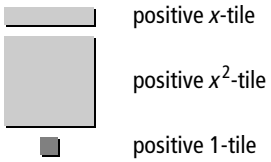
Chapter 5 Polynomials

5.1 Multiplying Polynomials

KEY IDEAS

- You can use the distributive property to multiply polynomials. Multiply each term in the first polynomial by each term in the second polynomial. Then, collect like terms.
- You can use algebra tiles to model algebraic expressions.



The same tiles not shaded represent negative quantities.

Example

Use the distributive property to determine the product of $(2a + 4)$ and $(a^2 + 5a + 7)$.

Solution

Multiply each term in the first polynomial by each term in the second polynomial.

$$(2a + 4)(a^2 + 5a + 7) = (2a)(a^2 + 5a + 7) + (4)(a^2 + 5a + 7)$$
$$= 2a^3 + 10a^2 + 14a + 4a^2 + 20a + 28$$

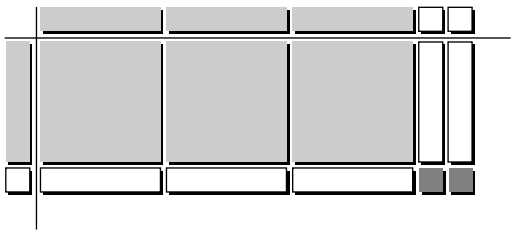
Collect the like terms and arrange them in descending order of the power of a.

$$= 2a^3 + 10a^2 + 4a^2 + 14a + 20a + 28$$
$$= 2a^3 + 14a^2 + 34a + 28$$

A Practise

1. Determine the product that each algebra tile model shows. Then, state the dimensions.

a)



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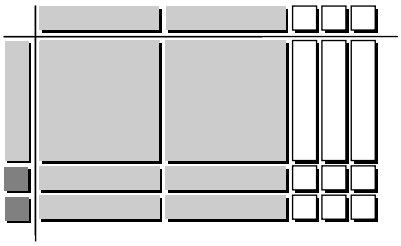


2. Determine each product using algebra tiles.

a) $(x + 3)(x + 4)$

b) $(x - 2)(x - 5)$

c) $(2x - 1)(3x + 2)$

d) $(4x + 3)(x + 2)$

e) $(x - 4)^2$

f) $(x - 3)(3x + 4)$

3. Multiply.

a) $(x - 4)(2x + 4)$

b) $(t + 5)(t + 4)$

c) $(3w + 2)(2w - 9)$

d) $(z + 2)(z - 2)$

e) $(a + b)^2$

f) $(5e + 5)(6e - 1)$

4. Match each binomial on the left with a trinomial on the right.

a) $(x - 2)(x - 1)$	**A** $x^2 + 10x + 21$
b) $(x + 5)^2$	**B** $x^2 - 8x + 16$
c) $(x + 7)(x + 3)$	**C** $x^2 - x - 56$
d) $(x + 6)(x - 2)$	**D** $x^2 - 7x + 12$
e) $(x - 4)^2$	**E** $x^2 - 3x + 2$
f) $(x - 8)(x + 7)$	**F** $x^2 + 11x + 10$
g) $(x - 3)(x - 4)$	**G** $x^2 + 4x - 12$
h) $(x + 1)(x + 10)$	**H** $x^2 + 10x + 25$

5. Choose the trinomial that is the product of the binomials.

a) $(x + 2)(x - 3)$

A $x^2 - x - 6$

B $x^2 + x - 6$

C $x^2 + x - 1$

D $x^2 - x + 1$

b) $(x - 4)(x - 1)$

A $x^2 - 4x - 4$

B $x^2 - 4x + 4$

C $x^2 + 4x + 5$

D $x^2 - 5x + 4$

c) $(x + 5)^2$

A $x^2 + 10x + 10$

B $x^2 + 10x + 5$

C $x^2 + 10x + 25$

D $x^2 + 5x + 25$

d) $(x - 6)(x + 3)$

A $x^2 + 3x - 18$

B $x^2 - 3x - 18$

C $x^2 - 6x - 18$

D $x^2 - 6x + 18$

e) $(x - 7)^2$

A $x^2 - 14x - 14$

B $x^2 - 14x + 49$

C $x^2 - x + 49$

D $x^2 - 7x + 49$

f) $(x + 1)(x + 10)$

A $x^2 + 11x + 10$

B $x^2 + 10x + 11$

C $x^2 + x + 10$

D $x^2 + x + 1$

6. Use the distributive property to determine each product.

a) $(d + 3)(2d^2 + 5d - 2)$

b) $(4s - 5)(s^2 - 9s - 1)$

c) $k(5k^2 - k + 7)$

d) $(3c + 6)(c^2 + 4c + 7)$

e) $(5y^2 - y)(2y^2 + 2y - 6)$

☆**f)** $(r^2 - 5r - 3)(3r^2 - 4r - 5)$

7. Simplify.

a) $4(5y + 3)(2y - 3)(3y + 1)$

b) $(3a + 9) + (2a - 5)(4a - 7) + (6a + 3)$

c) $(2d - e)(3d - 5e) + (6d + 5e)(d - 4e)$

d) $(5n + 4)^2 - (2n + 7)(8n - 6)$

e) $(3w^2 + w + 4)(2w^2 - 5w - 6)$

☆**f)** $2(4t + 5s)(2t - 3s) - (5t - s)$

8. Multiply. Then, combine like terms.

a) $(3a + 7) + (4a - 3)(2a + 2)$

b) $(b + 2)(3b + 6) + (b - 3)^2$

c) $(2x - y)(x - 4y) + (x + y)(3x + y)$

d) $4(6a + 2c)(a - 3c) - (a + 2c)^2$

e) $(x^2 - 2x + 3)(2x^2 + 3x - 4)$

f) $(4b - d)^2 - 2(2b + 3d)(b + d)$

B Apply

9. An error was made in each of the following solutions.

- Write the step number that contains the error.
- Determine the correct solution.

a) $(4t - 5)(7t + 2) - (6t - 3)$

Step 1: $4t(7t + 2) - 5(7t + 2) - (6t - 3)$

Step 2: $28t^2 + 8t - 35t - 10 - 6t - 3$

Step 3: $28t^2 + 8t - 35t - 6t - 10 - 3$

Step 4: $28t^2 - 33t - 13$

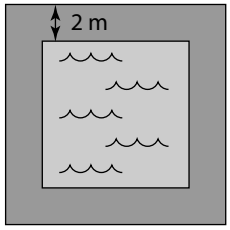
b) $x(2y^2 + y - 3) + x^2y$

Step 1: $2xy^2 + xy - 3x + x^2y$

Step 2: $2xy^2 + x^2y + xy - 3x$

Step 3: $4x^2y + xy - 3x$

10. The width of a deck that surrounds a square wading pool is 2 m.



a) Write a polynomial expression that represents the area of the pool and the pool deck.

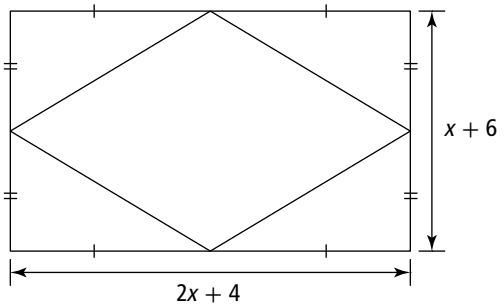
b) What is the area of the deck and the pool, if the pool has an area of 49 m^2?

