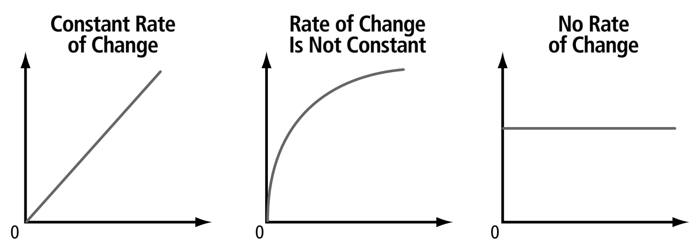
Chapter 6 Linear Relations and Functions

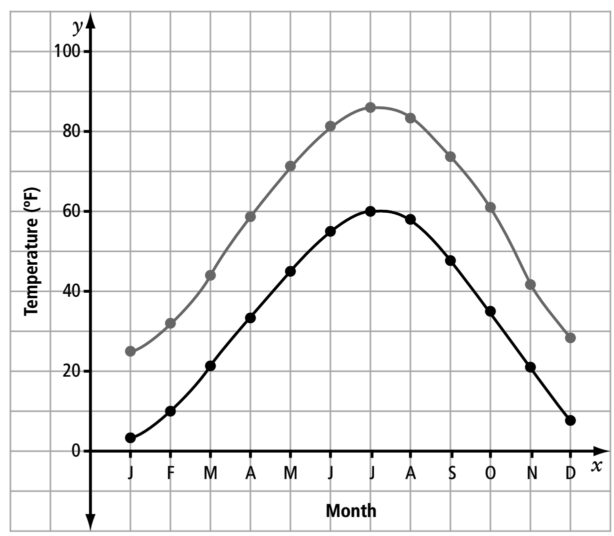
6.1 Graphs of Relations

KEY IDEAS

- When comparing two quantities, straight lines are used to indicate a constant change in the relationship. Curves are used when the rate of change is not constant. Horizontal lines are used if one quantity is not changing relative to a change in the other quantity.



Example



Travel guides often provide climate information, such as the graph shown here. Travellers can use this information to plan their trips.

a) What might the curves represent? Can you think of more than one explanation? What would the points on the curves represent for each of your explanations?

b) What assumptions can you make about the climate and location based on the shapes of the curves?

Solution

a) There are several possible explanations for having two curves:
- Both lines could represent temperatures for the *same* location. The upper curve would represent the average high temperature and the lower curve would represent the average low temperature. The dots on the curve would represent the average high and low temperature for each month.
- The curves could represent the average monthly temperatures for two *different* locations. Again, the dots would represent the average temperature for the month in each location.
- The curves could represent the record high and low temperatures in a single location, or compare record temperatures in two locations. The points on the curve would represent the record temperatures each month. But this explanation is unlikely. It is doubtful that the record temperatures would follow such a consistent pattern, represented by a smooth curve.

b) We can make several assumptions about the location from the shapes of the curves:
- If the curves represent two locations, the shapes and curves are similar enough that the two locations are probably both in the same area or in a similar climate zone. The location represented by the lower curve would be in a cooler location, perhaps further north or at a higher elevation.
- Considering that the highest temperatures occur in July, the location(s) are in the northern hemisphere.
- The slopes of the curves show a significant difference between winter and summer temperatures. But the differences are not extreme, as they might be in many places on the Canadian prairies. The average temperature curves here would be steeper. Similarly, in locations on the west coast, such as Victoria, British Columbia, where there is a more moderate climate, the curves would be flatter.

A Practise

1. Describe in words how each of the following situations can be drawn on a graph where time is on the horizontal axis and distance is on the vertical axis.

a) walking at a slow, constant rate away from the school

b) running at a constant rate toward the school

c) walking quickly away from the school, but constantly slowing the rate

d) standing still a distance from the school

e) running away from the school at a constant rate, and then suddenly turning around and walking toward the school at a constant rate

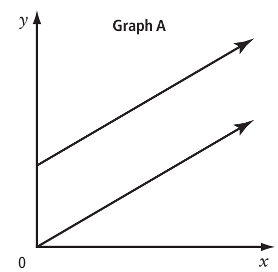
f) walking away from the school to a point, and then walking in a perfect circle around the school, all the while keeping a constant pace

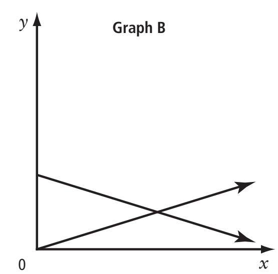
2. For the pairs of lines in each graph

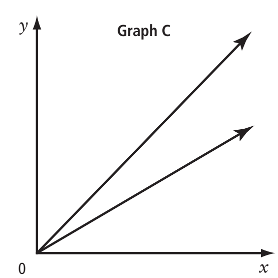
a) state what is similar

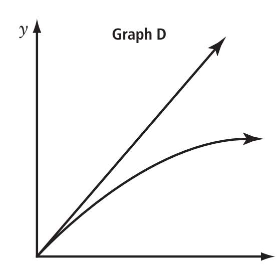
b) state what is different

c) give a real-life scenario that could be represented by the graph









☆**3.** For each scenario, draw a graph that is appropriate to the context, representing the activity in relation to time. Label the axes and provide a scale, if possible.

a) eating a bowl of cereal for breakfast

b) reading a novel from start to finish

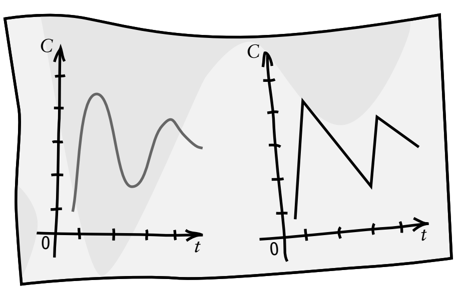
c) washing a load of laundry

d) flying from Calgary to Edmonton

4. **a)** Give two scenarios in which you would use only line segments to represent the relation.

b) Give two scenarios in which you would use only curves to represent the relation.

5. A student is drawing a graph, plotting time, t, in relation to cost, C, as shown. On his first attempt, the student produced a curve. After reconsidering the graph, he changed it to produce a line segment.



a) Explain the similarities that you see between the two relations.

b) Why might the student have made this change?

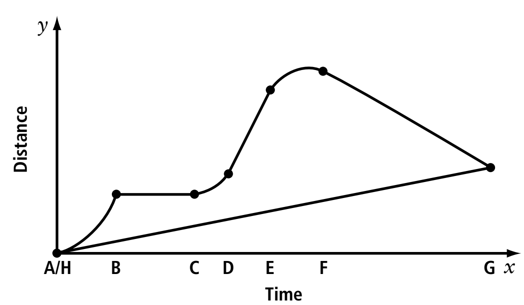
B Apply

☆**6.** The directions for cooking a particular type of rice say to add 1 cup of rice to 2 cups of boiling water, and then simmer the rice until the water is absorbed. Graph the cooking process from the time the pot is put on the stove, to the time the rice is cooked.
Hint: Put time along the horizontal axis, and choose what will be represented on the vertical axis.

☆**7.** In long track speed skating, skaters race around an oval track. Skaters start the race at the beginning of a long straightaway. They then race around the track for a number of laps, finishing near the end of the straightaway on which they started the race. Create a graph showing time versus distance from the finish line for a skater who skates at a constant pace for 2 laps. Explain why the symmetry occurs in your graph.
Hint: Draw a diagram of this situation and trace the path of a skater.

8. Kari drew the distance-time graph below to represent a scenario described by their teacher:

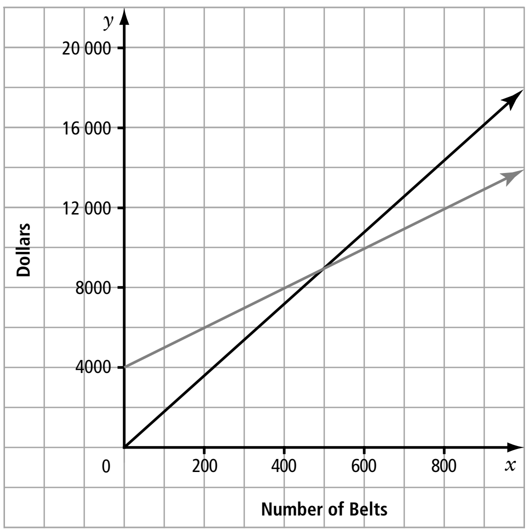
- time A to B: drive away from home accelerating to 50 km/h
- time B to C: continue away from home driving at 50 km/h
- time C to D: continue away from home accelerating to 100 km/h
- time D to E: continue away from home driving at 100 km/h
- time E to F: continue away from home decelerating to 0 km/h
- time F to G: spend 30 minutes in the mall
- time G to H: drive straight home at 40 km/h



a) For sections that you think are drawn incorrectly, state what you think is wrong. Then, draw the graph correctly.

b) Draw a speed-time graph for the scenario.

