1 000 000 cm³ = 1 m³.

$x = 126\,000 \text{ cm}^3$

The balloons will occupy 0.126 m³ or 126 000 cm³ of space.

A Practise

*Refer to the table of conversion factors to help answer #1 and #2.*

| Imperial Unit	SI Unit
1 in.	2.54 cm
1 ft	0.3048 m
1 yd	0.9144 m
1 mi	1.609 km

**1.** Determine the following areas to the indicated unit. Express the answers to the nearest hundredth of a square unit where necessary.

a)

$A = ?$ cm²   4 in.
6 in.

b)

$A = ?$ m²   2 ft
2.5 ft

c)

42 cm
$A = ?$ m²   18 cm

d)

$A = ?$ ft²   1.2 m
2.5 m
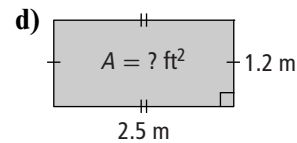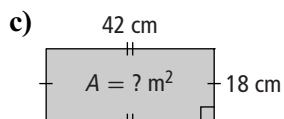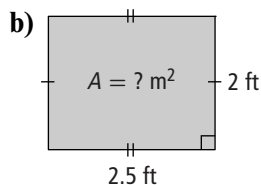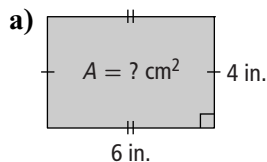
**2.** Determine possible dimensions for each area. Then, use the dimensions to calculate the area to the indicated equivalent, to the nearest tenth of a square unit.

a)

$A = 500$ cm²
$A = ?$ mm²

b)

$A = 15$ ft²
$A = ?$ cm²
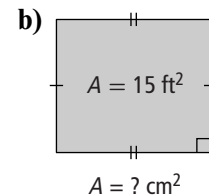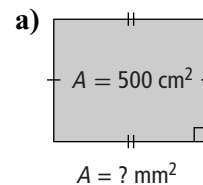
3. What is each volume in cubic centimetres? Express the answer to the nearest tenth of a cubic centimetre.

a)

10 in.
10 in.
10 in.

b)

1.25 ft
3 ft
6.5 ft
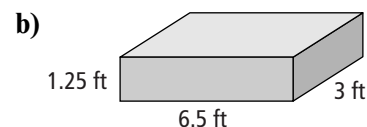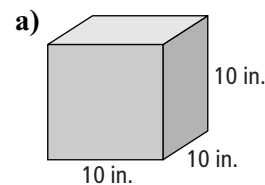
4. Calculate the volume of each prism to the indicated unit. Express the answer to the nearest hundredth of a cubic unit.

a)

30 cm
30 cm
30 cm
$V = ?\ \text{m}^3$

⭐ **b)**

16 cm
25 cm
7 cm
$V = ?\ \text{in.}^3$
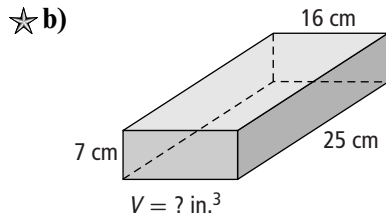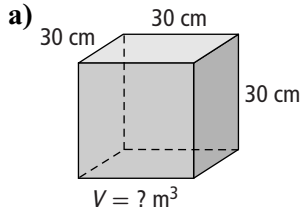
B Apply

Unless otherwise indicated, express the answers to the nearest tenth of a unit.

5. A utility trailer has the measures shown. What is the volume of the box in cubic metres?

6 ft
18 in.
5 ft
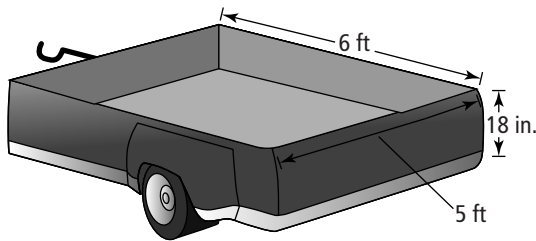
6. Shannon is painting a mural on one wall in her bedroom. The wall measures 10′ 6″ by 8′.

a) She plans to paint a primer coat. What is the total area to be painted in square metres?

b) One can of paint is expected to cover 15 m². How many cans of paint will Shannon need to buy?

7. The drama club sells advertising space in their playbill. The cost of an ad depends on the size of the ad. Each square centimetre costs $0.25. If the available space on each page of the playbill is 8″ by 10.5″, what is the cost of a full-page ad?

8. For a graduation ceremony, the floor of a school gymnasium is covered with a tarp to protect its finish. The gym measures 110 ft by 80 ft. What is the area of the tarp in square metres?

9. A recycling business provides customers with containers in which to place recycled materials. The business charges for recycling based on the size of the container. The largest container available measures 20 in. by 21 in. by 1 yd. Determine the volume of the container in cubic feet, to the nearest hundredth of a cubic foot.

⭐**10.** A hotel manager plans to replace the carpet in a meeting room with hardwood. The meeting room measures 6 m by 10 m.

a) Determine the area of the room in square feet.

b) The hardwood is available in strips measuring 4 in. by 4 ft. Determine the area of each strip in square feet.

c) The hotel manager was advised to buy extra hardwood. If she needs to buy 12% more than the area of the floor, how many strips of hardwood should she order?

11. A community is setting up a WiFi network. Each tower can reliably transmit a signal for a circular area of radius 5 mi. What is this area in SI units, to the nearest square unit?

12. For each container, determine the unknown dimension.

a) A rectangular prism has volume 504 cm³ and base 12 cm by 14 cm. Determine the height of the prism.

b) A box with a square base has height 3 in. and volume 243 in.³. Determine the dimensions of the base.

13. a) A square table has an area of 16 ft^2. Determine the side lengths of the table in centimetres, to the nearest centimetre.

b) A window has an area of 1728 in.^{2}. If the width of the window is 3 ft, what is its length in feet?

