Chapter 9 Solving Systems of Linear Equations Algebraically

9.1 Solving Systems of Linear Equations by Substitution

KEY IDEAS

- You can solve systems of linear equations algebraically using substitution.
  - Isolate a single variable in one of the two equations.
  - Where possible, choose a variable with a coefficient of 1 or −1.
  - Solve the linear system.

    $5x + y = 11$ (Equation 1)

    $2x + 3y = 7$ (Equation 2)
  - Isolate the variable y in Equation 1 since its coefficient is 1.

$$\begin{aligned} 5x + y &= 11 \\ 5x + y - 5x &= 11 - 5x \\ y &= 11 - 5x \end{aligned}$$

  - Substitute the expression for y in Equation 2.

$$\begin{aligned} 2x + 3y &= 7 \\ 2x + 3(11 - 5x) &= 7 \\ 2x + 33 - 15x &= 7 \\ 33 - 13x &= 7 \\ -13x &= 7 - 33 \\ -13x &= -26 \\ \frac{-13x}{-13} &= \frac{-26}{-13} \\ x &= 2 \end{aligned}$$

  - Substitute the solution for the first variable into one of the original equations. Solve for the remaining variable.

$$\begin{aligned} 5x + y &= 11 \\ 5(2) + y &= 11 \\ 10 + y &= 11 \\ 10 + y - 10 &= 11 - 10 \\ y &= 1 \end{aligned}$$

    The solution to the system is $x = 2$ and $y = 1$.
  - Check your answer by substituting into both original equations.

$$\begin{aligned} 5x + y &= 11 & 2x + 3y &= y \\ 5(2) + 1 &= 11 & 2(2) + 3(1) &= y \\ 10 + 1 &= 11 & 4 + 3 &= 7 \\ 11 &= 11 & 7 &= 7 \end{aligned}$$

    Therefore, $x = 2$ and $y = 1$ is the correct solution.

Example

Ryan bought 3 tickets and 6 hot dogs at an Edmonton Rush lacrosse game. The total cost of his purchases was $153. His friend Carrie bought 1 ticket and 3 hot dogs and spent $54. What was the price of a single ticket? What was the price of a hot dog?

Solution

Let T represent the price of a ticket.
Let D represent the price of a hot dog.
Write an equation that represents Ryan's purchases.
$3T + 6D = 153$ (Equation 1)
Write an equation that represents Carrie's purchases.
$1T + 3D = 54$ (Equation 2)
Isolate a variable in one of the equations. Isolate T in Equation 2.

$$\begin{aligned} 1T + 3D &= 54 \\ T + 3D - 3D &= 54 - 3D \\ T &= 54 - 3\mathrm{D} \end{aligned}$$

Substitute this expression for T into Equation 1. Solve for D.

$$\begin{aligned} 3T + 6D &= 153 \\ 3(54 - 3D) + 6D &= 153 \\ 162 - 9D + 6D &= 153 \\ 162 - 3D &= 153 \\ 162 - 162 - 3D &= 153 - 162 \\ -3D &= -9 \\ \frac{-3D}{-3} &= \frac{-9}{-3} \\ D &= 3 \end{aligned}$$

Therefore, the cost of a hot dog is $3. Substitute the cost of a hot dog into one of the equations. Solve for T.

$$\begin{aligned} T + 3D &= 54 \\ T + 3(3) &= 54 \\ T + 9 &= 54 \\ T + 9 - 9 &= 54 - 9 \\ T &= 45 \end{aligned}$$

Therefore, the cost of a ticket is $45.
Verify by substituting 45 for T and 3 for D into each equation.

$$\begin{aligned} 3T + 6D &= 153 \\ 3(45) + 6(3) &= 153 \\ 135 + 18 &= 153 \\ 153 &= 153 \end{aligned} \qquad \begin{aligned} T + 3D &= 54 \\ 45 + 3(3) &= 54 \\ 45 + 9 &= 54 \\ 54 &= 54 \end{aligned}$$

Therefore, $T = 45$ and $D = 3$ is the correct solution.
The cost of a ticket is $45 and the cost of a hot dog is $3.

A Practise

1. Solve the following linear systems of equations by substitution.

a) $x = y + 2$
$x + y = 25$

b) $y = 4x$
$x - y = 33$

c) $y = 18 - 2x$
$3x + y = 17$

d) $x - 2y = 8$
$3x + y = 3$

e) $2x + 5y = 5$
$8x - y = 41$

f) $\frac{1}{2}x - y = -2$
$x + y = 5$

★2. Solve the following system of linear equations by isolating x. Then, solve by isolating y. Which method do you prefer? Explain why.

$2x - y = -5$
$5x + y = -2$

★3. Determine algebraically if the point (2, 4) is the solution to the system $3x - y = 2$ and $x + y = 5$. Explain your answer.

B Apply

4. Seventy-five hundred fans attended a rock concert. Ticket prices were $60 for adults and $35 for students. If the total revenue from ticket sales was $300 000, how many students attended the concert?

5. Meredith invested $4000 in the stock market. She used some of the money to buy stock worth $2.50 a share and invested the rest in a stock worth $4.50 a share. If Meredith purchased a total of 1280 shares, how many shares of each type did she buy?

★6. Joel's hockey team collected a total of 900 aluminum cans and plastic bottles for recycling. The team received 10¢ for each can and 25¢ for each bottle. If the team received a total of $145.20, how many cans did they bring in?

7. Canada won 26 medals at the 2010 Winter Olympic Games, including 7 silver medals. The number of gold medals was 4 more than twice the number of bronze medals. How many gold medals did Canada win?

8. A satellite radio station plays 108 new wave and hip-hop songs in a 6-hour time slot. If the station plays 5 times as many new wave songs as hip-hop songs, how many songs of each type does the station play in 1 hour? What assumption(s) did you make?

9. The sum of Jane's age and Tim's age is 40. Four years from now, Jane's age will be 6 years less than twice Tim's age at that time. How old are Jane and Tim now?

10. A bag contains a total of 71 marbles, 14 of which are red. Each remaining marble is either black or white. If the number of black marbles is 3 less than 3 times the number of white marbles, how many marbles are black?