11. A mirror mounted horizontally on a wall has a width to height ratio of 5 : 2. The mirror frame adds 6 in. to the width and 4 in. to the height.

a) Write a polynomial expression that represents the total area of the mirror, including the frame. Multiply and combine like terms.

b) If the dimensions of the mirror are $5x$ by $2x$, calculate the total area when $x = 8$ in.

12. A diamond is drawn within a rectangle that has a width of $(x + 6)$ units and a length of $(2x + 4)$ units. The diamond touches the centre point of each side of the rectangle, as shown.

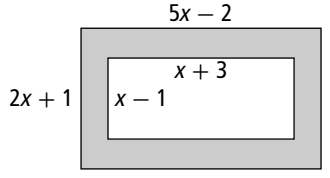


a) Write an expression to represent the area of the diamond. Multiply and combine like terms.

b) What is the relationship between the area of the diamond and the area of the rectangle?

C Extend

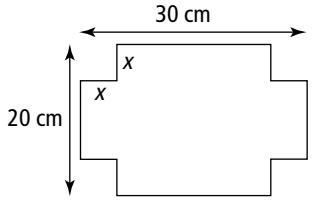
13. a) Write the expressions to calculate the area of the larger rectangle.



b) Write the expression to determine the area of the smaller rectangle.

c) Write the expression(s) for the area of the shaded region. Then, simplify.

14. An open-top box is made from a rectangular sheet of thin cardboard. The corner pieces are cut out as shown in the diagram. A metal corner piece reinforces the corner.



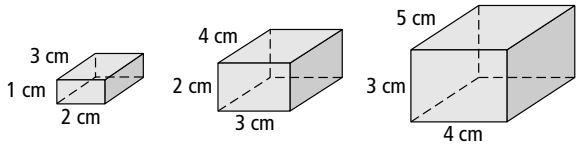
a) Write the expressions for the dimensions of the box.

b) Write the expression to calculate the volume of the box.

c) Simplify this expression.

D Create Connections

15. The diagram shows the first three rectangular prisms in a pattern.



a) Write the dimensions and the volume of the 4th prism.

b) Write the unsimplified expression that shows the volume of the n^{th} prism.

c) Show two different ways to determine the volume of the 10th prism.

16. a) Choose three consecutive even numbers. Multiply the first and third numbers. Then, calculate the square of the middle number. Repeat this multiplication and squaring with several different groups of three consecutive even numbers. What pattern do you notice?

b) Let x represent your middle number. What algebraic expressions represent the first and last numbers?

c) Use algebraic multiplication to show that your pattern in part a) is always true.

17. a) Copy and complete the tables.

Table A	
Numbers	Total
6, 7	42
–	56
–	–
–	–
–	–

Table B				
Numbers				Total
5	25	15	2	42
–	36	–	2	–
7	–	–	–	72
–	–	–	–	–
–	–	–	–	–

b) Write two binomials and their equivalent trinomial to explain why the totals in Table A are equal to the totals in Table B.

18. Consider the following list of expressions.
$(4 \times 3) - (2 \times 1)$
$(5 \times 4) - (3 \times 2)$
$(6 \times 5) - (4 \times 3)$
$(7 \times 6) - (5 \times 4)$

a) If the least number in each group of four numbers is n, write the unsimplified expression that determines the difference of the products in each case.

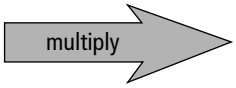
b) Simplify the expression.

c) Show how the simplified expression matches the differences of the products.

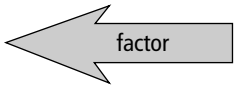
5.2 Common Factors

KEY IDEAS

- Factoring is the reverse of multiplying.



$5(x + 2) = 5x + 10$



To find the greatest common factor (GCF) of a polynomial, find the GCF of the coefficients and variables.

- To factor a GCF from a polynomial, divide each term by the GCF.
- Polynomials can be written as a product of the GCF and the sum or difference of the remaining factors.

$$2m^3n^2 - 8m^2n + 12mn^4 = 2mn(m^2n - 4m + 6n^3)$$

- A common factor can be any polynomial, such as a binomial.

$a(x + 2) - b(x + 2)$ has a common factor of $x + 2$.
The final factored form of this polynomial is $(a - b)(x + 2)$.

Example

Write $12x^4y - 28x^3y^2 + 8x^2y^3$ in factored form.

Solution

The GCF of 12, 28, and 8 is 4.
The GCF of x^4y, x^3y^2, and x^2y^3 is x^2y.
So the GCF for the whole polynomial is $4x^2y$.
Mentally divide the polynomial by the GCF to determine the other factor.
So the polynomial in factored form is $4x^2y(3x^2 - 7xy + 2y^2)$. Note that this GCF is a monomial. A GCF *may* be a binomial.

A Practise

1. List the factors of each number in each pair. Then, identify the GCF.

a) 10 and 15

b) 24 and 36

c) 16 and 48

d) 40 and 60

e) 18 and 45

f) 14 and 24

2. List the prime factors of the coefficients and of the variables for each term.

a) $6x^2$, $12x$

b) $20c^2d^3$, $30cd^2$

c) $4b^2c^3$, $6bc^2$

d) $18xy^2z$, $24x^2y^3z^2$

e) $5m^3n$, $20mn^2$

3. State the common prime factors and identify the GCF for each pair of terms in question 2.

4. Determine the GCF of the following sets of terms.

a) $14a$, $21b$

b) $-5n^2$, $-10n$

c) $3rs$, $7t$

d) $12f^2g^3$, $16fg^2$, $32f^3g^2$

e) $-15d^2e^3$, $-30cd^2e$, $-45cde$

f) $-18j^3k$, $27j^2kl$, $36j^2k^2l^2$

5. Identify the least common multiple for each of the following sets of numbers or terms.

a) 16 and 20

b) 15, 30, and 40

c) $6x$ and $9x$

d) $2t$, $3t^2$, and $4t^3$

e) $4ab^3$, $6a^2b^2$, and $10a^3b$

f) $8cde$, $14c^3de^2$, and $18c^2d^2e^3$

6. Factor the following polynomials. Then, use multiplication to check each answer.

a) $6s + 30$

b) $4t + 28$

c) $5a - 5$

d) $16r^2 - 12r$

e) $7xy + 14xy - 49xz$

f) $3c^3 - 9c^2 - 27d^2$

7. State the missing factor.

a) $15w^2 - 5w = 5w$ (_____)

b) $4a^2 - 6a^3 =$ _____ $(2 - 3a)$

c) $10x^2y^2 - 50xy = 10y$ (_____)

d) $2g^2 + 4g = 2g$ (_____)

e) $35x^2y + 15x^2y^2 + 5xy =$ _____ $(7x + 3xy + 1)$

f) $2r^2 + 6r^3s - 4rs^2 =$ _____ $(r + 3r^2s - 2s^2)$

8. Identify the GCF for each pair of terms.

a) $x(x - 6)$ and $4(x - 6)$

b) $a(a + 3)$ and $-7(a + 3)$

c) $d(d - 9)$ and $-6(d - 9)$

d) $ab(b + 2)$ and $a^2b(b + 2)$

e) $(x^2 + 2x)$ and $xy(x + 2)$

f) $4m(n - 1)$ and $(2m^2n^3 - 2m^2n^2)$

9. Write each expression in factored form.

a) $s(s + 5) - 2(s + 5)$

b) $r(r - 7) - 4(r - 7)$

c) $g(g + 6) + 9(g + 6)$

d) $p^2 + 3p + 4p + 12$ (Hint: Group like terms to find a common factor.)

e) $b^2 - 7b - 3b - 21$

f) $r^2 - 3r + 2rs - 6s$

B Apply

☆**10.** The Mount Baker girls' basketball team is planning a spaghetti dinner fund raiser. Table decorations will include floral centrepieces. The team decides to use roses, tulips, and daffodils. The local florist has 36 roses for $2.50 each, 48 daffodils for $1.70 each, and 60 tulips for $1.50 each.

a) If each centrepiece is to have the largest possible number of each type of flower, with the same number of flowers in each centrepiece, how many of each type of flower will the centrepieces contain?

b) To recover the money spent on the flowers, the centrepieces will be sold for the cost of the flowers. What will each centrepiece cost?

