Chapter 4 Exponents and Radicals

4.1 Square Roots and Cube Roots

KEY IDEAS

- A perfect square is the product of two equal factors. One of these factors is called the square root.
  36 is a perfect square: $\sqrt{36} = 6$ because $6^2 = 36$. The symbol for square root is $\sqrt{\ }$.

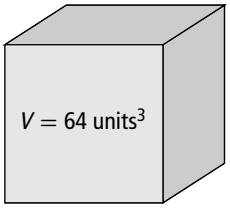
- A perfect cube is the product of three equal factors. One of these factors is called the cube root.
  – 125 is a perfect cube: $\sqrt[3]{-125} = -5$ because $(-5)^3 = -125$. The symbol for cube root is $\sqrt[3]{\ }$.

- Some numbers are both perfect squares and perfect cubes.
  15 625 is a perfect square: $125^2 = 15\,625$
  15 625 is a perfect cube: $25^3 = 15\,625$

- You can use diagrams, prime factorization, or a calculator to solve problems involving square roots and cube roots. Prime factorization involves writing a number as the product of its factors.

Determine the cube root of 64.

– Use a diagram.

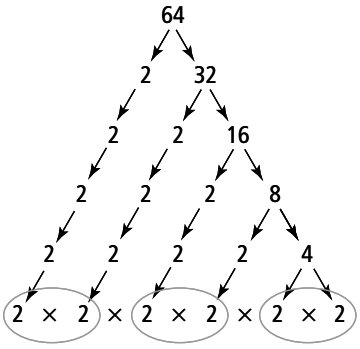


The edge lengths represent the cube root: (4)(4)(4) = 64.

– Use a calculator.

C 64 2nd $\sqrt[x]{y}$ 3 = 4.

– Use *prime factorization*.



There are three equal groups of 4.
Therefore, the cube root of 64 is 4.

Example

Identify each number as a perfect square or a perfect cube.

a) 256

b) 3375

Solution

Method 1: Use Guess and Check

a) Perfect square:
Since (13)(13) = 169, you could try 14.
$14^2 = (14)(14) = 196$ Too low
$16^2 = (16)(16) = 256$ Correct!
Therefore, 256 is a perfect square.

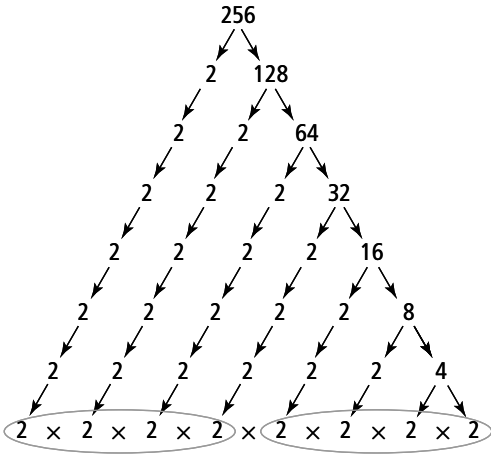
Perfect cube:
Since 256 ends with a 6, you could try 6.
$6^3 = (6)(6)(6) = 216$ Too low
$7^3 = (7)(7)(7) = 343$ Too high
No whole number cubed results in a product of 256.
Therefore, 256 is not a perfect cube.

b) Perfect square:
Since 3375 ends with a 5, you could try numbers that end with a 5.
$55^2 = (55)(55) = 2025$ Too low
$65^2 = (65)(65) = 4225$ Too high
No whole number squared results in a product of 3375.
Therefore, 3375 is not a perfect square.

Perfect cube:
Since 3375 ends with a 5, you could try numbers that end with a 5.
$25^3 = (25)(25)(25) = 15\,625$ Too high
$15^3 = (15)(15)(15) = 3375$ Correct!
Therefore, 3375 is a perfect cube.

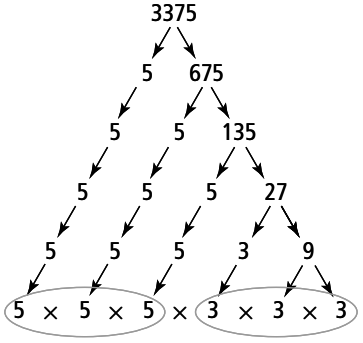
Method 2: Use Prime Factorization

a)



There are two equal groups of 2s.
Therefore, the square root of 256 is (2)(2)(2)(2) = 16.

b)



There is one group of 5s and one group of 3s.
Therefore, the cube root of 3375 is (5)(3) = 15.

Method 3: Use a Calculator

a) C 256 $\sqrt{x}$ = 16.
perfect square

C 256 2nd $\sqrt[x]{y}$ 3 = 6.349604
Since the cube root is not an integer, 256 is not a perfect cube.

b) C 3375 $\sqrt{x}$ = 58.09475
Since the square root is not a whole number, 3375 is not a perfect square.

C 3375 2nd $\sqrt[x]{y}$ 3 = 15.
perfect cube

A Practise

1. What is the value of each expression? Express the answers as integers or fractions.

a) 9^2 **b)** $(-15)^2$

c) -25^2 **d)** $\frac{4}{3^2}$

e) $-\frac{5^2}{8}$ **f)** $\left(\frac{-6}{7}\right)^2$

2. Evaluate. Express the answer as an integer or a fraction.

a) 9^3 **b)** $(-3)^3$

c) -6^3 **d)** $\frac{4^3}{8}$

e) $\frac{-9}{3^3}$ **f)** $\left(\frac{5}{7}\right)^3$

3. What is the value of each expression?

a) $\sqrt{25}$ **b)** $\sqrt{196}$

c) $\sqrt{(49)(16)}$ **d)** $\frac{18}{\sqrt{81}}$

e) $\frac{\sqrt{64}}{12}$ **f)** $\sqrt{\frac{64}{196}}$

g) $\frac{\sqrt{16}}{\sqrt{144}}$ **h)** $\sqrt{36x^2}$

i) $\frac{\sqrt{49a^2}}{\sqrt{169b^2}}$

4. Evaluate.

a) $\sqrt[3]{8}$ **b)** $\sqrt[3]{27}$

c) $\sqrt[3]{1728}$ **d)** $\sqrt[3]{(64)(125)}$

e) $\frac{\sqrt[3]{216}}{2}$ **f)** $\frac{15}{\sqrt[3]{15\,625}}$

g) $\sqrt[3]{\frac{8}{343}}$ **h)** $\sqrt[3]{125y^3}$

i) $\sqrt[3]{729a^3}$

☆**5.** Identify each number as a perfect square, a perfect cube, or both. Support your answer using a diagram or a factor tree.

a) 8 **b)** 512

c) 15 625 **d)** 196

e) 46 656 **f)** 729

6. State whether each number is a perfect square, a perfect cube, both, or neither.

a) 169 **b)** 225

c) 64 **d)** 256

e) 117 649 **f)** 133 642

7. Determine if each number is a perfect square or a perfect cube using prime factorization. Explain the process.

a) 16 **b)** 27

c) 1000 **d)** 324

e) 441 **f)** 2917

8. Calculate.

a) $\sqrt{289}$ **b)** $\sqrt{529}$

c) $\sqrt[3]{2744}$ **d)** $\sqrt[3]{10\,648}$

e) $\sqrt[3]{29\,791}$ **f)** $\sqrt[3]{19\,683}$

9. Bill is designing a cube-shaped storage container to store his hockey equipment. The container will have a volume of 2.744 m^3. What will the dimensions of the container be?

10. Sharon plans to build a square patio in a sunny area in her yard. If the patio has an area of 529 ft^2, what is its side length?

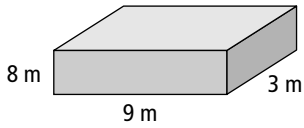
B Apply

☆**11.** Belle noticed that the water tap in the kitchen leaked. She decided to use a cylinder and collect the water drips for 24 h. She collected 5.88 cm^3 of water. Belle determined that at this rate her family would waste 2146.2 cm^3 of water per year. What would be the edge length of a cube that would contain this amount of water? Express the answer to one decimal place.

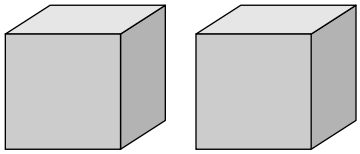
12. The Henderson family plans to build a square double garage. The floor plan shows that the garage will have an area of 576 ft^2. What are the side lengths of the garage?

13. If the area of John's square bedroom is 156.25 ft^2, what do the side lengths measure?

14. A right prism is shown. What would be the dimensions of a cube with the same volume?



15. The surface area of two dice is 1452 mm^2. What is the volume of each die?

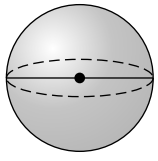


16. A grade 10 class collects scrap metal as a fundraiser. The students calculate that the scrap metal they collected occupies a volume of 238 m^3. If this metal were compressed into a cubic bale, what would its edge lengths be? Express the answer to the nearest tenth of a metre.

