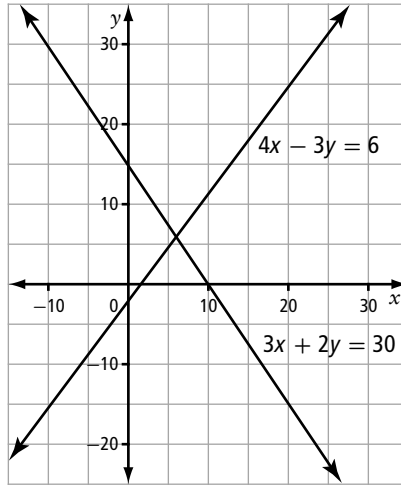# Chapter 8 Solving Systems of Linear Equations Graphically

8.1 Systems of Linear Equations and Graphs

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⟨⟨ **KEY IDEAS** ⟩⟩

- Systems of linear equations can be modelled numerically, graphically, or algebraically.

x	y
3	7
5	11
7	15
9	19
11	23

$4x - 3y = 6$

$3x + 2y = 30$

$6a + 2b = 0$
$2a + 5b = -13$

- The solution to a linear system is a pair of values that occurs in each table of values, an intersection point of the lines, or an ordered pair that satisfies each equation.

- One way to solve a system of linear equations is to graph the lines and identify the point of intersection on the graph.

- A solution to a system of linear equations can be verified using several methods:
  – Substitute the value for each variable and evaluate the equations.
  – Create a graph and identify the point of intersection.
  – Create tables of values and identify the pair of values that occurs in each table.

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Example

Consider the system of linear equations $y = 3x + 2$ and $2x - y = -4$.
  a) Rewrite the equations in slope-intercept form.
  b) Use technology to graph the equations together and identify the point of intersection.

Solution

  a) Rearrange each equation into slope-intercept form by isolating y. Identify the y-intercept and slope to draw the graph.
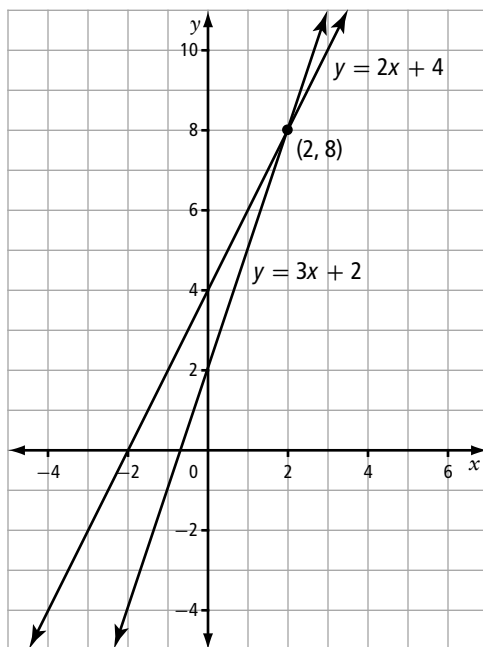
$y = 3x + 2$ is in slope-intercept form already.

$$2x - y = -4$$
$$2x - y + y = -4 + y$$
$$2x = -4 + y$$
$$2x + 4 = -4 + y + 4$$
$$2x + 4 = y$$
$$y = 2x + 4$$

The y-intercept is 2. The slope is 3.

The y-intercept is 4. The slope is 2.

• Using the Y= screen of your calculator, enter $y = 3x + 2$ as Y1 and $y = 2x + 4$ as Y2. Set the viewing window as X [3, 5, 1] and Y [−2, 12, 2]. The graph of the two lines will appear in the form shown. Use the 2nd/Calc/Intersect functions to identify the ordered pair at the point of intersection of the two lines. The point of intersection (2, 8) is the solution to the linear system.

A Practise

1. Rewrite the equations in slope-intercept form. Then, use technology to graph each pair of equations and determine the point of intersection.

   a) $x + y = 6$
   $2x - 3y = 2$

   b) $x - 4y = -2$
   $y = -x - 5$

   c) $3x + 4y = 0$
   $2x - 2y + 14 = 0$

2. Determine graphically whether each given point is a solution to the system of linear equations.

   a) $y = 4x - 9$
   $y = -2x + 3$
   $(2, -1)$

   b) $x + y = 7$
   $3x - 2y = -3$
   $(2, 5)$

3. For each pair of tables of values, determine two equations in slope-intercept form. Then, use technology to graph the equations and identify the point of intersection.

   a)

x	0	−1	2	3
y	5	7	1	−1

x	0	−2	1	4
y	2	3	1.5	0

   b)

Time (s)	0	2	5	10
Distance (m)	0	7	17.5	35

Time (s)	0	2	4	10
Distance (m)	10	13	16	25

★**4.** Solve each system of linear equations by creating a table of values and graphing with pencil and graph paper. Then, verify you solution for each case.

a) $y = 8x - 3$
$y = 2x + 3$

b) $x - y = -2$
$4x + y = 12$

c) $y = \frac{1}{2}x - 6$
$3x - y = -4$

B Apply

5. Alan has $10 and saves $0.50 each day. Vanessa has $5 and saves $1 each day.

a) Create a system of linear equations to model the amount of money, M, in dollars, that each of Alan and Vanessa has in terms of days, d.

b) Use a graph to determine when Alan and Vanessa will have the same amount of money. How much money will each of them have on that day?

★**6.** Two large tanks of oil are being drained. The first tank contains 125 m³ of oil and is being emptied at a rate of 2.5 m³ per minute. The second tank contains 80 m³ of oil and is being drained at a rate of 1 m³ per minute.

a) Create a system of linear equations to model the amount of oil, A, remaining in each tank in terms of time, t.

b) Graph the equations together to identify the point of intersection.

c) What does the point of intersection mean in the context of the problem?

d) Use your graph to determine which tank will be empty first.

7. A theatre production sold tickets at a price of $15 for adults and $10 for children. The total revenue from the sale of 69 tickets was $900.

a) Write a system of linear equations to model revenue from ticket sales, where a is the number of adults' tickets sold and c is the number of children's tickets sold.

b) Rewrite the equations in slope-intercept form.

c) Graph the equations together and determine the point of intersection.

d) Explain the meaning of the point of intersection in the context of the problem.

C Extend

8. 200 L of oil in a cylindrical tank are being transferred to a second container that is empty. The oil is being pumped at a rate of 8 L per minute.

a) Write an equation to model the volume, V, in litres, of oil in each tank in terms of time, t, in minutes.

b) Graph the two equations together and determine the point of intersection.

c) What is true about the volumes of oil in both tanks at this point?

d) How long will it take to empty the first tank?

e) Does the shape of the second container affect your answers?