☆**14.** David installed a rain barrel to store rainwater for use in his garden. The barrel has a height of 34 in. and a diameter of 23.5 in. Determine the volume of rain that the barrel can hold in cubic metres, to the nearest hundredth of a cubic metre.

C Extend

☆**15.** Amy is organizing a day for volunteers to clean up litter in a park.

a) She estimates that each volunteer will fill one large garbage bag. Each bag measures 19 in. by 11 in. by 34 in. Determine the capacity of each bag in cubic feet.

b) Amy needs to rent a dumpster for the litter. She can choose either a dumpster with a capacity of 10 yd^3 or one with a capacity of 15 yd^3. If 80 volunteers have signed up, which dumpster should she rent? Justify your answer mathematically.

16. In the late 1800s, in Western Canada, farmers were offered a quarter section of land to grow crops and raise livestock. This is equal to $\frac{1}{4}$ mi^2.

a) Each quarter section usually formed a square. Determine the length of each side of the section, in miles and in kilometres.

b) What is $\frac{1}{4}$ mi^2 to the nearest tenth of a square kilometre?

c) Today, the average farm in Saskatchewan is about $2\frac{1}{4}$ mi^2. Assuming that the land for an average farm forms a square, determine the length of each side, in miles.

d) How many times larger is the area of a farm today than a farm in the 1800s?

e) How many times longer is the length of each side of a farm today than a farm in the 1800s?

17. Wascana Park, which is located in Regina, SK, is one of the largest urban parks in Canada. The area of the park is about 2325 acres.

a) One acre is equal to 43 560 ft^2. What is the area of Wascana Park in square feet?

b) Determine the area of the park in square metres, to the nearest square metre.

c) The SI unit for large areas is the hectare. One hectare is equal to 10 000 m^2. What is the area of Wascana Park in hectares, to the nearest hectare?

D Create Connections

18. a) Identify an item you prefer to measure using imperial units.

b) Choose a quantity for the item in part a). Show how to convert the quantity from imperial to SI measure.

c) What is a quick estimate you could use for the conversion in part b)?

19. a) Identify an item you prefer to measure using SI units.

b) Choose a quantity for the item in part a). Show how to convert the quantity from SI to imperial measures.

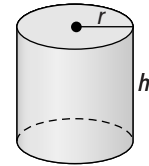**c)** What is a quick estimate you could use for the conversion in part b)?

2.2 Surface Area

• The surface area of a right cylinder and of a right prism can be calculated using the area of the bases (top and bottom) plus the lateral area.
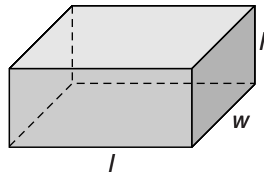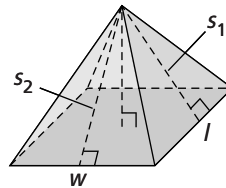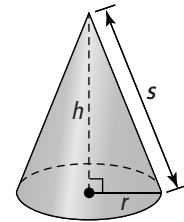
$SA_{prism} = 2lw + 2lh + 2wh$ $SA_{cylinder} = 2(\pi r^2) + 2\pi rh$

• The surface area of a right pyramid and of a right cone can be calculated using the area of the base plus the lateral area.
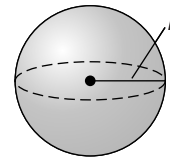
$SA_{pyramid} = lw + 2\left[\frac{1}{2}ls_1\right] + 2\left[\frac{1}{2}ws_2\right]$ $SA_{cone} = \pi r^2 + \pi rs$

• The surface area of a sphere depends on the radius only.

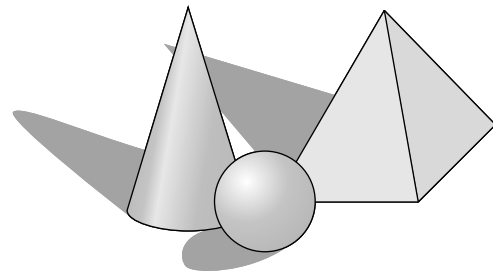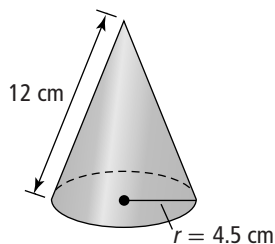$SA_{sphere} = 4\pi r^2$

Example

Colby is taking a pottery class. He sketches a set of geometric objects as shown.

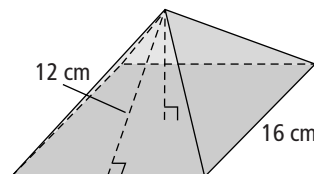Before he makes clay models, he adds dimensions to each object.

Given the measurements, determine the surface area of each object. Express the answers to the nearest tenth of a square centimetre.
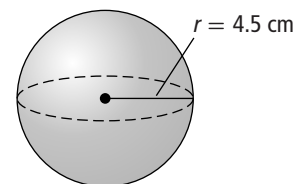
a) The right cone has radius 4.5 cm and slant height 12 cm.

12 cm

$r = 4.5$ cm

b) The right pyramid has a square base with side length 16 cm and slant height 12 cm.

12 cm

16 cm

c) The sphere has radius 4.5 cm.

$r = 4.5$ cm

Solution

a) The surface area of a right cone is the sum of the area of the base and the lateral area.

$SA = \text{base} + \text{lateral area}$

$SA = \pi r^2 + \pi rs$

$SA = \pi(4.5)^2 + \pi(4.5)(12)$

$SA = 233.2632...$

The surface area of the cone is approximately 233.3 cm^2.

b) The surface area of a right pyramid is the sum of the area of the base and the lateral area.

$SA = \text{base} + \text{lateral area}$

$SA = (\text{length})(\text{width}) + 2\left[\frac{1}{2}(\text{length})(\text{slant height}_1)\right] + 2\left[\frac{1}{2}(\text{width})(\text{slant height}_2)\right]$

$SA = (\text{length})(\text{width}) + 4\left[\frac{1}{2}(\text{length})(\text{slant height})\right]$

In this case, why can you multiply by 4?