17. The Dice House is a zero carbon home designed by Sybarite, a British architecture firm. This cubic house has a volume of 729 m^3. What are the dimensions of the Dice House?

18. A sphere has the surface area shown. What is the length of the diameter of the sphere? Hint: $SA = 4\pi r^2$.

$SA = 803.84\ cm^2$



C Extend

19. Given the equation $y = x^2 - 4$, determine the value of y when

a) $x = 8$

b) $x = 14$

20. Given the equation $y = x^2 - 4$, determine the value of x when

a) $y = 32$

b) $y = 525$

☆**21.** Sonja owns a helium tank that holds 54 ft^3 of gas. She rents out the helium tank for parties and sells balloons with a 6-in. radius. How many balloons will a full helium tank inflate?

22. A sphere has a volume of 1296 cm^3. Determine the surface area of the sphere. Express the answer to the nearest square centimetre.

Hint: $V = \frac{4}{3}\pi r^3$.

23. Evaluate each square root.

a)

$\sqrt{25}$	
$\sqrt{2.5}$	
$\sqrt{0.25}$	
$\sqrt{0.025}$	
$\sqrt{0.0025}$	
$\sqrt{0.00025}$	

b)

$\sqrt{81}$	
$\sqrt{8.1}$	
$\sqrt{0.81}$	
$\sqrt{0.081}$	
$\sqrt{0.0081}$	
$\sqrt{0.00081}$	

c) What can you conclude about the square root of decimal numbers?

D Create Connections

☆**24.** Explain why $\sqrt{-25}$ has no solution and $\sqrt[3]{-27}$ has a solution.

25. a) What happens to the area of a square when you double the length of each side? triple the length of each side?

b) What happens to the volume of a cube when you double the length of each edge? triple the length of each edge?

4.2 Integral Exponents

KEY IDEAS

- A power with a negative exponent can be written as a power with a positive exponent.
  - $a^{-n} = \frac{1}{a^n}, a \neq 0$ $\quad 2^{-5} = \frac{1}{2^5}$ $\quad \frac{1}{a^{-n}} = a^n, a \neq 0$ $\quad \frac{1}{2^{-5}} = 2^5$
- You can apply the above principle to the exponent laws.

Exponent Law	Example
Note that a and b are rational or variable bases and m and n are integral exponents.	
Product of Powers $(a^m)(a^n) = a^{m+n}$	$(3^{-2})(3^4) = 3^{-2+4}$ $= 3^2$ or 9
Quotient of Powers $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{x^3}{x^{-5}} = x^{3-(-5)}$ $= x^8$
Power of a Power $(a^m)^n = a^{mn}$	$(0.75^4)^{-2} = 0.75^{(4)(-2)}$ $= 0.75^{-8}$ or $\frac{1}{0.75^8}$
Power of a Product $(ab)^m = a^m b^m$	$(4z)^{-3} = \frac{1}{(4z)^3}$ or $\frac{1}{64z^3}$
Power of a Quotient $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$	$\left(\frac{t}{3}\right)^{-2} = \left(\frac{3}{t}\right)^2$ $= \frac{3^2}{t^2}$ or $\frac{9}{t^2}$
Zero Exponent $a^0 = 1, a \neq 0$	$(4y^2)^0 = 1$ $-(4y^2)^0 = -1$

Example

Write each expression as a power with a single, positive exponent. Then, evaluate where possible.

a) $\left(\frac{0.4^{-2}}{0.4^2}\right)$ **b)** $(6^4)(6^{-2})$ **c)** $\left[(3x)^{-2}\right]^{-3}$

Solution

a) Method 1: Subtract the Exponents

Since the bases are the same, you can subtract the exponents.

$$\begin{aligned}\left(\frac{0.4^{-2}}{0.4^2}\right) &= 0.4^{(-2-2)} \\ &= 0.4^{-4} \\ &= 39.0625\end{aligned}$$

Method 2: Use Positive Exponents

Convert the negative exponent to a positive exponent. Then, add the exponents when multiplying.

$$\begin{aligned}\left(\frac{0.4^{-2}}{0.4^2}\right) &= \left(\frac{1}{0.4^2}\right)\left(\frac{1}{0.4^2}\right) \\ &= \left(\frac{1}{0.4^{2+2}}\right) \\ &= \left(\frac{1}{0.4^4}\right) \\ &= 39.0625\end{aligned}$$

b) Method 1: Add the Exponents

Since the bases are the same, you can add the exponents.

$$\begin{aligned}(6^4)(6^{-2}) &= 6^{4+(-2)}\\ &= 6^2\\ &= 36\end{aligned}$$

Method 2: Use Positive Exponents

Convert the negative exponent to a positive exponent. Then, subtract the exponents when dividing.

$$\begin{aligned}(6^4)(6^{-2}) &= (6^4)\left(\frac{1}{6^2}\right)\\ &= \frac{6^4}{6^2}\\ &= 6^{4-2}\\ &= 6^2\\ &= 36\end{aligned}$$

c) Method 1: Multiply the Exponents

Raise the power to the exponent. Then, multiply the exponents.

$$\begin{aligned}\left[(3x)^{-2}\right]^{-3} &= (3x)^{(-2)(-3)}\\ &= (3x)^6\\ &= 729x^6\end{aligned}$$

Method 2: Use Positive Exponents

Convert the negative exponent to a positive exponent. Convert twice. Then, multiply the exponents.

$$\begin{aligned}\left[(3x)^{-2}\right]^{-3} &= \left[\frac{1}{(3x)^2}\right]^{-3}\\ &= \left[(3x)^2\right]^3\\ &= (3x)^{(2)(3)}\\ &= (3x)^6\\ &= 729x^6\end{aligned}$$

Hint: When an expression has a coefficient and a variable, apply the exponent law to each one. $(2b)^3 = (2^3)(b^3) = 8b^3$

A Practise

1. Write each expression with positive exponents.

a) 4^{-2}

b) $3x^{-3}$

c) $(5x)^{-2}$

d) $6a^{-3}b^{-2}$

e) $-5a^{-4}$

f) $-4a^4b^{-5}$

g) $\left(\frac{2}{3}\right)^{-3}$

h) $\frac{-3x^2}{y^{-4}}$

i) $\frac{6a^{-3}}{b^4}$

2. Shelby rewrote the expression $\left(\frac{y^3}{4x^5}\right)^{-2}$ as $\frac{8x^{10}}{y^6}$. Is her answer correct? Justify your answer.

3. Simplify, then evaluate. Express your answers to four decimal places, if necessary.

a) 1.4^{-3}

b) $\left(\frac{-4^2}{2^3}\right)^{-3}$

c) $\left[(2^{-2})(2^4)\right]^{-2}$

d) $\left(\frac{-5^3}{5^3}\right)^{-3}$

e) $\left(\frac{4}{4^3}\right)^{-3}$

f) $\left(\frac{4^{-2}}{3^{-3}}\right)^2$

4. Simplify each expression by restating it using positive exponents only.

a) a^4b^{-5}

b) $\frac{-2}{a^3b^{-2}}$

c) $\left[(p)^{-6}(p)^2\right]^{-3}$

d) $\frac{12s^3}{4s^{-7}}$

e) $(6x^{-4})^{-2}$

f) $\left(\frac{t^{-3}}{t^5}\right)^{-2}$

g) $\left[(n^3)(n^{-5})\right]^2$

h) $(xy^{-3})^{-2}$

☆**5.** Simplify each expression. State the answer using positive exponents.

a) $(6)^{-3}(6)$ **b)** $\frac{(-2)^{-6}}{(-2)^{-3}}$

c) $\frac{3^3}{3^{-2}}$ **d)** $\left(\frac{4^0}{4^{-2}}\right)^2$

e) $(6^{-4})^2$ **f)** $-(3^4)^{-3}$

g) $[(2^4)(2^{-7})]^{-3}$ **h)** $\left(\frac{3^3}{4^3}\right)^{-2}$

i) $(4a^{-3})^{-2}$ **j)** $-3[(2^4)(2^{-3})]^{-2}$

6. The students in a grade 10 class were investigating the algae growth rate on the surface of a local lake. When they began, 425 cm^2 of the surface area of the lake was covered with algae. The amount of surface area covered with algae doubles each month. The students modelled this situation using the formula $SA = 425(2)^n$, where SA is the surface area of the lake covered in algae after n months. If conditions remain constant, how much of the lake will be covered in algae

a) after 6 months?

b) after 2 years?

7. A biologist is monitoring the population growth of caribou in a national park. There were 1400 caribou in 2010. The caribou population increases at a growth rate of 1.04% per year. The growth rate can be modelled using the formula $P = 1400(1.04)^n$, where P is the projected population after n years. Assuming that the growth rate remains constant, what would be the estimated caribou population in 2014?