9. The graph represents a system of linear equations.
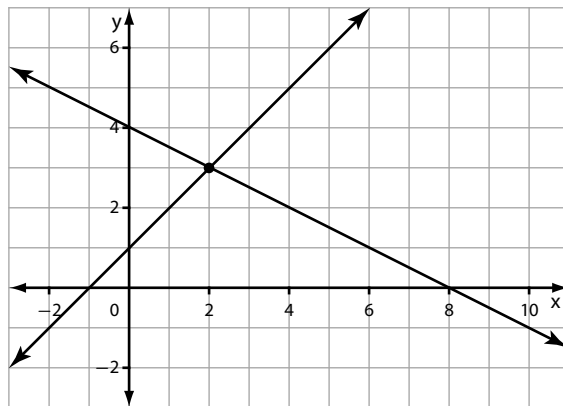

For each line, determine the

a) x-intercept

b) y-intercept

c) slope

d) point of intersection with the other line

e) equation

10. Mr. Darwal told his students that his daughter was getting married. When the students expressed surprise that he had a daughter old enough to be engaged, Mr. Darwal said: "Our ages have a sum of 62 and half of my age is one year more than my daughter's age."

a) Create a system of linear equations for the relationship between the two ages.

b) Solve the system graphically to determine the ages of Mr. Darwal and his daughter.

11. When an airplane is flying with a tailwind, the plane's speed is equal to its airspeed plus the speed of the wind. When a plane is flying into a headwind, its speed is equal to its airspeed minus the speed of the wind. Suppose that the airspeed of an airplane is 180 km/h and that the wind speed is 30 km/h.

a) Create a system of linear equations to model the distance, d, that the plane travels in terms of time, t, when the aircraft is flying with the wind and when it is flying against the wind.

b) What is the point of intersection of the two lines representing these equations?

c) How much farther can the airplane travel in 4 h with a 30-km/h tailwind than with a 30-km/h headwind?

★**12.** At what point will a line that passes through (1, 1) and (4, 7) intersect a second line that passes through (1, 6) and (3, 0)? Solve algebraically.

13. To convert temperatures expressed in degrees Celsius, C, to temperatures expressed in degrees Fahrenheit, F, you can use the equation $F = \frac{9}{5}C + 32$. To convert temperatures from the Fahrenheit scale to the Celsius scale, you can use the equation $C = \frac{5}{9}(F - 32)$. At what point are temperatures on the two scales equal?

14. Ferdinand lives in the mountains and rides his bicycle to and from school in the valley. Over level ground, Ferdinand rides at an average speed of 20 km/h. He travels at twice that speed coming down the mountain, and pedals back up the trail at half that speed. It takes Ferdinand a total of 1 hour to complete a ride to and from school.

a) Write a system of linear equations to model Ferdinand's ride in each direction in terms of distance travelled, d, in kilometres and time, t, in hours.

b) Use graphing technology to solve your system. What does the point of intersection represent?

c) Assuming that Ferdinand rides at a constant speed in each direction, how long does it take for him to travel from home to school? from school to home?

d) How far from school does Ferdinand live?

D Create Connections

15. Is it possible for two lines to have no point of intersection? Explain.

16. You are starting a website creation company. Your start-up costs are $200. For each website you create you must pay a fee of $25. If you charge customers $50 each to create a website for them, how many customers will it take for you to break even?

★**17.** For the parallelogram shown, determine the values of x and y.
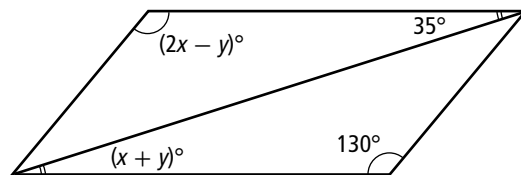
8.2 Modelling and Solving Linear Systems

《 **KEY IDEAS** 》

- When modelling word problems, assign variables that are meaningful to the context of the problem.

- To assist in visualizing or organizing a word problem, you can use a diagram or a table of values, or both.

- If a situation involves quantities that change at constant rates, you can represent it using a system of linear equations.

- If you know the initial values and rates, you can write the equations directly in slope-intercept form because the initial value is the y-intercept and the rate of change is the slope. Otherwise, you can determine the rate of change using start and end values.

- You can interpret information from graphs of linear systems.

Example

Sean is comparing the costs that two computer repair companies charge for home visits. Company A charges a rate of $45 per hour. Company B charges a flat rate of $50, plus $25 per hour for labour.

a) Create a system of linear equations to model the rates that both companies charge.

b) Solve the system of linear equations graphically. Explain what the solution represents.

Solution

a) Let h represent the number of hours for a home visit and let C represent the cost, in dollars. For Company A, the equation to express its rate is $C = 45 \times h$. For Company B, its rate may be modelled by the equation $C = 50 + 25 \times h$. These two equations form a linear system.