$SA = (16)(16) + 4\left[\frac{1}{2}(16)(12)\right]$

$SA = 640$

The surface area of the pyramid is 640.0 cm^2.

c) The formula for the surface area of a sphere is $SA = 4\pi r^2$.
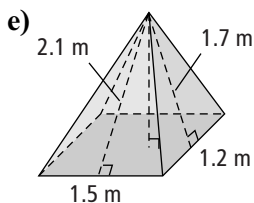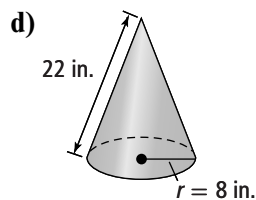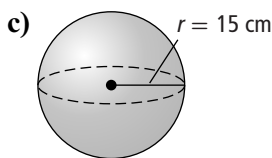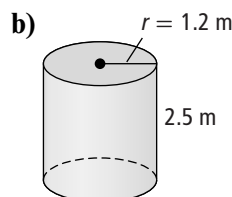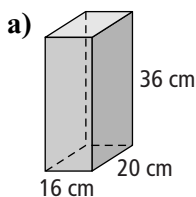
$SA = 4\pi(4.5)^2$

$SA = 254.469$

The surface area of the sphere is approximately 254.5 cm^2.

A Practise

Express your answers to the nearest hundredth of a unit where necessary.

1. Determine the surface area of each of the following.

a)
36 cm
20 cm
16 cm

b) $r = 1.2$ m
2.5 m

c) $r = 15$ cm

d) 22 in.
$r = 8$ in.

e) 2.1 m 1.7 m
1.2 m
1.5 m

2. What is the surface area of each of the following?

a) 10 in.
10 in.
10 in.

b) $d = 65$ cm

c) $d = 12$ cm
80 cm

d) 1.5 ft
$d = 1$ ft
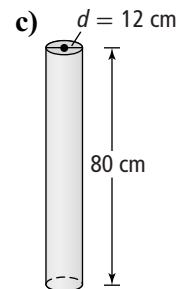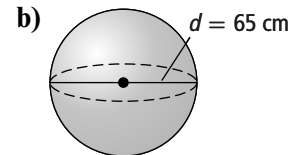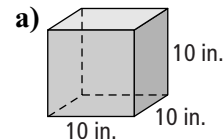
3. A jewellery box has a surface area of 148 in.^{2}. The base of the box is 6 in. by 4 in. Sketch a diagram to help determine the height of the box.

4. What is the surface area of the right pyramid?

44 ft

31 ft
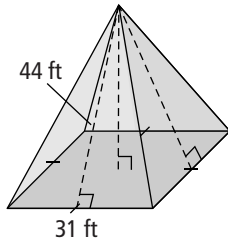
5. The surface area is given for each object. Determine the unknown dimension.

☆**a)** $SA = 4$ m²

$d = 1.2$ m

h

b) $SA = 84.5$ in.²

r

c) $SA = 5.7$ cm²

s

$r = 0.8$ cm

d) $SA = 2443.8$ ft²

l

15 ft

24 ft
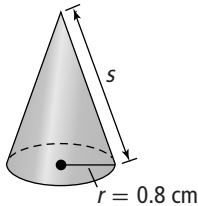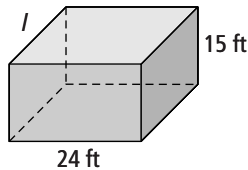
6. Determine the surface area of each composite object.

a) $d = 25$ cm
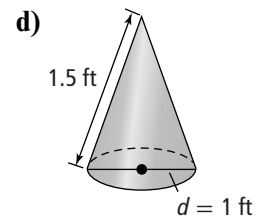
50 cm

$d = 25$ cm

☆**b)**

13 m   15 m

38 m

10 m

18 m

B Apply

Express your answers to the nearest tenth of a unit where necessary.
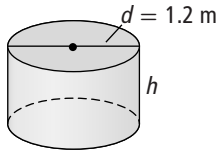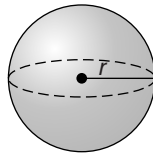
7. Andrea wants to make a leather case for her new smartphone. The phone measures 55 mm by 12 mm by 114 mm.
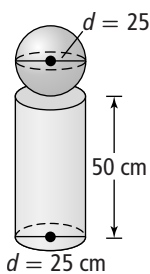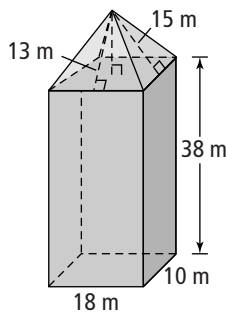
a) How much leather is needed for a case that completely encloses the smartphone?
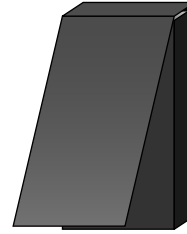
b) How much leather is needed for a case that is open at one end? Hint: Do not include the overlap of the leather pieces when they are sewn together.

8. The diameter of Earth is approximately 12 740 km. Determine the surface area of Earth to the nearest square kilometre.

9. A skylight has the shape of a right pyramid. The square base measures 1.2 m by 1.2 m and the height is 1.6 m.

a) Determine the slant height of each face of the skylight.

b) Determine the surface area of the pyramid that represents the skylight.

c) How much glass is needed for the skylight? Hint: There is no glass in the base of the pyramid.

1.6 m

s

1.2 m

1.2 m

10. A manufacturer is designing a new cylindrical paint can with diameter 20 cm. He plans to use 2200 cm^2 of material to make the can. What is the height of a can that uses all available material?

11. Obelisks are sometimes used to mark the border between Canada and the United States. One stone obelisk is a right prism topped by a right pyramid. The obelisk has a base 15 cm by 15 cm by 95 cm and a top in the shape of a pyramid with height 10 cm. Determine the surface area of the obelisk.