11. State whether each polynomial is factored fully and correctly. If it is not, write the correct and fully factored form.

a) $12x - 6 = 3(4x - 2)$

b) $-10w^2 - 10w = 10w(-w - 1)$

c) $10c^4d - 20c^3d^2 + 15c^2d^2 = 5c^2d(2c^2 - 4cd + 3d)$

d) $x^2 + 2xy + 3x + 6y = x(x + 2y) + 3(x + 2y)$

e) $a^2 - 3ab - 2ab + 6b^2 = (a - 2b)(a - 3b)$

f) $t^2 - 4t + 5t - 5 = t(t - 4) + 5(t - 1)$

12. a) Write a polynomial with two terms that have a GCF of $4x$.

b) Write a polynomial with two terms that have a GCF of $3rs$.

c) Write a polynomial with two terms that have a GCF of $5m^2n^2$.

d) Write a polynomial with three terms that have a GCF of ab.

e) Write a polynomial with three terms that have a GCF of $2c^2d^2$.

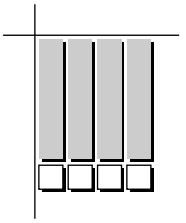
f) Write a polynomial with four terms that have a GCF of $2e$.

g) Write a polynomial with four terms that have a common factor of $c + 4$.

13. The models show rectangles of algebra tiles. Answer the following questions for each rectangle.

- What are the dimensions for each model?
- Write an expression for each model, using your dimensions.

a)



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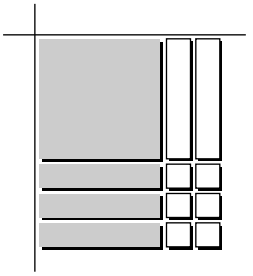

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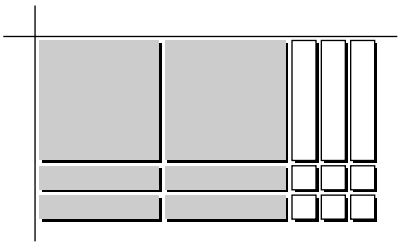
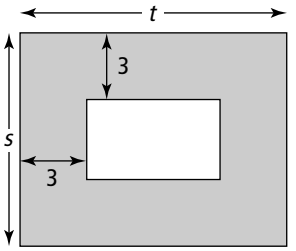


☆**14. a)** The white rectangle in the diagram is centred within the larger rectangle. State the dimensions of the white rectangle.



b) If $t = 10$ cm and $s = 8$ cm, express the area of the white rectangle as a product of binomials.

15. You have two dishes of lasagna. One dish measures 6″ by 15″. The second dish measures 9″ by 12″.

a) You want to cut the lasagna into equal-size servings with nothing left over. What is the largest size serving you can cut?

b) Suppose there are 20 students in your class. What is the largest size square you can cut to ensure that each student receives at least one serving and all servings are of equal size?

C Extend

16. List the pairs of numbers less than 100 that have a GCF of 15 and a product of 2700. How do you know that you have all of the possible pairs?

17. A school receives a shipment of notebooks. The unopened and equal packages of notebooks are put into three stacks. The three stacks have 365, 525, and 595 notebooks, respectively. What is the largest possible number of notebooks in each package?

18. The diameter of the bottom tier of a three-tiered round cake is 6 cm greater than the diameter of the middle tier and 12 cm greater than the diameter of the top tier.

a) Write an algebraic expression for the total top surface area of the three tiers.

b) Multiply and then simplify.

19. Write a polynomial that satisfies the following clues:
- It is a trinomial.
- Each term has the same variable.
- The exponents are odd integers.
- The GCF is $8x$.
- The greatest exponent is 5.
- The coefficients in one term of the polynomial's factored form are 1, 2, and 3.

20. The greatest common factor of two numbers is 487. Both numbers are even. Neither is divisible by the other. What are the smallest two numbers they could be?

D Create Connections

21. a) Draw a diagram to illustrate the largest circle that can be contained within a square with side length s.

b) Write an expression for the area of this circle. Use s as a variable in your expression.

c) Write the expression for the area of the square not contained by the circle. Develop an appropriate polynomial. Then, factor the polynomial.

22. The height of a basketball thrown vertically can be modelled by the expression $v_0t - 5t^2 + h_0$, where v_0 is the initial velocity of the basketball, t is the time, in seconds, that the ball is in the air, and h_0 is the initial height, in metres, of the ball.

a) If the initial height of the ball is 2 m, and the initial velocity of the ball is 15 m/s, how long will the ball be in the air before it comes back down to its initial height?

b) Factor the expression $15t - 5t^2$. At which times will the product be 0? Explain. Does this simplify the process for seeking the answer for part a)?

23. The height of a rectangular prism is $2x$. The width of the prism is 1 unit more than one and one half times the height.

a) Determine the length of the prism if the volume is $30x^3 + 10x^2$.

b) If the height is 6 cm, determine the width, length, and volume of the prism.

5.3 Factoring Trinomials

KEY IDEAS

- To factor a trinomial of the form $x^2 + bx + c$, first find two integers with
  – a product of c
  – a sum of b
  For $x^2 + 12x + 27$, find two integers with
  – a product of 27
  – a sum of 12
  The two integers are 3 and 9.
  Therefore, the factors are $x + 3$ and $x + 9$.

- To factor a trinomial of the form $ax^2 + bx + c$ (where b and c are integers), first factor out the GCF, if possible. Then, find two integers with
  – a product of $(a)(c)$
  – a sum of b
  Finally, write the middle term as a sum. Then, factor by grouping.
  For $8k^2 - 16k + 6$, the GCF is 2, so
  $8k^2 - 16k + 6 = 2(4k^2 - 8k + 3)$
  Identify two integers with
  – a product of $(4)(3) = 12$
  – a sum of -8
  The two integers are -2 and -6. Use these two integers to write the middle term as a sum. Then, factor by grouping.
  $2(4k^2 - 2k - 6k + 3) = 2(2k - 3)(2k - 1)$

- You cannot factor some trinomials, such as $x^2 + 3x + 5$ and $3x^2 + 5x + 4$, over the integers.

Example

Factor $6a^2 + 11a - 10$, if possible.