b) Use technology to graph the equations.


```
Intersection
X=2.5          Y=112.5
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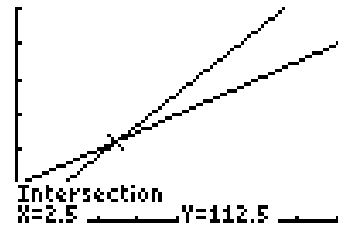
The point of intersection is (2.5, 112.5). This indicates that the charges of the two companies are equal, at $112.50, when 2.5 hours of labour are needed for a computer repair.

A Practise

1. Model each situation using a system of linear equations.

a) Shandra is three times as old as Cory. In four years she will be twice as old as Cory will be.

b) One vehicle has 5 L of fuel in its tank and is being filled at a rate of 0.9 L/s. A second vehicle has 3 L of fuel in its tank and is being filled at a rate of 1.2 L/s.

★**2.** The sum of two numbers is 168. Their difference (subtracting the second number from the first) is 18.

a) The table shown has x- and y-values adding to 168. Calculate the difference between x and y and put these values in the table. If you want the difference to be 18, what inferences can you make about the size of x and the size of y?

x	y	$x - y$
30	138	−118
50	118	
70	98	
80	88	
90	78	
100	68	

b) Create a system of equations to model the relationships between the two numbers.

c) Rewrite the equations in slope-intercept form.

d) Graph the equations to determine the intersection point.

e) Does the intersection point confirm the inferences you made using the table in part a)?

3. Josee invests a total of $15 000 in two different investments. The first amount is put into a long-term account that pays interest at a rate of 6.5% per year. The second amount is put into a short-term account earning interest at a rate of 5% per year. Josee's investments earn a total of $885 in interest in one year.

a) Write equations to represent the total amount invested and the total interest earned.

b) Rewrite the equations in slope-intercept form.

c) Use technology to graph the two equations and determine the point of intersection. Explain how this point relates to the investments.

B Apply

★**4.** A chemist wants to make 5 L of bromine solution with 32% concentration. The chemist has two available bromine solutions of concentrations 40% and 25%. She needs to determine the amount of each solution to mix to obtain the final amount in the desired concentration.

a) Write an equation to express the final amount of solution.

b) Write a second equation using the concentrations and amounts of the available solutions.

c) Rewrite the equations in slope-intercept form, if necessary, and graph them using technology.

d) Use the point of intersection to determine the amount of each bromine solution that the chemist needs to use.

5. A rectangle with a perimeter of 72 m has a length that is three times its width.

a) Create a table of values for each equation and determine five ordered pairs to satisfy each equation.

$l = 3w$

w					
l					

$2l + 2w = 72$

w					
l					

b) Plot the ordered pairs on graph paper with axes labelled w and l. Connect each set into a line, and estimate the intersection point of the lines.

c) Check your estimate using technology.

d) Explain the meaning of the point of intersection.

6. A food company wants to produce 0.5-kg bags of a mixture of cashews and peanuts to sell for $5.00. The company pays $12/kg for cashews and $3/kg for peanuts. The company needs to determine the amount of each to put into the bags to at least break even.

   a) Create an equation for the amount of cashews, *c*, and peanuts, *p*, in each bag. Create a second equation for the total cost of the different nuts and the mixture.

   b) Rewrite the equations in the form $y = mx + b$ and graph them on a graphing calculator.

   c) Solve the system of linear equations.

7. Aircraft are landing and taking off on parallel runways at a busy airport. On its approach, one aircraft descends from an altitude of 1200 m to an altitude of 500 m in 35 s. During the same time, a departing aircraft climbs from an altitude of 200 m to an altitudes of 1250 m.

   a) Write a system of linear equations to model the altitudes of the aircraft.

   b) When are the aircraft at the same altitude? What is that altitude?

C Extend

8. The highest point in Alberta is Mount Columbia. The highest point in Saskatchewan is Cypress Hills. Their elevations are related by the system of linear equations $a - s = 2279$ and $a + s = 5215$, where *a* is the height, in metres, of Mount Columbia above sea level, and *s* is the height, in metres, of Cypress Hills above sea level.

   a) Explain the meaning of each equation.

   b) Rewrite each equation to isolate the variable *s*.

   c) Graph the equations together and determine the heights of Mount Columbia and Cypress Hills.

9. Janna and Jordan are planning a birthday party and are comparing prices from two restaurants. Both restaurants have a flat rate for renting a banquet hall for a maximum of 100 guests, as well as a set meal price per guest.

   a) The prices that the restaurants charge can be modelled by the equations $C = 175 + 20n$ and $C = 100 + 22.5n$. Explain what each equation means.

   b) Graph the equations together and determine the point of intersection.

   c) For how many guests is the first restaurant the less expensive choice? For how many guests is the second restaurant the less expensive choice? Express each answer as a range of values.

☆10. The tables of values model the relationship between time and distance for two drivers who are travelling from Calgary in the same direction but who leave at different times.

   Driver A

Time (h)	0	1	2	5	6
Distance (km)	0	75	150	375	450

   Driver B

 | Time (h) | 0 | 1 | 2 | 3 | 6 |
   |---|---|---|---|---|---|
   | Distance (km) | 0 | 0 | 95 | 190 | 475 |