12. Daniel needs to paint four cylindrical columns that support the ceiling in the hall of his home. The columns have height 10 ft and diameter 10 in. Determine the total surface area, in square feet, requiring paint.

13. The surface area of a cone is 3520 cm^2. If the radius of the cone is 20 cm, determine the slant height, to the nearest centimetre.

☆**14.** Breanne makes and sells bracelets. She uses a bracelet mandrel to shape, stretch, and size the bracelets. The mandrel is a cone with the circumference marked at different locations on the lateral face of the cone. The mandrel is 6 in. tall and has a base circumference of 12 in. Determine the surface area of the mandrel.
Hint: $C = 2\pi r$.

15. a) Determine the edge length of a cube with surface area 62 m^2.

b) Write a formula for determining the edge length of a cube, when the surface area is known.

C Extend
16. A sheet of aluminum measures 4 ft by 8 ft.

a) If David uses all of the aluminum, what is the diameter of the largest sphere that he can make?

b) If he uses all of the aluminum, what are the dimensions of the largest cube that he can make?

c) Is it realistic for David to expect that all of the aluminum will be used? Explain why or why not.

17. A farmer is filling a bin with grain. The grain forms a cone near the top of the bin.

a) The grain cone has height 5.2 m and circumference 32 m. Determine the slant height of the grain, to the nearest tenth of a metre.

b) The farmer plans to treat the outside surface of the grain pile to reduce rot. Determine the area to be treated, to the nearest square metre.

D Create Connections
18. A formula for the surface area of a cylinder is $SA = 2\pi r^2 + 2\pi rh$.

a) Write a formula that you can use to determine the surface area of a cylinder using the diameter instead of the radius.

b) Write a formula that you can use to determine the surface area of a cylinder using the circumference instead of the radius.

c) For parts a) and b), identify a situation in which each form of the formula would be more useful.

2.3 Volume

- The volume of a right cone is found by calculating one third of the volume of its related right cylinder.

$$V_{cone} = \frac{1}{3}\pi r^2 h$$

- The volume of a right pyramid is found by calculating one third of the volume of its related right prism.

$$V_{pyramid} = \frac{1}{3}lwh$$

- The volume of a sphere is found by using the formula

$$V_{sphere} = \frac{4}{3}\pi r^3$$

- If you know the volume of an object, you can calculate an unknown dimension.

- The volume of the right pyramid with square base w is 384 ft^3. Find the dimensions of the base.

$$V = \frac{1}{3}Bh$$
$$384 = \frac{1}{3}w^2(8)$$
$$3(384) = 3[\frac{1}{3}w^2(8)]$$
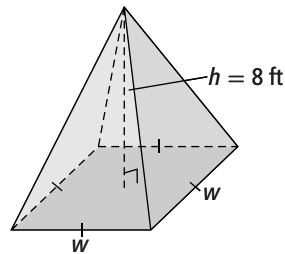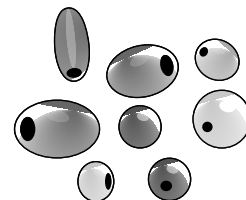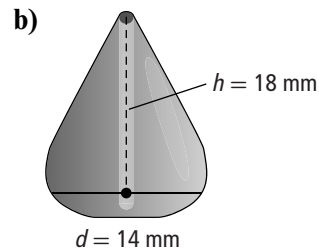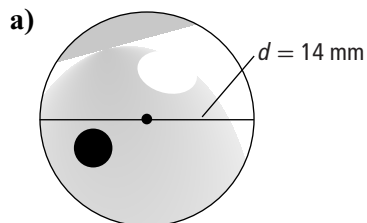$$1152 = w^2(8)$$
$$\frac{1152}{8} = w^2$$
$$144 = w^2$$
$$12 = w$$

The dimensions of the base are 12 ft × 12 ft.

$h = 8$ ft

w

w

Example

Julie makes glass beads and is experimenting with two new designs: a conical bead and a spherical bead. Determine how much glass is needed to make each bead. Express the answers to the nearest tenth.

a)

$d = 14$ mm

b)

$h = 18$ mm

$d = 14$ mm

Solution

a) For the sphere,

$r = 14 \div 2$

$= 7$

Substitute into the formula $V = \frac{4}{3}\pi r^3$.

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(7)^3$

$V \approx 1436.755\,04...$

The volume of glass needed per spherical bead is approximately 1436.8 mm³.

b) For the cone,

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(7)^2(18)$

$V \approx 923.628\,24...$

The volume of glass needed per conical bead is approximately 923.6 mm³.

A Practise

Express your answers to the nearest hundredth of a unit where necessary.

1. Determine the volume of each solid.

a)

26.8 in.

17 in.

10.2 in.

b)

$r = 2$ m

3.3 m

c)

$r = 8.5$ cm

d)

$h = 5$ ft

$r = 12$ ft

e)

94 mm

65 mm

65 mm

2. Calculate the volume of each solid.

a)

$h = 150$ in.

$s = 160$ in.

3.2 in.