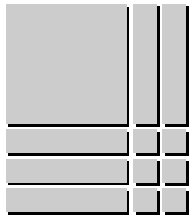
Solution

$\underline{6}a^2 + \underline{\underline{11}}a \underline{- 10}$	Ask: Are there two integers that when multiplied together equal -60 and when added together equal 11? Numbers that multiply to make 60. 1, 60 4, 15 2, 30 5, 12 3, 20 6, 10 Numbers that could add to make 11. 4, 15
$6a^2 + 15a - 4a - 10$	Then, break up the middle term with these integers.
$3a(2a + 5) - 2(2a + 5)$	Factor by grouping.
$(3a - 2)(2a + 5)$	

A Practise

1. State the trinomial represented by each rectangle of algebra tiles. Then, determine the dimensions of each rectangle.

a)



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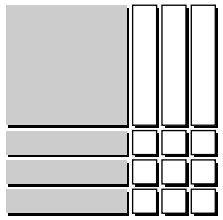
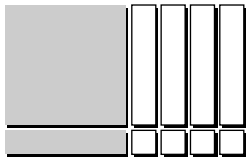


c)



d)

2. Use algebra tiles or a diagram to factor each trinomial.

a) $2x^2 + 3x + 1$

b) $3x^2 + 5x - 2$

c) $6x^2 - 13x + 6$

d) $2x^2 + 5x - 12$

e) $4x^2 - 18x - 10$

f) $3x^2 + 17x - 28$

3. If possible, identify integers with the given product and sum.

	Product	Sum
a)	12	8
b)	15	−3
c)	−4	−3
d)	24	−11
e)	−42	19
f)	12	−10

4. Factor, if possible.

a) $y^2 + 8y + 12$

b) $x^2 + 10x + 21$

c) $a^2 - 19a + 90$

d) $y^2 - 4y - 6$

e) $m^2 - mn - 42n^2$

f) $b^2 + 19b + 34$

5. Factor, if possible.

a) $g^2 - 10g + 24$

b) $n^2 - 15n + 26$

c) $c^2 - 15c + 56$

d) $s^2 - 7st + 10t^2$

e) $f^2 - 6f + 12$

f) $3v^2 + v - 2$

6. Factor, if possible.

a) $2r^2 + 11r + 14$

b) $2l^2 + 11l + 12$

c) $3w^2 + 9w + 6$

d) $10b^2 + 8b + 2$

e) $y^2 + 5yz + 6z^2$

f) $12a^2 + 19a + 4$

7. Factor, if possible.

a) $2f^2 + 7f - 15$

b) $r^2 + r - 110$

c) $6b^2 + 6b - 3$

d) $10m^2 - 17mn + 3n^2$

e) $x^2 - x + 56$

f) $9g^2 - 9gf + 2f^2$

g) $6l^2 + 32l + 42$

h) $5a^2 - 52a + 63$

B Apply

8. Determine at least two values of d that allow each expression to be factored.

a) $a^2 + da + 6$

b) $w^2 + dw - 15$

c) $y^2 - dy + 18$

d) $r^2 - dr - 14$

9. Determine two values of h that allow each expression to be factored.

a) $6p^2 + hp - 1$

b) $d^2 + hd + 8$

c) $t^2 - ht + 56$

d) $s^2 - hs - 20$

10. Determine two values of p that allow each expression to be factored.

☆**a)** $c^2 - pc - 10$

b) $x^2 + pxy + 3y^2$

c) $a^2 + pab + 14b^2$

d) $v^2 - pvw + 35w^2$

11. Identify one value of r that will allow each expression to be factored.

a) $10b^2 + 14b - r$

b) $rs^2 + 19st + 3t^2$

c) $d^2 - 8de + re^2$

d) $5y^2 - 32y - r$

e) $2x^2 - 11x - r$

f) $3x^2 - 3x - r$

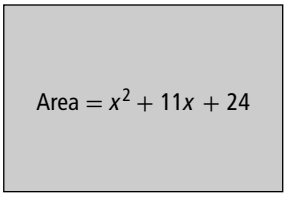
12. The penalty area on a soccer field can be represented by the trinomial $6x^2 - 2x - 48$.

a) Factor the trinomial to determine a binomial that represents the width and the length of the area.

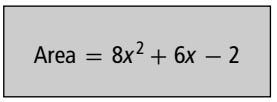
b) The unit used for soccer fields is the yard. What are the dimensions of the area if $x = 12$ yd?

☆**13.** Determine the binomials that represent the width and length of each rectangle. Then, calculate the dimensions if $x = 12$ cm.

a)



b)



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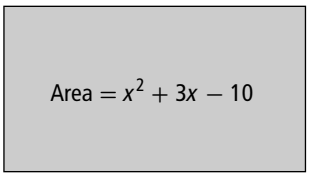


14. Carol throws a ball that will move through the air in a parabolic path due to gravity. The height, h, in feet, of the ball above the ground after t seconds can be modelled by the expression $h = -6t^2 + 27t + 15$.

a) Write the formula in factored form.

b) What is the height of the ball above the ground 4 s after it is thrown?

15. a) The area of a parallelogram is $A = x^2 + 13x + 42$. Determine the binomials that represent the height, h, of the parallelogram and the length, b, of its base. Then, calculate the dimensions of the parallelogram if $x = 18$ cm.

b) Suppose the area of the parallelogram in part a) is $A = 6x^2 + 7x - 3$. What are the binomials that represent the height and length of the parallelogram? Determine the dimensions if $x = 18$ cm.

C Extend

☆**16.** The area of a rectangle can be represented by the expression $35 - 8x - 3x^2$, where x represents a positive integer. What are the possible values for the width and the length of the rectangle?

17. Determine one value of c that allows the trinomial $cy^2 + 36y - 18$ to be factored over the integers.

18. a) What shape might have an area represented by the expression $16s^2 - 48s + 36$?

b) What in the expression indicates that shape?

c) What are the factors?

19. The area of a certain shape can be represented by the expression $x^2 + 6x + 9$.

a) Identify a possible shape.

b) Write expressions for the possible dimensions of the shape you identified in part a).

c) Suppose you have a second figure in the same shape as the shape you identified in part a) except that its area can be represented by the expression $4x^2 + 24x + 36$. Explain how you can use mental math to determine the dimensions of the second figure.

D Create Connections

20. A classmate is able to factor trinomials such as $n^2 + 7n - 44$ or $n^2 - 20n - 44$, but not trinomials such as $6n^2 + 13n - 5$ or $4n^2 - n - 3$. Explain the similarities and differences in factoring these two types of trinomials in sufficient detail that your classmate is then able to factor both types.

21. Write the completed statements after determining the answer in the blanks.

a) When factoring a trinomial of the form $x^2 + bx + c$, such as $x^2 + 5x + 6$, one can ask, "What two integers have a sum of ____ and a product of ____?"

b) The general form to show why part a) works, with each of m and n being any integer, is

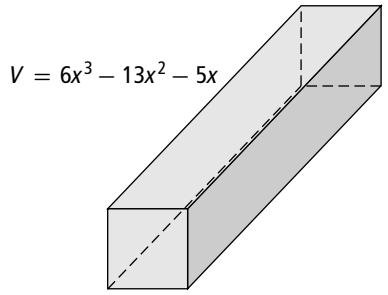
$$(x + m)(x + n) = x^2 + nx + \_\_ + mn$$
$$= x^2 + (n + \_\_)x + mn$$

c) Consider a trinomial of the form $ax^2 + bx + c$, such as $2x^2 + 13x + 15$. When factoring a trinomial of this form by grouping to break up the middle term, one can ask, "What two integers have a product of ____ and a sum of ____?"

d) The general form to show why part c) works, with each of a, m, and n being any integer, is

$$(ax + m)(x + n) = ax^2 + anx + \_\_ + mn$$
$$= ax^2 + (an + \_\_)x + mn$$

22. A rectangular prism has the volume as shown.



a) Factor the expression that represents the volume to determine the length of each of the sides of the prism.

b) If $x = 5$ cm, determine the lengths of the sides and the volume of the rectangular prism.

5.4 Factoring Special Trinomials

KEY IDEAS

- Some polynomials are the result of special products. When factoring, you can use the pattern that formed these products.