9. Les is the marketing manager at Noble Leather Goods in Winnipeg, Manitoba. His company is considering a new line of belts. Les's boss has asked him to present the business model for the new line. Les produced a simple model for cost (amount of money paid out) and revenue (amount of money received) relative to the number of belts made and sold. The graph shows his predictions for the first batch of belts to be produced.



a) Les forgot to label the lines on his graph. Which of the lines represents revenue and which represents cost? Explain.

b) What is the initial cost (the amount of money to start the production)?

c) How many belts does the company have to make and sell to break even (revenue exactly equals cost)?

d) What are the cost and revenue at the breakeven point?

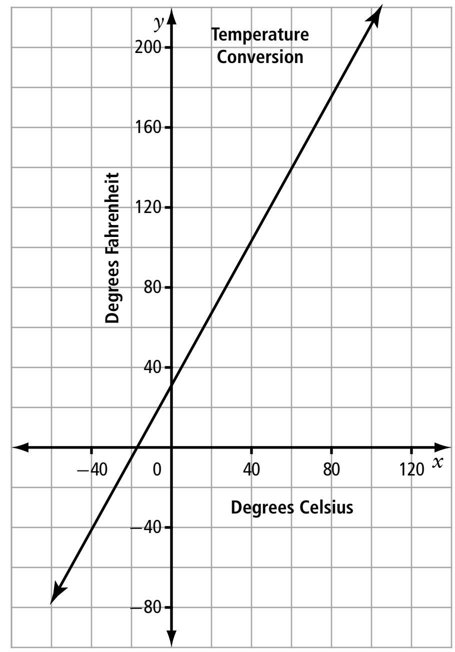
e) What is the predicted profit (the difference between revenue and cost) if the company makes and sells 1000 belts?

f) How much would the company lose if, after setup, management decide not to produce any of these belts?

g) What is the projected selling price of one belt?

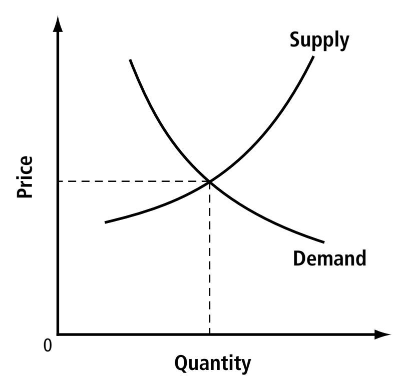
C Extend

10. Use the graph below to discover or verify facts regarding the relationship between temperatures measured in Celsius and those measured in Fahrenheit.



D Create Connections

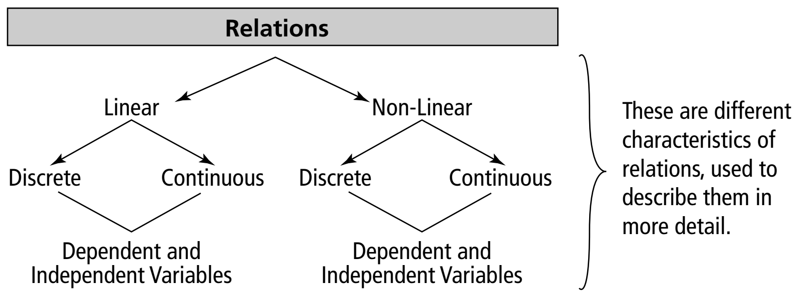
11. Supply and demand is a model that economists use to show how prices are determined for goods and services. This graph represents this model. The point where the two curves intersect represents where supply is equal to demand. Many factors affect the location and steepness of the curves, and where they intersect. Research supply and demand and try to determine three of these factors.



6.2 Linear Relations

KEY IDEAS

- Relations can be represented in a variety of ways. You can use words, equations, tables of values, ordered pairs, or graphs.



Example

Consider the following two relations:

a) Amir's height above the water after springing from a diving board

b) the cost for downloading music from a site that charges $1.25 per song

For each relation, decide
- if it is linear or non-linear
- if it is discrete (made up of values that are not connected on a graph) or continuous (made up of values that are connected by a line on a graph)
- what the dependent and independent variables are
- the best way to represent it (words, an equation, an ordered pair, a table of values, or a graph)

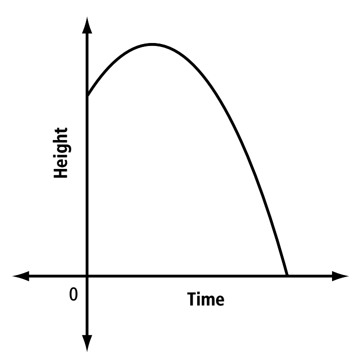
Solution

a) Amir's height above the water is non-linear over time. His height increases when he springs from the board, and then decreases as he plunges toward the water.

The data for this relation is continuous because for every instant of time that Amir is in the air there is a corresponding positive height above the water.

Amir's height above the water depends on how long he has been in the air. So, time is the independent variable and height is the dependent variable.

A graph is the best means of representing this situation. The dependent variable is always on the y-axis, so height is on the y-axis and time is on the x-axis.



b) The cost per song is constant, so this is a linear relationship.

This is a discrete relation because you cannot purchase part of a song. Each additional song purchased results in a unique purchase price.

The cost for the songs depends on the number of songs you download. So, the independent variable is number of songs, and the dependent variable is cost.

You could represent this relationship with a series of ordered pairs, a table of values, or a graph. Each would be clear. The equation $C = 1.25(n)$, where C is the cost and n is the number of songs downloaded, is probably most effective in this case. This equation makes it easy to determine the cost of any number of downloads.

A Practise

1. Convert each relation to a set of ordered pairs and a graph

a)

x	2	3	5	8	12
y	0	2	4	6	8

b)

a	10	8	6	4	2
b	2	4	6	8	10

☆**2.** Convert each relation to an equation and a table with 4 pairs of values. Is the relation discrete or continuous? Explain why.

a) the cost of buying concert tickets priced at $23.00 each

b) two positive numbers whose product is 24. Hint: Don't forget decimals. How would your answer change if you considered whole numbers only?

3. Given the following tables of values, determine which relations are linear and which are non-linear. Describe each relation in words.

a)

x	−3	−2	−1	0	1	2	3
y	6	2	0	0	2	6	12

b)

x	−3	−2	−1	0	1	2	3
y	−5	−3	−1	1	3	5	7

c)

x	−3	−2	−1	0	1	2	3
y	6	5	4	3	4	1	0

d)

x	−3	−2	−1	0	1	2	3
y	3	2	1	0	−1	−2	−3

☆**4.** Determine if each formula is linear or non-linear by representing it in any of the other forms of a relation. Indicate which variable is independent, and which is dependent.

a) the area, A, of a circle with radius r: $A = \pi r^2$

b) the perimeter, P, of an equilateral triangle with side s: $P = 3s$

c) the number of diagonals, d, of a polygon with n sides: $d = \frac{n(n-3)}{2}$

5. Predict which of the following are linear relations. Use a graphing calculator or graphing software to check your prediction.

a) $y = \pi x$

b) $y = \frac{x}{9}$

c) $y = x\sqrt{x}$

d) $y = (x + 1)(x - 1)$

B Apply

6. A video store charges $4.50 to rent a new release movie. The store's owner wants to put up a poster to make it easy for customers to determine the cost of renting multiple movies.
   - **a)** Name the independent and dependent variables in this situation.
   - **b)** Describe the pricing policy in words.
   - **c)** Write an equation to represent the cost of renting 1 through 5 movies.
   - **d)** Show a set of ordered pairs for renting 1 through 5 movies.
   - **e)** Make a table of values that shows the cost of renting 1 through 5 movies.
   - **f)** Make a graph for renting 1 through 5 movies. Does it make sense to show the cost of renting zero movies?
   - **g)** From the 5 ways you represented the relation, which do you think would be the best way for the owner to present the information on the poster? Explain.

7. A pizza restaurant sells 4 sizes of pizza: 8 in., 10 in., 12 in., and 14 in. All the pizzas are round and the size is the pizza's diameter.
   - **a)** Make a table of values comparing diameter to the area of the pizza.
   - **b)** Which is the independent variable?
   - **c)** Graph the four ordered pairs.
   - **d)** Is this data linear or non-linear?
   - **e)** Is this data continuous or discrete?

8. The pizza restaurant owner from question 7 charges $10 for his 8 in. pizza. He wants to price the others so that the relation between the *area* of the pizza and the price is linear.
   - **a)** What are the radii of the four different pizzas?
   - **b)** Determine the areas of the four pizzas, rounded to the nearest square inch. Hint: Remember that $A = \pi r^2$.
   - **c)** What is the price per square inch of the 8 in. pizza?
   - **d)** What should the owner charge for each of the other pizzas, rounded to the nearest $0.10?
   - **e)** Do you think it is a good idea for the restaurant owner to price his pizzas this way?
   - **f)** Is the relation between the area of the pizza and the price linear or non-linear? discrete or continuous?
   - **g)** Is the relation between the diameter of the pizza and the price linear or non-linear?