   a) Which driver left one hour later than the other?

   b) Which driver is travelling at a faster rate?

   c) What happens when 6 h have elapsed?

   d) Plot the points in the tables of values on graph paper, using time and distance axes, and estimate the number of hours it takes for the drivers to travel the same distance.

D Create Connections

11. A boat travels 50 km along a river in 2.5 h when it is moving downstream with the current at a constant speed. When moving upstream against the current, it takes 4 h for the boat to cover the same distance.

a) Write equations to model the travel of the boat in each direction, using *s* for the constant speed of the boat and *c* for the constant speed of the current, each in kilometres per hour.

b) Rewrite the equations and graph them together to determine the point of intersection.

c) What does the point of intersection indicate about the boat's travel on the river?

☆**12.** The graph shows the change in volume of water in two tanks over time. *V* represents volume, in litres, and *t* represents time, in minutes.
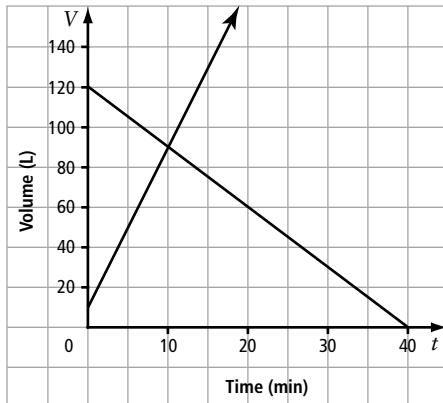

a) How much water is in each tank at $t = 0$?

b) What is different about how the volume of water is changing in the two tanks?

c) When do the tanks have the same volume of water and how much water is that?

d) At what rate is the volume of water changing for each tank?

e) Determine a system of linear equations to express the relationship between volume and time that matches the graphs.

13. The graph shows the charges, *C*, in dollars, of two appliance repair companies, A and B, in terms of hours of labour, *t*.

a) What is the point of intersection of the lines on the graph?

b) Which company charges less if a repair takes several hours?

c) Create a system of linear equations to match the graphs shown.
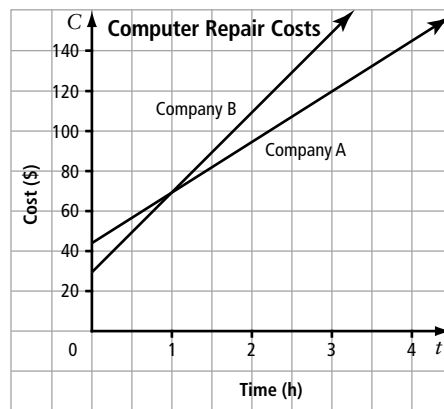
14. Create a problem or situation that could be modelled by each linear system.

a) $C = 100 + 18.5t$
$C = 75 + 20t$

b) $a + b = 900$
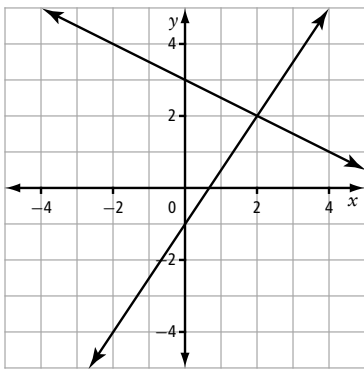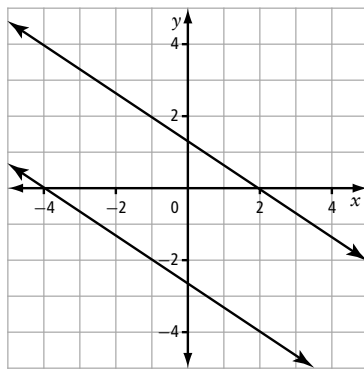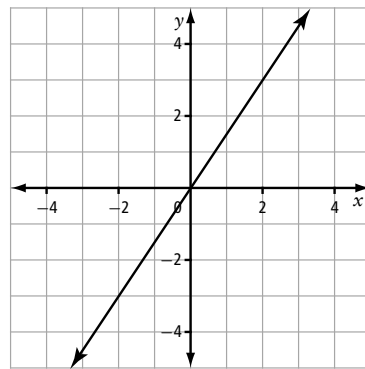$0.05a + 0.045b = 74$

c) $y = x + 8$
$5x + 9y = 100$

15. Create a problem involving a system of linear equations that has a solution of (3, 24).

8.3 Number of Solutions for Systems of Linear Equations

KEY IDEAS

- A system of linear equations can have one solution, no solution, or an infinite number of solutions.

- Before solving, you can predict the number of solutions for a linear system by comparing the slopes and y-intercepts of the equations.

Intersecting Lines	Parallel Lines	Coincident Lines
one solution	no solution	an infinite number of solutions
different slopes	same slope	same slope
y-intercepts can be the same or different	different y-intercepts	same y-intercept

- For some linear systems, reducing the equations to lowest terms and comparing the coefficients of the x-terms, y-terms, and constants may help you predict the number of solutions.

Consider the system of linear equations
$3x - y - 4$ and $6x - 2y = -9$.
Rewrite the equations in slope-intercept form.

$y = 3x - 4$ and $y = 3x + \frac{9}{2}$

The lines both have a slope of 3. Since they have different y-intercepts, the lines are parallel. Parallel lines have no intersection, so this system of linear equations has no solution.

Graphing the equations together shows that the lines are parallel and will not generate any solutions.
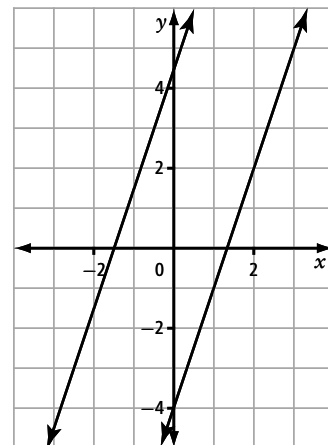
Example

Four students are going to type the same lengthy essay. Shonna begins first. Anna and Kristian start together, a short time later, and type at an equal speed that is faster than Shonna's rate. James begins last and types at the same speed as Shonna. If the number of words typed by each student as a function of time could be represented by a linear equation, how many solutions would there be for each system of linear equations?

a) Shonna and Anna

b) Anna and Kristian

c) Shonna and James

Solution

a) Although Anna started to type after Shonna did, Anna types more quickly. In time, Anna will catch up to and pass Shonna in terms of the number of words typed. There will be one point at which the numbers of words typed by Shonna and Anna are equal. Therefore, there is one solution.

b) Since Anna and Kristian start at the same time and are typing at the same speed, they will have typed the same number of words at all times. Therefore, there is an infinite number of solutions.

c) James is typing at the same speed as Shonna, but because he started typing after she did, he will never catch up. Therefore, there is no solution.

A Practise

1. Predict the number of solutions for each system of linear equations. Justify your answers.

   a) $y = 4x - 1$
   $y = 4x + 7$

   b) $y = 3x + 5$
   $y = -3x + 5$

   c) $x + 2y = 7$
 $5x + 10y = 35$

   d) $y = 2x + 3$
   $y = 2x - 7$

   e) $2y = \frac{x}{2} + 3$

   $3y = \frac{3x}{2} + \frac{9}{2}$

2. One equation of a linear system is $2x - 3y = 8$. Write a second equation so that the linear system will have

   a) no solution

   b) one solution

   c) an infinite number of solutions

3. The four lines on the graph intersect to produce parallelogram ABCD. Indicate the number of solutions for each system of linear equations.

   a) AB and CD   **b)** AC and BD

   c) AB and AC   **d)** BD and CD

   e) AD and BC
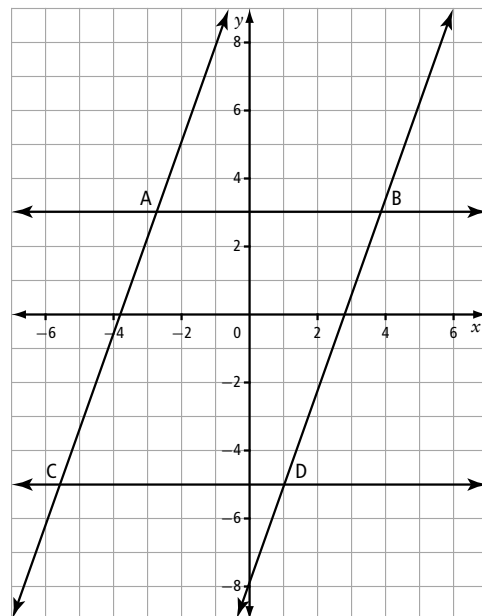
4. Graph each system of linear equations and indicate the number of solutions.

a) $6x + 2y = 10$
$y = -3x - 1$

b) $x + y = 9$
$x - y = 9$

c) $3y = x + 6$
$6y - 2x = 12$

5. Explain in words a method to correctly predict the number of solutions of a system of linear equations simply by looking at the equations.