Difference of Squares:
The expression is a binomial.
The first term is a perfect square.
The last term is a perfect square.
The operation between the terms is subtraction.
The two binomial factors will be the square roots of the squares, connected by "+" and "–" signs.

$x^2 - 25 = x^2 - 5$
$= (x - 5)(x + 5)$

Perfect Square Trinomial:
The first term is a perfect square.
The last term is a perfect square.
The middle term is twice the product of the square root of the first term and the square root of the last term.
The trinomial is of the form $(ax)^2 + 2abx + b^2$ or $(ax)^2 - 2abx + b^2$.

$x^2 + 16x + 64 = x^2 + 8x + 8x + 64$
$= x(x + 8) + 8(x + 8)$
$= (x + 8)(x + 8)$

Example

Factor $y^2 + 28y + 196$, if possible.

Solution

$\underline{1y^2} + \underline{28}y + \underline{196}$

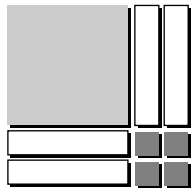
$\sqrt{1} = 1$
$\sqrt{196} = 14$
$14 + 14 = 28$
This trinomial is a "perfect square." The first and last terms are squares, and the middle term is double the product of their square roots.

$y^2 + 14y + 14y + 196$
$y(y + 14) + 14(y + 14)$
$(y + 14)(y + 14)$
$(y + 14)^2$

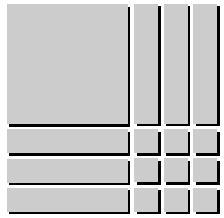
A Practise

1. State the factors of the polynomial shown by each algebra tile model.

a)



b)



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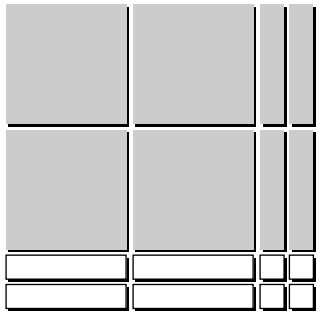
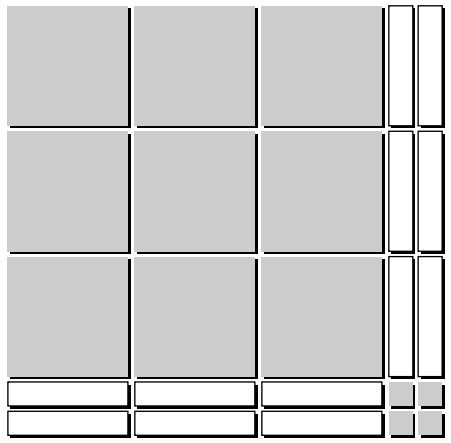


d)



2. Solve.

a) $(x + 5)(x - 5)$

b) $(3r - 4)(3r + 4)$

c) $5(w - 6)(w + 6)$

d) $(2b - 7c)(2b + 7c)$

e) $4(2x - 3y)(2x + 3y)$

f) $2y(x + 3)(x - 3)$

3. Determine the product.

a) $(y + 5)^2$

b) $(3d + 2)^2$

c) $(4m - 5p)^2$

d) $2(e - 6f)^2$

e) $3(2z - 4)^2$

f) $(2x - 3y)^2$

4. Determine the missing terms that complete the factors or products.

a) $n^2 - \_\_\_ + 25 = (n - \_\_\_)^2$

b) $r^2 - \_\_\_ = (r + \_\_\_)(r - s)$

c) $-16d^2 = (3c - \_\_\_)(3c + \_\_\_)$

d) $4s^2 + \_\_\_ + 36 = (\_\_\_ + \_\_\_)^2$

e) $4x^2 + \_\_\_ + 4 = (2x + \_\_\_)^2$

f) $(4x - \_\_\_)^2 = 16x^2 - \_\_\_ + 4$

5. Factor each binomial, if possible.

a) $a^2 - 100$

b) $t^2 - 49$

c) $x^2 + 4$

d) $64 - h^2$

e) $50g^2 - 72h^2$

f) $9p^2 - 15r^2$

g) $s^2 + 144$

h) $72g^2 - 32h^2$

6. Factor each trinomial, if possible.

a) $y^2 + 12y + 36$

b) $x^2 - 6x + 9$

c) $2z^2 + 12z + 18$

d) $a^2 + 5a + 25$

e) $144 - 48b - 4b^2$

f) $9s^2 + 48s + 64$

g) $25n^2 - 110n + 121$

h) $5t^2 - 60t + 108$

7. Factor completely.

a) $16d^2 - 64e^2$

b) $27m^2 - 48$

c) $-2k^2 - 24k - 72$

d) $3c^3 + 51c^2 + 147c$

e) $100a^2 - 25b^2$

f) $s^3t - 18s^2t + 81st$

g) $81g^4 - 16$

h) $12lm^2 + 12lmn + 3ln^2$

B Apply

8. An error was made in factoring the following trinomials or binomials. Identify the error. Then, factor correctly.

a) $4a^2 - b^2 = (2a - b)(2a - b)$

b) $9x^2 + 6x + 1 = (3x + 1)(3x + 2)$

c) $216 - 9y^2 = (16 - 3y)(16 + 3y)$

d) $d^2 - 4e^2 = (d + 4e)(d - e)$

e) $49 - 14h + h^2 = (h - 7)^2$

9. Determine the value(s) of c so that each trinomial is a perfect square.

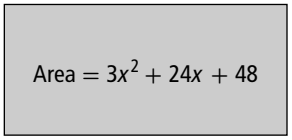
a) $w^2 + cw + 1$

b) $9b^2 + cb + 16$

c) $25 - cs + 36s^2$

d) $16g^2 + cgh + 36h^2$

☆**10.** The area of a rectangle can be represented by the trinomial $3x^2 + 24x + 48$.



a) Factor the trinomial completely.

b) If the length of the rectangle is triple the width, use the factors in part a) to represent the length and width.

c) If x represents 5 cm, what are the length and the width of the rectangle?

d) Calculate the area of the rectangle and check your answer.

11. Using the difference of squares model, $a^2 - b^2 = (a - b)(a + b)$, use mental math to make the following calculations. Record your reasoning.

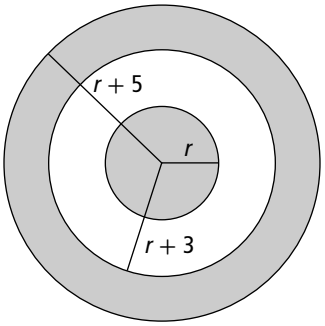
a) $16^2 - 4^2$

b) $7^2 - 27^2$

c) $45^2 - 15^2$

d) $113^2 - 13^2$

12. The diagram shows three concentric circles with radii r, $r + 3$, and $r + 5$.



a) Write an expression for the total area of the shaded regions.

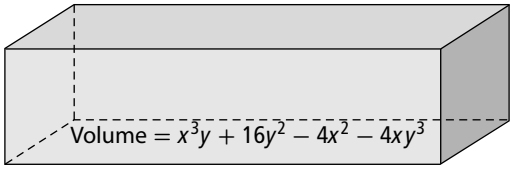
b) Factor this expression completely.

b) If $r = 2$ cm, calculate the total area of the shaded regions. Give your answer to the nearest tenth of a square centimeter.

C Extend

☆**13.** Factor $2r^5 - 4r^3 + 2r$ completely.

14. The volume of a rectangular prism is $x^3y + 16y^2 - 4x^2 - 4xy^3$.



Determine expressions for the dimensions of the prism.