☆9. Jogging is usually defined as running at approximately 10 km/h, or 6 min/km.
   - **a)** Is this a linear or non-linear relation?
   - **b)** Make one table of values with time in hours versus distance for running up to 5 hours. Make another table of values with distance versus time in minutes for running up to 8 km. Which table gives a better representation of jogging speed?
   - **c)** What would be the independent variable for graphing a jogging speed of 10 km/h?
   - **d)** Make a graph for jogging 10 km at a 6 min/km.
   - **e)** Is your graph continuous or discrete? Why?

10. The advertising manager for a small Alberta computer company believes that in any given month there is a linear relation between the amount of money spent on advertising, a, and the number of computers sold, n. In September, the company spent \$8000 on ads and sold 12 000 computers. Then, in October, it increased the advertising budget to \$20 000 and sold 18 000 units.

a) Set up a grid and plot the two points in the question. Hint: Determine the independent and dependent variables to decide what each axis represents. Mark both axes from 0 to 40 000.

b) Draw a line through your two points and use the graph to predict the number of computers the company would sell if the advertising budget were increased to \$40 000.

c) How much would the company need to spend on ads to sell 21 000 units?

d) How many computers would the company sell if it did no advertising?

C Extend

11. The length from your wrist to your elbow is approximately the same as the length of your foot. Measure 5 of your friends' arms and feet to determine if this is true.

a) Place this data in a table of values.

b) Graph the 5 pairs of data.

c) In this question, which measurement is independent? dependent?

d) Is the data linear or non-linear?

e) Is the data discrete or continuous?

12. When grocery shopping, the costs of many items can be represented by a linear relationship.

a) List 5 grocery items having costs that are linear in nature.

b) What is the dependent variable in all of these examples?

c) Are your examples discrete or continuous?

d) Name one item from a grocery store for which the cost would always be discrete. Name one for which the cost would always be continuous.

e) Name a grocery item that may have non-linear pricing structure.

☆**13.** Make a table of values for each of the following using $x = \{0, 1, 2, 3, 4, 5\}$.

- $y = 3x(x - 2)$
- $y = 5(x - 2)$
- $y = \frac{(x - 2)}{2}$

Predict whether each equation is a linear function by considering whether the y-value is increasing or decreasing by a constant amount. Then, graph each equation using technology and explain how the graphs support your predictions.

D Create Connections

14. When looking at a given relation, describe a way that you can predict whether the relation is linear or non-linear if the relation is

a) an equation

b) a table of values

c) a set of ordered pairs

d) a graph

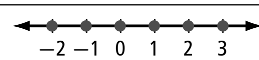
e) given in words

15. When you are graphing a relation, is there a simple way to decide whether to connect the data points to make the graph continuous or to leave the points discrete? Discuss with a classmate, and then explain in your own words.

6.3 Domain and Range

KEY IDEAS

- The domain of a relation is the set of all numbers for which the independent variable is defined.
- The domain of a relation may also be described as:
  – the set of first coordinates in a set of ordered pairs
  – the possible values in the first column of a table of values
  – the possible values on the horizontal axis of a graph.
- The range of a relation is the set of all numbers for which the dependent variable is defined.
- The range of a relation may also be described as:
  – the set of second coordinates in a set of ordered pairs
  – the possible values in the second column of a table of values
  – the possible values on the vertical axis of a graph.
- The domain and range can be expressed in different ways.

Words	All integers equal to or greater than −2 and less than or equal to 3
Number Line	
Interval Notation	$[-2, 3]$
Set Notation	$\{n \mid -2 \leq n \leq 3, n \in I\}$
A List	$\{-2, -1, 0, 1, 2, 3\}$

Example

The annual ecoENERGY for Vehicles Awards, administered by Natural Resources Canada's Office of Energy Efficiency, are presented for the most fuel-efficient vehicles for the current model year. One popular midsized car has a 77 L fuel tank. Its fuel efficiency is listed as 7.0 L/100 km in the city, and 5.5 L/100 km on the highway.

a) State the domain for the distance this car can travel for both city and highway driving. Express the domains in words and in set notation.

b) State the range of the fuel tank.

c) Draw one graph with two consumption lines on it: one for city driving and another for highway driving. Provide appropriate labels and scaling for the axes.

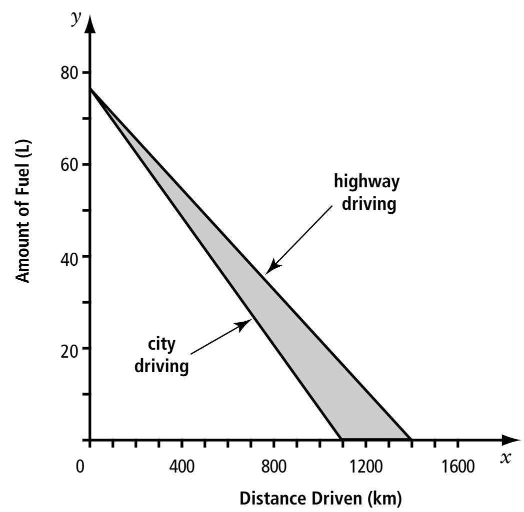
d) What does the portion of the graph between the two lines represent?

Solution

a) For city driving, the vehicle can travel 100 km for every 7.0 L of fuel. On a full tank of fuel it can travel $\frac{77}{7} \times 100$, or 1100 km. So, the domain of distance travelled for city driving is from 0 km, if the car is simply left idling, to 1100 km, or $\{d \mid 0 \leq d \leq 1100\}$.
For highway driving, the vehicle can travel 100 km for every 5.5 L of fuel. On a full tank of fuel it can travel $\frac{77}{5.5} \times 100$, or 1400 km. So, the domain of distance travelled for highway driving is from 0 km to 1400 km, or $\{d \mid 0 \leq d \leq 1400\}$.

b) The range of the fuel tank is from 0 L to 77 L of fuel, or $\{L \mid 0 \leq L \leq 77\}$.

c) The independent variable, plotted on the x-axis, is distance driven. The dependent variable, plotted on the y-axis, is the amount of fuel used.



d) The shaded area between the two lines represents all of the possible distances and litres of fuel consumed by a mixture of city and highway driving, if the car is always kept within the predicted consumption rates.

A Practise

1. Draw a number line to represent each set of numbers. Hint: On a number line a solid dot means that the value is part of the set and an open circle means that the value is not part of the set.

a) {−2, 0, 2, 4, 6, 8, 10}

b) $\{x \mid x < 5\}$

c) your age from grade 1 until now

d) all the factors of 15

e) the square roots of all perfect squares from 1 to 100

f) (−3, 4]

2. For each relation, state the domain and range.

a) buying less than 5 cans of soup that cost $0.38 each

b) listing all coin names from $0.01 to $2.00 and their value

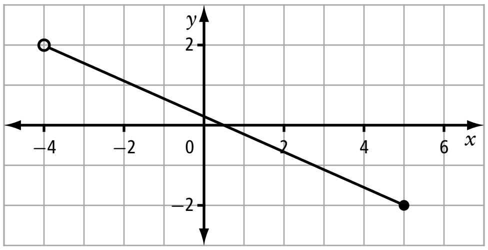
c) the squares of numbers 1 through 10

d) the cost for you and up to 5 of your friends to attend a concert at $35.00 per ticket

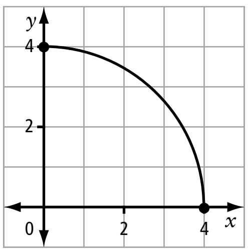
e) the granola bars you can buy with a $10.00 bill, at $1.50 per bar

3. Give the domain and range of each graph. Use both set notation and interval notation.

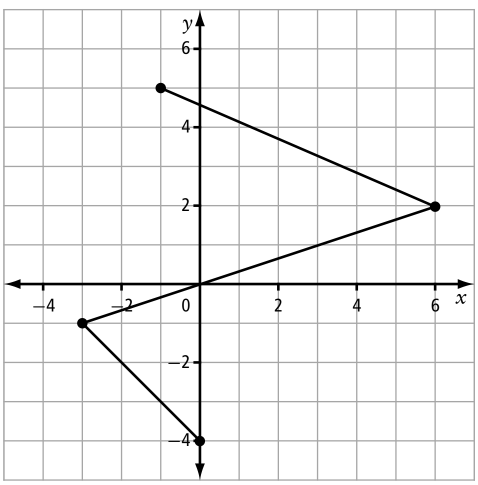
a)



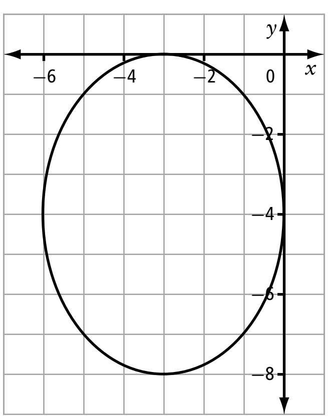
b)