B Apply

8. A culture of bacteria in a lab contains 400 bacterium cells. The number of cells doubles every hour. This situation can be modelled by the equation $B = 400(2)^h$, where B is the estimated number of bacteria and h is the time in hours. How many bacteria were present

a) after 3 h?

b) after 24 h?

c) 3 h ago?

☆**9.** Without using a calculator, evaluate $\left[\left((2^{-1})^2\right)^3\right]^{-1}$.

10. Kevin simplified $(2^3)(3^2)$ as 6^5. Is he correct? Justify your answer.

11. A radioactive element has a half-life of one month. The amount of the element remaining is given by the formula $A = 400\left(\frac{1}{2}\right)^n$, where n is the number of months. Today there are 400 g of the element.

a) How much will remain after 4 months?

b) How much was there a month ago?

☆**12.** The formula $d = \frac{1}{2}gt^2$ can be used to determine how long it takes an object to fall a certain distance from rest. In the formula, d is the distance the object falls, in metres, g is the acceleration due to gravity at 9.8 m /s^2, and t is the time it takes to fall, in seconds. Express each answer to one decimal place.

a) From what height does a penny fall if it takes 12.4 s to reach the ground?

b) How long does a penny take to fall from a height of 28.5 m?

c) How long does a penny take to reach the ground from a height of 248 m?

13. The population of Earth reached 6.8 billion people in 2009. Assume that the population increases by a growth rate of 1.8% per year and that the rate remains the same. The rate of growth can be modelled using the formula $P = [(6.8)(10^9)](1.018)^n$, where P is the estimated population and n is the number of years. Determine the projected population

a) by the end of 2015

b) by the end of 2020

14. In 2010, there were approximately 34 million people living in Canada. Assume that Canada's overall population growth rate is 0.9% per year and that the growth rate remains constant. The population can be estimated using the formula $P = [(3.4)(10^7)](1.009)^n$, where P is the estimated population and n is the number of years. What is the projected population

a) in 2018?

b) in 2021?

C Extend

☆**15.** Suppose you win the opportunity to receive a cash prize of $15 000 or double your money each year for a period of 25 years starting with an initial payment to you of $0.01. The value of your winnings can be determined using the formula $A = 0.01(2)^n$, where A is the payment at the end of n years.

a) What is the value of the payment you would receive after 3 years? after 10 years? after 25 years?

b) Which offer would you accept? Explain why.

c) If you received a cheque each year, how much money would you have received in total over the 25-year period?

16. The amount of sodium-24 remaining in a sample that started at 86 g can be represented by the equation $N = 86(0.5)^{\frac{t}{15}}$, where t is time, in hours. Determine the amount of sodium-24 remaining after each of the following time periods. Express the answers to two decimal places, if necessary.

a) after 30 h

b) after 90 h

c) after 120 h

17. Determine the value of x that makes each statement true.

a) $\left(\frac{4}{5}\right)^x = \frac{625}{256}$

b) $-3^x = -729$

c) $x^{-3} = \frac{27}{8}$

d) $2(6^x) = 432$

18. A scientist discovered a new isotope and called it mathodium-334. In the formula $A_f = A_i\,(3)^{-t}$, A_f represents the amount of the isotope remaining, A_i is the initial amount, in grams, and t is the time in days.

a) If a sample started at 85 g, how much would remain after 4 days? Express the answer to two decimal places.

b) The amount of mathodium-334 remaining after 6 h is 0.165 g. Calculate the amount of the original sample. Express the answer to two decimal places.

D Create Connections

19. Is $[(2^3)^4]^2$ equal to $[(2^4)^2]^3$? Justify your answer.

☆**20.** What value of x makes the following statement true?

$2^x + 2^x + 2^x + 2^x = 256$

21. Without using a calculator, show that $2^2 + 2^3 + 2^4$ is not equal to $(2^2)(2^3)(2^4)$. Explain why the answers are not the same.

22. Describe a real-life situation in which a positive exponent and a negative exponent can be used to model a problem.

a) Give an example of what the positive exponent represents.

b) Give an example of what the negative exponent represents.

4.3 Rational Exponents

KEY IDEAS

- A power with a negative exponent can be written as a power with a positive exponent.

  – $a^{-n} = \frac{1}{a^n}, a \neq 0$ $\quad 9^{-1.3} = \frac{1}{9^{1.3}}$ $\quad – \frac{1}{a^{-n}} = a^n, a \neq 0$ $\quad \frac{1}{2^{-3.2}} = 2^{3.2}$

- You can apply the above principle to the exponent laws.

Exponent Law	Example
Note that a and b are rational or variable bases and m and n are integral exponents.	
Product of Powers $(a^m)(a^n) = a^{m+n}$	$\left(x^{\frac{3}{5}}\right)\left(x^{\frac{6}{5}}\right) = x^{\frac{3}{5}+\frac{6}{5}}$ $= x^{\frac{9}{5}}$
Quotient of Powers $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{4s^{2.5}}{12s^{0.5}} = \frac{1}{3}s^{(2.5-0.5)}$ $= \frac{1}{3}s^2$ or $\frac{s^2}{3}$
Power of a Power $(a^m)^n = a^{mn}$	$(t^{3.3})^{\frac{1}{3}} = t^{(3.3)\left(\frac{1}{3}\right)}$ $= t^{1.1}$
Power of a Product $(ab)^m = a^m b^m$	$\left(8x^{\frac{1}{2}}\right)^{\frac{2}{3}} = (2^3)^{\frac{2}{3}}\left(x^{\frac{1}{2}}\right)^{\frac{2}{3}}$ $= 4x^{\frac{2}{6}}$ or $4x^{\frac{1}{3}}$
Power of a Quotient $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, \neq 0$	$\left(\frac{x^3}{y^6}\right)^{\frac{1}{3}} = \frac{(x^3)^{\frac{1}{3}}}{(y^6)^{\frac{1}{3}}}$ $= \frac{x}{y^2}$
Zero Exponent $a^0 = 1, a \neq 0$	$(-2y^2)^0 = 1$ $-(2y^2)^0 = -1$

- A power with a rational exponent can be written with the exponent in decimal or fractional form. $\quad x^{\frac{3}{5}} = x^{0.6}$

Example

Write each product or quotient as a power with a single positive exponent. Then, evaluate where possible.

a) $\left(7^{\frac{1}{2}}\right)(7^3)$ **b)** $\frac{9^{1.25}}{9^{\frac{3}{4}}}$ **c)** $(16x^6)^{\frac{1}{4}}$ **d)** $\left(\frac{3^{0.25}}{3^{\frac{3}{4}}}\right)^3$ **e)** $\left(\frac{27}{8}\right)^{-0.4}$

Solution

a) Since the bases are the same, you can add the exponents. Remember to determine the lowest common denominator when adding fractions.

$$\begin{aligned}\left(7^{\frac{1}{2}}\right)(7^3) &= \left(7^{\frac{1}{2}}\right)\left(7^{\frac{6}{2}}\right)\\ &= 7^{\left(\frac{1}{2}+\frac{6}{2}\right)}\\ &= 7^{\frac{7}{2}}\end{aligned}$$

b) Convert the rational exponents so both are fractions or decimal numbers. Then, since the bases are the same, you can subtract the exponents.

Method 1: Convert to Fractions

$$\begin{aligned}\frac{9^{1.25}}{9^{\frac{3}{4}}} &= \frac{9^{\frac{5}{4}}}{9^{\frac{3}{4}}}\\ &= 9^{\left(\frac{5}{4}-\frac{3}{4}\right)}\\ &= 9^{\frac{2}{4}}\\ &= 9^{\frac{1}{2}}\\ &= 3\end{aligned}$$

Method 2: Convert to Decimals

$$\begin{aligned}\frac{9^{1.25}}{9^{\frac{3}{4}}} &= \frac{9^{1.25}}{9^{0.75}}\\ &= 9^{(1.25-0.75)}\\ &= 9^{0.5}\\ &= 3\end{aligned}$$

c) Raise each term to the exponent. Then, multiply the exponents.

$$\begin{aligned}(16x^6)^{\frac{1}{4}} &= (16)^{\frac{1}{4}}\, x^{(6)\left(\frac{1}{4}\right)}\\ &= 2x^{\frac{6}{4}}\\ &= 2x^{\frac{3}{2}}\end{aligned}$$

d) Method 1: Subtract the Exponents

Convert the rational exponents to fractions or decimal numbers. Since the bases are the same, you can subtract the exponents. Raise the result to the exponent 3. Then, multiply.