11. Your neighbourhood music store is having a sale. You spend a total of $81.50 to buy 5 music videos and 4 compact discs. Your friend spends a total of $42.25 to buy 3 videos and 1 compact disc. All music videos cost the same amount, and all compact discs cost the same amount.

a) What is the price for one video?

b) What is the price for one CD?

12. Teams in the girls' school basketball league are awarded 2 points for a win and 1 point for a loss. At the end of the season, Team A has 36 points and Team B has 24 points. Deborah, the league statistician, notices that the record of Team A is the reverse of the record of Team B: the number of Team A's wins equals the number of Team B's losses, while the number of Team A's losses is the same as the number of Team B's wins. What is the win-loss record of each team?

13. A student is given the following problem.

> A vending machine in the school cafeteria accepts only \$1, \$2, and 25¢ coins. When it was emptied, the machine contained 125 coins, 45 of which were loonies. If the total value of the coins was \$184, how many coins of each type were in the machine?

The student wrote the following system of linear equations to solve the problem.

Let T represent the number of \$2 coins.
Let Q represent the number of 25¢ coins.
$T + Q + 45 = 125$
$27 + 0.25Q = 184$

Will this linear system provide the correct solution? If so, solve it. If not, write a correct system and solve it.

14. In which step is the first error made in the partial solution of the linear system $3x + y = 2$ and $2x + 5y = 23$?

Step 1: $2x + 5(-3x + 2) = 23$

Step 2: $2x - 15x + 2 = 23$

Step 3: $-13x = 21$

Step 4: $x = \frac{13}{21}$

Correct the error and determine the solution.

C Extend

☆**15.** Mandy drove from Prince George to Kamloops, a distance of approximately 400 km. It took her 4.5 hours to complete the trip. For part of the trip, she drove at an average speed of 80 km/h. For the remaining part, she travelled at an average speed of 100 km/h. For what distance did she drive at the higher speed?

16. $3x + 2y = 2$ and $4x + 5y = 12$ form a linear system in which part of the solution is $x = -2$. Determine the y-value of the solution.

17. Consider the system of linear equations $5x - 4y = 0$ and $x + 3y = 15$.

a) Without solving the system, determine whether there is one solution, an infinite number of solutions, or no solution. Explain your thinking.

b) If there is one solution, in which quadrant will the lines intersect? Show how you determined your answer.

18. Use a substitution method to solve this system of linear equations involving three variables.

$2x - y + 2z = -7$
$4x - 3y + 4z = -33$
$-x + y + z = -3$

D Create Connections

19. Consider the system of linear equations $x - 2y = 10$ and $3x - y = 0$.

a) Solve the system by graphing.

b) Solve the system by using substitution.

c) How are the methods similar? How are they different?

d) Which method do you prefer for solving this system of equations? Explain.

9.2 Solving Systems of Linear Equations by Elimination

KEY IDEAS

- A table can help you organize information in a problem. This can help you to determine the equations in a linear system.
  - You can solve a linear system by elimination.
    $3x + 2y = 2$
    $4x = 12 - 5y$
  - If necessary, rearrange the equations so that like variables appear in the same position in both equations. The most common form is $ax + by = c$.
    $3x + 2y = 2$ (Equation 1)
    $4x = 12 - 5y$ (Equation 2)
    $4x + 5y = 12 - 5y + 5y$
    $4x + 5y = 12$
  - Determine which variable to eliminate. If necessary, multiply one or both equations by a constant to eliminate the variable by addition or subtraction.

    Eliminate the variable y.
  - The lowest common multiple of 2 and 5 is 10. Multiply Equation 1 by 5 and multiply Equation 2 by 2 so that the coefficients of the terms involving y add to zero.
    $5(3x + 2y) = 5(2)$ $\qquad$ $2(4x + 5y) = 2(12)$
    $15x + 10y = 10$ $\qquad$ $8x + 10y = 24$

    Subtract the second equation from the first equation to eliminate y.
    $$\begin{array}{r} 15x + 10y = 10 \\ -(8x + 10y = 24) \\ \hline 7x = -14 \end{array}$$
  - Solve for the remaining variable.
    $\frac{7x}{7} = \frac{-14}{7}$
    $x = -2$
  - Solve for the second variable by substituting the value for the first variable into one of the original equations.
    $$\begin{aligned} 3x + 2y &= 2 \\ 3(-2) + 2y &= 2 \\ -6 + 2y &= 2 \\ -6 + 6 + 2y &= 2 + 6 \\ 2y &= 8 \\ \frac{2y}{2} &= \frac{8}{2} \\ y &= 4 \end{aligned}$$
  - Check your solution by substituting each value into both original equations.
    $3x + 2y = 2$ $\qquad$ $4x = 12 - 5y$
    $3(-2) + 2(4) = 2$ $\qquad$ $4(-2) = 12 - 5(4)$
    $-6 + 8 = 2$ $\qquad$ $-8 = 12 - 20$
    $2 = 2$ $\qquad$ $-8 = -8$

    The solution is $x = -2$ and $y = 4$.

Example

The Fête au Village is an annual festival in Legal, Alberta, to celebrate the French culture of Legal's pioneers. Two popular events are the tug o' war and the demolition derby. A tug o' war team consists of 8 people. The entry fee is \$20 per team. Each demolition derby team has 2 members. The entry fee is \$100 per team. If a total of 94 people enter both events and \$1660 is collected in fees, how many teams are entered in each event?

Solution

Let W represent the number of teams entered in the tug o' war.
Let D represent the number of teams entered in the demolition derby.
Write an equation that represents the total number of people entered in both events.
$8W + 2D = 94$ (Equation 1)

Write an equation that represents the total entry fees collected.
$20W + 100D = 1660$ (Equation 2)

Solve the system by elimination.
Eliminate variable W.