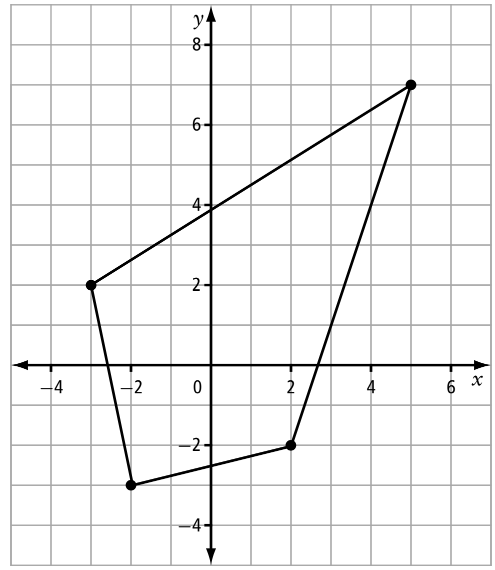
c)



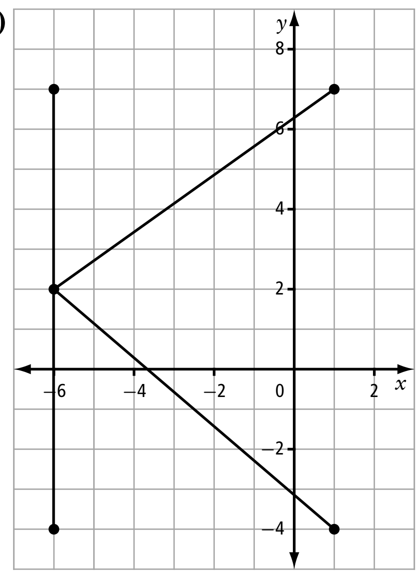
d)



e)



f)



☆**4.** The cost, C, of filling up a car with gasoline and buying an \$8.00 car wash can be expressed by the equation $C = 0.92n + 8.00$, where n is the number of litres of gasoline purchased. The car has a gas tank capacity of 40 L.

a) What is the domain of this equation?

b) What is the range of costs for this problem?

c) Which is the independent variable? Explain why.

B Apply

5. The table presents the internal temperatures that particular foods must reach in order to achieve the specified doneness and be considered safe to eat.

Food	Internal Temperature
Beef, veal, and lamb—medium-rare	63 °C (145 °F)
Beef, veal, and lamb—medium	71 °C (160 °F)
Beef, veal, and lamb—well done	77 °C (170 °F)
Pork (pieces and whole cuts)	71 °C (160 °F)
Poultry—pieces	74 °C (165 °F)
Poultry—whole	85 °C (185 °F)
Egg dishes	74 °C (165 °F)

a) State the range of temperatures for safe cooking of the foods in the table.

b) Is it better to use set notation or a list for the range, considering the context of the information?

c) What would be the danger of using either set notation or a list for the temperatures?

6. Draw a graph with each domain and range in the table. The graph can be made up of line segments or curves. Hint: Remember that a round bracket means that the value is not part of the set, while a square bracket means that the value is part of the set.

	Domain	Range
A	(−3, 5)	(1, 5)
B	(−3, 5]	[1, 5)
C	[−3, 5]	[1, 5]
D	[−3, 5)	(1, 5]

7. The domain of a relation is given as (−8, 6), while its range is $\{y \mid -4 \leq y < 5\}$. Set up a grid with the x-axis and y-axis marked from −10 to 10. Draw a rectangle that the relation would lie within when graphed. When drawing the rectangle, use a solid line if the graph could be on it, and a dashed line if the graph only comes up to it, but does not include it.

8. Which among the items listed could have a domain or range of either [−12, ∞) or (−∞, 0]?
Hint: Remember that ∞ is used when there is no end point.
- a line
- a line segment
- a ray
- an oval

C Extend

9. Draw a graph of any relation that has

a) an identical domain and range

b) only one domain element

c) only one range element

10. Does a relation always need a domain? Explain.

D Create Connections

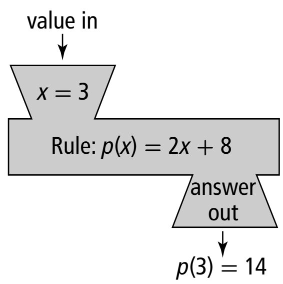
11. Create a relation where the range is $\{0 \text{ cm} \leq y \leq 20 \text{ cm}\}$.

12. In professional sports, there are many relationships that have an independent and dependent connection. For example, the players in a golf tournament represent a domain, and the scores they post for each round or for the tournament are the range. Provide two or more different examples, stating what the domain and range would be for each example.

6.4 Functions

KEY IDEAS

- All functions are relations, but not all relations are functions.
- A relation is classified as a function if each value in the domain corresponds to exactly one value in the range.
- Each function has its own formula, or rule, which is often given using a special notation, called function notation. For example, $p(x) = 2x + 8$ shows that the function p takes an input value, multiplies it by 2, adds 8, and outputs the answer.



Example

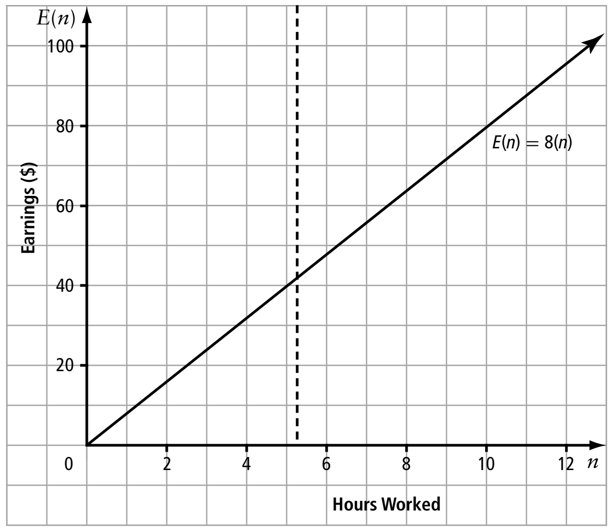
To distinguish between relations and relationships that are functions, and those that simply relate in an input/output manner, you must consider all the possibilities of the domain. If it is possible to have more than one output for one unique input, the relation is not a function.

Carefully consider the relationships listed in the table. State whether the relation is a function or not, and explain why. Sketch a graph of a function and a non-function from the questions. Show how the vertical line test supports your answer.

Relationship	Domain Input	Range Output
a) Working for $8/h	Hours worked in one week	Amount earned in that week
b) Putting footwear on before leaving the house	Choosing a pair of shoes	Leaving with something on your feet
c) Baking bread	Temperature of the oven	Height loaf rises
d) Studying for a math exam	Number of hours	Mark on exam
e) Squaring a number, and then adding 3	Any number you choose	Example: 28

Solution

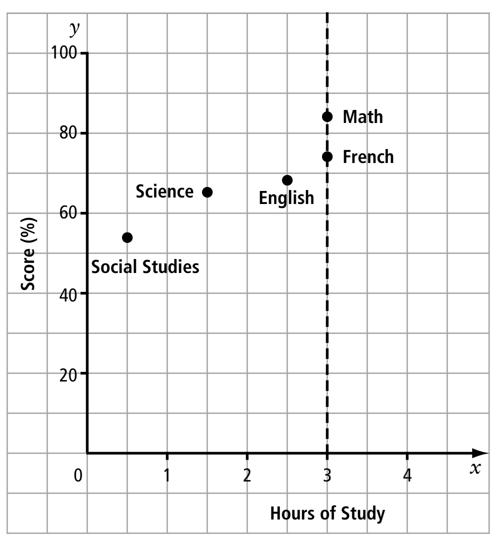
a) Function: it can be represented by $E(n) = 8(n)$, where E is earnings, and n is the number of hours worked. Each unique number of hours worked generates only one amount of money earned. This is supported by the vertical line test. Any vertical line, such as the one shown, will pass through one and only one point on the graph.



b) Function: you could wear runners, sandals, or any other footwear, and you would have something on your feet. Having footwear on produces only one output.

c) Not a function: the loaf may rise the same amount using different oven temperatures. Also, baking at the same temperature could produce a loaf of different heights.

d) Not a function: you could have studied for 3 hours for several exams and received different marks on some or all of them. The vertical line test shows that this is not a function because the vertical line at $x = 3$ passes through the mark received in both Math and French.



e) Function: for $S(n) = n^2 + 3$, both 5 and −5 produce 28. It is still a function if two different inputs produce the same output. It is not a function only if one input produces more than one output.

