Chapter 7 Linear Equations and Graphs

7.1 Slope-Intercept Form

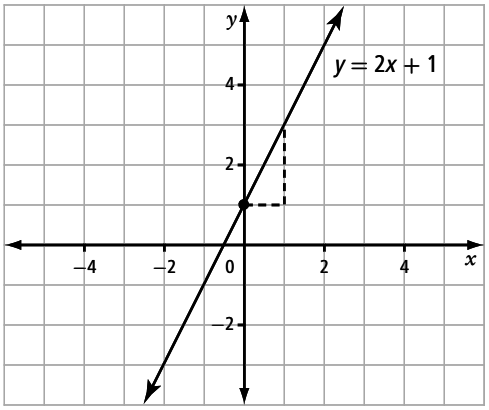
KEY IDEAS

- The slope-intercept form of a linear equation is $y = mx + b$, where m represents the slope and b represents the y-intercept.

$y = 2x + 1$

slope $= 2$ $\qquad$ y-intercept $= 1$

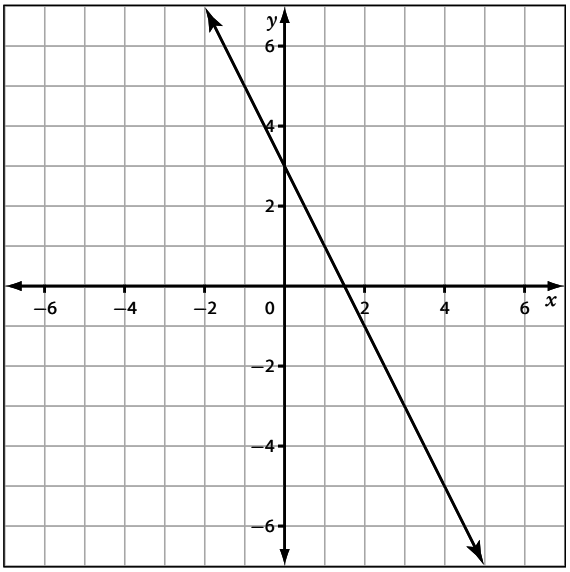
$\frac{\text{rise}}{\text{run}} = \frac{2}{1}$ $\qquad$ The graph passes through (0, 1).



Example

Consider the given graph.

a) What is the slope of the line?
b) What is the y-intercept?
c) What is the equation of the line in slope-intercept form?



Solution

a) Using the points (0, 3) and (1, 1), the slope is

$m = \frac{\text{rise}}{\text{run}}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{1 - 3}{1 - 0}$

$m = \frac{-2}{1}$

$m = -2$

The slope is -2.

b) The line crosses the y-axis at the point (0, 3). Therefore, $b = 3$.

c) Substitute the values $m = -2$ and $b = 3$ into the slope-intercept form of an equation.

$y = mx + b$

$y = -2x + 3$

A Practise

1. What are the slope and y-intercept of each line?

a) $y = \frac{1}{2}x - 2$

b) $y = -4x + 3$

c) $y = x$

d) $y = 0.75x + 3.5$

☆**2.** Convert each of the following into slope-intercept form. Then, state the slope and y-intercept.

a) $x + y = 7$

b) $y - 4x = 12$

c) $5x + 2y = 10$

d) $x - 3y - 12 = 0$

3. Given the slope and y-intercept, write an equation of the line in slope-intercept form.

a) $m = 4; b = -1$

b) $m = -\frac{1}{2}; b = 7$

c) $m = \frac{2}{3}; b = -2$

d) $m = 0.5; b = 0$

e) $m = -5; b = 1$

f) $m = 1; b = \frac{4}{5}$

4. Draw the graph of each line using the slope and y-intercept. Use graphing technology to check your graphs.

a) $y = 2x + 5$

b) $y = 3x - 1$

c) $y = x + 6$

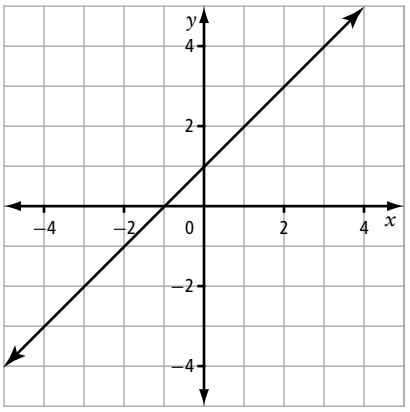
d) $y = -x$

e) $x - 3y - 9 = 0$

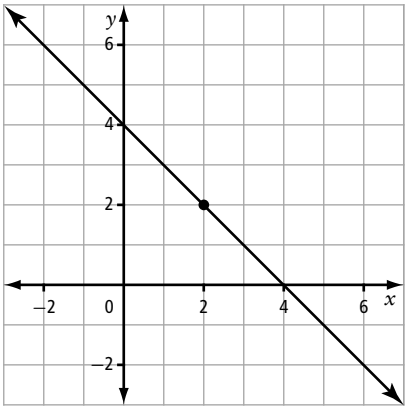
f) $y + 4 = 5x$

5. What are the slope and y-intercept of each line? Write the equation of each line in slope-intercept form.

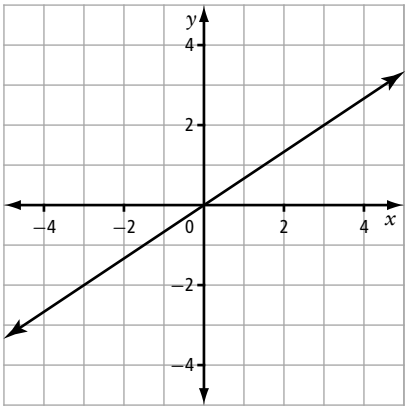
a)



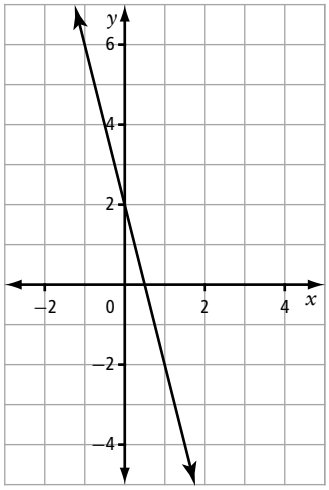
b)



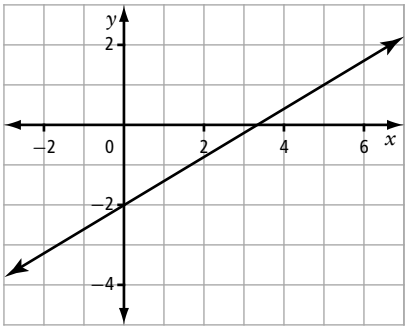
c)



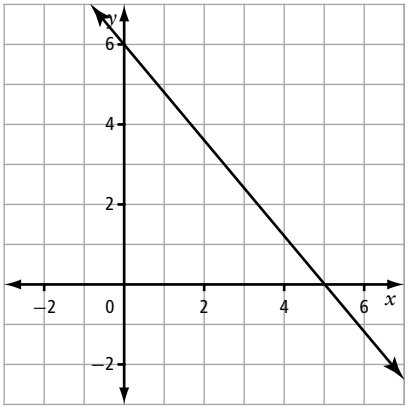
d)



e)



f)



☆**6.** An equation of a line is $y = \frac{1}{2}x + b$. What is the value of b if the line passes through the given point?

a) (12, 8) **b)** $\left(-3, \frac{1}{2}\right)$

7. An equation of a line is $y = mx - 8$. What is the value of m if the line passes through the given point?

a) (4, 0) **b)** (−3, 4)

B Apply

8. Mr. Wong's class is holding a raffle to raise money for earthquake relief efforts. The class buys a pair of Edmonton Oilers hockey tickets for $250 as the prize. The raffle tickets are going to be sold for $2 each.

a) Write a linear equation to represent the money raised based on the ticket sales, x, and the cost of the prize.

b) What is the slope of the line? What does it represent?

c) What is the y-intercept? What does it represent?

d) How many tickets does the class need to sell if they want to raise $300.00?

9. The Rabbit Hill Snowboard and Ski Resort is sponsoring a freestyle snowboard competition. Each competitor pays an entry fee of $75. The winner gets $600.

a) Write a linear equation, in slope-intercept form, to show the relationship between the number of contestants, x, and the money generated from the competition, y.

b) How much money will organizers make if 5 contestants enter? 15? 25?

c) How many competitors need to enter for the organizers to break even?