Convert to fractions:

$$\begin{aligned}\left(\frac{3^{0.25}}{3^{\frac{3}{4}}}\right)^3 &= \left(\frac{3^{\frac{1}{4}}}{3^{\frac{3}{4}}}\right)^3\\ &= \left[3^{\left(\frac{1}{4}-\frac{3}{4}\right)}\right]^3\\ &= \left(3^{\frac{-2}{4}}\right)^3\\ &= \left(3^{\frac{-1}{2}}\right)^3\\ &= \left(3^{\frac{-3}{2}}\right)\\ &= \frac{1}{3^{\frac{3}{2}}}\end{aligned}$$

Method 2: Apply Power of a Power

Raise each power to the exponent 3. Next, convert the rational exponents to fractions or decimal numbers. Then, subtract the exponents of the resulting powers.

$$\begin{aligned}\left(\frac{3^{0.25}}{3^{\frac{3}{4}}}\right)^3 &= \left(\frac{3^{\frac{3}{4}}}{3^{\frac{9}{4}}}\right)\\ &= 3^{\frac{3}{4}-\frac{9}{4}}\\ &= 3^{\frac{-6}{4}}\\ &= \frac{1}{3^{\frac{3}{2}}}\end{aligned}$$

e) Convert the bases to a single exponent. Then, raise the result to the exponent −0.4.

$$\begin{aligned}\left(\frac{27}{8}\right)^{-0.4} &= \left[\frac{(3^3)}{(2^3)}\right]^{-0.4}\\ &= \left[\left(\frac{3}{2}\right)^3\right]^{-0.4}\\ &= \left(\frac{3}{2}\right)^{-1.2}\\ &= \left(\frac{2}{3}\right)^{1.2}\end{aligned}$$

A Practise

1. Use the exponent laws to simplify each expression. Where possible, compute numerical values.

a) $(a^6)\left(a^{\frac{3}{2}}\right)$ **b)** $\left(y^{\frac{1}{3}}\right)\left(y^{\frac{1}{2}}\right)$

c) $(x^{0.4})\left(x^{\frac{1}{2}}\right)$ **d)** $(a^{0.2})^3$

e) $\left(x^{\frac{2}{3}}\right)^{-6}$ **f)** $\left(81^{\frac{1}{4}}\right)^2$

g) $\left(\frac{-64x^{\frac{3}{4}}}{27x^{\frac{1}{2}}}\right)^{\frac{1}{3}}$ **h)** $\left(-5a^{\frac{1}{2}}\right)\left(2a^{\frac{3}{5}}\right)$

i) $(256a^6)^{0.25}$

2. Use the exponent laws to simplify each expression. Leave your answers with positive exponents.

a) $(a^{-2})\left(a^{\frac{3}{4}}\right)$ **b)** $(16^{-0.25})^2$

c) $\frac{\left(y^{\frac{2}{3}}\right)^{-2}}{\left(y^{\frac{1}{2}}\right)^{-4}}$ **d)** $\left(a^{\frac{3}{4}}\right)^{-0.5}(a^2)^{-0.25}$

e) $\left[\frac{(a^2b)}{(ab)^3}\right]^{-1.5}$ **f)** $\left(\frac{25x^{-2}}{16x^{\frac{-1}{2}}}\right)^{-1.5}$

g) $(4x^3)^{\frac{-1}{2}}(27y^2)^{\frac{1}{3}}$ **h)** $\left(\frac{81x^{\frac{2}{3}}}{625y^{\frac{3}{5}}}\right)^{0.25}$

★3. Use the exponent laws to help identify a value for q that satisfies each equation.

a) $\left(x^{\frac{2}{3}}\right)^q = x^{\frac{4}{3}}$ **b)** $\left(x^{\frac{-2}{3}}\right)(x^q) = x^{\frac{-1}{6}}$

c) $\frac{y^{\frac{2}{3}}}{y^q} = y^{\frac{11}{12}}$ **d)** $(27x^2)^{\frac{1}{3}}\left(qx^2\right)^{\frac{-1}{2}} = \frac{3}{2x^{\frac{1}{3}}}$

e) $(5^q)(-3^{-q}) = \frac{-125}{27}$

4. Evaluate without using a calculator. Leave the answers as rational numbers.

a) $16^{\frac{3}{4}}$ **b)** $-243^{\frac{2}{5}}$

c) $8^{\frac{-5}{3}}$ **d)** $\left(\frac{49}{9}\right)^{\frac{3}{2}}$

e) $\left(\frac{125x^2}{8y^3}\right)^{\frac{2}{3}}$ **f)** $\frac{5^{-2}}{25^{\frac{-3}{2}}}$

5. Evaluate using a calculator. Express the answers to four decimal places, if necessary.

a) $(9^{-0.5})^3$ **b)** $\left(64^{\frac{1}{2}}\right)^3$

c) $(3^{1.2})(3^{2.4})$ **d)** $\frac{16^{\frac{2}{3}}}{16^{-0.2}}$

e) $\left(\frac{3^{\frac{-1}{2}}}{81^{0.25}}\right)^2$ **f)** $\left(\frac{8}{7^{\frac{1}{2}}}\right)^{\frac{5}{3}}$

6. Mid Lake, in Manitoba, is stocked with rainbow trout annually. The population grows at a rate of 11.5% per month. The number of trout stocked is given by the expression $623(1.115)^n$, where n is the number of months since the start of the trout season. Determine the number of trout after

a) 3 months **b)** $7\frac{1}{2}$ months

c) $3\frac{1}{2}$ months **d)** $6\frac{1}{2}$ months

B Apply

★7. For each solution, identify the step where an error was made. What is the correct answer? Compare your corrections with those of a classmate.

a) $\frac{a^{\frac{2}{3}}}{a^{\frac{1}{4}}} = a^{\frac{2}{3}-\frac{1}{4}}$
$= a^{-1}$
$= \frac{1}{a}$

b) $(16y^{-6})^{-0.5} = (16)^{-0.5}(y^{-6})^{-0.5}$
$= 8y^3$

8. Karen has saved $1500 for college. She deposits this amount into a 3-year term deposit that earns 3.25% interest per year. The formula for calculating the value of her investment is $A = P(1 + i)^n$, where A is the amount of money at the end of the term, i is the interest rate as a decimal number, and n is the number of years the money is invested. How much will her investment be worth at the end of

a) 3 years?

b) $2\frac{1}{2}$ years?

9. A species of bacteria increases in number by 50% every 25 min. The growth of the bacteria can be modelled using the equation $N = 1000(1.5)^{\frac{t}{25}}$, where N is the number of bacteria after t min.

a) What does the value 1.5 in the formula represent? the value 1000?

b) How many bacteria are present after 1 h?

c) How many bacteria were present 30 min ago?

10. In June 2009, there were approximately 33.985 million people living in Canada. Assume that Canada's natural growth rate is 0.3% per year and that this growth rate remains constant. The natural growth rate represents the number of births and number of deaths in a population and does not take immigration and emigration into account. Canada's growth rate can be modelled using the formula $P = 33.985(1.003)^n$, where P is the population in millions and n is the number of years since 2009.

a) What is the projected population in 15.5 years? Express the answer to three decimal places.

b) What was Canada's population in March 2001?

☆**11.** Bismuth-214 has a half-life of approximately 20 min. The amount of bismuth-214 remaining in a sample that began at 28 g can be represented by the formula $A = 28(0.5)^{\frac{t}{20}}$, where A is the amount remaining after t min. Determine the amount of bismuth-214 remaining after each of the following periods of time. Express each answer to the nearest hundredth of a gram.

a) after 45 min

b) after 2 h

c) after $3\frac{1}{4}$ h

12. In the formula $C = 50(0.5)^{\frac{t}{3}}$, C is the remaining concentration of a particular medication in the bloodstream, in milligrams, and t is the time, in hours.

a) Determine the missing values in the table.

Time (t)	0	3	6	9	12
Concentration (C)					

b) Graph the data in part a). Let t represent the x-axis and C represent the y-axis.

c) If $C = 0.195$ mg, what is the value of t?

d) If $t = 42$ h, what is the value of C?

C Extend

13. Phosphorus-32 has a half-life of 14 days. If 2.56 g of a sample of phosphorus-32 remain after 70 days, what was the original mass of the sample? Use the formula $A_f = A_i(0.5)^{\frac{t}{14}}$, where A_f is the final amount, A_i is the initial amount, and t is the time in days.

14. Michelle invested money in a mutual fund. By the end of 3 years, she had lost 8% of the original value of her investment. Her account balance was \$2672.57.

a) How much did Michelle originally invest? Use the formula $A_f = A_i(1 - r)^{\frac{t}{12}}$, where A_f is the final value of her investment, A_i is the original amount invested, r is the percent of the value lost, and t is the time, in months.

b) How much money did she lose in her investment?