2.5 in.

b)

$d = 54$ mm

$s = 95$ mm

$h = 75$ mm
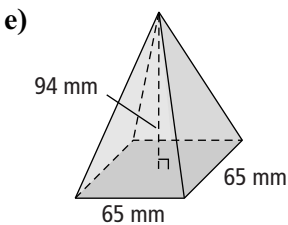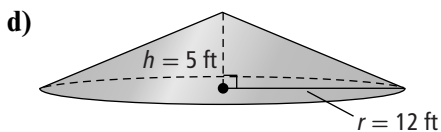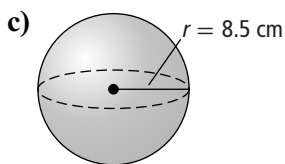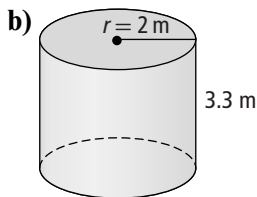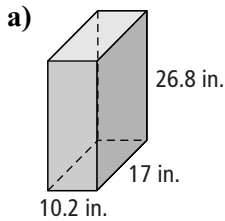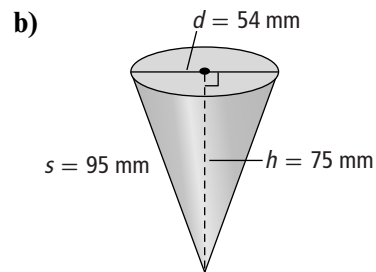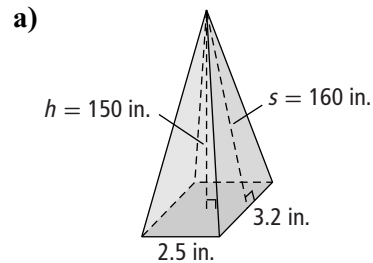
3. Determine the unknown dimension of each solid.

a) $V = 150$ in.³   ☆**b)** $V = 36\pi$ m³

h

4.1 in.

4.3 in.

r

c) $V = 7$ m³

h

$d = 1.5$ m

d) $V = 2.2$ ft³

$r = 0.5$ ft

h
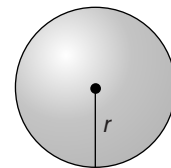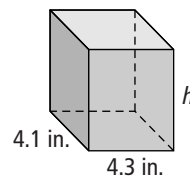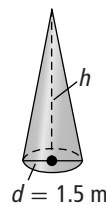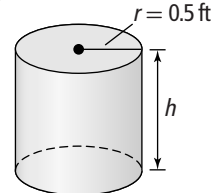
4. What is the volume of each solid, in cubic centimetres?

a)

1.42 m

$d = 13$ cm

b)

$s = 0.5$ m

$h = 0.46$ m

$d = 54$ cm
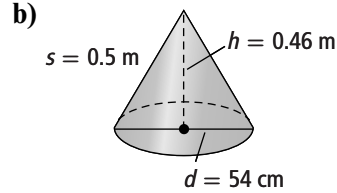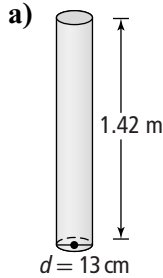
5. Determine the volume of each composite object, in cubic centimetres.

a)

75 cm

58 cm

20 cm

22 cm

b)

$d = 30$ cm

30 cm

$d = 30$ cm
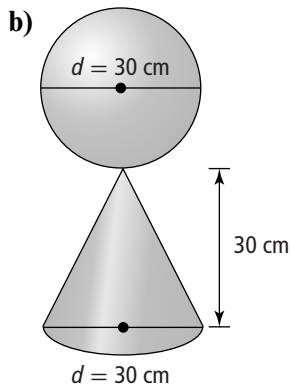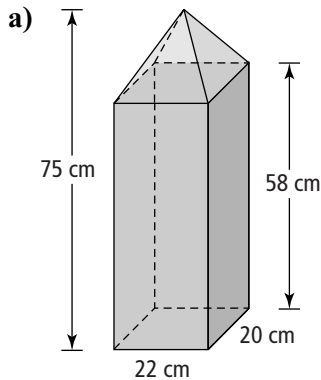
B Apply

Express your answers to the nearest tenth of a unit where necessary.

6. Earth has a diameter of approximately 12 740 km. What is Earth's volume?

7. Kyle plans to operate a snow cone stand. He determines that his right conical cups need a capacity between 75 cm³ and 100 cm³. Will either of the following cups meet this standard? Justify your answer mathematically.

   a) a conical cup with height 10 cm and diameter 6 cm

   b) a conical cup with height 6 cm and diameter 10 cm

8. Sketch the object and determine the volume.

   a) a right cone with height 44 in. and radius $\frac{1}{4}$ of the height

   b) a right pyramid with base 12.3 cm by 12.3 cm and height 18.8 cm

   c) a right pyramid with base 68 cm by 85 cm and height 1 m

   d) a sphere with diameter 1.1 m

9. A can of juice has height 11.5 cm and diameter 5 cm.

   a) What is the volume of the can?

   b) The can is labelled 225 mL. What is the relationship between millilitres and cubic centimetres?

☆**10.** The right pyramid shown has height 24 cm and slant height 25 cm. What is its volume?

$s = 25$ cm

$h = 24$ cm
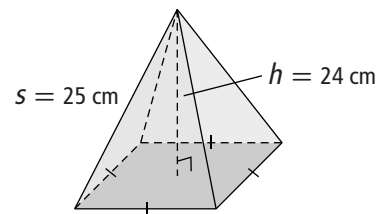
11. A cylindrical water bottle must hold 1000 cm³ and can be no more than 25 cm high. Determine the required radius. Is this the minimum radius or the maximum radius? Justify your answer.

12. The volume of each solid is given. Sketch the object and determine the unknown dimension.

a) A cylinder has a volume of 700 m³ and a height of 16 m. Calculate its radius.

b) A cone has a capacity of 250 cm³ and a diameter of 15 cm. Determine its height.

c) A square-based pyramid has a volume of 0.015 m³. The height of the pyramid is 50 cm. Determine the dimensions of the base.

13. The roof of a house is shaped like a right pyramid with a square base. The base of the pyramid measures 32 ft by 32 ft. The roof must enclose a volume of at least 7300 ft³ of air. What is the minimum height for the apex of the roof?

14. Lori designed a cylindrical can that must have a capacity of 250 cm³. Her can has a diameter of 5 cm and a height of 10 cm.

a) Calculate the capacity of the can.

b) Does it meet the design requirements? Explain.

☆**c)** Modify the height of Lori's can so that it will have the correct capacity. Justify your answer mathematically.