10. Ernesto needs to rent a paint sprayer. His friend Daniela rented one and paid $15/h plus a fixed charge. Daniela could not remember the fixed charge, but remembered that she rented the sprayer for 4 hours and paid $85.

a) What is the fixed charge?

b) Write an equation in slope-intercept form to represent the cost, y, for x hours to rent a paint sprayer.

c) What is the y-intercept? What does it represent?

d) Describe the graph.

☆**11.** The following table relates the number of litres left in a car's fuel tank to the distance travelled.

Distance (km)	Fuel (L)
0	60
50	56
100	52
150	48
200	44

a) Draw a graph of the relation.

b) What is the slope of the line? What is the y-intercept?

c) Write the equation in slope-intercept form.

d) What does the y-intercept represent?

e) After how many kilometres will the tank be empty?

12. The cost of printing programs for the school play, y, is a fixed charge of $200 for the artwork, plus $0.25 for each program.

a) Build a table of values to represent the cost of printing 50, 100, 150, 200, and 250 programs.

b) Draw a graph of the relation.

c) What is the slope? What does it represent?

d) What is the y-intercept? What does it represent?

e) Write the equation in slope-intercept form.

f) How many programs can be printed if the school wants to spend $350 on programs?

C Extend

☆**13.** The following table shows the linear relationship between temperatures in degrees Celsius and temperatures in degrees Fahrenheit.

°C	°F
−50	−58
−10	14
5	41
20	68

a) Sketch the graph of the line through the points.

b) What is the slope?

c) What is the y-intercept? What does it represent?

d) Write the equation in slope-intercept form, where x represents degrees Celsius and y represents degrees Fahrenheit.

e) Write the inverse of your equation in part d), where x is replaced with y, and y is replaced with x. Write this equation in slope-intercept form.

f) Calculate the conversions using your graph or equation.

−40 °C = _____ °F

100 °F = _____ °C

0 °C = _____ °F

14. Maureen is hosting a party and needs to choose a hall. Clarksdale Hall charges $200 for hall rental and $12/person for food and drinks. Lane Hall charges $320 for hall rental and $10.50/person for food and drinks. Use linear equations to determine the number of people that would make the costs for both halls the same.

D Create Connections

15. Given the linear equation $4x - y + 12 = 0$,

a) graph the line by building a table of values

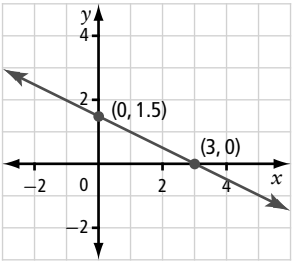
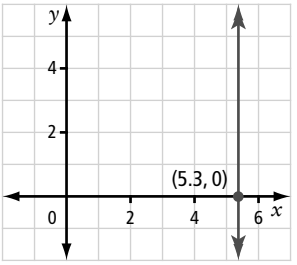
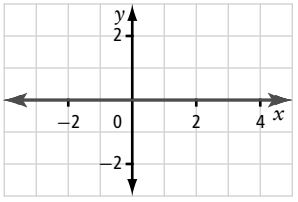
b) graph the line by using the slope and y-intercept of the line

c) Which method do you prefer? Explain.

7.2 General Form

KEY IDEAS

- The general form of a linear equation is $Ax + By + C = 0$, where A, B, and C are real numbers, and A and B are not both zero. By convention, A is a whole number.
- To graph an equation in general form, determine the intercepts, then draw a line joining the intercepts; or convert to slope-intercept form.
- To determine the x-intercept, substitute $y = 0$ and solve. To determine the y-intercept, substitute $x = 0$ and solve.
- A sketch of a linear relation may have one, two, or an infinite number of intercepts. A line that represents an axis has an infinite number of intercepts with that axis. A horizontal or vertical line that does not represent an axis has only one intercept.

Equation	x-Intercept(s)	y-Intercept(s)	Graph
$x + 2y - 3 = 0$	$x + 2y - 3 = 0$ $x + 2(0) - 3 = 0$ $x = 3$	$x + 2y - 3 = 0$ $(0) + 2y - 3 = 0$ $2y = 3$ $y = 1.5$	
$x = 5.3$	$x = 5.3$	no y-intercept	
$3y = 0$	infinite number of x-intercepts	$3y = 0$ $y = 0$	

Example

Consider the linear equation $y = -\frac{3}{2}x - 3$, which is in slope-intercept form.

a) Write the equation in general form, $Ax + By + C = 0$.
b) How many intercepts will this graph have? Explain how you know.
c) Sketch the graph using the x-intercept and y-intercept.
d) Explain how the equation $3x + 6 = 0$ differs from the linear equation in part a). Predict how this difference will be reflected in the graph of the line. Sketch the graph to check your prediction.
e) Explain how the equation $2y + 6 = 0$ differs from the linear equation in part a). Predict how this difference will be reflected in the graph of the line. Sketch the graph to check your prediction.

Solution

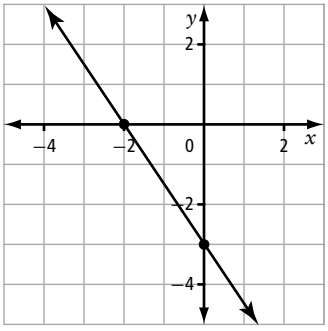
a)
$$y = -\frac{3}{2}x - 3$$
$$(2)y = (2)\left(-\frac{3}{2}x - 3\right) \qquad \text{Multiply by 2 to get rid of the fraction.}$$
$$2y = -3x - 6$$
$$2y + 3x + 6 = -3x + 3x - 6 + 6 \qquad \text{Move all terms to one side of the equal sign.}$$
$$3x + 2y + 6 = 0$$

b) This equation has both an Ax-term and a By-term, which means that it can be solved in terms of both x and y. Hence, the equation has two intercepts.

c) Determine the x-intercept and y-intercept and draw a line passing through the two.

To determine the x-intercept, replace y with 0, and solve for x.
$$3x + 2y + 6 = 0$$
$$3x + 2(0) + 6 = 0$$
$$3x + 0 + 6 = 0$$
$$3x + 6 = 0$$
$$3x + 6 - 6 = 0 - 6$$
$$3x = -6$$
$$\frac{3x}{3} = \frac{-6}{3}$$
$$x = -2$$
The x-intercept is $(-2, 0)$.

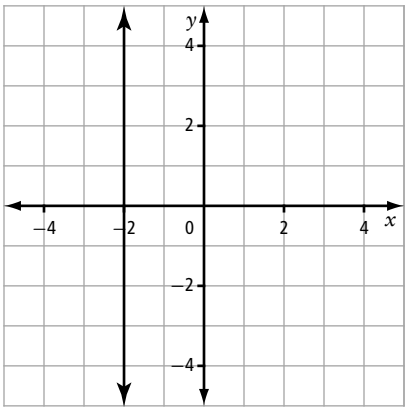
To determine the y-intercept, replace x with 0 and solve for y.
$$3x + 2y + 6 = 0$$
$$3(0) + 2y + 6 = 0$$
$$0 + 2y + 6 = 0$$
$$2y + 6 = 0$$
$$2y + 6 - 6 = 0 - 6$$
$$2y = -6$$
$$\frac{2y}{2} = \frac{-6}{2}$$
$$y = -3$$
The y-intercept is $(0, -3)$.



d) The equation $3x + 6 = 0$ is in general form, but has no By-term. Since the equation cannot be solved in terms of y, there is no y-intercept. This graph must be a vertical line passing through the x-intercept.

$$3x + 6 = 0$$
$$3x + 6 - 6 = 0 - 6$$
$$3x = -6$$
$$\frac{3x}{3} = \frac{-6}{3}$$
$$x = -2$$

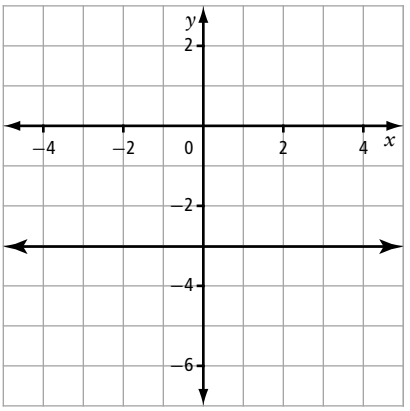
The x-intercept is $(-2, 0)$.



e) The equation $2y + 6 = 0$ is in general form, but has no Ax-term. Since the equation cannot be solved in terms of x, there is no x-intercept. This graph must be a horizontal line passing through the y-intercept.

$$2y + 6 = 0$$
$$3y + 6 - 6 = 0 - 6$$
$$2y = -6$$
$$\frac{2y}{2} = \frac{-6}{2}$$
$$y = -3$$

The y-intercept is $(0, -3)$.



