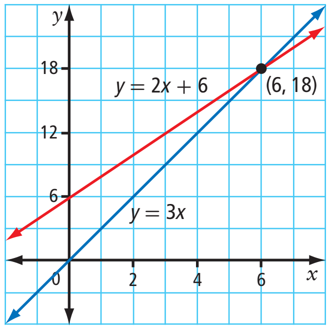
Chapter 8

8.1 Systems of Linear Equations and Graphs, pages 426 to 431

1. Yes. Example: The table of values shows that at $x = 1$, both outputs are 3. The graph shows that the point of intersection is (1, 3).
2. No. Example: The calculator screen shows that (5.2, 3) is a solution to $7x - 2y = 30.4$ because the left side equals the right side. However, it is not a solution to $4x + y = 25.1$ because (5.2, 3) produced 23.8 and not 25.1.

3. a)

x	$y = 2x + 6$	$y = 3x$
0	6	0
1	8	3
2	10	6
3	12	9
4	14	12
5	16	15
6	18	18
7	20	21



b) Example: The solution is (6, 18). From the table of values, both values of y are 18 when $x = 6$. From the graph, the point of intersection is (6, 18).

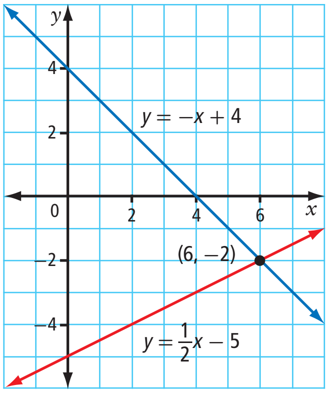
c) Since $2(6) + 6 = 18$ and $3(6) = 18$, the ordered pair (6, 18) satisfies both equations and is the solution.

4. a) Example:

x	$y = -x + 4$	$y = \frac{1}{2}x - 5$
0	4	−5
2	2	−4
4	0	−3
6	−2	−2
8	−4	−1
10	−6	0

The table of values shows that when $x = 6$, $y = -2$ in each of the equations. Therefore, (6, −2) is the solution.

b)



c) Example: The solution is the ordered pair common to both equations and the point of intersection of the two linear graphs.

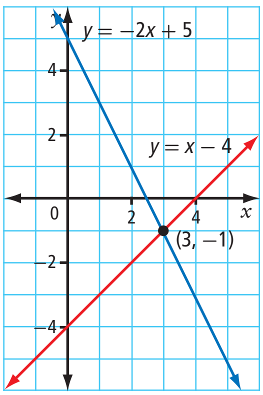
5. a) Yes. Example: The point with coordinates (4, 7) satisfies both equations. (4, 7) is the solution to the linear system.

b) No. Example: The point with coordinates (−1, 3) satisfies the equation $4x + 3y = 5$, but not the equation $x + 4y = 13$. (−1, 3) is not a solution to the linear system.

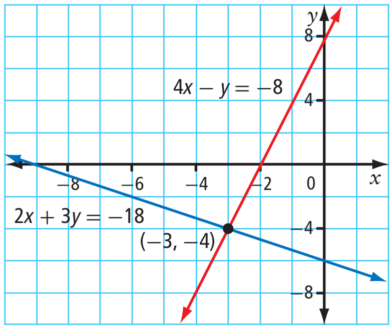
c) Yes. Example: The point with coordinates (−6, −10) satisfies both equations. (−6, −10) is the solution to the linear system.

d) No. Example: The point with coordinates (1.2, 2.4) satisfies the equation $y = 4.5x - 3$, but not the equation $12x - 3y = 7$. (1.2, 2.4) is not a solution to the linear system.