The lowest common multiple of 8 and 20 is 40. Multiply Equation 1 by 5 and multiply Equation 2 by 2.

$$5(8W + 2D) = 5(94) \qquad 2(20W + 100D) = 2(1660)$$
$$40W + 10D = 470 \qquad 40W + 200D = 3320$$

Subtract Equation 1 from Equation 2.

$$\begin{array}{r} 40W + 200D = 3320 \\ -(40W + 10D = 470) \\ \hline 190D = 2850 \end{array}$$

Solve for D.

$$\frac{190D}{190} = \frac{2850}{190}$$
$$D = 15$$

Substitute $D = 15$ into Equation 1. Solve for W.

$$\begin{aligned} 8W + 2D &= 94 \\ 8W + 2(15) &= 94 \\ 8W + 30 &= 94 \\ 8W + 30 - 30 &= 94 - 30 \\ 8W &= 64 \\ \frac{8W}{8} &= \frac{64}{8} \\ W &= 8 \end{aligned}$$

Check the answer by substituting $D = 15$ and $W = 8$ into both equations.

$$8W + 2D = 94 \qquad 20W + 100D = 1660$$
$$8(8) + 2(15) = 94 \qquad 20(8) + 100(15) = 1660$$
$$64 + 30 = 94 \qquad 160 + 1500 = 1660$$
$$94 = 94 \qquad 1660 = 1660$$

Therefore, there are 8 teams entered for the tug o' war and 15 teams entered for the demolition derby.

A Practise

1. Solve, using elimination.

a) $2x - y = -5$
$5x + y = -2$

b) $4x + 2y = 6$
$4x - 3y = 1$

c) $3x + 7y = 17$
$2x - 3y = -4$

d) $2x - 3y = 20$
$2x + 2y = 12$

e) $7x + 2y = 3$
$4x - 3y = -48$

2. Rearrange the terms of the equations to the form $ax + by = c$.

a) $x + 3y = -1$
$2x + 4y - 12 = 0$

b) $3y = -2x + 1$
$4x = 2y + 10$

c) $3x - 5 = 2y$
$4y - 1 - 5x = 0$

d) $x - 3y = -4$
$4x + 2y - 12 = 0$

e) $2y = -3x - 9$
$3y + 2x - 9 = 0$

3. Solve the equations in question 2 by elimination.

4. Solve the following systems of linear equations by the elimination method.

a) $y = \frac{1}{2}x + 2$
$x + y = 5$

☆**b)** $\frac{1}{2}x - \frac{1}{3}y = 1$
$x + \frac{1}{4}y = 2$

c) $\frac{2}{3}x + \frac{1}{5}y = -2$
$\frac{1}{3}x - \frac{1}{2}y = -7$

5. Solve the following systems of linear equations by the elimination method. For each system, explain the result and determine the solution by another method.

a) $x + 3y = -1$
$2x + 6y + 2 = 0$

b) $3x - 2y = 5$
$-6x + 4y = 1$

B Apply

Solve problems 6 to 14 by the elimination method. Check your answers.

6. Mrs. Chan's Math class contributed $2 coins and $1 coins to an earthquake relief fund. The number of $1 coins contributed was 8 less than 5 times the number of $2 coins contributed. If the class raised a total of $160, how many coins of each type were collected?

7. A sports club charges an initiation fee and a monthly fee. At the end of 5 months, Christelle had paid a total of $170. Her friend, Keaton, had paid $295 at the end of 10 months. What is the initiation fee and what is the monthly fee?

8. Huyen paid $124 to rent a car for 3 days and drove a total distance of 160 km. When she rented the same car for 5 days and drove 400 km, it cost Huyen $240. What was the rental charge per day and what was the charge per kilometre?

9. Robyn wishes to invest $660 so that the income from an investment paying interest at 10% per annum is equal to the interest from a bond paying 12% annual interest. How much should Robyn invest at each rate?

10. A 500-space parking lot is filled with motorcycles and passenger cars, with only one vehicle in each space. How many motorcycles and cars are there if the total number of tires on the parked vehicles is 1650?

11. General admission tickets to the Calgary Zoo cost a total of $109 for a group of 4 adults and 3 children. Tickets for a group of 2 adults and 5 children cost a total of $93. What is the cost for one adult ticket? What is the admission price for one child?

12. John lives 1.2 miles from his school. Each morning, he walks the entire distance in a time of 24 min. On his return each afternoon, John runs part of the way so that it takes him only 15 min to reach home. If John runs twice as fast as he walks, how far does he run on his way home from school?

13. The Golden Ears Bridge is a toll bridge across the Fraser River in British Columbia. It connects Pitt Meadows and Maple Ridge on the north side to Langley and Surrey on the south side. The regular toll for passenger cars is $3.95. The fee is reduced to $2.80 if a vehicle is equipped with an electronic transponder to pay the toll automatically. One day, 8200 cars crossed the bridge. If the total of the tolls paid was $30 032.50, how many vehicles had a transponder?

☆**14.** Shanice purchased a total of 50 oranges and granola bars as a snack for the girls' soccer team. Oranges cost $2.40 per dozen and granola bars cost $3.25 for a 5-bar box. If Shanice paid a total of $19, how many boxes of granola bars did she buy?

C Extend

15. Cashews and peanuts are mixed together and sold by the pound. You can buy 3 lb of peanuts and 4 lb of cashews for $14.90. You can also buy 5 lb of peanuts and 2 lb of cashews for $12.70. What is the price per pound of each of cashews and peanuts?

☆**16. a)** The sum of the digits of a two-digit number is 14. The number formed by reversing the digits is 36 more than the original number. What is the original number?

b) The sum of the digits of a two-digit number is 11. If the digits are reversed, the second number is 9 more than the original number. What is the original number?

17. For what values of m and n is (2, 5) the solution of the linear system $mx + y = 19$ and $nx - 2y = -6$?

18. Using the equation $2x + y = 3$, write a second equation to form a linear system that has
a) an infinite number of solutions
b) no solution
c) one solution

19. If (2, −5) and (−5, −2) both satisfy the equation $Ax + By = -29$, what is the value of A?

D Create Connections

20. Consider the system of linear equations $3x + 4y = 1$ and $5x - 3y = -8$.

a) Solve the system by substitution.

b) Solve the system by elimination.

c) Which method do you prefer? Explain.

d) What do you need to consider when choosing whether to use the substitution method or the elimination method to solve a system of linear equations?

9.3 Solving Problems Using Systems of Linear Equations

KEY IDEAS

- Systems of linear equations can be solved
  – graphically
  – algebraically by substitution or elimination

Method	Advantages	Disadvantages
Graphical	• provides a visual that can show how two variables relate • can be done with or without a graphing calculator • can result in an accurate and quick solution when using a graphing calculator	• can be time-consuming • may not provide an exact solution
Algebraic	• allows for an exact solution relatively quickly • can be done using more than one method (substitution and elimination)	• does not provide any visual insight into how the two variables relate • can result in an incorrect answer due to a minor arithmetic error

- It may be better to use a graphical approach to solve linear equations when you wish to see how the two variables relate, such as for cost analysis and speed problems.