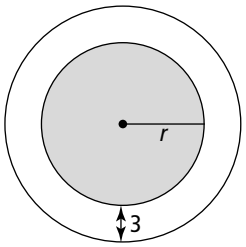
15. To determine the product of two numbers that differ by 6, square their average and then subtract 9.

a) Use this method to determine the following products.
(17)(23) = ____
(25)(31) = ____

b) Explain this method using a difference of squares.

16. The area of a square of side length a may be expressed as $A = 9b^2 - 12b + 4$. What is the area of a rectangle in terms of b if the length of the rectangle is $(a + 2)$ and the width of the rectangle is $(a - 2)$?

☆**17.** Many road intersections use roundabouts to handle traffic flow. Some roundabouts contain a circular area with plants surrounded by a cement walkway.



a) Write an expression that represents the area of the garden.

b) Write an expression that represents the area of the walkway and the garden. Then, expand that expression.

c) Using your answer from part a), write a simplified expression to determine the area of the walkway.

d) If $r = 8$ m, calculate the area of the walkway to the nearest tenth of a square metre.

D Create Connections

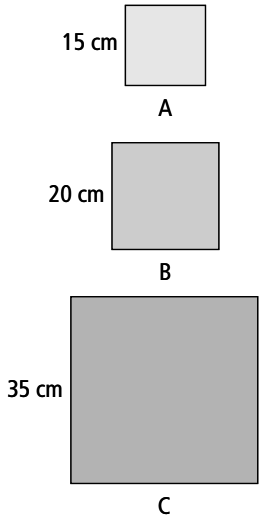
18. a) Continue the pattern to complete the table below.

$11^2 = 121$	$10 \times 12 = 120$
$12^2 =$ ____	$11 \times 13 =$ ____
$13^2 =$ ____	$12 \times 14 =$ ____
$14^2 =$ ____	$13 \times 15 =$ ____
_____	_____

b) How does the squared number compare with the product of the factors that are 1 less and 1 greater than the squared number?

c) Write and simplify algebraic expressions to show why this is the case.

19. Consider three squares having the dimensions shown.



a) How much greater is the area of square C than the combined areas of square A and square B?

b) Explain how the answer can be calculated mentally.

Chapter 5 Review

5.1 Multiplying Polynomials

1. Draw an algebra tile diagram to model each product.

a) $(x + 6)(x - 2)$

b) $(s - 4)^2$

2. Determine the product and then combine like terms.

a) $(a + 5)(a + 7)$

b) $(y + 8)(y - 8)$

c) $(2v + 4w)(5v + 6w)$

d) $(2c - 1)(2c + 1)$

e) $-2(r - 3s)(r + 3s)$

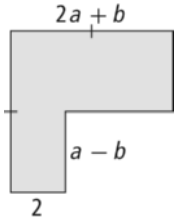
f) $-(g + 4h)^2$

3. Multiply and then combine like terms.

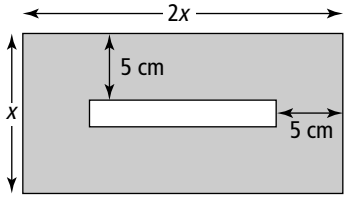
a) $(r + 4)(r^2 - 7r - 8)$

☆**b)** $3p(4p - 5)(p - 7) - 5p(6p + 2)(2p - 8)$

4. Write a simplified expression to represent the area.



5. What expression represents the area of the white rectangle?



5.2 Common Factors

6. Determine the GCF of each set of terms.

a) 30 and 45

b) 84 and 112

c) $72y$, 90, and $108y$

d) $4d$ and $10d^2$

e) $34a^2b$ and $51ab$

f) $10rst$ and $15r^2s^2t^2$

7. Identify the LCM of the following pairs of numbers.

a) 25 and 15

b) 32 and 128

8. Identify the GCF of each set of terms.

a) 18 and 27

b) 13, 26, and 39

c) $8ab^3$ and $12a^2b$

d) $48xy^3z$ and $36x^2y^2z^4$

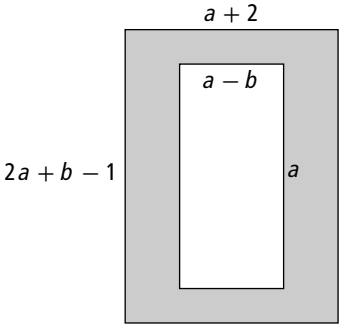
e) $11m^6n^5$, $-22m^3n^9$, and $14m^5n^6$

9. Use algebra tiles or a diagram to factor each polynomial.

a) $2x^2 + 4x$

b) $x^2 + 3x$

10. Write an expression in fully factored form for the shaded area.



5.3 Factoring Trinomials

11. Use algebra tiles or a diagram to factor each trinomial.

a) $x^2 + 2x - 8$

b) $x^2 - 5x + 6$

c) $2x^2 - 10x + 12$

d) $4x^2 + 4x - 3$

12. Factor, if possible.

a) $x^2 - 6x + 8$

b) $x^2 - x - 20$

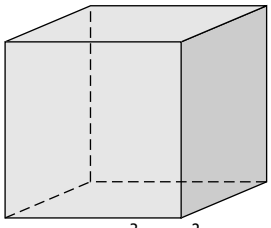
c) $9x^2 - 12x - 5$

d) $8x^2 - 10x - 2$

e) $-6x^2 + 45x - 81$

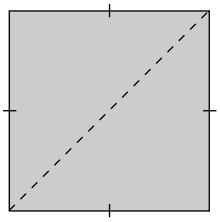
f) $-12x^3 + 2x^2 + 4x$

13. Given the volume of the rectangular prism as shown in the diagram, write the algebraic expressions that represent its dimensions. Then, calculate the dimensions of the rectangular prism if $x = 5$ cm.



Volume $= x^3 - 2x^2 - 3x$

☆**14.** The expression for a square field's area is as shown in the diagram. A fence borders the field, and also partitions it in half by running diagonally from corner to corner.



Area $= 9x^2 - 42x + 49$

a) Write a factored and simplified expression to determine the perimeter of the field.

b) If $x = 20$ m, what is the length of the fence, to the nearest tenth of a metre?

5.4 Factoring Special Trinomials

15. Factor fully.

a) $s^2 - 64$

b) $d^2 - 121$

c) $4h^2 - 25$

d) $9n^2 - 81$

e) $144 - 4b^2$

f) $98c - 18cd^2$

16. Verify that each trinomial is a perfect square. Then, factor.

a) $b^2 + 14b + 49$

b) $144 + 24w + w^2$

c) $16 - 24g + 9g^2$

d) $64s^2 - 208st + 169t^2$

17. Factor fully.

a) $81 - x^2$

b) $10x^4y - 10y$

c) $9x^2 + 30x + 25$

d) $16x^2 - 100y^2$

e) $x^4 - 16x^2 + 64$

f) $-8x^2y - 24xy - 18y$

18. None of the following can be factored over the integers. In each case, explain why this is so.

a) $s^2 - 12 - 36$

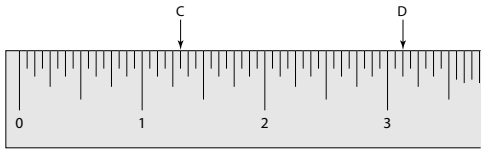
b) $16m^2 + 25$

c) $3y^2 - 30y + 25$

d) $x^2 - 14x + 40$

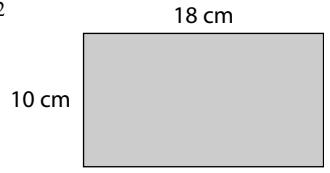
Chapters 1–5 Cumulative Review

1. State the reading for point D on this imperial ruler as a mixed number in lowest terms. What is the distance from C to D? Show two ways to determine the answer.