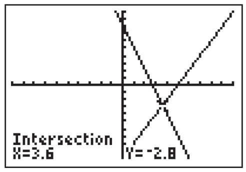
6. a) (3, −1)



b) (−3, −4)

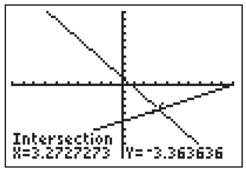


7. a)



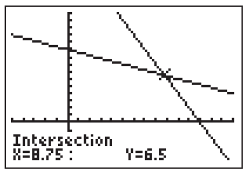
(3.6, −2.8)

b)



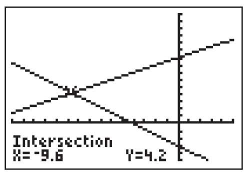
$\left(\frac{36}{11}, -\frac{37}{11}\right)$ or about (3.27, −3.36)

c)



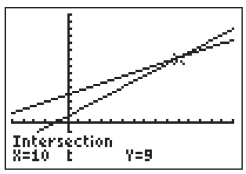
(8.75, 6.5)

d)



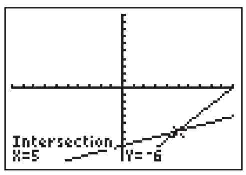
(−9.6, 4.2)

8. a)



(10, 9)

b)



(5, −6)

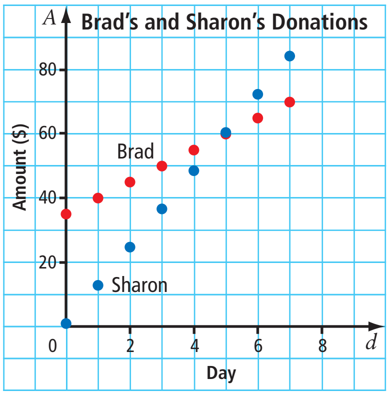
9. a) No. Example: The point (2, 5) lies on the graph of $x + 4y = 22$ but not on the graph of $3x - y = 2$. It is not a solution to the linear system.

b) Yes. Example: The point $(-3, -2)$ lies on the graph of both lines as the point of intersection. It is the solution to the linear system.

10. a)

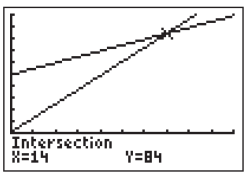
Brad	
Day	Amount ($)
0	35
1	40
2	45
3	50
4	55
5	60
6	65
7	70

Sharon	
Day	Amount ($)
0	0
1	12
2	24
3	36
4	48
5	60
6	72
7	84



b) (5, 60); It would take 5 days for both Brad and Sharon to collect $60.

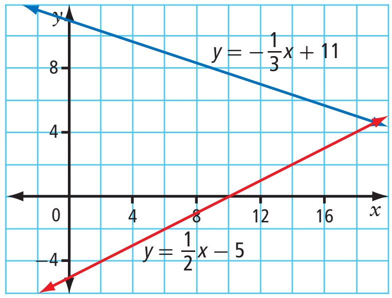
11. a)



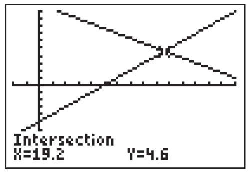
b) 14 necklaces

c) From the graph, the cost for 20 necklaces is $99 and the revenue is $120. The profit is $120 − $99, or $21. This is the vertical gap between the graphs of the two lines at 20 necklaces.

12. a) (19.2, 4.6)



b)

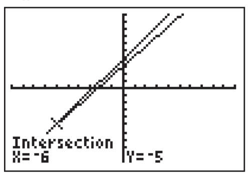


(19.2, 4.6)

c) Example: Technology can give you a more accurate answer if the numbers are not integers, and is usually faster. However, sometimes you need to adjust the viewing window to find the solution. Alternatively, the x-intercept and y-intercept method may be easier to graph manually than manipulating the equations into the form $y = mx + b$ for the graphing calculator.

13. Example: Yes. While the table of values does not show the same value of $f(x)$ for a given value of x, the pattern of changes shows that (4.5, 32) would be halfway between 4 and 5 for both equations. Therefore, this is the solution.

14.



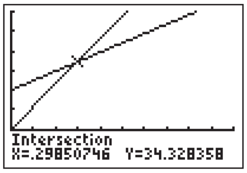
The coordinates of the point $(-6, -5)$ satisfy both equations, $3x - 2y = -8$ and $4x - 3y = -9$. Example: The two equations are almost coincident, so your pencil lines would need to be very accurate to distinguish where the point of intersection is. A graphing calculator does not have this problem.