15. Bree has 1000 cm³ of melted wax to make a candle.

a) Sketch a cylindrical candle with height 20 cm. Determine the radius.

b) Sketch a cylindrical candle with height 15 cm. Determine the radius.

c) Suppose Bree makes a right conical candle with height 25 cm. Sketch the candle and determine the radius of its base.

d) Suppose Bree makes a candle shaped like a pyramid with a square base and a height of 20 cm. What is the area of the base of the pyramid?

16. The Luxor Hotel in Las Vegas, Nevada, is a square-based pyramid with a height of about 350 ft and a slant height of about 447 ft. Determine the volume of the hotel.

☆**17.** A ceramic bead is half a sphere with a diameter of 1.85 cm. Determine the volume of the bead.

C Extend

18. Stan's company is designing a container that must have a volume of 750 cm³.

a) What is the height of a cylindrical container with a diameter of 10 cm?

b) What is the height of a right prism with a square base measuring 10 cm by 10 cm?

c) Determine the surface area of the container in parts a) and b).

d) Which container do you recommend that Stan's company produce? Explain why.

19. Determine the volume of the building block.

r = 4 cm
12 cm
25 cm
30 cm
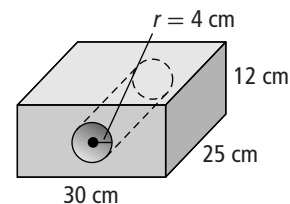
20. a) What are the dimensions of a cube with a surface area numerically equal to its volume?

b) Determine the radius of a sphere with a surface area numerically equal to its volume.

D Create Connections

21. Suppose that each dimension of a solid is doubled. Which will change more— the surface area or the volume of the object? Explain your reasoning using an example.

Chapter 2 Review

2.1 Units of Area and Volume

Express your answers to the nearest hundredth of a unit where necessary. You may need to refer to the table of conversion factors on page 21.

1. Calculate each area using the indicated unit.

a)

8.5 in.

11 in.

$A = ?$ cm^2

b)

0.6 m

0.6 m

$A = ?$ ft^2

2. Calculate each volume using the indicated unit.

a)

3 ft

1 ft

1.5 ft

$V = ?$ m^3

b)

$r = 2.5$ in.

5 in.

$V = ?$ cm^3

3. Cassidy wants to replace her kitchen countertop. The dimensions are 65 cm by 1.5 m. The building supply store sells countertop material by the square foot. Determine the area of her countertop, in square feet.

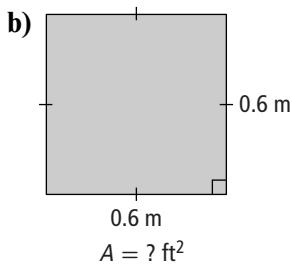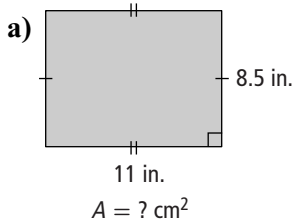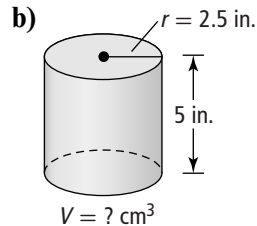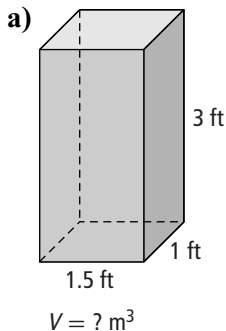⭐4. Ken is researching the cost of topsoil for his lawn. One supplier quotes a price of $50 per cubic yard, and another quotes a price of $62 per cubic metre. Which supplier offers a better price? Justify your answer mathematically.

2.2 Surface Area

Express your answers to the nearest hundredth of a unit where necessary.

5. Sketch each solid and determine its surface area.

   a) A right cone has radius 3.5 m and slant height 12 m.

   b) A right cone has diameter 12 cm and slant height 10 cm.

   c) A sphere has diameter 8.5 in.

   d) A pyramid has a square base with sides 4 ft and slant height 5.5 ft.

6. Calculate the unknown dimension in each of the following.

   a) A sphere has surface area 450 in.^{2}.

   b) The base of a right pyramid has sides 12 cm by 10 cm. The slant height of the face with base 10 cm is 10 cm. The surface area is 333.2 cm^2.

   c) A right cone has surface area 20 ft^2 and radius 2 ft.

   d) A right cone has surface area 500 m^2 and the base of the cone has area 314 m^2.

⭐7. The floor of a storage shed has sides 8 ft by 10 ft. The height of the walls is 7 ft. The roof is shaped like a pyramid. The slant height of the face with the shorter side of the shed is 5.4 ft. The slant height of the face with the longer side is 4.5 ft. Sketch the shed and determine the total surface area of the shed.

8. A concrete pillar has a diameter of 12 in. If the pillar is 10 ft tall, what is its surface area?

9. The surface of a sphere with diameter 50 cm is composed of small mirrors. The construction company purchased 5% extra material to cover the sphere. If the mirrors cost $10 per square foot, how much did it cost to cover the sphere?

10. A conical sculpture has diameter 10 ft and height 12 ft.

   a) Sketch and label a cross-section of the cone.

   b) Determine the radius and slant height of the cone.

   c) Determine the surface area of the cone.

2.3 Volume

Express your answers to the nearest hundredth of a unit where necessary.

11. Calculate the volume of each solid.

   a) A right cylinder has radius 9 in. and height 4 ft.

   b) A right pyramid has a base with sides 2 m by 2.5 m and a height of 3.2 m.

   c) A sphere has diameter 1 m.

   d) A right cone has height 18 cm and radius 6.5 cm.

12. What is the unknown dimension of each solid?

   a) A square-based prism has volume 33 750 m^3 and height 50 m.

   b) A right cone has height 12.5 cm and volume 325 cm^3.

   c) A sphere has volume 905 $in.^3$.

   d) A right pyramid has a base with sides 1 ft by 1.5 ft and a volume of 3 ft^3.