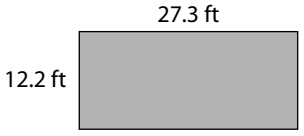


2. Calculate the area using the indicated unit. Round each answer to the nearest hundredth of a unit where appropriate.

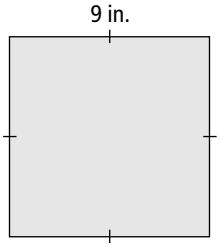
a) unit: mm^2



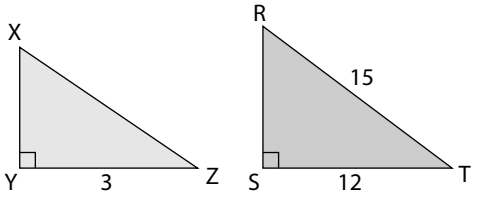
b) unit: m^2



c) unit: cm^2



3. Triangles XYZ and RST are similar triangles. Calculate the lengths of the unknown sides.



4. Calculate.

a) $\sqrt{169}$

b) $\sqrt[2]{(18)(8)}$

c) $\sqrt[3]{\frac{512}{27}}$

d) $\sqrt[3]{2744x^3}$

5. Determine the product and then combine like terms.

a) $(x + 4)(x + 2)$

b) $(b + 3)(b - 3)$

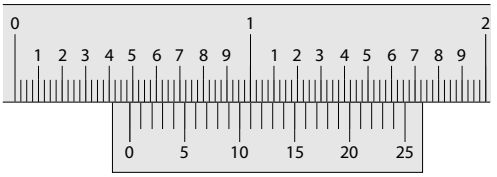
c) $(y - 8)(y + 8)$

d) $(4a + 7b)(2a - 3b)$

e) $-4(5x + 3y)^2$

f) $-(x + y)(x - y)$

6. What reading is shown on this imperial caliper? Name an object that could be this length.



☆**7.** Quentin wants to replace his shed. The current shed has a floor area of 11′ 8″ by 7′ 2″. The floor area of the shed Quentin wants to buy is 9.6 m^2. Which shed has the larger floor area? by what percent?

8. Draw and label a right triangle to illustrate each ratio. Then, calculate the measure of each angle to the nearest degree.

a) $\tan \beta = \frac{4}{3}$

b) $\tan \theta = \frac{2}{5}$

c) $\sin A = \frac{1}{3}$

d) $\cos X = \frac{7}{9}$

9. Simplify each expression. State the answer using positive exponents.

a) $[(-3x^{-7})(2x)]^3$

b) $\left[\frac{(5b)^2}{(4b)^{-2}}\right]^{-1}$

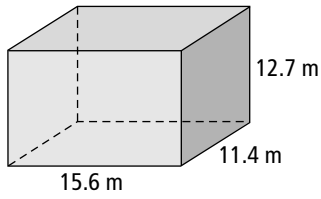
c) $\left[\frac{(-5)^{-1}}{(-5)^0}\right]^2$

d) $5[(6)^{-1}(6)^{-3}]^{-1}$

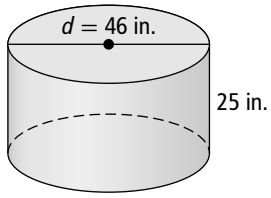
10. What is the circumference of the largest circle you could cut from a sheet of paper measuring 40 cm by 45 cm? What area of paper would you cut away? Round your answer to the nearest hundredth of a unit.

11. Calculate the volume, to the nearest hundredth of a unit, for each of the following.

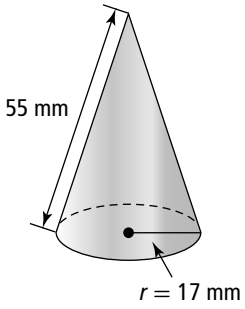
a)



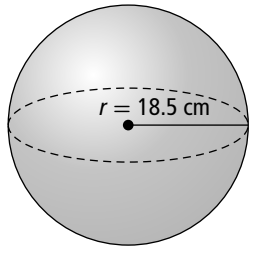
b)



c)



d)



12. Evaluate each trigonometric ratio to four decimal places.

a) cos 30°

b) cos 48.6°

c) sin 90°

d) sin 45°

e) tan 72°

13. For each of the following, use the exponent laws to help identify a value for p that satisfies the equation.

a) $(x^p)^{\frac{-3}{2}} = x^6$

b) $\frac{b^p}{b^{-4}} = b^{\frac{3}{5}}$

c) $\left[\frac{16a^{-6}}{121}\right]^p = \frac{4}{11a^3}$

14. Factor the following polynomials.

a) $2x + 10$

b) $7z + 8z^2$

c) $a^3q + a^3r + a^3s$

d) $4m^5n - 12mn^3$

15. Kelsey wants to repaint her room. The walls are all 9′ high. The floor space measures 11′ 6″ by 10′ 9″. The room has one window measuring 4′ 6″ by 6′ and a door measuring 7′ by 3′.

a) Assuming that the room has no baseboards, what is the wall space Kelsey will need to paint?

b) Kelsey wants to apply two coats of paint. Each can of paint covers 32 m^2. How much paint will she need?

c) A can of paint costs $35.95. How much will it cost to paint the room?

16. A tree casts a shadow 11 m long. The angle measured to the top of the tree from the end of the shadow is 58°. What is the height of the tree? Express your answer to the nearest tenth of a metre.

17. From the beginning of 2001 to the beginning of 2009, the population of British Columbia increased at an average annual rate of 1.6%. This situation can be modelled with the equation $P = 3.9077(1.016)^n$, where P is the population, in millions, and n is the number of years since 2001.

a) What do you think the number 3.9077 represents?

b) Assuming the growth rate continues, what will be the population of British Columbia after 17.5 years?

c) Assuming the growth rate was the same prior to 2001, what was the population of British Columbia at the beginning of 1997?

18. Factor, if possible.

a) $x^2 + 6x - 27$

b) $-2x^2 - 6x + 36$

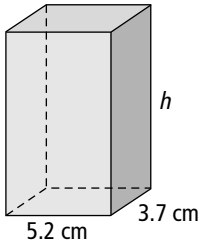
c) $-4x^2 + 30x - 14$

d) $9x^2 + xy - 12y^2$

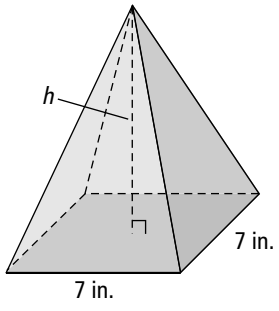
e) $x^3 - 6x^2 - 7x$

19. Calculate the missing dimension for each. Where appropriate, round to the nearest hundredth of a unit.

a) $V = 175.54 \text{ cm}^3$



b) $V = 149.8 \text{ in.}^3$



20. Solve for all unknowns for each triangle. Round your answers to the nearest hundredth of a unit.

a) ΔABC, BC = 5.3 km, $\angle B = 90°$, and $\angle A = 52°$

b) ΔDEF, $\angle E = 90°$, DE = 12 m, and EF = 16 m

21. Express each radical as a power.

a) $\sqrt[3]{4^5}$

b) $\sqrt{(xt)^3}$

c) $-2\sqrt[6]{a^{-5}}$

22. Convert each measurement to the unit specified.

a) The distance from Banff to Medicine Hat is 227.8 mi. (kilometres).

b) The height of the wooden roller coaster on the grounds of the Pacific National Exhibition in Vancouver is 75 ft (metres).