A Practise

☆**1.** For each relation, state whether it is a function. For those that are not functions, indicate where or explain why it is not a function. Where possible, use the vertical line test as part of your explanation.

a) (1, 3) (2, 4) (3, 5) (4, 3) (2, 1)

b) (5, 1) (4, 1) (3, 1) (2, 1) (1, 1)

c) (9, 3) (4, 2) (1, 1) (9, −3) (4, −2) (1, −1)

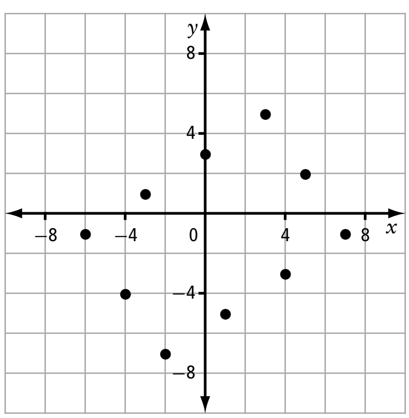
d)

Name	Shoe Size
Andrew	10
Nathan	11
Joel	12
Aaron	13
Simeon	12

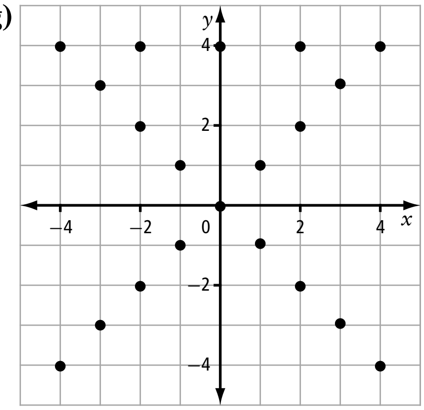
e)

Name	Sibling
Anika	Jared
Anika	Joel
Anika	Nathan
Carolyn	Aaron
Carolyn	Simeon

f)



g)



2. The formula for calculating the value of \$500.00 deposited into an account earning 8% compounded annually for n years is $A = 500(1 + 0.08)^n$. Write this formula using function notation.

3. Anika is helping her parents make plans for her grandparents' 50th wedding anniversary. The cost for the banquet is given by the function $W(p) = 26p + 1200$, where W is the cost in dollars, and p is the number of people attending. Write this function as a formula in two variables.

4. If $z(a) = -3a + 7$, determine the following:

a) $z(-3)$

b) $z(2)$

c) a, if $z(a) = 7$

5. If $t(n) = 5 + (n - 1)(4)$, determine the following:

a) $t(1)$

b) $t(20)$

c) n if $t(n) = 41$

B Apply

☆**6.** For a single membership to FITFIT Health Club, you pay a \$55 initiation fee upon enrollment and then \$35 a month. The cost of belonging to the club is represented by the function $P(m) = 35m + 55$.

a) What is the independent variable in this relation and what does it represent?

b) What would it cost for you to belong to this health club for one year?

c) After how many months of membership would you have spent \$1000?

d) The cost of belonging to one of FITFIT's competitors is represented by the function $P(w) = 10w + 100$, where w represents the number of weeks you are enrolled. Which club would be cheaper to belong to for one year?

7. For each function, calculate $f(5)$.

a) $f(x) = 3x - 6$

b) $f(x) = -2x + 11$

c) $f(x) = \frac{(x + 5)}{2}$

d) $f(x) = \frac{1}{4}(3 - x)$

8. Determine the value of x for the functions in question 7, when $f(x) = -15$.

9. Graph the following functions for the given domain. For what x-value does the graph go through $(x, 7)$?

a) $g(x) = 3x - 5$, for the domain $\{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

b) $h(x) = -2x + 7$, for the domain $\{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

c) $j(x) = 7(x - 4)$, for the domain $\{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

C Extend

10. Graphing calculators or software can find many function values quickly. Enter the function $f(x) = 3x - 11$ into either of these technologies. Determine the smallest value of x that produces

a) a prime number

b) a multiple of 8

c) a number larger than 100

d) the largest negative number

11. A science teacher asks students to write an equation to represent the temperature, T, measured in °C, of a liquid cooling on a laboratory table. The equation is to be in the form $T(m) = (A)m + (B)$, where m is the number of minutes since the liquid was placed on the table. Determine the values for A and B so that the equation produces the values in the chart.

Time (min)	Temperature (°C)
4	74
7	62
13	38
16	26

D Create Connections

12. Using the Internet, find the addresses in Canada associated with the following postal codes:

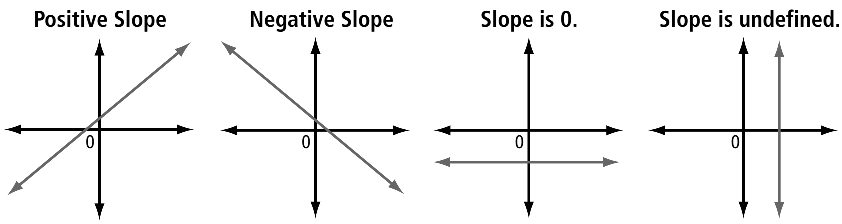
- V0B 2P0
- R2J 3E7
- T0H 2P0

a) Are any of these postal codes and the address(es) associated with them a function? Explain.

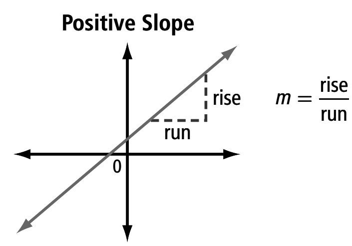
b) Use the Internet to search your own postal code. Is your postal code and address a function? Explain.

6.5 Slope

KEY IDEAS

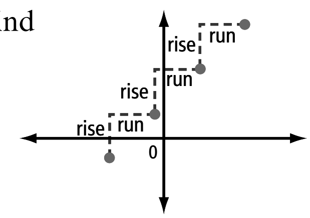
- 

- The slope of a line is the ratio of the rise to the run.



- The slope of a line can be determined using two points on the line, (x_1, y_1) and (x_2, y_2).
$m = \frac{y_2 - y_1}{x_2 - x_1}, x_2 \neq x_1$

- If you know one point on the line, you can use the slope to find other points on the line.



- The slope gives the average rate of change.

Time (s)	Distance (m)
1	4
2	7
3	10
4	13
5	16
6	19
7	22

Rate of change $= \frac{\Delta d}{\Delta t}$

Rate of change $= \frac{3}{1}$

The average rate of change is 3 m/s.

Time (s)	Distance (m)
1	4
3	10
5	16
7	22

Rate of change $= \frac{\Delta d}{\Delta t}$

Rate of change $= \frac{6}{2}$

Rate of change $= \frac{3}{1}$

The average rate of change is 3 m/s.

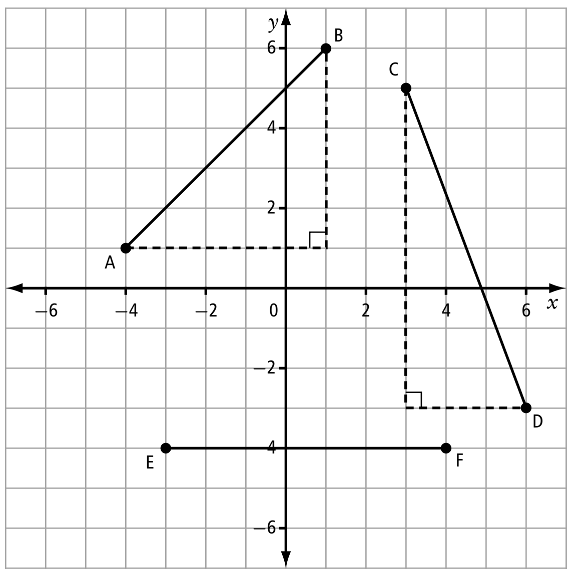
Example

Use two methods to determine the slope of each of the following line segments:

- A(−4, 1) and B(1, 6)
- C(3, 5) and D(6, −3)
- E(−3, −4) and F(4, −4)

Solution

Method 1: Graph each line segment. Then, for each segment, connect the two endpoints by drawing a right triangle.



Slope, m, is equal to $\frac{\text{rise}}{\text{run}}$. Determine the *rise* by counting the units along the height of the triangle, and the *run* by counting the units along the base of the triangle.

For AB:

$$m = \frac{5}{5} = 1$$

For CD:

$$m = \frac{-8}{3}$$

For EF:

$$m = \frac{0}{7} = 0$$

Method 2: Use the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, $x_2 \neq x_1$.

Substitute values to determine each slope.

For AB:

$$m = \frac{(6 - 1)}{(1 - (-4))} = \frac{5}{5} = 1$$

For CD:

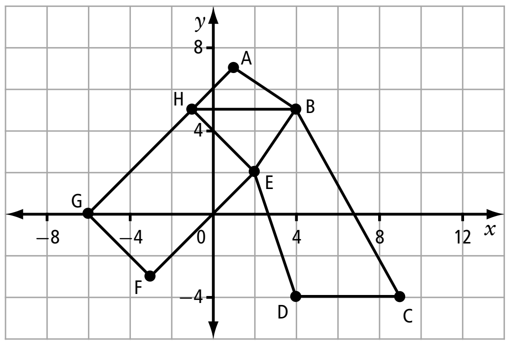
$$m = \frac{(-3 - 5)}{(6 - 3)} = \frac{-8}{3}$$

For EF:

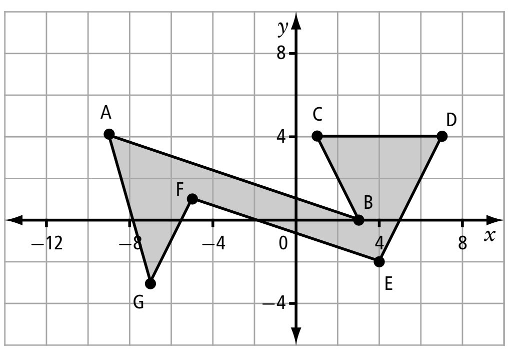
$$m = \frac{(-4 - (-4))}{(4 - (-3))} = \frac{0}{7} = 0$$

A Practise

1. Create a three-column table with the headings Positive Slope, Negative Slope, and Zero Slope. Place each line segment in the diagram in the appropriate column.