- It may be better to use an algebraic approach when
  – you need only the solution (intersection point)
  – it is unclear where to locate the solution on a coordinate plane

Example

An aircraft travels 5432 km from Montreal to Paris in 7 h with a tailwind and returns in 8 h against the wind. Determine the wind speed and the speed of the aircraft in still air. What assumption(s) are you making?

Solution

Assume that the wind speed and the aircraft's speed in still air are constant throughout both trips.
Let W represent the wind speed.
Let A represent the aircraft's speed in still air.

Organize the information about distance, D, aircraft groundspeed, S, and time, T.

	D (km)	S (km/h)	T (h)
With a tailwind	5432	$A + W$	7
Against the wind	5432	$A - W$	8

Write a system of linear equations from the information in the chart.
$D = ST$
$5432 = (A + W)(7)$
$5432 = (A - W)(8)$

Choose a method to solve the system. For example, use the algebraic method involving elimination.

Expand each equation using the distributive property.
$5432 = 7A + 7W$
$5432 = 8A - 8W$

Choose a variable to eliminate. Eliminate the variable W by multiplying the first equation by 8 and the second equation by 7.
$8(5432) = 8(7A + 7W)$
$7(5432) = 7(8A - 8W)$

Rewrite the first equation so the like variables in the equations line up. Then, add the equations.

$$\begin{array}{r} 43\,456 = 56A + 56W \\ +\ 38\,024 = 56A - 56W \\ \hline 81\,480 = 112A \end{array}$$

$$81\,480 = 112A$$
$$\frac{81\,480}{112} = \frac{112A}{112}$$
$$727.5 = A$$

Substitute $A = 727.5$ into the first equation and solve for W.

$$\begin{aligned} 5432 &= 7A + 7W \\ 5432 &= 7(727.5) + 7W \\ 5432 &= 5092.5 + 7W \\ 5432 - 5092.5 &= 5092.5 - 5092.5 + 7W \\ 339.5 &= 7W \\ \frac{339.5}{7} &= \frac{7W}{7} \\ 48.5 &= W \end{aligned}$$

Verify the solutions by substituting $W = 48.5$ and $A = 727.5$ into both original equations.

$5432 = 7(A + W)$	$5432 = 8(A - W)$
$5432 = 7(727.5 + 48.5)$	$5432 = 8(727.5 - 48.5)$
$5432 = 7(776)$	$5432 = 8(679)$
$5432 = 5432$	$5432 = 5432$

Therefore, the wind speed was 48.5 km/h and the aircraft's speed in still air was 727.5 km/h.

A Practise

1. Solve each system of linear equations by an algebraic method. Verify your answer graphically.

a) $x - 2y = 10$
$3x - y = 0$

b) $3x + 2y = 6$
$3x - 5y = -15$

c) $5x + 4y = 2$
$2x - 3y = 10$

d) $3x - 2y = 10$
$4x + y = -5$

e) $2x + 4y = 12$
$\frac{x}{5} + 3y = 17$

2. Solve each system of linear equations by a method of your choice. Leave any non-integer answers in fraction form. Explain your choice of method.

☆**a)** $2x - 5y = -18$
$8x - 13y = -58$

b) $5x = y$
$-x + 3y = 3$

c) $\frac{1}{3}x + \frac{1}{2}y = -\frac{1}{2}$
$\frac{1}{5}x - \frac{1}{3}y = \frac{8}{5}$

B Apply

To solve the following problems, write a system of equations and use a method of your choice to solve the system.

3. The perimeter of a rectangular field is 6400 m. Two times the width is 40 m more than the length. Determine the dimensions of the field.

4. Jimal plans to invest $12 000 in two types of bonds which yield 9% and 11% annually. If he wants to earn a total of $1200 annually, how much should Jimal invest in each type of bond?

5. A boat travelling against a current took 3 h to travel 36 mi. Travelling with the current, the boat took 2 h for the return trip. Determine the speed of the boat in still water.

6. Liam paid a total of $27 for 6 tennis balls and 8 golf balls. Jessica paid $43.50 for 14 tennis balls and 10 golf balls at the same store. What is the price of a tennis ball?

☆**7.** During the winter, Mason feeds his horse a daily diet of hay and a mixture of grain. The total weight of the hay and grain is 20 pounds. He recently ordered enough grain at $2.10/lb and hay at $0.08/lb to last 60 days and paid a total of $702.00. How much hay and how much grain does Mason feed his horse per day?

8. Raymark's basketball team scored a total of 93 points in its last game. A total of 49 baskets were made, consisting of free throws worth 1 point each, field goals worth 2 points each, and three-point shots worth 3 points apiece. If the team made 11 free throws, how many field goals and three-point shots did it make?

9. Devon has a summer job planting trees. He is paid $0.07 per tree when planting in ideal conditions and $0.30 per tree for planting in rocky or muskeg regions. One day, Devon planted 1750 trees and earned $180.00. How many trees did he plant in rocky or muskeg areas?

10. During a 20-day period, Tom spent 70 hours mountain biking and swimming. He burned 860 calories per hour when he swam and 730 calories per hour when he rode his bike. If Tom burned a total of 54 350 calories, how many hours did he spend doing each activity?

11. Diobel operates a student painting company with a crew of five painters. They are paid at two different rates—an hourly wage for outdoor work and an hourly wage for indoor work. During one week, Diobel's payroll was $3060 for 180 hours of indoor work and 20 hours of outdoor work. The following week, her payroll was $3555 for 15 hours of indoor work and 185 hours of outdoor work. What is a painter's pay per hour for indoor work? for outdoor work?

12. Company A rents cars at a price of $25 per day and 15 cents per kilometre. Company B charges a daily rental fee of $30 and 10 cents per kilometre.

a) Create a system of linear equations relating the cost of a rental (y dollars) to the distance driven (x kilometres).

b) How many kilometres do you have to drive for the total rental charges of Company A and Company B to be equal?