A Practise

1. Write each equation in the general form, $Ax + By + C = 0$.

a) $y = \frac{1}{3}x + 5$ **b)** $y = \frac{-2}{7}x$

c) $y = \frac{1}{8}$ **d)** $y = -0.2x + 1.2$

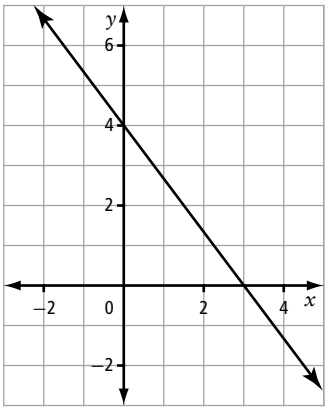
2. Determine the intercepts of each line. Graph each line.

a) $2x - y - 8 = 0$ **b)** $9x - 4y = 0$

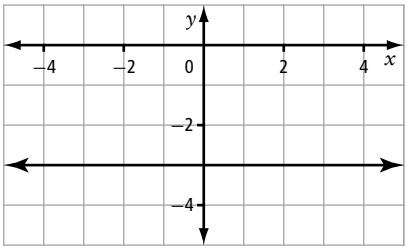
c) $5x - 20 = 0$ **d)** $8y + 4 = 0$

☆**3.** For each line, state the domain and range, slope, and any intercepts. Then, write the equation in general form.

a)



b)



B Apply

4. Write an equation in general form for each.

a) a line that does not have an x-intercept

b) a line that has an infinite number of intercepts

c) a line that does not have a y-intercept

d) a line for which both the x-intercept and y-intercept are 0

e) a line for which the x-intercept and y-intercept are the same, but are not 0

5. Determine the missing value, A, B, or C, in the following linear equations.

a) $6x - By + 1 = 0$, for the line that passes through the point $(-1, 5)$

b) $Ax + y - 10 = 0$, for the line that passes through the point $(3, -2)$

c) $9x - 5y + C = 0$, for the line that passes through the point $(0, 0)$

☆6. Josef is training for a race. His training consists of swimming and mountain biking. The table shows the number of calories burned per minute for a person of Josef's body mass.

Activity	Calories Per Minute
Swimming	14
Biking	12

a) Write a linear equation to show the number of minutes Josef would need to swim, x, and the number of minutes he would need to bike, y, to burn 4200 calories.

b) What are the intercepts of the line? What do they represent?

c) What are the graph's domain and range?

d) Suppose Josef bikes for 2 hours. How long would he need to swim to burn 4200 calories?

7. Jaden plants trees for the British Columbia government during the summer. The table shows how many trees she can plant per minute under different conditions.

Conditions	Number of Trees Planted Per Minute
Ideal	5
Rocky muskeg	2

a) If Jaden planted 2250 trees in one day, write an equation, in general form, showing the number of minutes she planted trees under ideal conditions, x, and the number of minutes she planted trees under rocky muskeg conditions, y.

b) For the linear equation, what are the slope, intercepts, and domain and range?

c) If Jaden planted trees under rocky muskeg conditions for 125 minutes, how long did she spend planting trees under ideal conditions?

C Extend

8. Graph the following on the same coordinate plane.
Line 1: $2x + 10 = 0$
Line 2: $4x - 5y + 30 = 0$
Line 3: $3x - 21 = 0$
Line 4: $-8x + 10y + 30 = 0$
Calculate the area of the region formed by their intersection.

D Making Connections

9. Write the coordinates of two points on a line that satisfies each given condition. Then, write the equation of each line in general form.

a) a line rises from left to right

b) a line is horizontal

c) a line falls from left to right

d) a line is vertical

7.3 Slope-Point Form

KEY IDEAS

- For a non-vertical line through the point (x_1, y_1) with slope m, the equation of the line can be written in slope-point form as $y - y_1 = m(x - x_1)$.

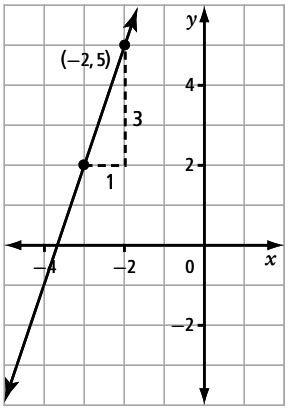
  A line through (−2, 5) has a slope of 3.

  The slope-point form of the equation of this line is

  $y - y_1 = m(x - x_1)$

  $y - 5 = 3[x - (-2)]$

  $y - 5 = 3(x + 2)$



- An equation written in slope-point form can be converted to either slope-intercept form or general form.
- Any point on a line can be used when determining the equation of the line in slope-point form.

Example

Consider a line passing through the points (−4, 5) and (6, 0).

a) Write the equation of this line in slope-point form.
b) Rewrite the equation in part a) in slope-intercept form.
c) Rewrite the equation in part a) in general form.
d) Sketch the graph.

Solution

a) Determine the slope.

$$m = \frac{y - y_1}{x - x_1}$$

$$m = \frac{0 - 5}{6 - (-4)}$$

$$m = \frac{-5}{10}$$

$$m = \frac{-1}{2}$$

Use either point (−4, 5) or (6, 0) to replace the point (x_1, y_1). Replace m with $\frac{-1}{2}$.

Using point (−4, 5):

$y - y_1 = m(x - x_1)$

$y - 5 = \frac{-1}{2}(x - (-4))$

$y - 5 = \frac{-1}{2}(x + 4)$

Using point (6, 0):

$y - y_1 = m(x - x_1)$

$y - 0 = \frac{-1}{2}(x - 6)$

$y = \frac{-1}{2}(x - 6)$

The slope-point form of the equation of the line passing through the points (−4, 5) and (6, 0) is $y - 5 = \frac{-1}{2}(x + 4)$ or $y = \frac{-1}{2}(x - 6)$.

b) The slope-point form can be changed to the slope-intercept form by solving for y.

For point $(-4, 5)$:

$$y - 5 = \frac{-1}{2}(x + 4)$$
$$y - 5 = \frac{-1}{2}x - 2$$
$$y - 5 + 5 = \frac{-1}{2}x - 2 + 5$$
$$y = \frac{-1}{2}x + 3$$

For point $(6, 0)$:

$$y = \frac{-1}{2}(x - 6)$$
$$y = \frac{-1}{2}x + 3$$

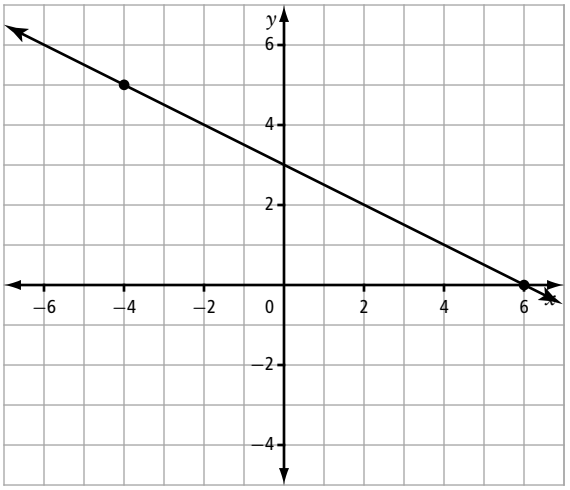
The slope-intercept form of the equation passing through the points $(-4, 5)$ and $(6, 0)$ is $y = \frac{-1}{2}x + 3$. The result is the same, regardless of which of the two points is used.

c) The slope-point form can be changed to general form.

$$y - 5 = \frac{-1}{2}(x + 4)$$
$$y - 5 = \frac{-1}{2}x - 2$$
$$2(y - 5) = 2\left(\frac{-1}{2}x - 2\right)$$
$$2y - 10 = -x - 4$$
$$2y - 10 + 4 = -x - 4 + 4$$
$$2y - 6 = -x$$
$$2y - 6 + x = -x + x$$
$$x + 2y - 6 = 0$$

The general form for the equation of the line passing through the points $(-4, 5)$ and $(6, 0)$ is $x + 2y - 6 = 0$.

d) Plot the points $(-4, 5)$ and $(6, 0)$ and draw a line passing through them. Or, plot the y-intercept, $(0, 3)$, and draw a line passing through it with a slope of $\frac{-1}{2}$.