6. A line is defined by the equation $7x - 3y = 12$. Determine the equation of a second line such that the system of linear equations has

a) no solution

b) an infinite number of solutions

c) one solution

B Apply

7. A real estate company is comparing the projected earnings, E, of several of its sales representatives. Projected earnings are based on current earnings plus a percent of sales, s, of each employee.

Employee	Current Earnings ($)	Percent of Sales (s)
Jocelyn	1200	3.00
Mario	1000	4.50
Kendra	2000	3.00
Pavel	2000	3.00

Write a linear equation for the projected earnings of each sales representative. For which pair(s) of sales representatives could you create a system of linear equations that has

a) no solution?

b) an infinite number of solutions?

c) one solution?

8. Service charges of two cell phone companies consist of a flat rate (a constant) and a rate per minute of use. If C represents the total cost and m represents the rate per minute, use values of your choice to create a system of linear equations that expresses the service charges of the companies where the system has

a) one solution

b) no solution

c) an infinite number of solutions

9. Two students are using graphing technology to solve the system of linear equations $y = \frac{3}{5}x + \frac{5}{2}$ and $y = \frac{2}{3}x$. The graphing calculator viewing window is set at X [0, 10, and 1] and Y [−2, 10, 1]. Antonio says that the system has no solution. Ling says it has one solution.

a) Which student is correct? Why?

b) How could the system of linear equations be solved without actually graphing the equations?

C Extend

10. The Gold Coast Fishery pays fishers $1.25/kg for Coho salmon and has a $40 processing fee. The Salmon House pays fishers $1.00/kg for Coho salmon and charges $25 for processing.

a) Determine a system of linear equations to model the earnings, E, in dollars, for fishers in terms of kilograms, k, of salmon delivered.

b) Predict the number of solutions to the system.

c) For what range of k-values should a fisher bring his or her salmon catch to Gold Coast Fishery? to The Salmon House?

d) How would your answer to part c) change if The Salmon House raised its rate to $1.25/kg to match its competitor?

11. Given the system of linear equations $3x - 5y = 30$ and $6x - 10y = C$, for what value(s) of C is there

a) no solution?

b) an infinite number of solutions?

12. Given the system of linear equations $2x + 6y = 12$ and $Ax + 3y = 6$, for what value(s) of A is there

a) an infinite number of solutions?

b) one solution?

13. Two 20-storey office towers are under construction. The first building is being erected at a rate of 2 storeys every 6 weeks. The height of each storey is 15 ft. Construction of the second tower began 4 weeks after the first. It is going up at a rate of 1 storey every 2 weeks. Each storey has a height of 12 ft.

a) Determine a system of linear equations to model construction of the towers where h is the height of each building, in feet, and w is the number of weeks since construction first started.

b) Predict the number of solutions to the system of linear equations.

c) What is the solution to the linear system? What does the solution represent?

D Create Connections

☆**14.** Consider the statement "Determine two natural numbers having a sum of 20 and a difference of 10."

a) If a system of linear equations can model the statement, how many solutions will the system have?

b) Compare the statement "Determine two natural numbers that have a sum of 10 and a difference of 20." If a system of linear equations can model this statement, how many solutions will the system have?

c) Graph each system of linear equations to confirm your answers for parts a) and b).

d) Why are the answers for parts a) and b) different?

15. There is an international dragon boat festival each year in Vancouver. Competitive racing takes place on False Creek over a course that is 500 m in length. Information about four dragon boats part way through one race is shown in the table of values.

Boat	Current Distance (m)	Current Speed (m/s)
A	220	3.1
B	206	3.4
C	198	3.6
D	230	3.2

Represent the data for each pair of boats using a system of linear equations, assuming that each boat continues at its current speed. What is the solution to each system? What does the solution represent?

a) A and C   **b)** A and D

c) B and D

16. The volume of water, V, in two different pools as they are being filled is given by the equations $V = 150 + 32.5t$ and $V = 175 + 35t$, where V is measured in litres and t is time, in minutes. Eva says that one pool has less water to start with and is filling more slowly, so the system of linear equations has no solution. Vince says that the lines have different slopes and must intersect, so there is one solution. Who is correct? Justify your answer.

17. A linear system of equations can be described in general as $ax + by = c$ and $dx + ey = f$, where coefficients $a, b, c, d, e, f \in I$. What is the relationship between the coefficients in the two equations such that the system has

**a)** no solution?

b) an infinite number of solutions?

Chapter 8 Review

8.1 Systems of Linear Equations and Graphs

1. Verify, without graphing, whether the given point is a solution for the system of linear equations.

   a) $y = 4x + 9$ and $y = -3x - 5$, $(-2, 1)$

   b) $5x - 3y = 17$ and $2x + 2y = 11$, $(4, 1)$

2. Use technology to solve each system of linear equations. Round your answer to two decimal places, if necessary.

   a) $y = \frac{3}{4}x - 3$ b) $5x + 6y = -35$
      $y = -\frac{2}{5}x + 1$ $3x + 8y = 10$

3. The lines on the graph form a system of linear equations.

   a) State the solution of the system.