D Create Connections

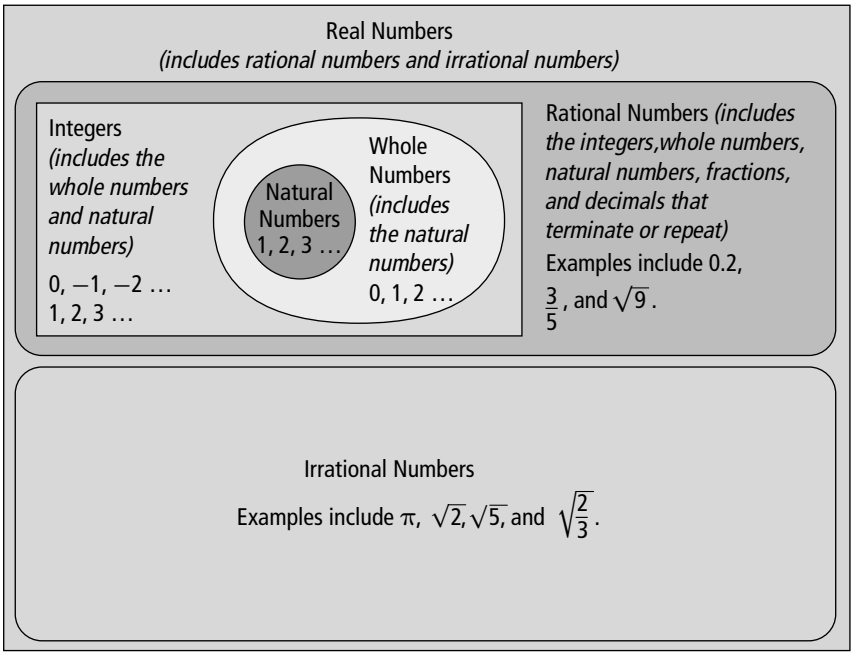
☆**15.** Solve for x using the exponent laws.

$4^{\frac{1}{2}} + 4^{\frac{1}{2}} + 4^{\frac{1}{2}} + 4^{\frac{1}{2}} = 4^x$

4.4 Irrational Numbers

KEY IDEAS

- Rational numbers and irrational numbers form the set of real numbers.



- Radicals can be expressed as powers with fractional exponents.
  $\sqrt[n]{x^m} = x^{\frac{m}{n}}$
  The index of the radical has the same value as the denominator of the fractional exponent.
  $\sqrt[3]{10} = 10^{\frac{1}{3}} \qquad \sqrt[5]{7^3} = 7^{\frac{3}{5}}$

- Radicals can be entire radicals such as $\sqrt{72}$, $\sqrt[5]{96}$, and $\sqrt[3]{\frac{54}{8}}$. They can also be mixed radicals such as $6\sqrt{2}$, $2\sqrt[5]{3}$, and $\frac{3\sqrt{2}}{2}$. You can convert between entire radicals and mixed radicals.

- You can order radicals that are irrational numbers using different methods:
  - Use a calculator to produce approximate values.
  - Express each irrational number as an entire radical.

Example

Convert each of the following as requested.
Express each power as an equivalent radical.

a) $32^{\frac{1}{2}}$ **b)** $16^{\frac{2}{3}}$ **c)** $(8x^3)^{\frac{1}{4}}$

Express each radical as a power with a rational exponent.

d) $\sqrt{6^3}$ **e)** $\sqrt[3]{5^2}$ **f)** $\sqrt[t]{8^3}$

Express each mixed radical as an entire radical.

g) $2.5\sqrt{4}$ **h)** $2\sqrt[3]{4}$ **i)** $-2\sqrt[5]{3}$

Express each entire radical as a mixed radical.

j) $\sqrt{112}$ **k)** $\sqrt[4]{96}$ **l)** $\sqrt{252}$

Solution

Write each power as a radical. Use the denominator of the exponent as the index.

a) $32^{\frac{1}{2}} = \sqrt{32}$ **b)** $16^{\frac{2}{3}} = \left(\sqrt[3]{16}\right)^2$ **c)** $(8x^3)^{\frac{1}{4}} = \sqrt[4]{8x^3}$

Write each radical as a power. Use the index as the denominator of the exponent.

d) $\sqrt{6^3} = 6^{\frac{3}{2}}$ **e)** $\sqrt[3]{5^2} = 5^{\frac{2}{3}}$ **f)** $\sqrt[t]{8^3} = 8^{\frac{3}{t}}$

The index is 3. Convert the whole number 2 to a radical using the fraction $\frac{3}{3}$.
$2^{\frac{3}{3}} = \sqrt[3]{(2^3)}$

Write each mixed radical as an entire radical.

g)
$$\begin{aligned} 2.5\sqrt{4} &= \sqrt{(2.5^2)}\sqrt{4} \\ &= \sqrt{(2.5^2)(4)} \\ &= \sqrt{(6.25)(4)} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

h)
$$\begin{aligned} 2\sqrt[3]{4} &= \left(2^{\frac{3}{3}}\right)\left(\sqrt[3]{4}\right) \\ &= \sqrt[3]{(2^3)(4)} \\ &= \sqrt[3]{(8)(4)} \\ &= \sqrt[3]{32} \end{aligned}$$

i)
$$\begin{aligned} -2\sqrt[5]{3} &= (-2)^{\frac{5}{5}}\left(\sqrt[5]{3}\right) \\ &= \sqrt[5]{(-2^5)(3)} \\ &= \sqrt[5]{(-32)(3)} \\ &= \sqrt[5]{-96} \end{aligned}$$

Express each entire radical as a mixed radical.

j)
$$\begin{aligned} \sqrt{112} &= \sqrt{(16)(7)} \\ &= \sqrt{16}\sqrt{7} \\ &= 4\sqrt{7} \end{aligned}$$

k)
$$\begin{aligned} \sqrt[4]{96} &= \sqrt[4]{(2)(2)(2)(2)(6)} \\ &= \sqrt[4]{(2^4)(6)} \\ &= 2\sqrt[4]{6} \end{aligned}$$

l) $\sqrt{252} = \sqrt{(36)(7)}$ *or* $\sqrt{252} = \sqrt{(4)(63)}$

$$\begin{aligned} &= \sqrt{36}\sqrt{7} && = \sqrt{4}\left(\sqrt{(9)(7)}\right) \\ &= 6\sqrt{7} && = (2)(\sqrt{9})(\sqrt{7}) \\ & && = (2)(3)(\sqrt{7}) \\ & && = 6\sqrt{7} \end{aligned}$$

A Practise

1. Express each power as an equivalent radical.

a) $5^{\frac{2}{3}}$ **b)** $8^{0.75}$

c) $6^{\frac{3}{5}}$ **d)** $81^{0.5}$

☆**e)** $\frac{1}{9^{\frac{5}{3}}}$ **f)** $(x^3)^{\frac{1}{4}}$

g) $\left(a^{\frac{1}{3}}\right)^2$ **h)** $\left[\frac{\left(x^{\frac{1}{3}}\right)}{\left(y^{\frac{1}{3}}\right)}\right]^2$

2. Express each radical as a power.

a) $\sqrt[4]{3^3}$ **b)** $\sqrt[3]{(5t)^4}$

c) $\sqrt[3]{x^2}$ **d)** $\sqrt[5]{\frac{a^2}{b^3}}$

e) $\sqrt[3]{y^{\frac{5}{2}}}$ **f)** $\sqrt[a]{2^3}$

3. Evaluate each expression. State the result to four decimal places, if necessary.

a) $\sqrt{0.25}$ **b)** $(64)^{\frac{1}{3}}$

c) $3\sqrt{12}$ **d)** $\sqrt{\left(\frac{5}{4}\right)^2}$

e) $4(1.2)^{\frac{3}{4}}$ **f)** $\frac{\sqrt[3]{16}}{\sqrt{12}}$

4. Express each mixed radical as an equivalent entire radical.

☆**a)** $4\sqrt{5}$ **b)** $3\sqrt{4}$

c) $5\sqrt{13}$ **d)** $6.2\sqrt{10}$

e) $3.3\sqrt{16}$ **f)** $\frac{1}{5}\sqrt{10}$

5. Express each mixed radical as an equivalent entire radical.

a) $3\sqrt[3]{5}$ **b)** $7\sqrt[3]{3}$

c) $5\sqrt[3]{6}$ **d)** $2\sqrt[4]{7}$

e) $\frac{1}{2}\sqrt[3]{5}$ **f)** $1.5\sqrt[4]{10}$

6. Express each entire radical as an equivalent mixed radical.

a) $\sqrt{32}$ **b)** $\sqrt{44}$

c) $\sqrt{90}$ **d)** $\sqrt{80}$

e) $\sqrt{360}$ **f)** $\sqrt{475}$

7. Express each entire radical as an equivalent mixed radical.

a) $\sqrt[3]{48}$ **b)** $\sqrt[3]{120}$

c) $\sqrt[3]{324}$ **d)** $\sqrt[4]{48}$

e) $\sqrt[4]{405}$ **f)** $\sqrt[4]{208}$

8. Order each set of numbers from greatest to least. Then, identify the irrational numbers.

a) $0.5\sqrt{2}$ $0.\overline{7}$ $\frac{3}{4}$ $\sqrt{0.49}$

b) $\frac{2}{3}$ $\sqrt[3]{0.343}$ $\sqrt{0.38}$ 0.62

9. Plot each set of numbers on a number line. Which of the numbers in each set is irrational?

a) $\sqrt[3]{435}$ $8.\overline{5}$ $4\sqrt{5}$ $\sqrt{64}$

b) $\frac{2\sqrt{85}}{3}$ $\sqrt[3]{216}$ $6\frac{9}{11}$ $3\sqrt{7}$

B Apply

10. Determine the diameter of a sphere that has a surface area of 320 cm^2. Use the formula $SA = 4\pi r^2$. Express the answer to three decimal places.