A Practise

1. Identify the slope and a point on each line.

a) $y + 7 = 4(x - 3)$ **b)** $y - 5 = \frac{1}{3}(x + 5)$

c) $y = -2(x - 6)$ **d)** $y + 1 = x - 3$

2. Rewrite the following in slope-intercept form, $y = mx + b$, and general form, $Ax + By + C = 0$.

a) $y - 3 = \frac{2}{3}(x + 1)$ **b)** $y + 4 = -2(x - 1)$

c) $y = \frac{3}{4}(x - 4)$ **d)** $y - 1 = 3(x + 6)$

3. From the information given, write the equation of the line in slope-point form, slope-intercept form, and general form.

☆**a)** $(-1, -5)$; $m = \frac{4}{3}$

b) $\left(\frac{-1}{2}, -3\right)$; $m = 1$

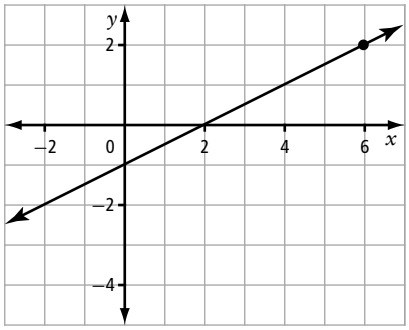
c) $(1, 4)$; $m = -1.5$

☆**d)** $(-5, -8)$ and $(-7, -9)$

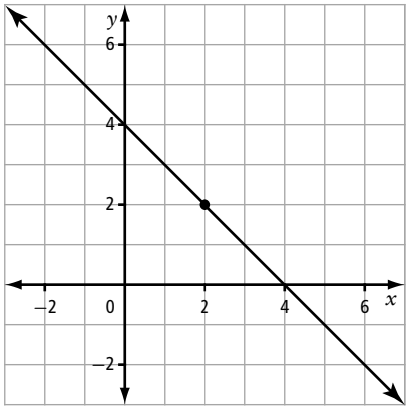
e) $(-1, -2)$ and $(3, 0)$

4. Write an equation in slope-point form for each graph.

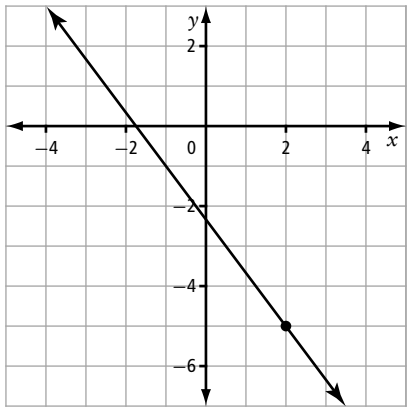
a)



b)



c)



B Apply

5. Write the equation of each line in slope-point form. Then, convert each equation to general form.

a) slope of 0 and passing through $(-3, 1)$

b) same slope as $y = 2x + 2$ and passing through $(-1, 8)$

☆**c)** same slope as $5x + 2y - 10 = 0$ and passing through $(-1, 4)$

d) same y-intercept as $3x - y - 1 = 0$ and passing through $(2, -6)$

e) x-intercept of -5 and y-intercept of 3

f) same slope as $3x + 2y + 6 = 0$, with an x-intercept of 0

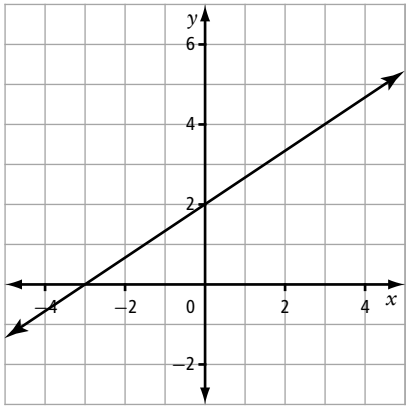
☆**6.** Show that the point $(-2, -6)$ lies on the line that has an x-intercept of 10 and a y-intercept of -5.

7. A rectangle has vertices A$(-3, 4)$, B$(-3, -1)$, C$(4, -1)$, and D$(4, 4)$. Plot the points on a grid and draw the rectangle. Then, draw the two diagonals and write an equation in general form for each.

8. Use graphing technology to identify the x-intercept and y-intercept of the line $2x - 3y + 12 = 0$. Use algebra to verify your answer.

9. Consider the linear equation $8x + ky - 6 = 0$. If the line passes through the point $(1, -2)$, what is the value of k?

10. Compare the following five lines to the line graphed below.
Line 1: $x + y + 3 = 0$
Line 2: $2x + 3y + 6 = 0$
Line 3: $2x - 3y + 18 = 0$
Line 4: $4x - 6y - 9 = 0$
Line 5: $2x - y + 2 = 0$



a) Which line(s) have the same slope as the graphed line?

b) Which line(s) have the same y-intercept as the graphed line?

c) Which line(s) have the same x-intercept as the graphed line?

11. The annual cost of operating a snowmobile depends on the distance driven plus a fixed cost, which includes maintenance, depreciation, and trail fees. The cost is $4000 for 1200 miles driven and $5625 for 2500 miles driven.

a) Sketch a line showing the relationship between distance and cost.

b) Calculate the slope of the line. What does the slope represent?

c) Determine the y-intercept. What does it represent?

d) Write an equation in general form for the cost of operating a snowmobile.

e) Use your equation to determine the cost of operating the snowmobile for 900 miles.

C Extend

12. A candle is lit at 1400 hours. At 1600 hours, it is 16 cm tall. At 2030 hours, it is 4.75 cm tall.

a) Write a linear equation, in general form, with the points representing (hours, height).

b) Use the equation to determine the rate at which the candle burns per hour and its height at 1400 hours.

c) What does the slope of the line represent?

d) What does the ***y***-intercept represent?

13. The following lines pass through the sides of a triangle:

$2x + 3y - 18 = 0$
$5x + y + 7 = 0$
$3x - 2y - 14 = 0$

Determine the vertices of the triangle.

14. Consider the linear equation $\frac{x}{-8} + \frac{y}{6} = 1$.

a) Write the equation in general form.

b) What are the x- and y-intercepts? How do they relate to the original form of the equation?

c) Predict the x- and y-intercepts of the equation $\frac{x}{3} - \frac{y}{5} = 1$. Verify your answer.

D Create Connections

15. In 2001, the pollution in a local lake was measured at 4.5 parts per million. In 2010, the level had decreased to 1.4 parts per million.

a) Write an equation in slope-point form, showing the relationship between time, x, and the pollution rate, y.

b) If the decrease in pollution continues at the same rate, in what year should the pollution level be 0 parts per million?

7.4 Parallel and Perpendicular Lines

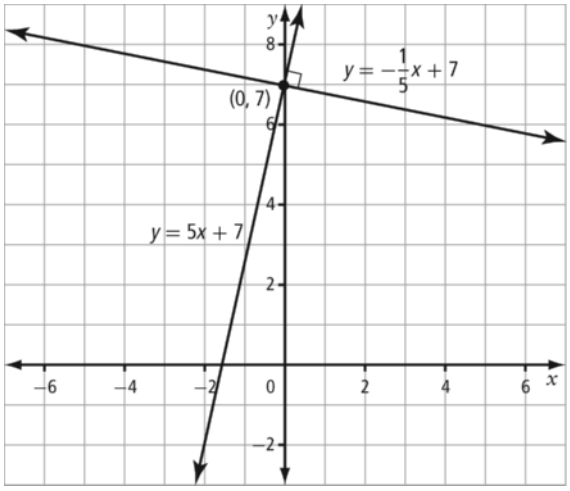
KEY IDEAS

- Parallel lines have the same slope and different intercepts. Vertical lines are parallel to each other, as are horizontal lines, if they have different intercepts.
- Perpendicular lines have slopes that are negative reciprocals of each other. A vertical line with an undefined slope and a horizontal line with a slope of zero are also perpendicular.
- The properties of parallel and perpendicular lines can give information about the slopes. Knowing the slopes can help you develop an equation.