   b) Determine the equation of each line in slope-intercept form.
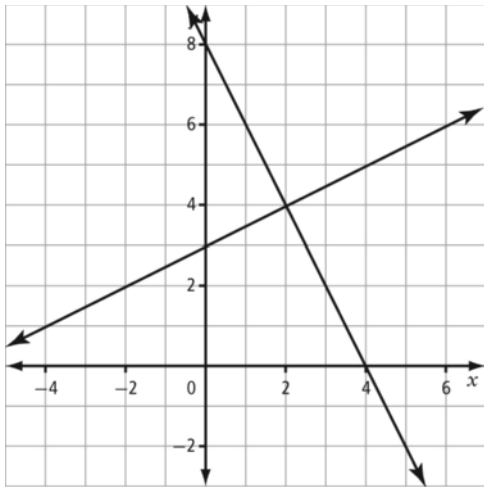
4. A cyclist is riding along a trail. Her distance travelled can be represented by the equation $d = 20t + 15$. A second cyclist's distance on the same trail is given by the equation $d = 25t + 5$. In both equations, d is distance, in kilometres, and t is time, in hours.

   a) Graph the system using technology to determine the solution for the system of linear equations.

   b) Explain what the solution represents in the context of the situation.

8.2 Modelling and Solving Linear Systems

5. Model each situation with a system of linear equations.

   a) One long distance phone plan charges $0.50/min with no sign-up fee. A second plan charges a $25 sign-up fee plus $0.25/min.

   b) A box contains 23 coins consisting of dimes and quarters. There is a total of $3.35 in the box.

   c) A bus leaves Regina, heading west at 85 km/h. A car leaves Regina 1 h later at 100 km/h, also heading west.

6. Jenny needs to rent a car for the day. The graph shows the daily cost, C, of renting a vehicle from each of two companies in terms of distance driven, d, in kilometres.

   a) How can you use the graph to determine the basic cost of a rental car (excluding distance charges) and the distance charge per kilometre for each company?

   b) Jenny thinks she will drive about 100 km. Which company should she choose?

   c) Under what circumstances should Jenny choose DirectCar?

   d) How does the point of intersection of the lines relate to the decision about which company to choose?
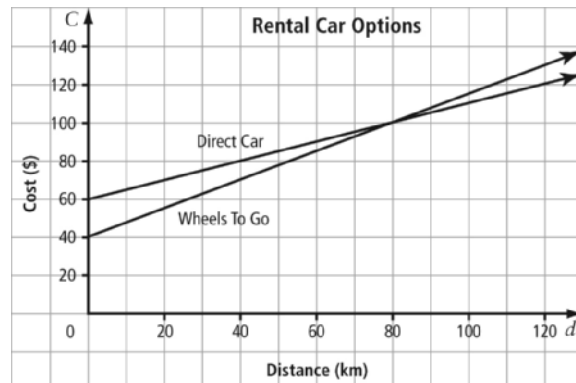
☆**7.** A load of 12.5 m³ of grain is being dumped from a truck into a bin at a rate of 1.4 m³ per minute.

   a) Write an equation to express the volume of grain in the truck and a second equation to represent the volume of grain in the bin, both in terms of time. In the equations, let V represent volume, in cubic metres, and let t represent time, in minutes.

   b) Graph the system of linear equations.

   c) Determine the point of intersection and explain its meaning in the context of the problem.

8. A desktop computer begins downloading an 885-megabyte (MB) file at 35 MB/s. At the same time, a laptop begins downloading a 1450 MB file at a rate of 60 MB/s.

   a) Create a system of linear equations for the amount, A, of each file still to be downloaded, in terms of time, t, in seconds.

   b) Graph the equations together and determine the point of intersection of the lines.

   c) Explain the meaning of the point of intersection in the context of the downloading of files to the two computers.

9. The sum of Bill's age and Nancy's age is 45. In three years, Bill will be three times as old as Nancy was four years ago.

   a) Create a system of linear equations to represent the relationship between Bill's age and Nancy's age.

   b) Solve the system graphically to determine how old Bill and Nancy are today.

8.3 Number of Solutions for Systems of Linear Equations

10. Predict the number of solutions for each system of linear equations. Justify your answers.

   a) $y = 3x + 7$ and $y = 3x - 7$

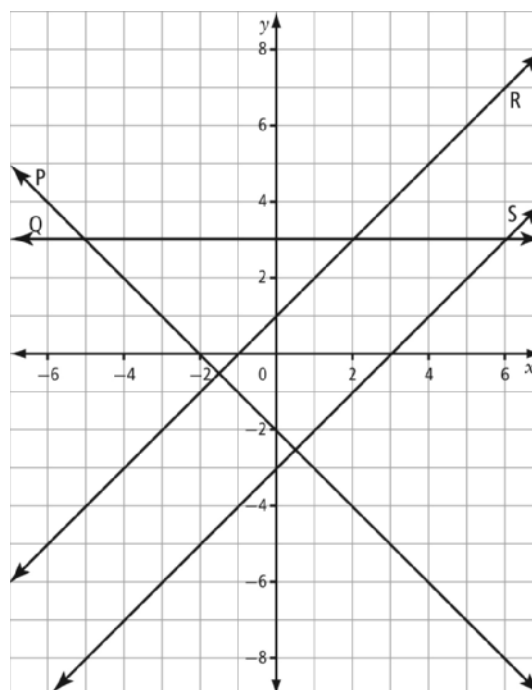   **b)** $x - 2y = -5$ and $4x - 8y = -20$

   c) $y = -6x + 1$ and $y = 6x + 1$

11. Without graphing, determine the number of solutions to the system of linear equations $2x - 5y = 18$ and $10y = 4x + 13$.

12. Which pair(s) of lines in the graph has (have)

   a) exactly one solution?

   b) no solution?

13. The South Edmonton Pet Shop has several parrots and dogs for sale. There are a total of 24 heads and 82 legs in the display cages.

   a) Write a system of linear equations to represent the number of parrots, p, and the number of dogs, d, for sale.
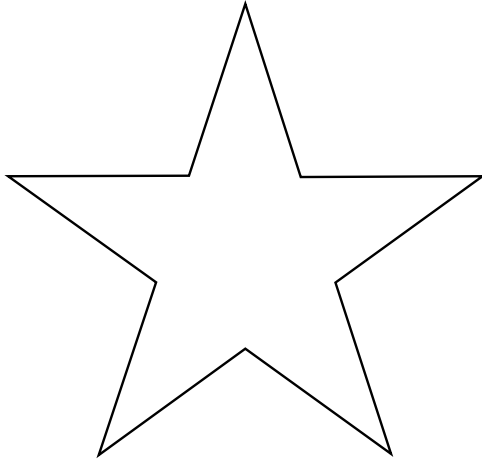
   b) Determine the solution to this system graphically.

   c) Explain why this system of linear equations would have no solution if the total number of legs is changed from 82 to 83.

   d) Why is your answer to part c) not related to the slopes of the two lines?

Chapters 1–8 Cumulative Review

1. Estimate the perimeter of the figure in an appropriate SI and imperial unit.

2. The infield of a baseball field is covered with a tarp to protect it from rain when the field is not in use. The tarp covers a square area that measures 30 m on each side. Determine the area of the tarp in square yards. Round your answer to the nearest tenth of a square yard.

3. Draw right $\triangle JKL$ in which $\angle K$ is the right angle.
  a) Label the leg opposite to $\angle J$ and the leg adjacent to $\angle J$ and the hypotenuse.
  b) State the tangent, sine, and cosine ratios of $\angle J$.

4. Which numbers are perfect squares? perfect cubes? both perfect squares and perfect cubes?
  a) 1024
  b) -8
  c) 216
  d) 1
  e) 4096
  f) 169

5. Simplify. Then, combine like terms.
  a) $2(-5n + 4)(3n - 2) + (n - 5)^2$
  b) $(6s^2 + s - 2)(-s^2 + 3s + 6)$

6. During the 2010 Olympic Winter Games, a zip line was erected over Robson Square in downtown Vancouver. The line stretched across a distance of 170 m at a height of six storeys. Create a speed–time graph for the following scenario:

  > Rebecca decides to take a ride on the zip line and climbs to the top of the tower. After climbing into safety gear, she jumps from the tower and accelerates to a speed of 11 m/s within the first 10 s of the ride. Rebecca stays at this top speed for another 10 s, until the line starts to curve upward and begins to slow her down. Rebecca lands at the opposite tower 30 s after she began her ride.

7. Write the equation of the line with each slope and y-intercept.
  a) slope is $-\frac{2}{5}$, y-intercept is $(0, -3)$
  b) slope is 0, y-intercept is $(0, 12)$