2. Determine the slope of each line segment in the diagram.



3. A 45° line has a slope of +1 or −1. Use the slope formula to determine the slope of a line passing through each pair of points. Is the line steeper or less steep than 45°?

a) (3, 5), (1, 4) **b)** (6, 5), (1, −3)

c) (2, −3), (−2, 6) **d)** (−4, 5), (−1, 4)

4. Graph a line from each given point back to the origin. Determine the slope of each line.

a) (4, −2) **b)** (−3, 0)

c) (7, 3) **d)** (−2, −5)

5. For each given point and slope, calculate what the next ordered pair *to the right* of the point will be. If possible, try to do this mentally, without graphing. The first one is done for you.

Given Point A(x, y)	Slope	Next Point to the Right of A
(3, 5)	$-\frac{1}{2}$	(5, 4)
(3, 5)	$\frac{2}{3}$	
(−3, 7)	$\frac{3}{7}$	
(2, −5)	$-\frac{4}{1}$	
(0, −4)	$\frac{5}{4}$	

B Apply

6. Building codes and safety concerns dictate slopes in structures. According to Canadian building codes, a wheelchair ramp cannot have a slope greater than $\frac{1}{12}$. When designing a mall, an architect has designed a central courtyard that is 84 cm higher than the corridor approaching it. How far away will a wheelchair ramp have to begin, in metres, if it is to have the steepest allowable slope?

7. Phoebe has asked her mother to help her save money for a vacation. At the end of each work week, Phoebe gives her mother the same amount of money to hold for her. At the beginning of last year, Phoebe had given her mother $255.00. At the end of the current year, the total had risen to $1035.00. Determine the average rate of change in her vacation savings. What does the number represent?

☆**8.** Snowcoach rides on the Columbia Icefields travel from a side moraine onto the glacier via a road that is constantly shifting due to ice movement and melting. The road is the steepest passenger route in the world, at a 32% grade.

a) If the elevation increase on this road is 130 m from start to finish, what would be the horizontal distance travelled (referred to as the shortest run)?

b) State the slope of the road in terms of $\frac{\text{rise}}{\text{run}}$.

c) Use the Internet to research how long the actual road is.

9. On the first day of school in September, Jodi measured the length of her hair. She found that her longest hair was 36 cm. At the end of the school year, in June, Jodi measured her hair again. Her longest hair at this time was 48.5 cm. What is the average rate of change of hair length per month?

10. Adult fingernails grow at a yearly rate of change. Lee Redmond set a Guinness world record when she grew her nails for about 29 years without cutting them. Added together, her nails were 8.65 m long. Assuming that she grew her nails for exactly 29 years, what was the average rate of change in her nails' length, per month, to the nearest centimetre?

C Extend

11. In 1981, the population of Saskatoon, Saskatchewan, was 154 210. In 2006, its population had grown to 202 340. Determine the average annual rate of growth and, assuming this rate of growth will continue, project Saskatoon's population in 2021. Research the most recent population of Saskatoon that you can, and see if it matches your projection.

☆**12.** A business wants to decide whether employees should use their own vehicle for company business or use a rental car. An employee uses a rental company that charges a daily rate plus a mileage charge, with no free kilometres.

	Trip A	Trip B
Days	3	3
Distance Driven (km)	425	680
Rental Charge	$301.25	$365.00

a) Determine the rate the car rental company charges per kilometre.

b) Determine the daily rate for a car rental.

c) Determine if it would be cheaper for the company to pay employees $0.50/km to use their own car.

d) Will your answer for part c) ever change if the trip is always 3 days long? Explain.

D Create Connections

13. Slope is defined as rise over run. Draw three different positive-sloped line segments on a piece of graph paper, with each one steeper than the previous. Make sure your lines go through ordered pairs that you can identify. For each segment, calculate the slope and then calculate the fraction $\frac{\text{rise}}{\text{run}}$. Explain why, as the line gets steeper, it becomes more apparent why the fraction is in the form $\frac{\text{rise}}{\text{run}}$.

14. Graph a series of lines that start at the origin and have the slope ratios found in the table. Notice that the numerator (rise) is consecutive even numbers and the denominator (run) is consecutive prime numbers. If you kept this pattern going, would you ever draw a line with a slope of 0? Explain.

Rise	2	4	6	8	10	12	14	16	18
Run	2	3	5	7	11	13	?	?	?

Chapter 6 Review

6.1 Graphs of Relations

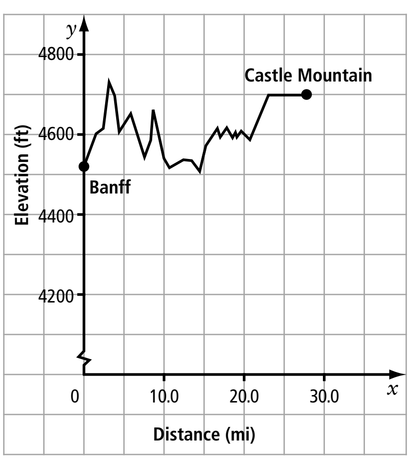
1. Darlene goes to the grocery store. She buys milk, granola, deli ham, bananas, and bread.

a) Draw a map of a grocery store and label the locations of the five items. Draw a dashed line representing the route Darlene would take to pick up these items.

b) What units would best be used to describe Darlene's speed and time as she walks around the store?

c) Draw a graph of Darlene's speed versus time from the time she enters the store until the time she leaves. Be sure to consider any stops she makes or time she spends waiting for service. On your graph, explain each section and label the point on the graph at which each item is picked up.

2. Serge and Colette plan to bike from Banff, Alberta to Castle Mountain along the Bow Valley Parkway. The graph shows the elevation change over the distance of their route.



a) Choose two 5-mile sections of the graph, and describe what the riding might be like in these sections.

b) Which 5-mile section is the easiest? most difficult? Explain your choice.

6.2 Linear Relations

3. For each of the following scenarios, would the graph of the relation be discrete or continuous? Explain.

a) the number of tickets written by a police officer

b) time on the ice during a hockey game for one player

c) number of characters used in a text message

4. Insurance for Priority Mail International parcels is calculated according to this table.

Fee	Insured Amount (not over)
\$1.75	\$50
\$2.25	\$100
\$2.75	\$200
\$4.70	\$300
\$5.70	\$400
\$6.70	\$500
\$7.70	\$600
\$8.70	\$675 max

a) Assign a variable to each quantity. Which is independent and which is dependent?

b) Is this relation linear or non-linear? Explain.

c) Graph the relation. Note that one fee charge covers a range of insured amounts.

6.3 Domain and Range

5. State the domain and range of each relation.

a) $\{(3, -7), (5, 5), (3, -4), (0, 0), (3, 11)\}$

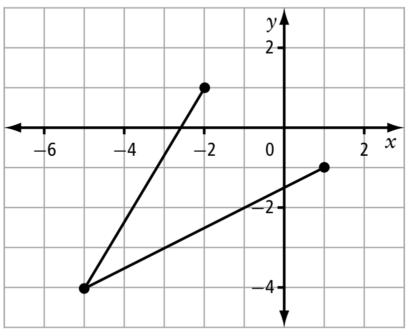
b) All the factors of 10. The answers when the factors are divided back into 10.

6. Change from the given notation to the other notation for each domain given.

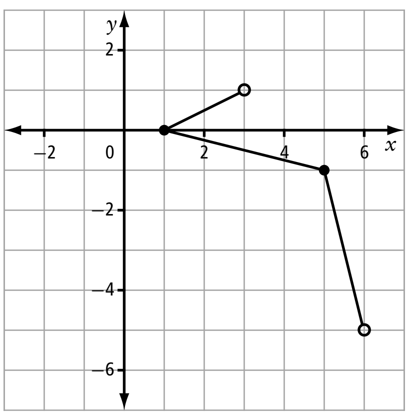
Set Notation	Interval Notation
$\{x \mid 3 < x < 7\}$	
	$[-5, 0]$
$\{x \mid -13 \leq x\ 27\}$	
	$(-\infty, 5]$

7. Provide the domain and range of each relation in set notation.

a)



b)



6.4 Functions

8. Consider the function $f(x) = 2x^2 - 2x + 1$.

a) Calculate $f(0)$.

b) Calculate $f(-1)$.

c) Calculate $f(3)$.

d) Considering that $f(-2)$ and $f(3)$ result in the same value, is $f(x)$ still a function? Explain.

e) Graph $f(x)$. Does the function pass the vertical line test?