15. a) Example: Brendan began with a 400-m head start but ran more slowly than Malcolm did. He took 9 min to complete 2800 m. Malcolm made up the difference by running 3200 m in 8 min. He passed Brendan 4.5 min into the run, around 1800 m from the start of the timing.

b) Example: Since each person is running at a constant speed during this portion of the run, the graph represents a system of linear equations.

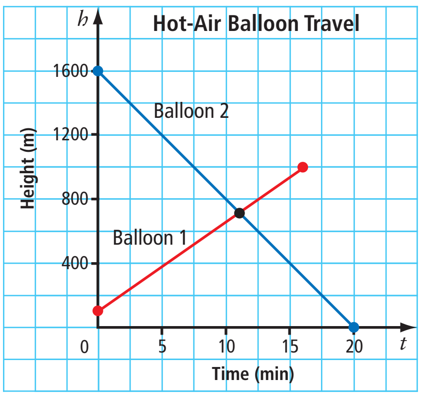
16. a) 20 km

b) At $t = 0$, the canvasback ducks have not started to fly, while the green-winged teals have already flown 20 km.



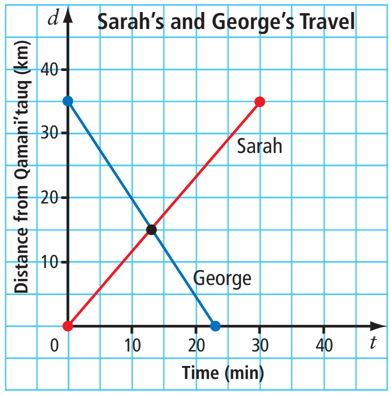
c) Example: It took the canvasback ducks about 0.3 h, or 18 min, to catch up to and pass the green-winged teals. Both types of duck were just over 34 km into the flight and 16 km from the water source. The canvasbacks arrived about 11 min before the teals despite the 20-km head start.

17.



At approximately 11 min, the two balloons pass one another, just over 719 m in the air.

18.

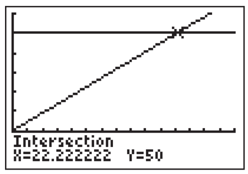


At approximately 13 min, Sarah and George pass one another, about 15 km from Qamani'tauq.

19. Example: If a person talks for over 300 min per month, that person would save money with Plan C. If a person talks for over 100 min per month but less than 300 min per month, Plan B is the least expensive. If a person talks for less than 100 min per month, Plan A is the least expensive.

20. 6.25 s

21. Example: I need to make a decision on transit passes. Do I pay \$2.25 per trip or spend \$50 per month for a pass? How many trips per month would I need to take to save money with a pass?



I would need to take over 22 trips a month to make a pass worthwhile.

22. Example: Solving a system of linear equations determines the coordinates of the point of intersection of the graphs of the equations. Verifying a solution to a system of linear equations is testing a point to see if it satisfies each equation. For example, the point of intersection of the graphs of $y = -2x + 2$ and $y = x - 7$ is the solution, $(3, -4)$. Substituting the coordinates of the point $(3, -4)$ into each equation results in a true statement.

23. a) The lines will be parallel, since the slopes are equal but the y-intercepts are different. There is no solution to this system of linear equations.

b) The lines will be perpendicular to one another, since the slopes are negative reciprocals. There is one solution to this system of linear equations.

c) The lines will intersect on the y-axis, since the slopes are different but the y-intercepts are the same. There is one solution to this system of linear equations.

d) The lines will be coincident, since the slopes and y-intercepts are the same. There are an infinite number of solutions to this system of linear equations.

8.2 Modelling and Solving Linear Systems, pages 440 to 445

1. **a)** Let C represent the cost, in dollars. Let s represent the number of songs downloaded. $C = 0.99s$ and $C = 0.79s + 11$

b) Let h represent the height above the ground, in metres. Let t represent time, in minutes. $h = 800 - 55t$ and $h = 80t$

c) Let R represent the material sorted, in tonnes. Let t represent time, in hours. $R = 100 + 20t$ and $R = 40t$

2. **a)** Let J represent Jamal's age, in years. Let M represent Maria's age, in years. $J = 3M$ and $J + 7 = 2(M + 7)$

b) Let C represent the temperature, in degrees Celsius. Let t represent time, in hours. $C = 2 - 2t$ and $C = -8 + 4t$