11. The volume of a cylinder is 312 cm^3 and its height is 6 cm. Determine the diameter of the cylinder. Use the formula $V = \pi r^2 h$. Express the answer to the nearest hundredth of a centimetre.

☆**12.** There are approximately 1.3 billion km^3 of water on Earth. What would be the length of the edge of a cube that contained Earth's estimated total volume of water? Express the answer to the nearest kilometre.

13. In the formula $r = \sqrt[3]{\frac{3V}{4\pi}}$, r represents the radius of a sphere, in centimetres, and V is the volume of the sphere, in cubic centimetres. What is the length of the radius of a sphere with each of the following volumes? Express the answers to two decimal places.

a) 132 cm^3 **b)** 1896 cm^3

14. A pendulum has a length of 6 ft. The formula $T = \sqrt{\frac{4\pi^2 l}{32 \text{ ft/s}^2}}$ represents the period of the pendulum. In this formula, T is the period of the pendulum, in seconds, and l is the length of the pendulum, in feet. Calculate the period of the pendulum. Express the answer to two decimal places.

15. A cone has a volume of 27 489 cm3 and a height of 14 cm. Using the formula $V = \frac{1}{3}\pi r2h$, determine the diameter of the cone. Express the answer to the nearest centimetre.

16. Chemical equilibrium applies to chemical reactions that can occur in two directions. When a chemical reaction reaches equilibrium, the rate of the forward reaction is equal to the rate of its reverse reaction. In the formula $Q = \frac{C}{A^2B^3}$, Q represents the solutions at equilibrium, and A, B, and C are three chemicals involved in a chemical reaction. Express each answer to two decimal places.

a) Determine the concentration of solution B if solution A has a concentration of 0.25 M, solution C has a concentration of 0.12 M, and the value of Q is 569.

b) Determine the concentration of solution A if solution B has a concentration of 0.32 M, solution C has a concentration of 0.45 M, and the value of Q is 26.

17. The national arena of Sweden, the Ericsson Globe, is considered the largest hemispherical building in the world. The interior of the Ericsson Globe has a volume of 696 910 m^3.

a) Determine the diameter of the arena using the formula $r = \sqrt[3]{\frac{3V}{4\pi}}$, where r is the radius of the arena, in metres, and V is the volume of the arena, in cubic metres. Express the answer to one decimal place.

b) Determine the surface area of the Ericsson Globe using the formula $SA = 4\pi r^2$, where SA represents the surface area in square metres. Express the answer to one decimal place.

☆**18.** The surface area of a cylinder given its volume can be calculated using the formula $SA = 2\pi\left[h\left(\sqrt{\frac{V}{\pi h}}\right) + \left(\frac{V}{\pi h}\right)\right]$. Determine the surface area of a cylinder with height 26 m and volume of 26 465 m^3. Express the answer to the nearest square metre.

C Extend

19. Without using a calculator, solve each of the following:

a) $\sqrt{\sqrt{16}}$

b) $\sqrt{\sqrt[3]{15\,625}}$

☆**c)** $\sqrt{4 + \sqrt{19 + \sqrt{36}}}$

☆**d)** $\sqrt[4]{13 + \sqrt[3]{22 + \sqrt[3]{125}}}$

20. Express as a power with a single rational exponent.

☆**a)** $\sqrt[3]{\sqrt{7}}$ ☆**b)** $\sqrt[4]{\sqrt[3]{5^2}}$

c) $\sqrt[5]{\sqrt{\frac{1}{8}}}$ **d)** $\sqrt[4]{\sqrt[3]{\left(\frac{2}{5}\right)^6}}$

D Create Connections

21. Does the expression $\sqrt[4]{x^3}$ always have a solution? Explain your reasoning.

22. Does the expression $\sqrt[3]{x^4}$ always have a solution? Explain your reasoning.

23. Copy the table and add rows to fill in the information for the first 20 whole numbers.

Number	Square Root	Answer
1	$\sqrt{1}$	1

Use your calculator to determine each square root and copy all of the digits showing on the calculator.

a) What do you notice about the square roots of numbers that are not perfect squares?

b) Are the square roots of all non-perfect squares irrational? Explain.

Chapter 4 Review

4.1 Square Roots and Cube Roots

1. Which of the following numbers are perfect squares, perfect cubes, or both?

a) 49 **b)** 343

c) 484 **d)** 1728

e) 1024 **f)** 15 625

2. Use prime factorization to evaluate

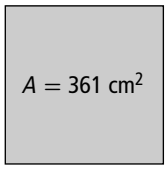
☆**a)** $\sqrt{196}$ **b)** $\sqrt[3]{512}$

3. Calculate.

a) $\sqrt{256}$ **b)** $\sqrt[3]{2197}$

c) $\sqrt[3]{27\,000}$

4. What are the dimensions of the square?



☆**5.** Christina wants to replace the flooring in her bedroom with square tiles. Each tile measures 6 in. by 6 in. The area of the floor is 9 ft by 9 ft.

a) How many tiles does Christina need?

b) Each tile costs $1.38 including taxes. How much will the tiles cost?

4.2 Integral Exponents

6. Write as a power with a positive exponent.

a) $(a^3)^{-2}$

b) $\dfrac{(3.5)^3}{(3.5)^{-4}}$

☆**c)** $\left(\dfrac{b^2}{b^{-5}}\right)^2$

7. Evaluate each expression. Express the answer to three decimal places, if necessary.

a) $(3^2)^{-2}$

b) $\left[\dfrac{5^2}{(2.5)^3(1.25)}\right]^3$

c) $(0.5^2)^{-3}(2.8^2)^2$

8. A radioactive element has a half-life of one month. The formula for the amount of the element remaining is $A = m\left(\frac{1}{2}\right)^n$, where m is the mass of the element, in grams, and n is the number of months. How much of a 740-g sample of the element

a) remains after 6 months? Express your answer to two decimal places.

b) remains after 14 months? Express your answer to three decimal places.

c) was there 4 months ago? Express your answer to the nearest gram.

9. Newfoundland has the highest population density of moose in North America. In 2009, there were approximately 135 000 moose on the island. Assuming a growth rate of 8.5%, this situation can be modelled using the formula $P = 135\,000(1.085)^n$, where P is the estimated moose population and n is the number of years since 2009. If the growth rate remains constant, how many moose will there be after

a) 1 year?

b) 2 years?

c) 5 years?

10. a) Using the information in #9, how many moose will there be in 2020?

b) Assume that the growth rate was the same before 2009. How many moose were there at the beginning of 2000?

4.3 Rational Exponents

☆**11.** Simplify each expression. Express each answer with a positive exponent.

a) $(5^{-0.5})^{\frac{3}{4}}$

b) $\dfrac{2.8^{0.4}}{2.8^{\frac{-1}{2}}}$

c) $(27x^{-2})^{\frac{-2}{3}}$

12. Without using a calculator, Victoria incorrectly simplified the following expression. What errors did she make? Determine the correct answer.

$$(27x)^{\frac{-1}{3}}(9x)^{\frac{1}{2}} = (243x)^{\left(\frac{-1}{3}+\frac{1}{2}\right)}$$
$$= (243x)^{\frac{1}{6}}$$

☆**13.** Without using a calculator, evaluate each expression.

a) $\dfrac{8^{\frac{5}{3}}}{4^2}$ **b)** $\dfrac{125^{\frac{2}{3}}}{5^{-1}}$

c) $\dfrac{9^{\frac{3}{2}}}{27^{\frac{1}{3}}}$ **d)** $\dfrac{8^{\frac{2}{3}}}{32^{\frac{4}{5}}}$

14. Evaluate each expression. Express each answer to four decimal places, if necessary.

a) $\left(20^{\frac{1}{4}}\right)\left(20^{\frac{2}{3}}\right)$ **b)** $(6^{-4})^{\frac{1}{3}}$

c) $\left(\dfrac{2.5^{\frac{3}{4}}}{2.5^{-0.5}}\right)^2$ **d)** $\dfrac{(25^3)}{(2^3)(10^2)}$

15. Jessica invested $1500 in an account that increases in value at a rate of 3.25% annually. The value of the account can be determined using the formula $A = 1500(1.0325)^t$, where A is the total value of the investment and t is the number of years. What is the value of Jessica's account at the end of three years?

16. The students in a grade 10 class are making T-shirts for a fundraiser. The cost of the ink needed to print T-shirts can be determined using the equation $C = 5.75n^{\frac{3}{4}} + 60$, where n is the number of T-shirts. Determine the cost of the ink needed to print 350 T-shirts.