A line perpendicular to $y = 5x + 7$ has the same y-intercept.

The line $y = 5x + 7$ has a slope of 5 and a y-intercept of 7.

The perpendicular line has a slope of $-\frac{1}{5}$ and a y-intercept of 7. So, the equation of the perpendicular line is $y = -\frac{1}{5}x + 7$.



Example

a) Write an equation, in slope-intercept form, for the line that has a y-intercept of -4 and is parallel to the line $5x + 3y - 6 = 0$. Graph the lines using graphing technology.

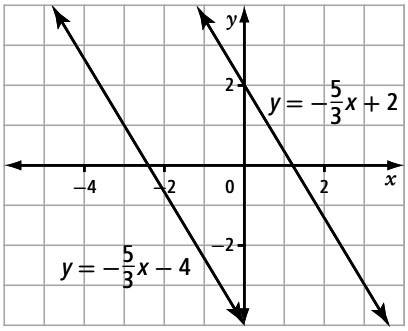
b) Write the equation, in slope-intercept form, for the line that has a y-intercept of -4 and is perpendicular to $5x + 3y - 6 = 0$. Graph the lines using graphing technology.

c) Write the equations for the lines you created in parts a) and b) in general form.

Solution

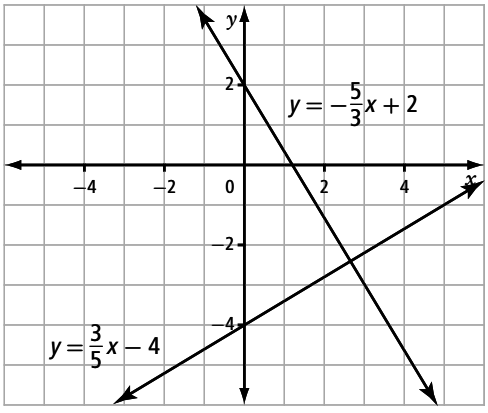
a) Since the new line is parallel to the line $5x + 3y - 6 = 0$, the two lines have the same slope.
Determine the slope of $5x + 3y - 6 = 0$ by rewriting it in slope-intercept form, $y = mx + b$.

$$\begin{aligned} 5x + 3y - 6 &= 0 \\ 5x + 3y - 6 + 6 &= 0 + 6 \\ 5x + 3y &= 6 \\ 5x - 5x + 3y &= -5x + 6 \\ 3y &= -5x + 6 \\ \frac{3y}{3} &= \frac{-5x}{3} + \frac{6}{3} \\ y &= \frac{-5x}{3} + 2 \end{aligned}$$



The slope of the first line is $\frac{-5}{3}$.
Since the second line has the same slope and a y-intercept of -4, the equation for the second line is $y = \frac{-5}{3}x - 4$.

b) A perpendicular line has a slope that is the negative reciprocal of the slope of the first line. Since the slope of the first line is $\frac{-5}{3}$, the slope of the perpendicular line is $\frac{3}{5}$. So, the equation of the perpendicular line running through $(0, -4)$ is $y = \frac{3}{5}x - 4$.



c) Rewrite the equations from part a) and b) in general form.

Parallel line: $y = \frac{-5}{3}x - 4$
Multiply each term by 3.

$$\begin{aligned} 3y &= 3\left(\frac{-5}{3}x - 4\right) \\ 3y &= -5x - 12 \end{aligned}$$

Bring all the terms to one side of the equal sign.

$$\begin{aligned} 3y + 5x + 12 &= -5x + 5x - 12 + 12 \\ 5x + 3y + 12 &= 0 \end{aligned}$$

The equation in general form is $5x + 3y + 12 = 0$.

Perpendicular line: $y = \frac{3}{5}x - 4$.
Multiply each term by 5.

$$\begin{aligned} 5y &= 5\left(\frac{3}{5}x - 4\right) \\ 5y &= 3x - 20 \end{aligned}$$

Bring all the terms to one side of the equal sign.

$$\begin{aligned} 5y - 3x + 20 &= 3x - 3x - 20 + 20 \\ -3x + 5y + 20 &= 0 \end{aligned}$$

The equation in general form is $3x - 5y - 20 = 0$.

A Practise

1. Given the slopes of two different lines, determine whether the lines are parallel, perpendicular, or neither.

a) $m_1 = \frac{1}{2}; m_2 = -2$

b) $m_1 = \frac{3}{4}; m_2 = \frac{6}{8}$

c) $m_1 = \frac{-1}{4}; m_2 = 4$

d) $m_1 = -0.5; m_2 = 2$

e) $m_1 = 1; m_2 = -1$

f) $m_1 = \frac{1}{4}; m_2 = 0.25$

2. For each given line, state the slope of a line that is parallel and the slope of a line that is perpendicular.

a) $y = -3x - 4$

b) $y = x$

c) $4x + y - 4 = 0$

d) $8y - 7 = 0$

e) $5x - 2y + 3 = 0$

3. The following are slopes of parallel lines. Determine the value of n.

a) $2, \frac{n}{2}$ **b)** $-3, \frac{6}{n}$

c) $\frac{2}{n}, \frac{4}{5}$ **d)** $\frac{-n}{3}, \frac{-2}{4}$

4. The following are slopes of perpendicular lines. Determine the value of r.

a) $3, \frac{r}{6}$ **b)** $\frac{r}{9}, \frac{-3}{5}$

c) $\frac{9}{2}, \frac{4}{r}$ **d)** $\frac{-1}{2}, \frac{r}{4}$

5. Write the general form of the equation of a line that is parallel to the given line and passes through the given point.

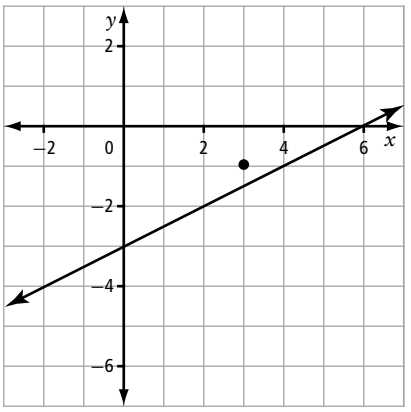
a) $5x + y + 8 = 0; (2, -3)$

b) $y = \frac{-1}{3}x + 2; (-4, 0)$

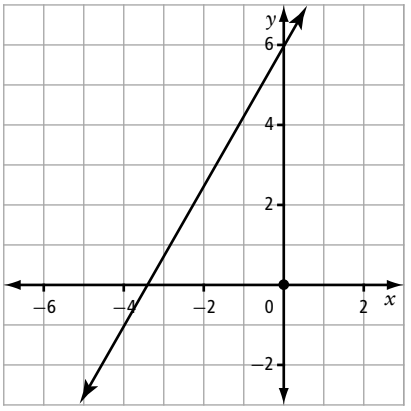
c) $y + 2 = -(x + 1); (-2, 6)$

6. Write the general form for the equation of a line that is perpendicular to the given line and passes through the given point.

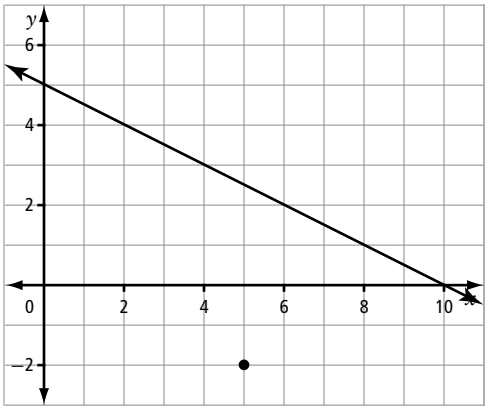
a)



b)



c)



B Apply

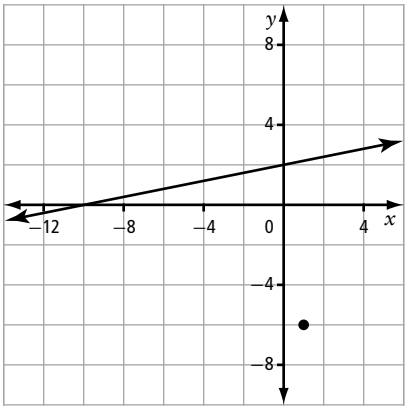
7. Consider lines $x - 2y + 10 = 0$ and $y = \frac{1}{2}x + 5$.

a) What are the slopes of the lines?