9. The formula for the volume of a cube is $V = s^3$. The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$.

a) Write each formula in function notation.

b) Complete the table for a sphere that fits exactly inside the given cube.

Side Length of Cube	Volume of Cube	Volume of Sphere
10 cm		
20 cm		
30 cm		
40 cm		

6.5 Slope

10. Find the slope between pairs of points. Before calculating, predict if the slope will be positive, negative, zero, or undefined.

a) P(11, 3), Q(2, 8)

b) M(−6, 0), N(0, 8)

c) J(−3, 5), K(−2, −7)

11. Jordan kept track of all his training distances while bike riding one week in July. His goal was to ride farther each day to build endurance.

	July Date	Distance (km)
Monday	4	60
Tuesday	5	80
Wednesday	6	110
Thursday	7	120
Friday	8	100

a) Plot this data on a graph.

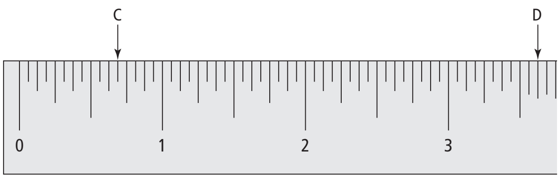
b) Calculate the slopes of the four line segments that connect these points.

c) What does the slope represent?

d) What does a negative slope mean?

Chapters 1–6 Cumulative Review

1. This ruler shows imperial units. State the reading for point D on this ruler as a mixed number in lowest terms. What is the distance from C to D? Show two ways to determine the answer.



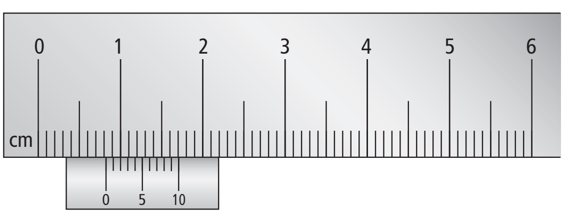
2. What are the dimensions of a cube that has a volume of 17 576 cm^3?

3. Multiply and then combine like terms.

a) $(a - 2)(a^2 + 7a + 4)$

b) $3b(4b - 2)(b + 3) - b(4b + 8)(3b - 1)$

4. What reading is shown on this metric caliper? Name an object that could be this length.



5. Jasmine is planning a family reunion. She rents a hall for $200 for the evening. She estimates that she will need $10 per person to pay for food and entertainment. Consider the relationship between the cost of the evening and the number of people in attendance.

a) Is this a linear or non-linear relationship? Explain how you know.

b) Assign a variable to represent each quantity in the relation. Which variable is the dependent variable? Which is the independent variable?

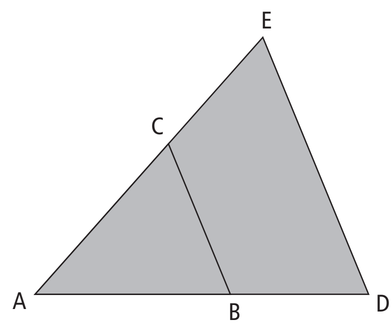
c) Create a table of values for the following number of guests: 0, 10, 20, 30, 40, 50, and 100.

d) Is the data discrete or continuous? Explain how you know.

e) Is this relation a function? Explain how you know.

f) Graph the relation.

6. Triangles ABC and ADE are similar. If AC = 5 cm, BC = 3 cm, and DE = 9 cm, determine the length of CE.



7. Lucy wants to invest in a guaranteed investment fund (GIF) that pays 6% interest per year, compounded monthly. To guarantee her interest rate, she must keep her money in the account for three years. This relationship can be modelled by the equation $A = P\left(1 + \frac{0.06}{12}\right)^{36}$, where P is the principal (money invested) and A is the amount of money after a certain length of time.

a) If Lucy invests $1500, how much money will she have after three years?

b) How much interest has Lucy earned in three years?

c) If Lucy reinvests the original principal plus the interest she earned, and leaves it in the GIC for another three years, how much will the investment be worth?

8. The average centre-to-centre distance from Earth to the moon is 384 403 km. It takes the moon 27.32 days to orbit Earth. In this time it will have travelled approximately 2 455 260 km.

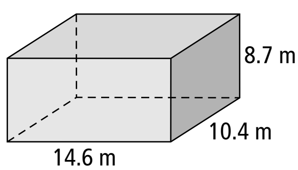
a) Draw and label a diagram of Earth and the path of the moon.

b) How far does the moon travel along its orbit in one day, to the nearest hundredth of a kilometre?

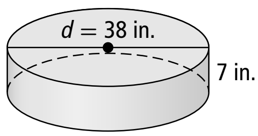
c) What is the speed of the moon's orbit, to the nearest hundredth of a kilometre per hour? Hint: Use the formula, speed = distance/time.

9. Calculate the volume and surface area of the following prisms, to the nearest hundredth of a unit.

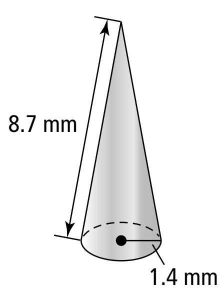
a)



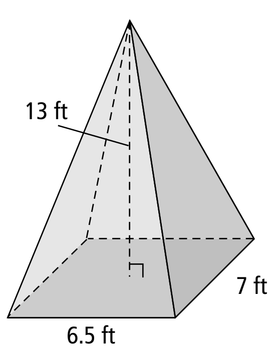
b)



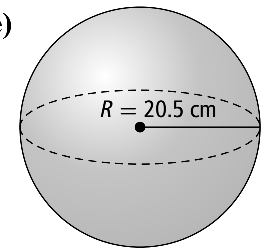
c)



d)



e)



10. Draw and label a right triangle to illustrate each ratio. Then, calculate the measure of each angle to the nearest degree.

a) $\tan \beta = \frac{9}{2}$ **b)** $\tan \theta = \frac{1}{6}$

c) $\sin A = \frac{8}{9}$ **d)** $\cos X = \frac{2}{7}$

11. Use the exponent laws to simplify each expression. Leave your answers with positive exponents.

a) $(x^4)\left(x^{-\frac{3}{4}}\right)$ **b)** $(16^{-0.25})^5$

c) $\dfrac{(k^{-3})^{\frac{1}{4}}}{\left(k^{\frac{2}{3}}\right)^{-5}}$ **d)** $\left(8p^3\right)^{-\frac{1}{3}}\left(p^{-\frac{4}{3}}\right)$

12. Factor the following polynomials.

a) $3x(x - 5) + 9(x - 5)$

b) $4yz - 2y + 10z - 5$

c) $14a^2b^2 - 21ab$

d) $12x^3y - 9x^2y + 3xy$

13. Factor completely.

a) $x^2 - x - 56$

b) $-4x^2 + 16x - 48$

c) $6m^2 + 7mn + 2n^2$

d) $35s^2 + 59s + 18$

14. Factor completely.

a) $c^2 - 121$

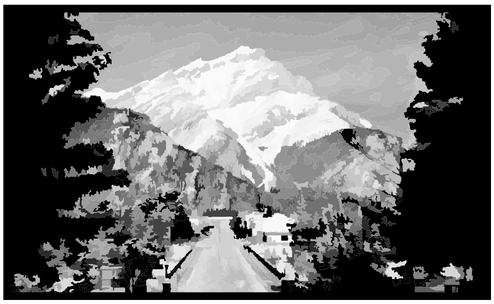
b) $1 - 81y^4$

c) $27h^2 - 147$

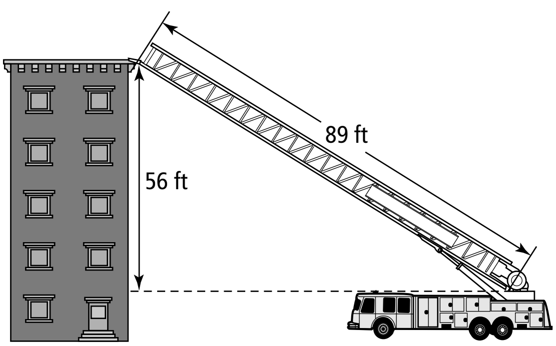
d) $y^2 + 4y + 4$

e) $121 - 110x + 25x^2$

15. An advertising company wants to put this picture on the front of a Banff brochure. The picture needs to be enlarged to 4 in. by 6 in. Is this possible? Explain, using measurements and ratios.



16. A ladder truck is needed for the fire department to reach the top of a building. If the ladder is 89 ft long and the top of the building is 56 ft higher than the bottom of the ladder, determine the angle of the ladder. Give your answer to the nearest degree.