8. For each system of linear equations, explain how you could verify whether the given point is a solution. Is the given point a solution?
  a) $y = x - 7$ and $y = -3x + 1$, $(2, -5)$
  b) $12x + 4y = 12$ and $-3x + 4y = 12$, $(3, -6)$

9. From 2006 to 2009, the population of Calgary increased at an average annual rate of 4.85%. This growth can be modelled using the formula $P = (1\ 079\ 310)(1.0485)^n$, where P is the estimated population and n is the number of years since 2006. Assume that the city's rate of growth is constant.
  a) What will be the population of Calgary in 2015?
  b) What was the population in 1988, the year that Calgary hosted the Olympic Winter Games?

10. Determine the measure of angle θ in each triangle. Express your answer to the nearest tenth of a degree.

a)

15.3
9
θ

b)

θ
15
8

c)

θ
25
7
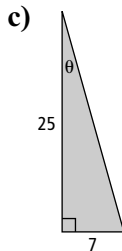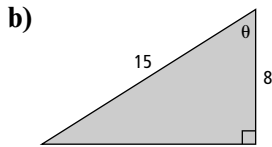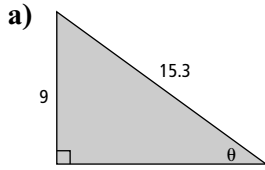
11. The population of a small college grows at a constant rate of 89 students per year. The college had 3420 students in 2006. Consider the relationship between the total number of students at the college and time, in years.

a) Assign variables to represent each quantity in the relation. Identify the dependent variable and the independent variable.

b) Is the graph of the relation linear or non-linear? Explain.

c) Write a formula to represent the relation.

d) How many students will the college have in 2017?

e) Use the formula to determine the year in which the college had 1017 students.

12. Solve each system of linear equations by graphing.

a) $y + 3x = 4$ and $4y - x = 3$

b) $y = \left(\frac{1}{3}\right)x - 5$ and $y = -\left(\frac{3}{2}\right)x + 6$

13. For each object, the surface area is given. Calculate the missing dimension to the nearest hundredth of a unit.

a) A cone has a surface area of 314.5 cm^2 and a radius of 8.5 cm.

b) A right pyramid has a surface area of 4080 cm^2 and a square base with area 1156 cm^2.
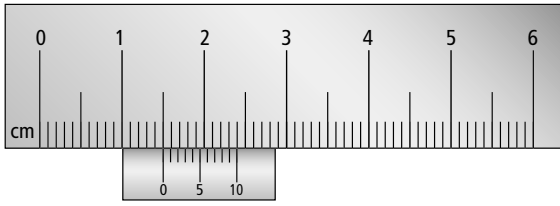
c) A sphere has a surface area of 245 cm^2.

14. Identify the GCF of each set of terms.

a) $12ab^2$ and $42ab$

b) $18x^3y^2$ and $108xy^4$

**c)** $13p^3q^5$, $-26p^5q^3$, and $3p^2q^2$

15. State the intercepts of each line as ordered pairs. Then, write the equation of each line in general form.

a)

b)
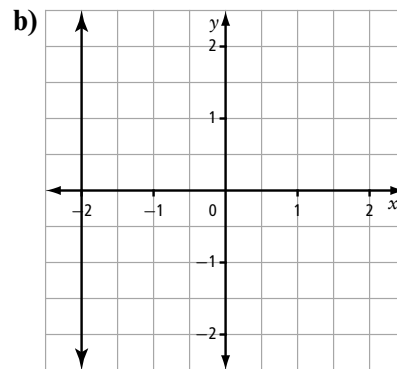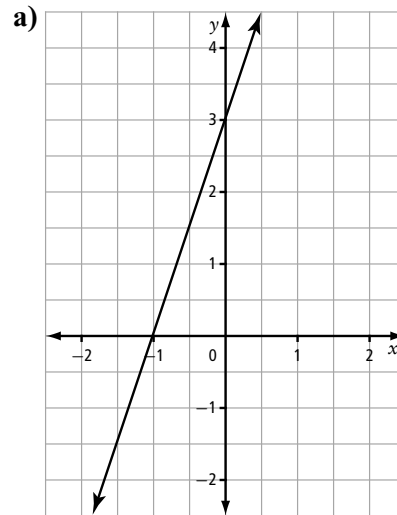
16. What reading is shown on this SI caliper? Name an object that could be this length.

17. A bongo drum has a diameter of $6\frac{1}{2}$ in. A second, smaller drum has a diameter of $5\frac{3}{8}$ in. Both drums are $\frac{7}{8}$ in. deep. What is the minimum amount of hide to cover only the tops and lateral surfaces of the drums? Express your answer to the nearest square inch.
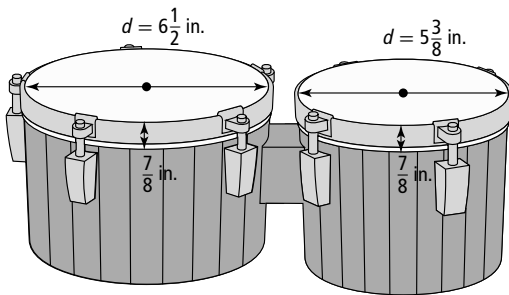
18. A boy at an amusement park is standing at a distance of 7 m away from a bouncy castle. He is looking up to the top at an angle of 40°. How high is the castle, to the nearest hundredth of a metre?

19. Evaluate each expression. Express the answers with positive exponents.

a) $\left(x^{\frac{1}{4}}\right)\left(x^{\frac{-3}{2}}\right)$

b) $(64h^9)^{\frac{-3}{2}}$

c) $\dfrac{12^{\frac{3}{5}}}{12^{\frac{-2}{7}}}$

d) $\left(\dfrac{x^{-2}}{0.125x^{\frac{-1}{3}}}\right)^4$

20. Factor completely.

a) $c^2 - 144$

b) $1 - 256y^4$

c) $54h^2 - 486f^2$

d) $x^4 - 13x + 36$

21. A relation is given by the formula $m = 9.3p - 4.7$. If the domain of the relation is $[-3, 18]$, what is the range?

22. Use slope-point form to write an equation of a line through each point with the given slope. Express each answer in slope-intercept form and in general form.

a) $(-4, 1)$ and $m = -2$

b) $(5, 9)$ and $m = \frac{1}{3}$

23. Factor completely.

a) $x^2 + 3x - 54$

b) $3x^2 + 8x - 3$

c) $20m^2 - 14mn + 2n^2$

d) $12x^2 + 16xy - 3y^2$

e) $3s^2t + 3st - 36t$

24. Consider two tanks of water. One tank, which holds 100 L, drains at a rate of 5 L per minute. The function $T(w) = 100 - 5w$ describes this draining pattern. The second tank holds 120 L of water and drains at a rate of 6 L per minute. The function $H(w) = 120 - 6w$ models the tank's draining pattern.

a) What does the variable w represent in each function?

b) Determine $T(4)$ and $H(4)$. Explain your answer.

c) Determine the value of w if $T(w) = 25$. Explain your answer.

d) Identify the domain and range of both functions.

25. Calculate the volume of each object, to the nearest hundredth of a unit.

a) A cylinder has a radius of 25 ft and height equal to its diameter.

b) A cone has a height of 24 cm and diameter 14.5 cm.

c) A rectangular pyramid has a base measuring 22 ft by 24 ft and a height of 28 ft.

d) A sphere has a radius of 14.1 cm.

26. A plane takes off from Calgary International Airport in the direction of Nose Hill Park at an angle that is to be kept constant until the aircraft passes the hill. At takeoff, the plane is a horizontal distance of 7.7 km from the peak of the hill, which is 1230 m in height. The pilot wants to clear the peak by 200 m. What is the angle of elevation of the plane to the nearest degree?

27. Police can estimate the speed of a car by the length of the skid marks made when the driver braked. The formula is $v = \sqrt{30df}$, where v is the speed of the vehicle, in miles per hour; d is the length of the skid marks, in feet; and f is the coefficient of friction. What was the speed of a vehicle if the length of the skid marks is 90 ft and the coefficient of friction is 0.9?

28. Write the equation of a line that passes through the point (9, 0) and is parallel to the line $2x + 6y - 7 = 0$.

29. Graph each line, given a point on the line and its slope.