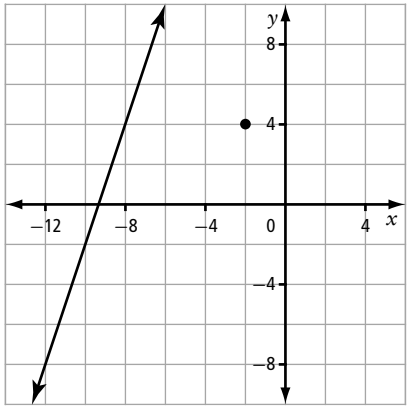
b) Are the lines parallel? Explain your answer.

8. Write the general form for the equation of a line passing through the given point and running parallel to the line.

a)



b)



9. Write the equation of a line that is perpendicular to the y-axis and has a y-intercept of 15.

10. Given three points, determine whether or not the points are vertices of a right triangle. Explain.

a) M(1, 1), N(−2, 5), C(3, −2)

b) D(2, 4), F(−2, 2), G(5, −2)

☆**11.** Consider the linear equation $4x + y - 11 = 0$.

a) Write an equation of a line parallel to the given line. How many such equations can be written?

b) Write an equation of a line perpendicular to the given line. How many such equations can be written?

12. Write the equation of the line perpendicular to $x - 12y + 15 = 0$ and having the same y-intercept as $7x + 4y - 12 = 0$.

13. Sketch the graph of a line parallel to the line $3x + 6y - 7 = 0$ and passing through the origin.

C Extend

14. The line passing through A(−2, 3) and B(0, 4) is perpendicular to the line passing through C(k, 4) and D(1, −6). What is the value of k.

15. What is the value of k if the lines $x - 2y + 6 = 0$ and $kx + 8y + 1 = 0$ are parallel?

16. For what value of k are the lines $3kx - 7y - 10 = 0$ and $2x + y - 7 = 0$ perpendicular?

☆**17.** Determine the value(s) of k for which the lines $kx - 2y - 1 = 0$ and $8x - ky + 3 = 0$ are

a) parallel

b) perpendicular

18. Write the equation of a line that passes through the point of intersection of the lines $5x + y - 11 = 0$ and $2x + 3y - 7 = 0$ and is

a) parallel to the x-axis

b) perpendicular to the line $2x - y + 4 = 0$

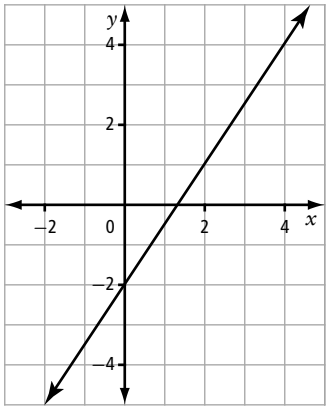
D Making Connections

19. Write the equations of lines that form the sides of a square or a rectangle so that no sides are vertical or horizontal lines.

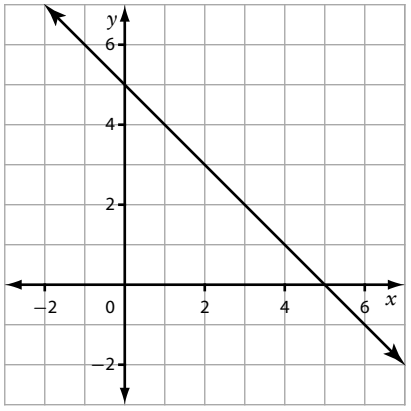
7.1 Slope-Intercept Form

1. For each line, state the slope and y-intercept. Then, write the equation of the line in slope-intercept form.

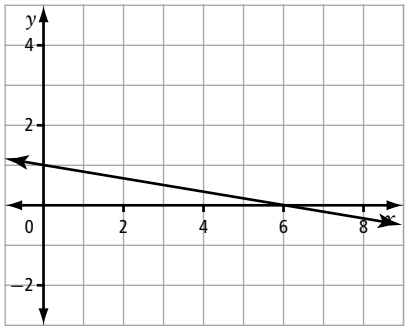
a)



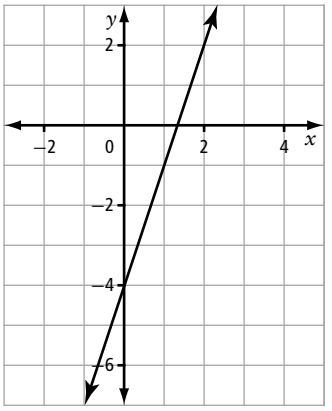
b)



c)



d)



2. The equation of a line is $y = \frac{1}{3}x + b$. What is the value of b if the line passes through the point $(-3, 7)$?

☆3. The equation of a line is $y = mx - 8$. Determine the value of m if the line passes through the point $(-2, 6)$.

4. Mandy's hockey team is trying to raise \$850 to attend a tournament. The team has raised \$500. They plan to raise the remainder of the money by collecting plastic 4-L milk jugs, which they redeem for \$0.25 per jug.

a) Write a linear equation to represent this situation, where y is the money they have raised.

b) Graph the line for the equation in part a).

c) State the slope and y-intercept of the line. What does each of these represent?

d) Determine from the graph how many jugs the team must collect to raise enough money to attend the tournament. Verify your answer algebraically.

7.2 General Form

5. Express each equation in general form.

a) $y = \frac{-1}{2}x - 9$

b) $\frac{x}{-3} + \frac{y}{2} = 1$

c) $y + 2 = -(x + 1)$

6. Determine the intercepts of each line. Then, graph each line.

a) $2x + y - 4 = 0$

b) $5x - 4y + 20 = 0$

7. Bamboo is the fastest growing plant on Earth. On average, it can grow 60 cm/day, depending on the soil and climatic conditions. Under less than ideal conditions, the growth rate averages 10 cm/day.

a) Write an equation, in general form, that represents the number of days of growth for ideal conditions, x, and the number of days of growth under less than ideal conditions, y, needed for a bamboo plant to reach a height of 42 m.

b) What are the intercepts? What do they represent?

c) What are the domain and range?

d) If it takes 55 days for a bamboo tree to reach 42 m under ideal conditions, how many days would it take to reach this height under less than ideal conditions?

7.3 Slope-Point Form

8. A line has a slope of 2 and passes through the point $(-6, -5)$. Write the equation of the line in slope-point form and in general form.

9. A line has an x-intercept of $\frac{-1}{2}$ and a y-intercept of 4. Write the equation of the line in slope-point form and in general form. Verify your answer using graphing technology.

★10. Robyn is skiing with friends at Marmot Basin in Jasper, Alberta. When he is at the top of Marmot Peak, which has an elevation of 8570 ft, he notes that the temperature is 3 °F. When he skis to the base, where the elevation is approximately 5570 ft, the temperature is 12 °F.

a) Assuming that the temperature change is constant as the altitude changes, write a linear equation in slope-point form showing the relationship between altitude, x, and temperature, y.

b) Use the equation to determine the temperature at the base of Eagle Chair, where the elevation is approximately 6500 ft.

7.4 Parallel and Perpendicular Lines

11. State whether each set of lines is parallel or perpendicular.

a) $6y + 2x - 4 = 0$
$y = 3x + 12$

b) $y = \frac{4}{3}x - 17$
$9y - 12x + 8 = 0$

c) $-9y = 3x - 12$
$12y - 4x = 14$

12. Write the equation of a line, in general form, that runs perpendicular to $2x - y + 8 = 0$ with a y-intercept of 6.

★13. A line runs through $(-1, 3)$ and $(2, 1)$ and has an x-intercept of -4. Write the equation of the line perpendicular at the x-intercept.