13. Hockey teams are awarded 2 points for winning a game and 1 point for tying a game by the end of regulation time. The Vancouver Giants have 26 points this season. Three times the number of ties is 1 more than the number of wins the Giants have.

a) Write a system of linear equations to represent the situation.

b) How many wins do the Giants have?

C Extend

14. A car averages 8.5 L/100 km in city driving and 6.4 L/100 km in highway driving. In a trip that covered a distance of 720 km, the car used 55.7 L of fuel.

a) How far was the car driven in the city?

b) How much fuel did the car consume in city driving?

15. Two snowmobile riders start at the same point heading in opposite directions, but leave at different times and travel at different speeds. The first rider leaves at 1:00 p.m. and travels east. The second rider leaves at 3:00 p.m., travelling west at a speed 15 km/h faster than the first rider. At 6:00 p.m., the riders are 365 km apart. What is the speed of each snowmobile rider? What assumption are you making?

16. A 95% sulfuric acid solution needs to be diluted with 60% solution to obtain a 70% solution. If 2700 mL of the new solution is required, how much of the 95% solution is needed?

☆**17.** On a multiple choice test with 5 possible answers for each question, students are penalized 0.2 points for each incorrect answer. If the test consisted of 76 questions and a student achieved a score of 58, how many questions did the student answer correctly? What assumption are you making?

18. The lines that enclose a triangle can be represented by graphs of the equations $x = -4$, $y = 3x + 4$, and $y = -2x + 9$. Use a system of linear equations to determine the area of the triangle.

D Create Connections

19. Create a system of linear equations.

a) Solve your system using a method of your choice.

b) Explain your method of choice. What did you consider when making your choice?

c) Try solving your system by another method. Which method do you prefer? Explain.

Chapter 9 Review

9.1 Solving Systems of Linear Equations by Substitution

1. Solve each system of equations by substitution.

a) $2x + 3y = 11$
$5x - y = -15$

b) $3m - n = 5$
$5m - 2n = 8$

c) $3a + b - 7 = 0$
$5a + 2b - 13 = 0$

d) $z - 1 = -3w$
$11 - 5w = 3z$

2. Determine the number of solutions for each system of equations. Identify the answers for the systems that have one solution.

a) $2x + y = 6$
$y - 8 = -2x$

b) $3x + y = 1$
$6x + 2y = 2$

c) $2x + y = -4$
$x + 2y - 6 = 0$

d) $7x + 4y = -9$
$2y - 6 = 0$

3. The world's longest suspension bridge, in Kobe, Japan, and the Capilano Bridge in Vancouver have a combined length of 2129 m. The bridge in Kobe is 74 m longer than 14 times the length of the Capilano Bridge. What is the length of each bridge?

4. A hockey team offers players two annual salary packages. Package A includes a base salary plus \$1000/goal. Package B includes a base salary of \$30 000 less than the base salary in package A plus \$1500/goal. How many goals must be scored for the packages to pay the same amount?

☆5. A board 180 cm long is cut into two pieces such that 3 times the length of the larger piece is 85 cm longer than 4 times the length of the smaller piece. How long are the two pieces of board?

6. Green Health Food store makes quarter-pound and half-pound veggie burgers. One month they sold 3 times as many half-pound burgers as quarter-pound burgers. If the total vegetable mixture used was 378 pounds, what was the total number of burgers sold that month?

9.2 Solving Systems of Linear Equations by Elimination

7. Solve using elimination.

a) $7x - 4y = 26$
$3x + 4y = -6$

b) $5x = 5 - 2y$
$23 + 3x = 4y$

c) $\frac{x}{3} + \frac{y}{2} = \frac{1}{6}$
$-6y + 8 = -x$

d) $8x + 2y = -10$
$3y + 2x = 5$

e) $5 - 3(y - x) + y = 13$
$2(x + y) - 3x + 5y = 10$

☆8. Wade pays a monthly charge for his cell phone plus a charge per text message sent. In January, when he sent 300 text messages, his bill was \$63. Wade missed his February payment, so in March he paid \$142.50 for 675 text messages and a \$12.00 late fee charge.

a) What is Wade's monthly cell phone charge?

b) What is the cost per text message sent?

9. For what values of m and n is $(5, -3)$ the solution to the system of linear equations $mx + ny = -11$ and $2mx - 3ny = 8$?

10. For which system of linear equations is $(-1, 1)$ a solution?

a) $5x + 6y = 1$
$5x + 2y = -3$

b) $3x + 4y = 1$
$5x - 3y = -8$

c) $7x - 3y = 10$
$6x + 5y = -1$

11. Yasmin invested \$2100 in high-yield investments. Part of the \$2100 was invested at 7% per annum and part at 10% per annum. After 1 year the total interest earned was \$166.50. How much did Yasmin invest at each rate?

12. The perimeter of a tennis court is 69 m. The length is 13 m longer than the width. What are the dimensions of the tennis court?

13. The Alberta ski resorts at Marmot Basin in Jasper and Sunshine Village in Banff have a total of 193 runs. If the number of runs at Sunshine is 64 more than half the number of runs at Marmot, how many runs are there at each resort?

9.3 Solving Problems Using Systems of Linear Equations

14. You are given this system of linear equations:

$2x - y = 9$ and $x - \frac{1}{2}y = -5$.

a) Solve the system graphically and algebraically.

b) Use the graphs of these two lines to explain the solution.

15. Michele competed in a running–swimming race of total length 16.5 km. If Michele ran at a speed of 12 km/h and swam at a speed of 3 km/h, how far did she run if she completed the race in 105 min?

★16. Avatar works at an electronics store. He is paid a fixed weekly wage and a commission on his sales. One week, his sales totalled \$15 500 and his take-home pay was \$1015. The following week, his sales were \$9800 and his pay was \$844. What is Avatar's fixed wage and what is his commission rate?

17. Xan rides her bike from home to school in 15 min. If she increases her speed by 5 km/h, she reduces her travel time by 5 min. How far from school does Xan live?

18. Two kayakers paddled downstream on the North Saskatchewan River. They travelled 15 miles in 90 minutes. The return trip took 165 minutes. What was the speed of the current in miles per hour?

19. Mackenzie paid \$187.50 for 2 concert tickets and 2 T-shirts. Awet paid \$408.75 for 5 tickets and 3 T-shirts. What was the cost of a concert ticket?