17. Iodine-131 has a half-life of 8 days. Iodine-131 has medical uses such as treating people with an overactive thyroid. A patient is given 9.5 mg of iodine-131. How much would remain in the patient's body after 30 days? Use the formula $A = 9.5(0.5)^{\frac{t}{8}}$, where A is the amount remaining in the patient's body and t is the time, in days. Express the answer to the nearest thousandth of a milligram.

4.4 Irrational Numbers

18. Write each power as an equivalent radical.

a) $x^{\frac{2}{5}}$ **b)** $(16s^3)^{\frac{3}{5}}$

c) $\left(\dfrac{a^5}{7}\right)^{0.75}$ **d)** $(5a^4)^{\frac{-1}{3}}$

19. Express each radical as a power.

a) $\sqrt{x^5}$ **b)** $\sqrt[4]{5^2}$

c) $4\sqrt[5]{x^3}$ **d)** $\sqrt[3]{(4y)^4}$

20. Convert each mixed radical to an equivalent entire radical.

a) $4\sqrt{7}$ **b)** $6\sqrt{5}$

☆**c)** $3\sqrt[3]{2}$ **d)** $-5\sqrt[3]{3}$

21. Express each entire radical as an equivalent mixed radical.

a) $\sqrt{252}$ **b)** $\sqrt[3]{384}$

c) $\sqrt[4]{48}$ **d)** $\sqrt[3]{405}$

22. Identify the irrational numbers in each set. Then, arrange the numbers from greatest to least.

a) $\sqrt[3]{216}$ $0.2\overline{3}$ $\dfrac{4\sqrt{5}}{2}$ $\sqrt{0.25}$

b) $\sqrt{0.81}$ $\sqrt[3]{32}$ $\dfrac{3\sqrt{25}}{4}$ $0.\overline{49}$

23. The volume of a sphere is given by the formula $V = \dfrac{4\pi r^3}{3}$, where r is the radius of the sphere.

a) What is the volume of a sphere with a radius of 25.4 cm? Express the answer to two decimal places.

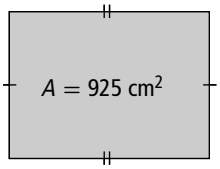
b) Determine the radius of a sphere with a volume of 384.66 cm^3. Express the answer to one decimal place.

Chapters 1–4 Cumulative Review

1. The orca is the motif for the 2010 Olympic gold medal. Assume that the image shows a reduction of 1 : 4. What is the diameter of the actual gold medal, to the nearest centimetre?

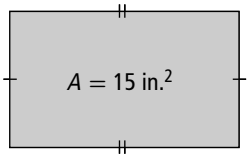
2. Determine possible dimensions for each area. Then, use your dimensions to calculate the area to the indicated equivalent.

a)



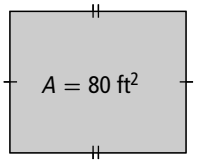
$A = ?\ mm^2$

b)



$A = ?\ m^2$

c)



$A = ?\ cm^2$

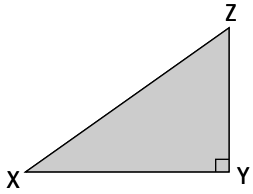
3. State whether each number is a perfect square, a perfect cube, both, or neither.

a) −1 **b)** 64 **c)** 19 683

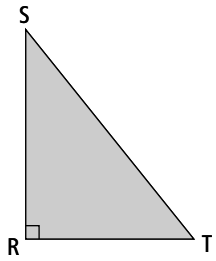
d) 625 **e)** 7650

4. Identify the hypotenuse, opposite, and adjacent sides associated with each specified angle.

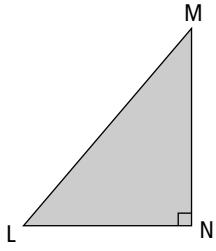
a) ∠Z



b) ∠S

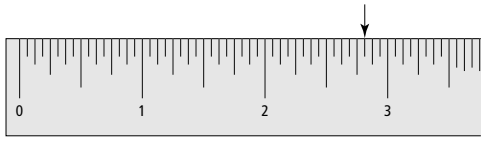


c) ∠M

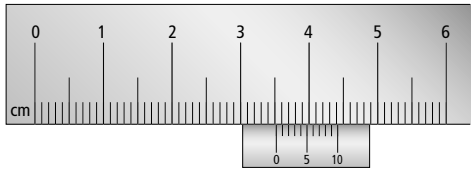


5. What is the reading represented on each measuring device? Estimate and then calculate each equivalent measurement in the other system (SI or imperial). Express the answer to the nearest tenth of a unit.

a) imperial ruler



b) SI caliper



6. Emma is planting a new lawn. The instructions on a 3-lb bag of grass seed say to apply 3 lb of seed per 1000 ft^2. The lawn has dimensions of 550 m^2.

a) How many pounds of grass seed will Emma require? Express the answer to the nearest hundredth of a pound.

b) How many bags of grass seed will she need to buy?

7. Draw and label a right triangle to show each ratio. Then, determine the measure of each angle, to the nearest degree.

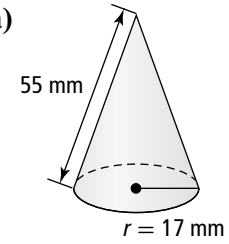
a) $\tan \beta = \frac{3}{2}$ **b)** $\tan \theta = \frac{1}{4}$

c) $\sin A = \frac{4}{7}$ **d)** $\cos C = \frac{3}{5}$

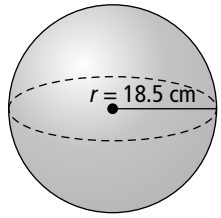
8. What are the dimensions of a cube with volume 2197 cm^3?

9. Calculate the surface area of each object, to the nearest hundredth of a unit.

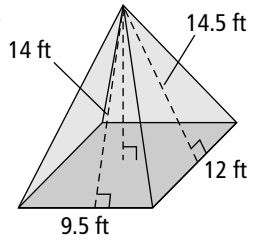
a)



b)



c)



10. Jim is standing in front of a 75-m tall building. The angle of depression from the top of the building to Jim is 35°.

a) How far is Jim from the building, to the nearest hundredth of a metre?

b) Jim moves so that the angle of depression is now 40°. Did he move toward or away from the building? Explain your answer.

11. Evaluate each trigonometric ratio to four decimal places.

a) cos 90° **b)** cos 67.2°

c) sin 18° **d)** tan 28°

12. Write as a power with a positive exponent.

a) $(x^{-5})^3$ **b)** $\frac{b^4}{b^{-4}}$

c) $\frac{(-5.6)^{-7}}{(-5.6)^{-2}}$

13. Stephen found a high wheel bike at an antique shop. The diameter of the front wheel is 54 in. and the diameter of the rear wheel is 18 in.

a) How many times does the rear wheel rotate for each rotation of the front wheel? Give the answer as a ratio of front wheel rotations to rear wheel rotations. Write the ratio in lowest terms.

b) How many times does the front wheel rotate when the bike travels 250 yd?

c) Suppose Stephen cycles for $1\frac{1}{2}$ mi. How many rotations will the front wheel make?

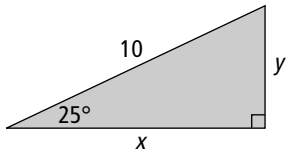
14. There is a relationship between the mass of an organism and the mass of its brain. The estimated mass of an organism's brain can be modelled using the formula $E = 0.025b^{\frac{2}{3}}$, where E is the estimated brain mass, in kilograms, and b is the body mass, in kilograms.

a) A bottlenose dolphin has a mass of 250 kg. What is the mass of its brain?

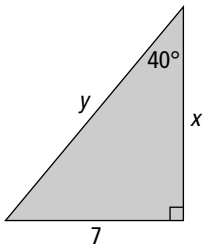
b) The average mass of a human adult male is 70 kg. What is the approximate mass of a human brain?

15. Solve each triangle, to the nearest tenth of a unit.

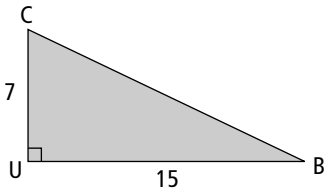
a)



b)

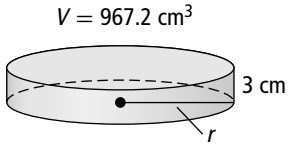


c)

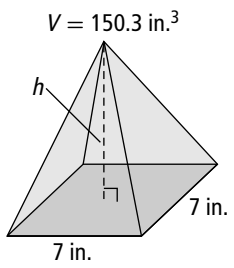


16. Determine the missing dimension for each of the following. Express the answer to the nearest hundredth of a unit, if necessary.