13. Mike is building a patio with an area of 700 ft^2. The concrete pad will have a thickness of 4 in.

   a) Determine the volume of concrete required for the patio, in cubic feet.

   b) Mike estimates that for every 35 ft^3, he will need 1 m^3 of concrete. How many cubic metres of concrete will he need? Convert from cubic feet to cubic metres and check the accuracy of his estimate.

☆14. Astrid has a cylindrical compost bin with a height of 1.1 m and a diameter of 1.25 m.

   a) Determine the volume of compost in a full bin.

   b) Astrid's garden is rectangular and measures 12 ft by 20 ft. If she spreads the compost uniformly on her garden, how deep will the compost be, in inches?

15. Liam made a ceramic mug with a diameter of 8 cm and a height of 10 cm. Determine the capacity of the mug.

16. Suppose Liam makes a cylindrical mug with twice the capacity of the mug in #15.

   a) If the diameter stays the same, predict the height of a mug with the desired capacity. Check your prediction.

   b) If the height stays the same, predict the diameter of a mug with the desired capacity. Check your prediction.

17. A pile of gravel is shaped like a cone. It has a diameter of 12 ft and a height of 4.5 ft.

   a) What is the volume of gravel in the pile?

   b) Gravel is often sold in cubic yards. If one cubic yard sells for $15, determine the value of the gravel pile.
   Hint: 1 yd^3 = 27 ft^3.

Chapter 1–2 Cumulative Review

Unless otherwise indicated, express the answers to the nearest hundredth of a unit. You may need to refer to the table of conversion factors on page 21.

**1.** Calculate each area using the indicated unit.

a)

1.6 m

0.5 m

$A = ?\ mm^2$

b)

$6\frac{3}{4}$ in.

$2\frac{1}{4}$ in.

$A = ?\ cm^2$
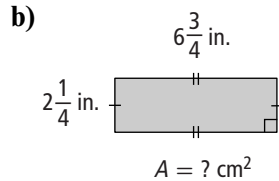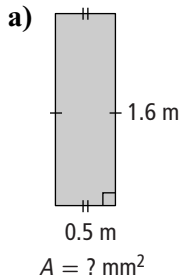
2. On a plain piece of paper, draw a letter O whose curve length you estimate to be each distance.

   a) 35 mm   **b)** 15 cm

3. a) Explain how you could measure the distance of each curved letter you drew for #2.

   b) Measure each O and compare your measurements with the required distances. If you are out by more than 5 mm for part a) or 2 cm for part b), try drawing the letter again.

4. Determine each volume using the indicated unit.

a) $r = 2.5$ ft

3.9 ft

$V = ?\ m^3$

b)

43.5 cm

21.5 cm

7 cm

$V = ?\ in.^3$
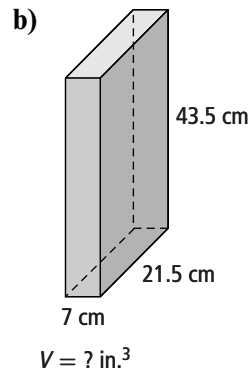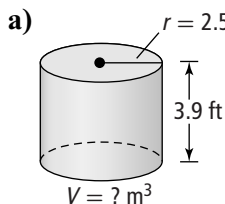
5. Work with a partner and create a list of unit conversions you have used in Chapters 1 and 2.

6. What reading is shown on each measuring device? Give each reading in both millimetres and centimetres.

a) SI ruler
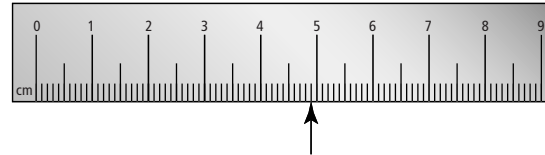
b) SI caliper
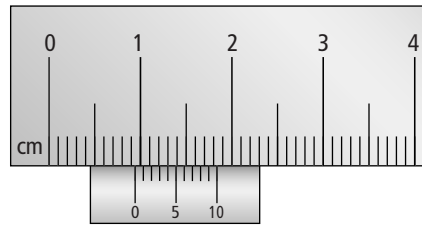
7. Chloe plans to build a chest with the dimensions shown. The store sells wood by the square foot. Determine the total amount of wood she needs to build the chest, including the base of the chest and the lid. Assume that there is no overlap and no waste. Express the answer to the nearest quarter of a square foot.

0.75 m

1.5 m

0.6 m
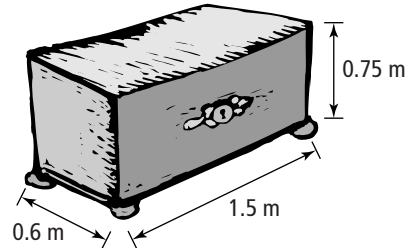
8. Suppose the actual height of the flag pole is 7.6 m. What are the dimensions of the flag? Hint: Measure the dimensions of the flag in the picture.

7.6 m
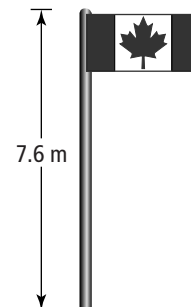
9. Describe how you would determine each volume in the indicated units. Calculate the volume to the nearest tenth of a cubic unit.

   a) a sphere with radius 6.2 cm

   b) a right pyramid with base 1.5 m by 0.7 m and height 2 m

   c) a right cone with radius 4 mm and height 7 mm

10. What reading is shown on this imperial ruler? Name one item that might be this length.

11. Sketch each solid and calculate the surface area, to the nearest tenth of a square unit.

   a) A right cone has slant height 17 in. and diameter 12 in.

   b) A pyramid has a square base with sides 14 cm and slant height 21 cm.

   c) A sphere has diameter 5 ft.