14. Explain how the following lines are related.

Line 1: $-2x = 3y$

Line 2: $y = \frac{-2}{3}x - 11$

Line 3: $8x + 12y + 7 = 0$

Line 4: $y - 6 = \frac{-4}{6}(x + 5)$

Chapters 1–7 Cumulative Review

1. What are the slope and y-intercept of each line?

a) $y = -7x + 10$ **b)** $9x + 2y - 15 = 0$

2. Calculate the following areas to the indicated SI unit. Express your answers to the nearest tenth of a square unit.

a) the area of a rectangle 18 in. by 11 in., in square centimetres

b) the area of a rectangle 15.5 ft by 40 ft, in square metres

3. Julie is biking to the grocery store. She is going to pick up her friend Fong, who lives about halfway the store. Julie starts off, but when she is about halfway to Fong's house, she realizes that she forgot her money. She returns home, finds her wallet, and then starts off again. Julie stops at Fong's house and they ride the rest of the way together. At the store, the girls meet their friend Zoe. The three decide to go to a juice bar, which is in the opposite direction from Julie's home. Julie and Fong walk their bikes because Zoe does not have hers with her. They buy a juice, then Julie suddenly remembers that she has frozen food in her shopping bag and has to leave. She jumps on her bike and rides straight home. Draw a distance–time graph for Julie's journey.

4. Calculate.

a) $\sqrt{64}$ **b)** $\sqrt[3]{729}$

c) $\sqrt[3]{27\,000}$

5. Factor completely.

a) $x^2 + 13x + 12$ **b)** $6m^2 - 9mn + 3n^2$

c) $15x^2 + 2xy - 1y^2$ **d)** $2s^2t - 2st - 12t$

e) $c^2 - 81$ **f)** $x^4 - 25x + 144$

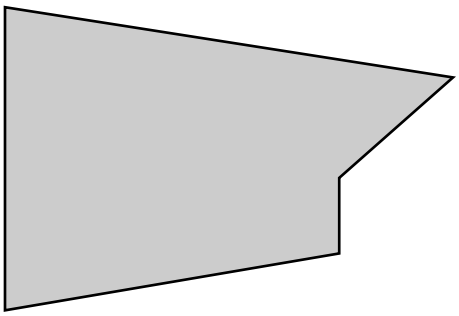
g) $1 - 16y^4$ **h)** $28h^2 - 847f^2$

6. Multiply and then combine like terms.

a) $3(5n + 6)(2n - 3) + (n - 2)^2$

b) $(4s^2 + 2s - 3)(-2s^2 + 5s + 1)$

7. Estimate the perimeter of the figure. Express your answer in appropriate SI units.



8. Fuel efficiency is the amount of gas it takes a vehicle to drive a certain distance. A truck has a fuel efficiency of 0.138 L/km. Consider the relationship between the amount of gasoline consumed, in litres, and the distance the truck travels, in kilometres.

a) Assign variables to represent each quantity in the relation. Identify the dependent variable and the independent variable.

b) Assume that the truck drives for 250 km. Create a set of ordered pairs for the relation.

c) Is the relation continuous or discrete? Explain.

d) Graph the relation.

e) Is the relation linear or non-linear? Explain.

9. What are the x-intercept and y-intercept of each line? Sketch the lines.

a) $x = -7$

b) $12x - 3y - 6 = 0$

10. Niobium is a radioactive element. A certain type of niobium has a half-life of 1 min. The formula for the amount of niobium remaining is $A = 900\left(\frac{1}{2}\right)^n$, where n is the number of minutes. Consider a 900-g sample of this element.

a) How many grams remain after 10 minutes? Answer to the nearest tenth of a milligram. Hint: There are 1000 mg in a gram.

b) How many grams were there 12 min ago? Answer to the nearest tenth of a kilogram.

11. State the missing factor.

a) $4ab^2c^3 + 18a^3c = (?)(2b^2c^2 + 9a^2)$

b) $5s - 20s^2 = (5s)(?)$

c) $9xyz^2 - 6x^2y = (3xy)(?)$

12. Solve for all unknowns. Round your answer to the nearest hundredth of a unit.

a) ΔABC, BC = 8.4 km, $\angle B = 90°$, and $\angle A = 27°$

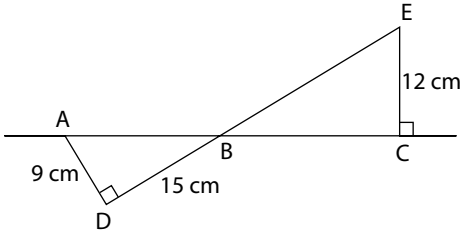
b) ΔDEF, $\angle E = 90°$, DE = 6.1 m, EF = 9.1 m

13. Use slope-point form to write an equation of a line through each point with the given slope. Express each answer in slope-intercept form and in general form.

a) $(-5, 4)$ and $m = -3$

b) $(-3, 2)$ and $m = -\frac{1}{2}$

14. In the figure, ΔADB and ΔBCE are similar triangles. Calculate the length of AC. Round your answer to the nearest tenth of a centimetre.



15. Evaluate each expression. Express the answers with positive exponents.

a) $\left(x^{\frac{1}{5}}\right)\left(x^{-\frac{1}{3}}\right)$ **b)** $(81h^8)^{-\frac{3}{4}}$

c) $\frac{8^{\frac{3}{4}}}{8^{-\frac{2}{5}}}$ **d)** $\left(\frac{x^{-3}}{0.25x^{-6}}\right)^3$

16. In the following paragraph, convert each imperial unit to an equivalent SI unit.

Mount Edziza, in northwestern British Columbia, is Canada's highest volcano at 9144 ft. Its summit comprises an ice-filled bowl, or caldera, which stretches for almost a mile. But some people argue that Edziza is not Canada's highest volcano. They say that Mount Silverthrone, also in British Columbia, is higher at 9400 ft. But since Silverthrone's peaks are always covered by snow and ice, it is difficult to determine if they are, in fact, volcanic rock.

17. Calculate the surface area of each to the nearest hundredth of a centimetre.

a) a cone with a radius of 8.5 cm and slant height of 29.2 cm

b) a square-based pyramid 41-cm-wide that has a slant height of 72 cm

c) a sphere with a diameter of 5.0 cm

d) a cylinder with a diameter of 11 cm and a height of 7 cm

e) a rectangular prism with a height of 2.3 cm, a length of 3.7 cm, and a width of 1.8 cm

18. A plane is at a cruising altitude of 10 000 m. It begins its descent to the airport at an angle of depression of 8°. What is the horizontal distance of the plane to the airport, to the nearest tenth of a mile?

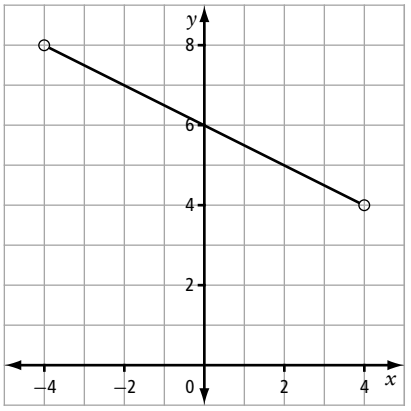
19. Express each power as an equivalent radical.

a) $3^{\frac{2}{3}}$ **b)** $16^{0.5}$

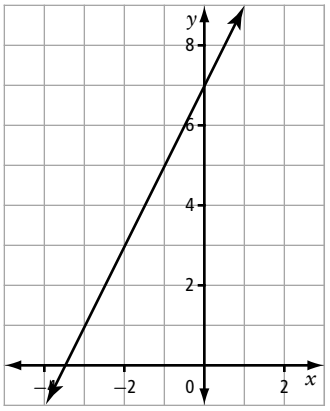
c) $\left(\frac{y^3}{x^5}\right)^{\frac{3}{2}}$

20. Give the domain and range of each relation. Use words, interval notation, and set notation to describe each.

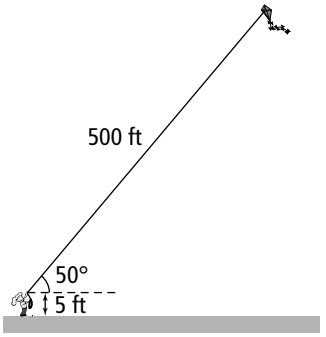
a)



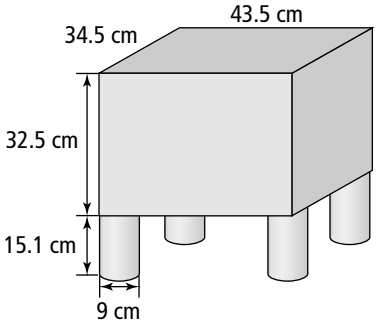
b)



21. Chloe is flying her kite. She lets out 500 ft of string. Chloe's hand holding the kite is 5 ft from the ground. The angle of elevation from Chloe's hand to the kite is 50°. How high is Chloe's kite from the ground, to the nearest tenth of a metre?