20. The cost of electricity is charged by usage as measured in kilowatt-hours (kWh). For usage under a fixed amount of kilowatt-hours, the cost is 6.25¢/kWh. For usage for any amount over the fixed amount, the cost is 8¢/kWh. One month Mrs. Richelieu paid \$92.25 for using 1350 kWh of electricity. At what number of kilowatt-hours of usage does the rate increase?

21. The total weight of Marvel's truck and a full load of recyclable asphalt is 45.5 tons. At the waste recycling centre, Marvel is charged \$17 per ton for the asphalt and a service fee of \$1.50 per ton based on the weight of her truck. If Marvel pays \$564.25 to dump a full load of asphalt, what is the weight of Marvel's truck?

Chapters 1–9 Cumulative Review

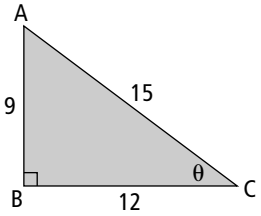
1. For each system of linear equations, explain how you could verify whether the given point is a solution. Is the given point a solution?

a) $y = 2x - 12$
$y = -3x + 13$
$(5, -2)$

b) $4x + y = 7$
$-x + 2y = -5$
$(1, 3)$

2. A semicircular window of radius 14 in. is to be laminated with a sun block coating. The coating costs 12 cents per square centimetre. What is the cost to laminate the window?

3. Write each trigonometric ratio in lowest terms.



a) sin A **b)** sin C **c)** cos A
d) cos C **e)** tan A **f)** tan C

4. What is the value of each expression? Express your answers as integers or fractions.

a) 4^3

b) -25^2

c) $(-7)^2$

d) $-\frac{3^3}{5}$

e) $\frac{3}{4^3}$

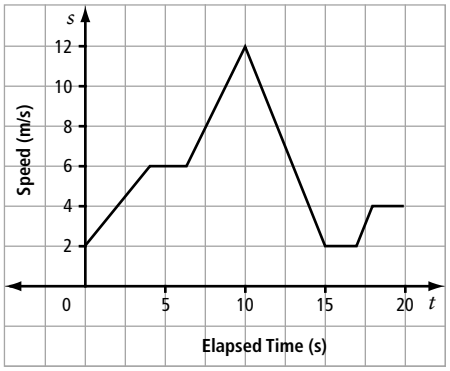
f) $\left(\frac{-1}{5}\right)^3$

5. Multiply. Then, combine like terms.

a) $6(-2n + 3)(5n - 2) + (2n - 3)^2$

b) $(4s^2 + 2s - 3)(-s^2 - s + 5)$

6. Describe a possible scenario for the following graph.



7. Solve each system of linear equations graphically. Verify your solution.

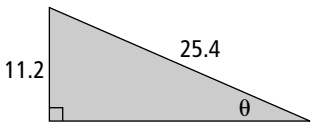
a) $y = -x - 4$
$y = -3x + 8$

b) $2x + y = -6$
$-x + 2y = 3$

c) $y = -2x - 5$
$y = -3x - 4$

8. Determine the measure of each angle θ to the nearest tenth of a degree.

a)



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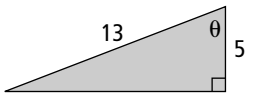
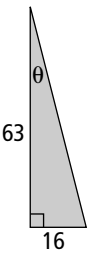


c)



9. What are the slope and y-intercept of each line?

a) $y = -5x + 8$

b) $3x - 7y - 10 = 0$

10. Calculate the surface area of each object. Round your answer to the nearest hundredth of a unit.

a) A right cone has a circular base with a radius of 18 cm and slant height of 29.3 cm.

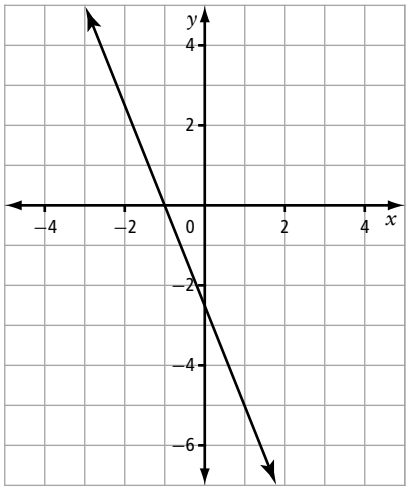
b) A right cone having a circular base has a slant height of 12 in. and a diameter of 5 in.

c) A pyramid has a square base with sides 54 cm and a slant height of 74 cm.

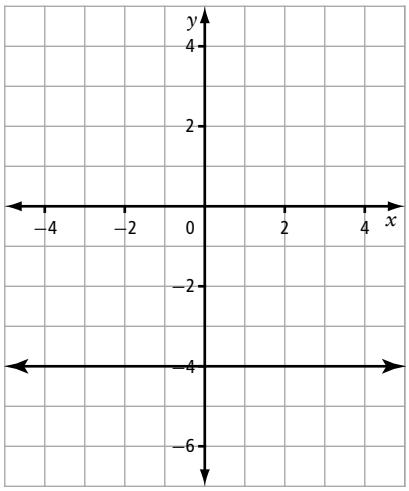
d) A sphere has a diameter of 31 cm.

11. For each line, state the domain, range, intercepts, and slope. What is the equation of each line in general form?

a)



b)



12. Identify the greatest common factor of each set of terms.

a) $18a^3b^2$ and $8ab^3$

b) $27x^4y^2$ and $63xy^4z$

c) $19p^4q$, $-9pq^6$, and $81p^3q^3$

13. Bees maintained by beekeepers in Alberta produce about 13 million kilograms of honey per year. Suppose the average price of honey is \$10.98 per kilogram. Consider the relationship between the total amount of money each beekeeper receives and the number of kilograms of honey the beekeeper sells.

a) Is this relationship linear or non-linear?

b) Assign a variable to represent each quantity in the relation. Which variable is the dependent variable? Which is the independent variable? How do you know?

c) Honey production in Alberta is 90 kg per hive. Create a table of values for the relationship.

d) Are the data discrete or continuous? Explain how you know.

e) Graph the relationship.