23. A right rectangular prism measures 20 cm × 7 cm × 8 cm. What is the radius of a sphere with the same volume? Round your answer to the nearest hundredth of a centimetre.

24. The angle of depression from the top of a 60-m lighthouse to a boat is 25°. Determine the distance from the base of the lighthouse to the boat to the nearest hundredth of a metre.

25. Convert each entire radical to an equivalent mixed radical.

a) $\sqrt{200}$

b) $\sqrt{1575}$

c) $\sqrt[3]{128}$

d) $\sqrt[4]{80}$

26. Determine two values of n that allow each polynomial to be a perfect square trinomial. Then, factor.

a) $x^2 + nx + 36$

b) $4x^2 + nx + 81$

Chapter 5 Extend It Further

For questions #1 to #5, choose the correct answer.

1. Simplify $(\sqrt{2} + \sqrt{6})(\sqrt{2} - \sqrt{6})$.

A -8

B -4

C 4

D 8

★2. The sum of the squares of 5 consecutive positive odd integers is 3685. What is the sum of the five numbers?

A 121

B 135

C 147

D 151

3. When $\frac{2}{5}$ is substituted into a polynomial, the expression is equal to zero. Which of the following is a factor of the polynomial?

A $5x - 2$

B $2x - 5$

C $2x + 5$

D $5x + 2w$

4. If x is a real number, what is the maximum value of $5 - (x - 7)^2$?

A there is no maximum

B 7

C 5

D 0

5. If x is a real number, what is the minimum value of the expression $(7 - x)^2 - 6$?

A -6

B 0

C 6

D there is no minimum

6. When $(6x^2 + kx + 13)$ is divided by $(3x - 4)$, the remainder is 1. Determine the value of k and the second factor.

7. Show that $(2m, m^2 - 1, m^2 + 1)$ is a Pythagorean triple, where m is an integer greater than 1.

★8. Calculate $\frac{2^{20} - 2^{16} + 15}{2^{16} + 1}$ without using a calculator.

9. Sammi simplified $\frac{x^2 - y^2}{(x + y)^2} = \frac{x - y}{x + y}$ by crossing out all the 2s. Even though her method was incorrect, show that she still has the right answer.

10. Identify the GCF and LCM of $a^2 - 6a + 9$ and $9 - a^2$.

11. Simplify $\frac{1 + a}{1 - \frac{1}{a^2}}$.

12. Without using a calculator, determine which is greater: $\sqrt{14} + \sqrt{12}$ or $\sqrt{15} + \sqrt{11}$.

13. $(x - 2)^2 - (x + 5)(x - 5) = \mathrm{A}(x - 1) + \mathrm{B}$. Solve for A and B.

14. Without using a calculator, explain how you can evaluate $\frac{2010}{(4321)(4319) - (4320)^2}$. What is the answer?

★15. If 1 is subtracted from the sum of the square of an odd number and the square of an even number, what is the greatest positive integer that will divide the difference?

16. Use the simplified expression for $\sqrt{(x + 1)^2 - (x - 1)^2}$ to evaluate $\sqrt{90\,001^2 - 89\,999^2}$. What is the value?

17. Determine all possible values of x for which $x^3 - x = 2009(x - 1)(x + 1)$.

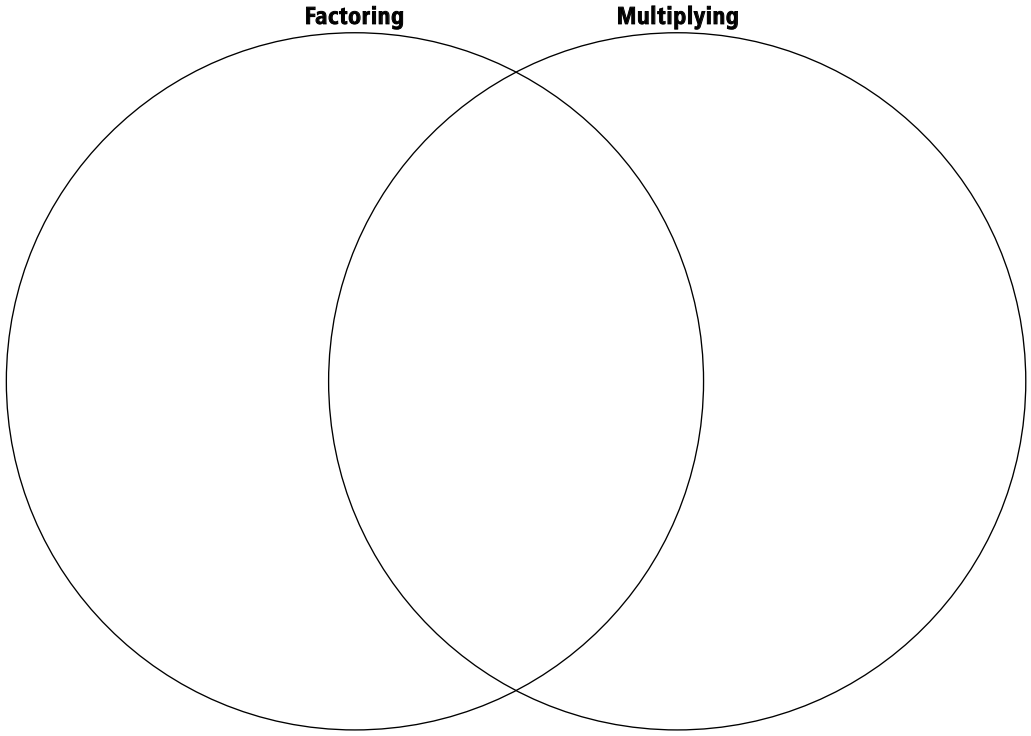
Chapter 5 Study Check

Use the chart below to help you assess the skills and processes you have developed during Chapter 5. The references in italics direct you to pages in *Mathematics 10 Exercise and Homework Book* where you could review the skill. How can you show that you have gained each skill? What can you do to improve?

Big Idea	Skills	This Shows I Know	This Is How I Can Improve
Multiply polynomials using algebra tiles, diagrams, and algebra patterns *pages 86–89, 102, 104*	✓ Use algebra tiles to multiply binomials *pages 87, 102*		
	✓ Use the distributive property to multiply polynomial measurements *pages 87–89, 102*		
	✓ Apply binomial multiplication *pages 87–89, 102, 104*		
Determine factors of whole numbers and algebraic expressions *pages 90–103, 105–107*	✓ Identify the GCF and LCM of polynomials *pages 90–93, 102, 107*		
	✓ Factor polynomials using algebra tiles, using a table, and by modelling with a rectangle *pages 92–93, 95–97, 99–100, 102–103, 105–106*		
	✓ Factor polynomials using patterns such as the difference of two squares *pages 94, 98–101, 103, 105–107*		
	✓ Apply factoring to solve problems *pages 92–93, 96–97, 100–103, 107*		

Organizing the Ideas

In the Venn diagram below, show examples of factoring and multiplying. Be sure to represent these processes concretely, pictorially, and algebraically. How can you use this Venn diagram to show how factoring and multiplying are related?



Study Guide

Review the types of problems you handled in Chapter 5. What do you need to remember to help you do similar problems?

Things to Remember About Polynomials

Factoring

Multiplying

GCF and LCM

Concrete, Pictorial, Algebraic, and Special Patterns