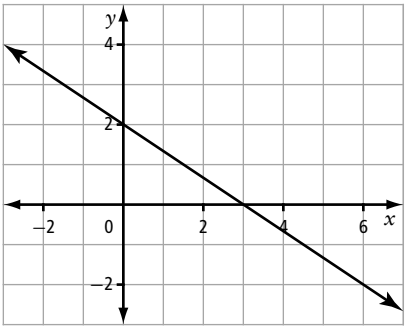


22. A box is sitting on four identical cylinders. Calculate the volume of the composite object, in cubic metres. Round to the nearest hundredth of a cubic metre.

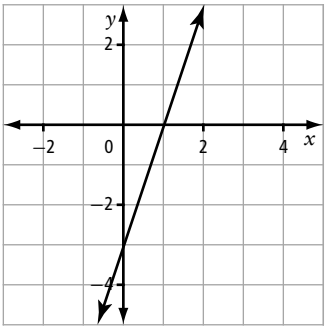


23. Determine the slope of each.

a)



b)



24. A wooden wedge is used to prop open a door. The wedge is 7 cm long, 5 cm high, and 2 cm deep. If the bottom of the door is 2.5 cm from the floor, what percent of the volume of the wedge is under the door?

25. Write an equation of a line through (8, 0) and perpendicular to $2x + 4y + 10 = 0$.

Chapter 7 Extend It Further

☆**1.** Which point on the line $2x + 5y = 10$ is closest to the origin?

A $x = \frac{17}{20}, y = \frac{33}{20}$ **C** $x = \frac{20}{23}, y = \frac{50}{23}$

B $x = \frac{17}{23}, y = \frac{33}{23}$ **D** $x = \frac{20}{29}, y = \frac{50}{29}$

2. For what value of k does the point $(-1, 5)$ lie on the graph of the equation $4x - 2ky = 3$?

A $\frac{-3}{10}$ **C** $\frac{7}{10}$

B $\frac{-7}{10}$ **D** $\frac{3}{10}$

3. For the linear equation $y = -2x + 5$, what is the decrease in y that results when x is increased by 1?

A 5 **C** 3

B 4 **D** 2

☆**4.** Prove that the midpoint, M, of the hypotenuse of a right triangle is equidistant from the three vertices.

5. Points (2, 1) and (−2, 4) lie on the graph of $Ax + By = 10$.

a) Identify the values of A and B.

b) Point (2010, s) also lies on the graph. Determine the value of s.

6. Describe the effect on the graph of $y = mx + b$ when

a) the coefficient of x increases

b) the coefficient of x decreases

c) the constant term increases

d) the constant term decreases

7. Water freezes at 0 °C, or 32 °F. Water boils at 100 °C, or 212 °F.

a) Use these two points to write an equation relating Celsius and Fahrenheit.

b) Use the equation you created in part a) to convert −40 °C to degrees Fahrenheit.

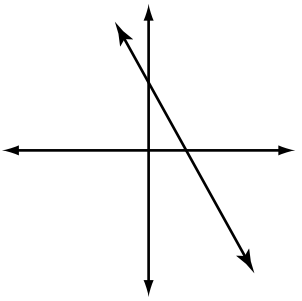
8. A 10-km taxi ride costs $9.75. At the same rate, a 22-km ride costs $18.75.

a) Write an equation to represent the distance travelled, x, and the cost of the fare, y.

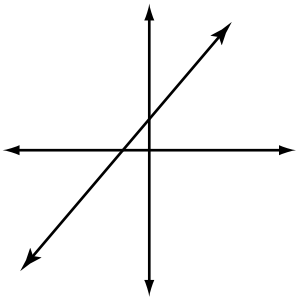
b) What does the y-intercept represent?

9. Consider the equation of each line in slope-intercept form: $y = mx + b$. Replace the question marks with $=$, $>$, or $<$.

a) m ? 0, b ? 0



b) m ? 0, b ? 0



10. Consider the equation of each of the lines in question 9 in general form: $Ax + By + C = 0$. Replace the question marks with $=$, $>$, or $<$.

a) A ? 0, B ? 0, $C > 0$

b) $A > 0$, B ? 0, C ? 0

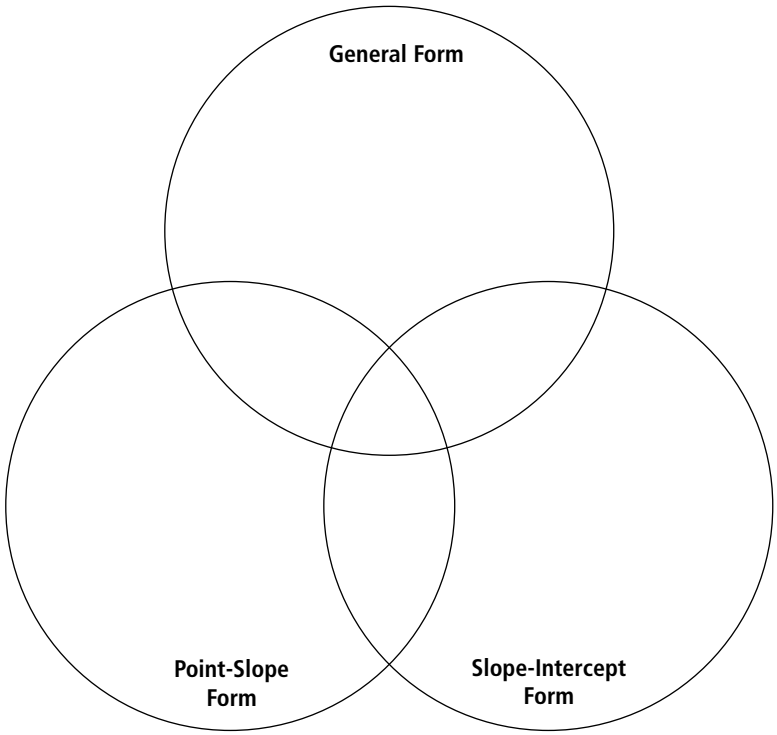
Chapter 7 Study Check

Use the chart below to help you assess the skills and processes you have developed during Chapter 7. The references in italics direct you to pages in *Mathematics 10 Exercise and Homework Book* where you could review the skill. How can you show that you have gained each skill? What can you do to improve?

Big Idea	Skills	This Shows I Know	This Is How I Can Improve
Relate the various forms of linear relations (slope-intercept, general, and slope-point) to their graphs *pages 142–145, 147–155, 157–160, 162–163*	✓ Given a graph of a linear relation, determine the slope and *y*-intercept *pages 142–145, 148, 152–153, 158–159, 162–163*		
	✓ Graph linear relations with and without technology *pages 143–145, 147–149, 151, 154–155, 157–159, 163*		
	✓ Change a linear relation given in one form to another form (e.g., slope-intercept to general form, point-slope form to slope-intercept form, point-slope form to general form) *pages 143–145, 147–155, 158–160, 163*		
Use the characteristics of a linear relation to determine the equation of the related linear relation *pages 150, 152–157, 159, 161–163*	✓ Given a graph, determine the equation of the relation *pages 152–153, 156–157*		
	✓ Determine the equation of a relation when given one of the following: a graph, the slope and *y*-intercept, two points on the line, the slope and one point on the line *pages 150, 152–153, 157, 159, 161, 163*		
	✓ Determine the equation for a relation given a point and the equation of a line parallel *or* perpendicular to the given line *pages 154–157, 159, 162*		
Apply linear equations to model and solve problems *pages 144–145, 149, 153, 157–159, 163*	✓ Use a linear relation to model a real-life situation *pages 144–145, 149, 153, 157–159, 163*		

Organizing the Ideas

Use the Venn diagram below to compare and contrast the three forms of the equation for a linear relation.



Study Guide

Review the types of problems you handled in Chapter 7. What do you need to remember to help you do similar problems?

Things to Remember

Graphing Linear Equations

Writing Equations

Changing From One Form of the Equation for a Relation to the Other

Applying Linear Equations to Solve Problems