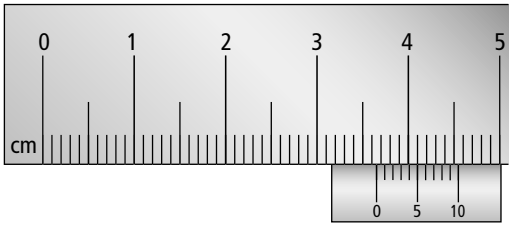
14. Determine whether each linear system has no solution, one solution, or an infinite number of solutions.

a) $y = \frac{2}{5}x + 9$
$y = \frac{5}{2}x + 9$

b) $2x - 6y + 17 = 0$
$-2x + 6y + 17 = 0$

c) $-5x - 8y = 12$
$10x + 16y = -24$

15. What reading is shown on this SI caliper? Name an object that could be this length.



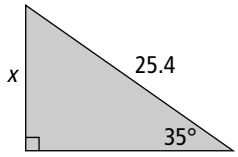
16. Solve by substitution.

a) $y = 3x - 2$
$x + y = 6$

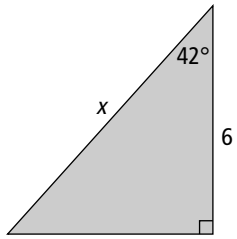
b) $3x - 2y = 10$
$4x + y = -5$

17. Determine the measure of each side length x. Express your answer to the nearest tenth of a unit.

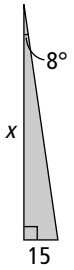
a)



b)



c)



18. Evaluate each expression. Use positive exponents to express the answers.

a) $(x^5)(x^{\frac{-2}{5}})$

b) $(256^{-0.25})^3$

c) $\dfrac{(z^3)^{\frac{3}{2}}}{(z^{\frac{1}{4}})^2}$

d) $(125p^3)^{\frac{-1}{3}}\,(p^{\frac{-3}{2}})$

e) $\left[\dfrac{x^{-3}}{(xy)^5}\right]^{-25}$

f) $\left[\dfrac{x^{-3}}{0.16(y^{-6})}\right]^{-\frac{3}{4}}$

19. Factor completely.

a) $4x^2 + 7x - 2$

b) $x^2 - 9x + 18$

c) $-24m^2 + 42mn - 9n^2$

d) $35x^2 - 11xy - 6y^2$

e) $12s^2t + 24st - 288t$

20. Solve using elimination.

a) $2x + y = 6$
$x + y = 2$

b) $y = 5 - 3x$
$4x + 3y = 20$

21. Read the following paragraph about Lake Athabasca. Convert each measurement to the unit specified.

> Lake Athabasca is the largest and deepest lake in both Alberta and Saskatchewan and the eighth largest in Canada. The lake is 283 km (miles) in length and has a maximum width of 31 miles (kilometres). The average depth of the lake is 20 m (feet). The surface elevation is 700 ft (metres).

22. Calculate the missing dimension in each of the following. Round each answer to the nearest hundredth of a unit.

a) A cylinder has a volume of 1 ft^3 and a height of 0.7 ft.

b) A right cone has a circular base with a radius of 35 cm and a volume of 7500 cm^3.

c) A square-based pyramid has a height of 91 cm and a volume of 5.7 m^3.

d) A sphere has a volume of 3476 cm^3.

23. Order each set of numbers from least to greatest. Then, identify the irrational numbers.

a) $\frac{4}{9}$, 0.77..., $\sqrt{82}$, $\sqrt[3]{634}$

b) $\sqrt[5]{67}$, $\sqrt{289}$, $14\frac{1}{3}$, $\sqrt[4]{1296}$

24. A relation is given by the formula $t = 6.3s - 5.8$. If the domain of the relation is $[-2, 12]$, what is the range?

25. Use slope-point form to write an equation of a line through each point with the given slope. Express each answer in slope-intercept form and in general form.

a) $(-2, -1)$ and $m = -3$

b) $(3, -9)$ and $m = \frac{3}{4}$

26. Factor completely.

a) $c^2 - 169$

b) $81 - 16y^4$

c) $175h^2 - 1008f^2$

d) $x^4 - 41x^2 + 100$

27. Twyla is collecting pledges as she prepares to enter a 20-km charity run. She donates $40 of her own money and collects pledges of $20 each. The function $P(n) = 20n + 40$ represents the total amount of money that Twyla raises.

a) Determine an appropriate domain and range. Then, use a table of values to graph the function.

b) Determine the value for $P(9)$. Explain the meaning of your answer.

c) Awards are presented to students who raise more than $520. How many pledges must Twyla collect to receive an award?

d) Explain why this situation depicts a function.

28. Write the equation of a line that passes through $(-2, 0)$ and is parallel to $5x - 4y + 8 = 0$.

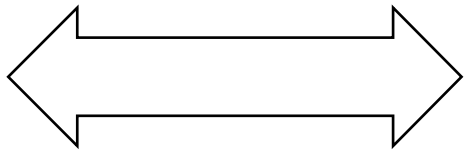
29. Verify that each trinomial is a perfect square. Then, factor.

a) $9s^2 - 54s + 81$

b) $16s^3 + 96s^2 + 144s$

c) $256 - 160y + 25y^2$

30. Use the figure below to answer the following questions.



a) What is the perimeter of the figure in centimetres? in millimetres?

b) What is the perimeter of the figure in inches?

c) Is it necessary to measure all the sides of the figure three times to answer parts a) and b)? Explain.

31. You are standing 15 ft from a three-storey building observing a worker on the roof. The angle of elevation of your line of sight to the top of the worker's head is 62°. If your eyes are 5.1 ft from the ground and the building is 27 ft high, how tall is the worker?

32. Express each entire radical as an equivalent mixed radical.

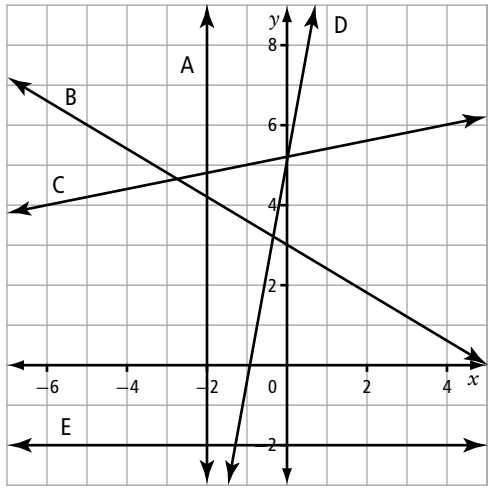
a) $\sqrt{40}$

b) $\sqrt{18}$

c) $\sqrt[3]{108}$

d) $\sqrt[4]{162}$

33. Determine the slope of each line.



Chapter 9 Extend It Further

For #1 to #4, choose the best answer.