12. Convert each measurement to the unit indicated.

   a) The world's tallest man is 8′ 11″. (yard)

   b) Mount Everest is 29 029 ft. (tenth of a mile)

   c) The world's tallest tree is 126.4 yd. (inch)

13. Determine the missing dimension.

   a) A right cone has surface area 16 ft^2 and radius 1.6 ft.

   b) The square base of a pyramid has area 20 cm^2 and surface area 948 cm^2.

   c) A sphere has surface area 56.4 m^2.

14. Tim needs to shingle the roof of his gazebo. The roof is a pyramid with a square base 4.1 m by 4.1 m and a slant height of 2.1 m.

   a) How much roofing material is needed for the roof? Express the answer to the nearest square metre.

   b) If the roofing tiles measure 265 mm by 165 mm, how many tiles will he need? Assume that there is no overlap.

15. What is the height of the cone funnel?
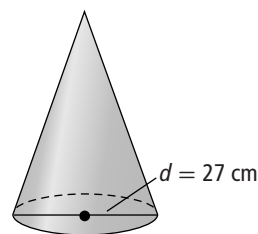
$V = 4019.2$ cm^3

16. Jon is driving home from Seattle, Washington, to Vancouver, British Columbia. A highway sign just outside Seattle indicates that the Canada–U.S. border is 138 mi. away. As Jon passes the border, he sees another sign indicating that Vancouver is 49 km away. What is the total driving distance between Seattle and Vancouver in SI units?

17. Charlotte wants to buy a bracelet made of rhodonite. The cost of the bracelet depends on the amount of rhodonite in the bracelet. If there are 30 beads and each bead is 6 mm in diameter, what is the amount of rhodonite in the bracelet, in cubic centimetres?
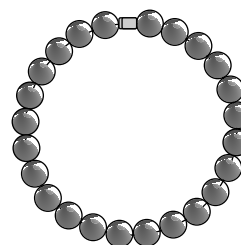
Chapter 2 Extend It Further

1. In 2009, the most expensive home in Canada was listed at $14 million CDN for 789 m². The world's most expensive home was listed at $2 billion US for 400 000 ft². Assume the exchange rate is $1 CDN = $0.9372 US. In terms of Canadian dollars per square foot, by how many times is the world's record home more expensive than Canada's record home?

A between 2 and 3 times

B between 3 and 4 times

C between 4 and 5 times

D between 5 and 6 times

⭐**2.** A cylindrical glass has an inner diameter of 6 cm and contains juice with a height of 10 cm. Two spherical scoops of ice cream are added to the glass. Each scoop has a radius of 2 cm. What is the new height of the juice?

A 1.7 cm   **B** 2.4 cm

C 11.7 cm   **D** 12.4 cm

3. The inner core of a toilet paper roll is shown. Point D is directly below point A. The paper core has diameter 4 cm and height 10 cm. Delia draws a curve along the surface of the roll starting at point D and wrapping around the roll twice before reaching point A. What is the length of the curve she draws?
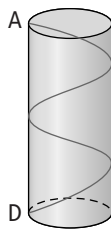
A 27 cm   **B** 30 cm

C 31 cm   **D** 34 cm

4. A cylinder that is 40 cm tall is inscribed in a cone that is 50 cm tall. The radius of the base of the cone is 30 cm. What is the lateral surface area of the cylinder?

A 360π cm²   **B** 480π cm²
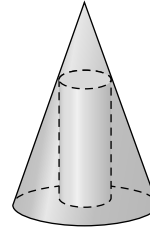
C 500π cm²   **D** 520π cm²

⭐**5.** A test tube is made of a cylindrical tube joined to a hemispherical bowl of the same radius. The capacity of the hemispherical part is $\frac{1}{6}$ that of the whole test tube. What is the ratio of the radius to the height?

⭐**6.** The pencil is composed of a cone, a cylinder, and a hemisphere, all of the same radius. If the volumes of the three components are equal, what is the ratio $x:y:z$?
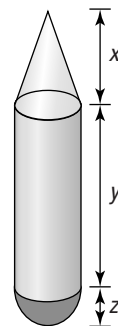
Chapter 2 Study Check

Use the chart below to help you assess the skills and processes you have developed during Chapter 2. The references in italics direct you to pages in *Mathematics 10 Exercise and Homework Book* where you could review the skill. How can you show that you have gained each skill? What can you do to improve?

Big Idea	Skills	This Shows I Know	This Is How I Can Improve
Solve surface area and volume problems that involve right cones, right cylinders, right prisms, right pyramids, and spheres, using SI and imperial units *pages 20–36*	✓ Determine surface area of 3-D objects *pages 20–27, 32–36*		
	✓ Determine volume of 3-D objects *pages 20–23, 28–36*		
	✓ Determine the volume or surface area of a composite 3-D object *pages 26–27, 30–32, 36*		
	✓ Given surface area and other dimensions, determine the unknown dimension of a 3-D object *pages 23, 25–27, 32, 35*		
	✓ Given volume and other dimensions, determine the unknown dimension of a 3-D object *pages 22, 29–31, 33, 35*		
	✓ Create diagrams to represent and solve problems involving 3-D objects *pages 22–23, 25–27, 30–36*		
	✓ Convert within and between SI and imperial units of area or volume *pages 20–23, 30–36*		
	✓ Use mental math to judge the reasonableness of answers *pages 21–23, 25–27, 29–36*		

Organizing the Ideas

Use the table below to make comparisons among the solids you have studied in Chapter 2. Compare the solids and how to calculate their surface area and volume.

Surface Area and Volume of 3-D Objects				
Characteristics	Right Cones	Spheres	Right Pyramids	
Surface Area				Similarities
				Differences
Volume				Similarities
				Differences

Study Guide

Review the types of surface area and volume problems you handled in Chapter 2. What do you need to remember to help you do similar problems? Develop a series of cards or file folders for each type of 3-D object you studied in Chapter 2.

Right Prisms
Things to Remember

Right Cylinders
Things to Remember

Right Cones
Things to Remember

Right Pyramids
Things to Remember

Spheres
Things to Remember