17. Write each power as an equivalent radical.

a) $c^{\frac{4}{3}}$

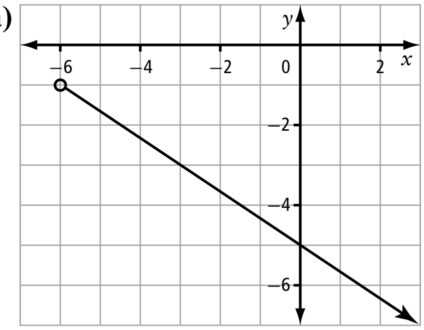
b) $(18t^3)^{\frac{2}{5}}$

c) $\left(\frac{m^2}{12}\right)^{0.25}$

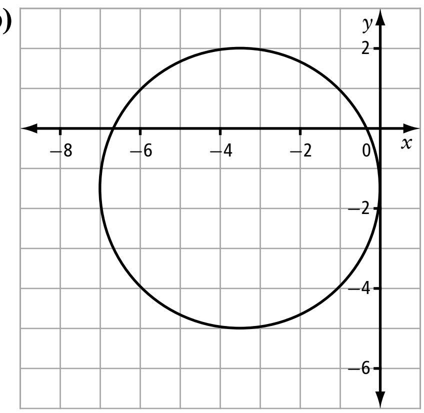
18. A balloonist is flying at an altitude of 1456 m. She measures her angle of depression to the landing spot to be 19.5°. If she begins her descent at that moment in a straight diagonal line, how far away, to the nearest hundredth of a kilometre, is she from her landing site?

19. Give the domain and range of each relation. Use words, interval notation, and set notation.

a)



b)



20. Solve for all unknown sides and angles for each. Round your answers to the nearest hundredth of a unit.

a) For ΔDEF, AB = 12 km, ∠B = 90°, and ∠A = 52°.

b) For ΔDEF, ∠E = 90°, DF = 18 m, EF = 11 m.

21. Use the slope formula to determine the slope of the line passing through each pair of points.

a) (−2, 6) and (12, 18)

b) (3, −5.6) and (−0.7, 1.8)

Chapter 6 Extend It Further

1. Point (4, −1) is one endpoint of the diameter of a circle with centre C(−3, 4). What are the coordinates of the other point?

A (−11, 8) **B** (−11, 7)
C (−10, 9) **D** (−9, 9)

2. Points (5, 0), (3, 8), and (−1, 4) are three vertices of a parallelogram. The fourth vertex is in quadrant II. What are its coordinates?

A (−1, 11) **B** (−3, 7)
C (−3, 12) **D** (−4, 12)

3. A linear function, f, includes the ordered pairs (−3, 2) and (3, 6). What is $f(-1)$?

A $3\frac{1}{3}$ **B** $2\frac{2}{3}$
C $1\frac{1}{3}$ **D** $1\frac{1}{6}$

4. What is $g(3)$ if g is a linear function, where $g(1) = 2$ and $g(-1) = 8$.

A 6 **B** 4
C 0 **D** −4

5. If $f(x)$ means the reciprocal of x, what is the value of x that satisfies $f(x) = f(2) + f(3) + f(6)$?

A 6 **B** 3
C 2 **D** 1

☆**6.** Two sides of a triangle are 12 cm and 19 cm. The length of the third side is a natural number, x. The perimeter of the triangle is represented by $T(x)$. Determine the domain and range.

7. Show that (−3, 0), (0, 6), and (0, −1.5) are the vertices of a right triangle.

8. If $g(x) = 10x + 2009$, determine the domain for which $g(x) \geq 0$.

9. A function is defined to be $f(ab) = f(a) + f(b)$. If $f(3) = x$ and $f(4) = y$, what is the value of $f(144)$ in terms of x and y?

10. What is the ordered pair of real numbers, (x, y), that satisfy $y = 6x - 12$, but do not satisfy $\frac{y}{x-2} = 6$?

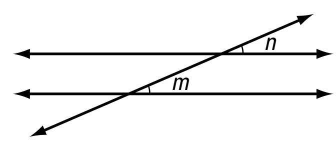
☆**11.** Draw the line $4x - 3y + 12 = 0$ using the domain $\{x \mid -4 \leq x \leq 2\}$. Calculate the area of the shape bounded by the line and the two axes.

12. Determine the slope of the straight line joining the two given points.

a) (a, b) and (ma, mb), where $m \neq 1$

b) (a, b) and (b, a)

13. Consider the lines in the figure.



a) Measure the angle m to the nearest degree, using a protractor. Create triangles and measure sides. Express the slope of the line as rise over run.

b) Explain why angle n has the same properties as angle m.

14. A circle has a radius of 5 units. $P(x, y)$ is a point on the circle.

a) Use a compass to construct the circle with (0, 0) as the centre.

b) If $x = 3$, find the value(s) of y.

c) If $y = -4$, find the value(s) of x.

d) Each point (x, y) on this circle is represented by the equation $x^2 + y^2 = 25$. Is this a relation or a function?

Chapter 6 Study Check

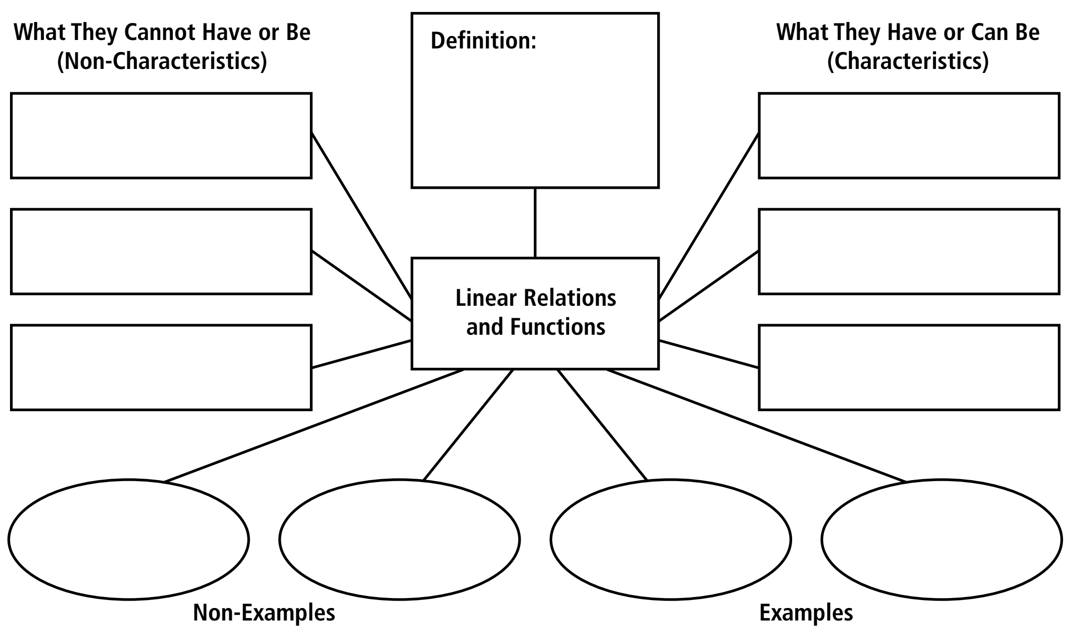
Use the chart below to help you assess the skills and processes you have developed during Chapter 6. The references in italics direct you to pages in *Mathematics 10 Exercise and Homework Book* where you could review the skill. How can you show that you have gained each skill? What can you do to improve?

Big Idea	Skills	This Shows I Know	This Is How I Can Improve
Make and explain graphs of data *pages 114–121, 134*	✓ Graph a set of data *pages 114–116, 120–121, 134*		
	✓ Interpret a graph of data *pages 114–121, 134*		
Identify, describe, and represent linear relations in a variety of ways, and determine and apply the characteristics of linear relations *pages 118–126, 130–136, 138–139*	✓ Represent relations in a variety of ways *pages 118–121*		
	✓ Justify whether or not a given relation is linear *pages 118–121, 134, 136*		
	✓ Differentiate between discrete and continuous data *pages 118–121, 134, 136*		
	✓ Identify the independent and dependent variables in a relation *pages 118–121, 124, 129, 134, 136*		
	✓ Determine and express the domain and range of a relation *pages 122–126, 134, 138–139*		
	✓ Determine and classify slopes of line segments *pages 131–132, 135, 138*		
	✓ Calculate and apply slope to solve problems and graph linear relations *pages 132–133, 135, 139*		

Big Idea	Skills	This Shows I Know	This Is How I Can Improve
Identify and explain the differences between relations and functions, and apply function notation to linear functions *pages 118–129, 135*	✓ Sort relations into functions and non-functions *pages 126–129, 135–136, 139*		
	✓ Apply function notation to determine range and/or domain values for a given function *pages 118–126, 135, 139*		
	✓ Apply function notation to graph linear functions *pages 127–129, 135–136, 139*		

Organizing the Ideas

In the concept definition map below, indicate the key characteristics of linear relations and functions. Show how to tell a linear relation from a non-linear relation. Show how function notation is used.



Study Guide

Review the types of problems you handled in Chapter 6. What do you need to remember to help you do similar problems? Use a file card to write a series of brief notes outlining what you need to remember about:

- graphs
- linear relations and functions
- function notation
- applications of slope