1. $Ax + By = C$ and $Dx + Ey = F$, where $\frac{A}{D} = \frac{B}{E} = \frac{C}{F}$. The system of linear equations has

A one solution

B two solutions

C an infinite number of solutions

D no solution

2. $Ax + By = C$ and $Dx + Ey = F$, where $\frac{A}{D} = \frac{B}{E} \neq \frac{C}{F}$. The system of linear equations has

A one solution

B two solutions

C an infinite number of solutions

D no solution

3. If $18x + 54y = 26$, what is the value of $19x + 57y$?

A $27\frac{1}{9}$

B $27\frac{4}{9}$

C $28\frac{1}{6}$

D $29\frac{2}{5}$

4. The point $(2, -6)$ lies on the graph of $3x + ky + 2 = 0$. The point $(m, 4)$ also lies on the graph. What is the value of m?

A $\frac{-22}{9}$

B $\frac{-3}{7}$

C $\frac{-22}{7}$

D $\frac{7}{3}$

5. Determine (x, y) such that $123x + 321y = 345$ and $321x + 123y = 543$.

6. What is the ordered pair of real numbers (x, y) for which $x + y = 80$ and $x^2 - y^2 = 80$?

7. At an aquarium, if hoses M and N are both used, it takes 4 h 48 min to fill a giant fish tank. If only hose M is used for the first 2 h, it takes another 3 h and 36 min to fill the tank. How long does it take to fill the entire tank if only hose N is used from the start?

8. Let $m = \frac{1}{x}$ and $n = \frac{1}{y}$ to solve for x and y in the following system of linear equations.

$\frac{6}{x} - \frac{4}{y} = -5$

$\frac{9}{x} + \frac{8}{y} = 26$

9. The cost of postage, C, in dollars, to mail a parcel is given by $C = a + bn$, where a and b are constants and n is the parcel's weight, in kilograms. If C equals \$8.05 when n is 1.5 kg and C equals \$20.74 when n is 6.2 kg,

a) Determine the constants a and b.

b) What is the cost of postage for a parcel that weighs 4.2 kg?

10. Robert and Maggie are running on a 400-m circular track. Starting at the same place and running in the same direction, Robert runs 1.5 laps when Maggie is completing her first lap. If they ran in opposite directions, Robert and Maggie would have met each other after 25 s. What is Robert's speed, in metres per second?

11. A six-digit number of the form $a2b407$ is divisible by 33. Identify all the possible solutions for the number.

12. If $(x + a)(x + b) = x^2 - 3x + 1$, calculate $(a + 1)(b + 1)$.

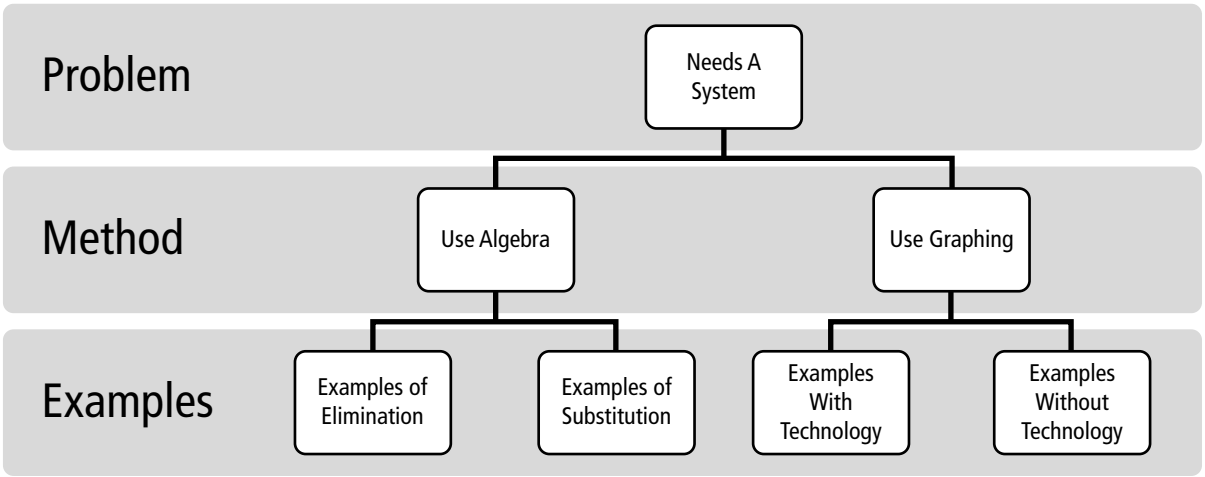
Chapter 9 Study Check

Use the chart below to help you assess the skills and processes you have developed during Chapter 9. The references in italics direct you to pages in *Mathematics 10 Exercise and Homework Book* where you could review the skill. How can you show that you have gained each skill? What can you do to improve?

Big Idea	Skills	This Shows I Know	This Is How I Can Improve
Solve a system of linear equations algebraically *pages 191–198, 203–204, 207, 209*	✓ Solve systems using substitution *pages 191–194, 198, 203, 209*		
	✓ Solve systems by writing equivalent equations or using a table to eliminate a variable *pages 195–198, 203–204, 207*		
Solve problems that involve systems of linear equations *pages 199–202, 204–205, 209*	✓ Select an algebraic method to solve a given problem *pages 199–202, 204–205, 209*		
	✓ Solve algebraically using substitution or elimination *pages 201–202, 204*		

Organizing the Ideas

How can you use this diagram to help you use systems of equations to solve problems? Draw an enlargement of this organizer on a fresh sheet of paper. Insert notes to help remind you of when a problem needs a system to help you solve it, how to decide which method to use, and an example of each method.



Study Guide

Review the types of problems you handled in Chapter 9. What do you need to remember to use each method to do similar problems?

Things to Remember

Elimination Method

Things to Remember

Substitution Method