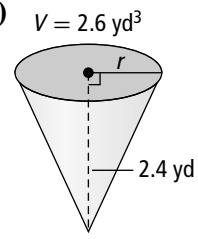
a)



b)



c)



17. Evaluate each expression. Express the answer to four decimal places, if necessary.

a) $\left(9^{\frac{3}{5}}\right)\left(9^{0.6}\right)$

b) $-\left(2^{\frac{-1}{4}}\right)^{3}$

c) $\left(\frac{0.25^{0.25}}{0.25^{-3}}\right)^{2}$

d) $\left(\frac{4^{-2}}{6^{-3}}\right)^{\frac{1}{2}}$

18. Earth has a diameter of approximately 12 756 km. Earth's moon has a diameter of approximately 3475 km. Assume that they are both spheres.

a) Calculate the approximate surface area of Earth.

b) Calculate the approximate surface area of the moon.

c) How much greater is Earth's surface area than the moon's surface area? Express the answer as a percent.

19. Convert each measurement to the unit indicated.

a) 1 ft $2\frac{11}{16}$ in. to the nearest quarter of an inch

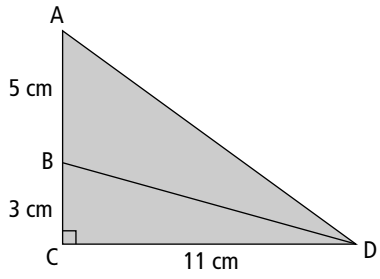
b) 8′2″ to the nearest quarter of a yard

c) 150 000 ft to the nearest tenth of a mile

d) 8 mi to the nearest foot

20. A right rectangular prism measures 16 cm by 8 cm by 4 cm. What would be the dimensions of a cube with the same volume?

21. Determine $\angle ADB$, to the nearest tenth of a degree.



22. Express each entire radical as an equivalent mixed radical.

a) $\sqrt{54}$

b) $\sqrt{512}$

c) $\sqrt[3]{135}$

d) $\sqrt[4]{144}$

23. Write each power as an equivalent radical.

a) $x^{\frac{2}{7}}$

b) $(13t^4)^{\frac{1}{5}}$

c) $\left(\frac{h^2}{12}\right)^{0.5}$

Chapter 4 Extend It Further

1. When a square sheet of paper is folded in half, its area is reduced by half. When folded again, the area is $\frac{1}{4}$ or 2^{-2} times as large. After the eighth fold, the area of the sheet of paper is 144 in.^{2}. What was the original size of the paper?

A 256″ by 256″ **B** 192″ by 192″

C 178″ by 178″ **D** 128″ by 128″

2. If $\sqrt{9^5}\, a^{-3} = 9$, what is the value of a?

A 1 **B** 2

C 3 **D** 9

3. If $a^{\frac{2}{3}} = 16$, determine the value of $\left(a^{\frac{2}{3}}\right)\left(a^{\frac{1}{6}}\right)$.

A 4 **B** 6 **C** 8 **D** 32

4. If $3^{2b} = 49$, determine the value of 3^{-3b}.

A $\frac{1}{3}$ **B** $\frac{1}{7}$

C $\frac{1}{49}$ **D** $\frac{1}{343}$

5. Without using a calculator, determine which set of numbers is written in descending order.

A $\sqrt[3]{3}, \sqrt[4]{5}, \sqrt{2}$

B $\sqrt[3]{3}, \sqrt{2}, \sqrt[4]{5}$

C $\sqrt[4]{5}, \sqrt[3]{3}, \sqrt{2}$

D $\sqrt{2}, \sqrt[2]{3}, \sqrt[4]{5}$

6. Without using a calculator, determine which set of numbers is written in ascending order.

A $(-3)^{42}, \left(\frac{1}{8}\right)^{-21}, (-5)^{28}, \left(\frac{1}{1024}\right)^{-7}$

B $\left(\frac{1}{1024}\right)^{-7}, (-3)^{42}, (-5)^{28}, \left(\frac{1}{8}\right)^{-21}$

C $\left(\frac{1}{8}\right)^{-21}, (-5)^{28}, (-3)^{42}, \left(\frac{1}{1024}\right)^{-7}$

D $(-3)^{42}, (-5)^{28}, \left(\frac{1}{8}\right)^{-21}, \left(\frac{1}{1024}\right)^{-7}$

7. Express $(\sqrt{5})(\sqrt[3]{7})$ as a single radical.

8. If a and b are both irrational numbers, are all a^b also irrational? Explain.

9. Simplify $\frac{3^{n+2} - 3^{n+1}}{3^{n+3}}$.

10. In order to double the size of a spherical lollipop, by what percent must the radius be increased?

11. For what real values of n, if any, is each statement true?

a) $\sqrt{n} = 8$

b) $\sqrt{-n} = 8$

c) $\sqrt{n} > 0$

d) $\sqrt{n} = -8$

e) $\sqrt{-n} = -8$

12. For what whole numbers n is $\sqrt{n}$ a rational number? Explain.

13. Solve for t if $512 = 4096^{-t}$.

14. Solve for x in $\sqrt{2009^{\ }} = 2009\sqrt{x}$.

15. How many pairs of positive integers (a and b) satisfy $(2^a)^b = 2^{32}$? Justify your answer.

☆**16.** James deposited $12 500 in a savings account. He earned $878.29 in interest after two years and nine months. What was the annual interest rate? Use the formula $A = P(1 + i)^n$, where A is the final value of his investment, P is the original amount invested, i is the interest rate, and n is time, in years.

Chapter 4 Study Check

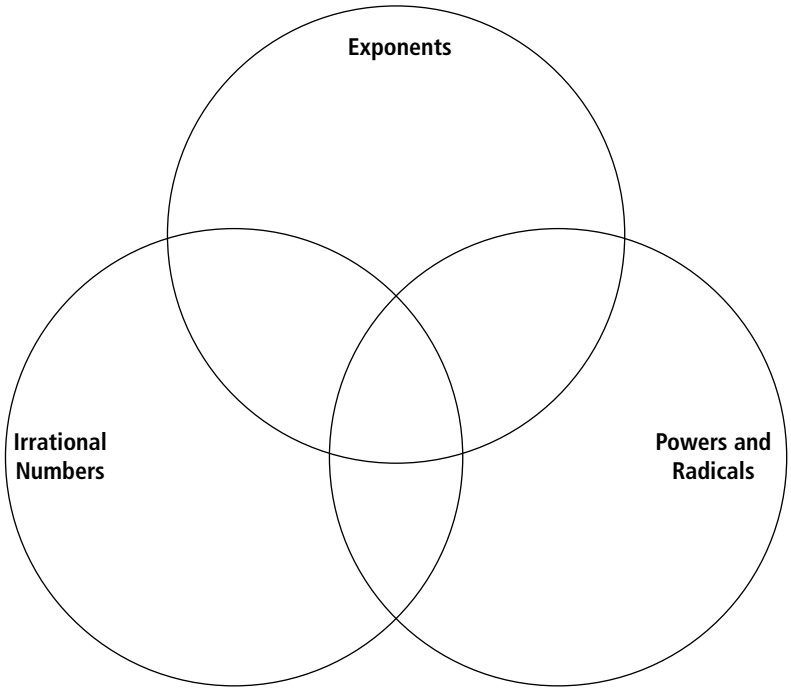
Use the chart below to help you assess the skills and processes you have developed during Chapter 4. The references in italics direct you to pages in *Mathematics 10 Exercise and Homework Book* where you could review the skill. How can you show that you have gained each skill? What can you do to improve?

Big Idea	Skills	This Shows I Know	This Is How I Can Improve
Solve problems involving square roots and cube roots *pages 62–65, 78, 80–81, 83*	✓Determine square roots of perfect squares *pages 63–65, 78, 80, 83*		
	✓Determine cube roots of perfect cubes *pages 62–65, 78, 80–81*		
Solve problems involving integral and rational exponents *pages 66–73, 79, 81–83*	✓Apply the exponent laws to expressions with integral exponents *pages 66–69, 78, 81, 83*		
	✓Convert a power with a negative exponent to an equivalent power with a positive exponent *pages 66–69, 73, 76,78–79, 81*		
	✓Apply the exponent laws to expressions with rational exponents *pages 70–73, 78–79, 81–83*		
Solve problems involving irrational numbers, including radicals *pages 75–77, 79, 82–83*	✓Represent, identify, simplify, and order irrational numbers *pages 75–77, 79, 82–83*		
	✓Convert between powers and radicals *pages 75–77, 79, 82*		
	✓Convert between mixed radicals and entire radicals *pages 75–77, 79, 82–83*		

Organizing the Ideas

In the Venn diagram below, show examples of each type of real number. Use the intersections to show any common features of individual subgroups.

How can you use this Venn diagram to help show the similarities and differences between types?



Study Guide

Review the types of problems you handled in Chapter 4. What do you need to remember to help you do similar problems?

Things to Remember

Determining Square Roots and Cube Roots	Applying the Exponent Laws	Working With Irrational Numbers and Radicals