a) $(-2, 3)$, $m = -\frac{2}{3}$

b) origin, $m = \frac{3}{5}$

c) $(5, -6)$, $m = $ undefined

d) $(7, 1)$, $m = 0$

30. Write an equation of a line that passes through $(-3, -4)$ and is perpendicular to the line $6x + 7y - 5 = 0$.

31. How many solutions does each linear system have? Justify your answers.

a) $x + 4y = 7$
$y = -\frac{x}{4} + 9$

b) $5x - 2y = 16$
$6x - 2y = 16$

c) $-7x - 3y = 5$
$-14x - 6y = 10$

32. Justin has 14 coins in his pocket. Some are quarters and the rest are dimes. Justin has a total of $2.90.

a) Write a system of linear equations to model the situation.

b) How many of each type of coin does Justin have?

33. Convert each measurement to the unit indicated.

a) The distance from Fort St. John, BC, to Prince George, BC, is 297 miles. (kilometres)

b) Lake Athabasca is the largest and deepest lake in both Alberta and Saskatchewan. It has a maximum depth of 124 m. (feet)

34. A container in the shape of a cylinder measures 6.6 cm in diameter and 12.1 cm in height. A box of 24 containers is arranged in four rows of six cans each. How much empty space is in the box, to the nearest hundredth of a cubic centimetre?

35. A house is sighted from a taller building that is 30 m in height. From the top of the taller building, the angle of depression to the bottom of the house is 40°. How far is the house from the building, to the nearest hundredth of a metre?

36. Express each radical as an equivalent power.

a) $\sqrt{(18x)^5}$

b) $\sqrt[3]{7^2}$

c) $\sqrt{z^{\frac{4}{5}}}$

d) $\sqrt{\frac{a^2}{b^2}}$

37. Verify that each trinomial is a perfect square. Then, factor.

a) $16s^2 - 24s + 9$

b) $108s^3 + 108s^2 + 27s$

c) $225 - 120y + 16y^2$

Chapter 8 Extend It Further

1. Zeno's paradox of Achilles' race with the tortoise was an impossible problem to solve at the time. The paradox states that Achilles gives the tortoise a head start of 100 units but runs at a speed 10 times as fast. Using an arbitrary speed for the tortoise, plot a distance–time graph on the same grid for each of Achilles and the tortoise. Then, determine the distance where Achilles catches up to the tortoise. Compare your answer with that of another person using a different speed. What do you notice?

2. Determine the values of m and n so that the system of linear equations $4x + 3y = 6$ and $2x + (m - 4)y = n + 1$ has

   a) no solution

   b) an infinite number of solutions

3. The sum of the digits of a two-digit number is 10. If the digits are reversed, the number decreases by 18.

   a) Write a system of linear equations to model the information.

   b) Represent the linear system graphically.

   c) What is the number?

4. Consider the rectangle shown.

2y + 1 (top), 2y − 3 (left), x + y − 1 (right), 8 + x (bottom)
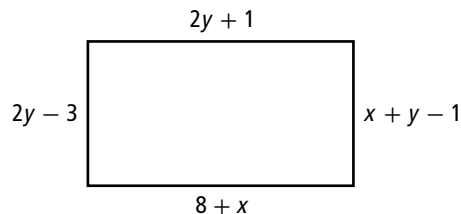
   a) Write a system of linear equations to represent the length and width of the rectangle.

   b) Represent the linear system graphically.

   c) Calculate the area of the rectangle.

5. Points (0, 0), (0, 6), and (4, 0) are three vertices of a trapezoid. The intersection of a vertical line passing through (4, 0) and a line passing through (0, 6) provides the coordinates of the last vertex. The area of the trapezoid is 15 square units.

   a) Determine the coordinates of the last vertex.

   b) Write the system of linear equations that gives the same intersection.

6. Given $Ax + 3Bx + 2A + B = 8 - x$

   a) Write a system of equations.

   b) Solve for A and B.

7. Determine the perimeter of the equilateral triangle.

$3x - 7$ (left side), $x + y - 5$ (right side), $2(y - 3)$ (bottom)
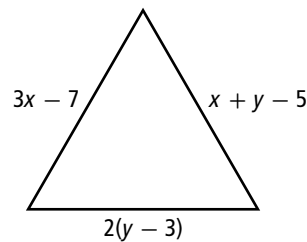
8. The ratio of the number of English books to Math books in the school library is $7:2$. After six English books are borrowed, the ratio of English books to Math books becomes $5:2$. How many books were there initially?

9. If $4^{2y + x - 3} = 1$ and $5^{x - 2y} = 25$, what is the value of $16^{3y}2^{2x}$?

10. If $x + y = 4$ and $x - y = 1$, what is the value of $2^{x^2 - y^2}$?

11. If $2009x + 2009y = 2010(x + y)$, $xy \neq 0$, what is the value of $\frac{x}{y}$?

Chapter 8 Study Check

Use the chart below to help you assess the skills and processes you have developed during Chapter 8. The references in italics direct you to pages in *Mathematics 10 Exercise and Homework Book* where you could review the skill. How can you show that you have gained each skill? What can you do to improve?

Big Idea	Skills	This Shows I Know	This Is How I Can Improve
Create and graph systems of linear equations with two variables to model and solve problems *pages 170–185, 187–188*	✓Create graphs of systems of linear equations with and without technology *pages 170, 173–177, 180–183, 185, 188*		
	✓Determine and verify the solution to a system of linear equations *pages 171–177, 180–184, 188*		
	✓Interpret information provided by a graph of a system of linear relations *pages 172, 174, 177, 179, 182–183*		
	✓Explain why systems of linear equations can have different numbers of solutions and identify the number of solutions for a given system *pages 178–181, 183, 187–188*		

Organizing the Ideas

Use the table below to compare solving linear equations with solving systems of linear equations.

Linear Equations Comparison Chart			
Characteristics	Solving Equations	Solving Systems of Equations	
Tables			Similarities
			Differences
Graphs			Similarities
			Differences
Possible Number of Solutions			Similarities
			Differences
Interpreting Results			Similarities
			Differences

Study Guide

Review the types of problems you handled in Chapter 8. What do you need to remember to help you do similar problems?

Writing

Graphing

Systems of Linear Equations

Solving

Verifying