3. Let G represent the number of goals. Let A represent the number of assists. $G + A = 32$ and $A = 3G$

4. **a)** $d + q = 50$ and $0.1d + 0.25q = 6.80$

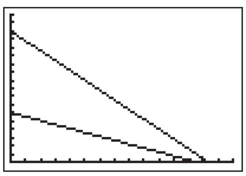
b)

Type of Coin	Value of One Coin (¢)	Number of Coins	Value of Coins (¢)
Dime	10	d	$10d$
Quarter	25	q	$25q$

$d + q = 50$ and $10d + 25q = 680$

5. **a)** Let V represent the volume of water remaining in each tank, in cubic metres. Let t represent time, in minutes. $V = 800 - 30t$ and $V = 300 - 12t$

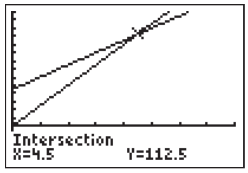
b)



c) Example: The larger tank starts with more water and drains in about 27 min. The smaller tank drains more slowly but starts with less water. It is empty in 25 min. The two graphs do not intersect until after 27 min, so they never contain the same amount of water until they are both empty.

6. **a)** Let L represent the length of sash woven, in centimetres. Let t represent time, in hours. $L = 45 + 15t$ and $L = 25t$

b)



c) Example: At 4 h 30 min (4.5 h), Kianna and Naomi both have 112.5 cm of sash woven. From this point on, Naomi's sash will be increasingly longer than Kianna's sash.

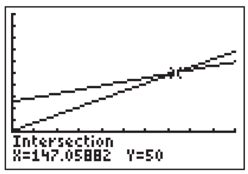
7. In 75 days, both oil wells will have produced 2625 m^3 of oil.

8. Example: If Megan is planning on driving less than 196 km in the day, she should go with the first option. If she will be driving over 196 km, then the flat rate would be more cost effective.

9. **a)** $C = 0.002(170n)$

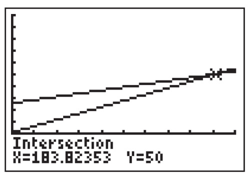
b) $C = 0.34n$ and $C = 25 + 0.17n$

c)



d) (147, 50); At about 147 showers, the cost of showering with either type of shower head is $50.

e) Example: It will take more showers to reach an equal cost.

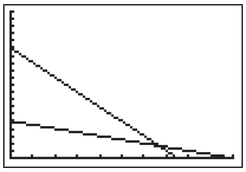


It would take almost 184 showers for the cost of showering with either type of shower head to be $50.

10. approximately 22.7 km

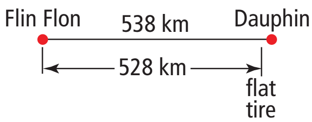
11. Yes. Example: The solution to the system of linear equations $d = 0.4 + 7.5t$ and $d = 10t$ is (0.16, 1.6). In 9.6 min (0.16 h), they will both be 1.6 km up the hill.

12. a) Let n represent the population of a species remaining. Let t represent time, in years. $n = 100 - 5t$ and $n = 300 - 20t$



b) Yes. In 13 years 4 months, there will be fewer than 34 of each species. This is the point where the graphs of the two linear equations intersect.

13. Let f represent the time driven at 90 km/h. Let s represent the time driven at 75 km/h. $f + s = 6$ and $90f + 75s = 538$; 528 km



14. They finish painting the fence in about 5.7 h, or 5 h 42 min. Chris paints almost $51\frac{1}{2}$ ft and Robert a little more than $68\frac{1}{2}$ ft.

15. In about 67 days, both trees will be approximately 153.3 cm tall.

16. Yes; about 22.9 kg

17. a) Let h represent the height above the ground, in metres. Let t represent time, in seconds. $h = 500 - 4t$ and $h = 200 + 5t$

b) In about 33.3 s, they will be approximately 366.7 m above the ground.

18. Let A represent the number of grapes that Andrea has. Let H represent the number of grapes that Hunter has. $A = 3H$ and $A = 2H + 6$; Hunter: 6 grapes, Andrea: 18 grapes

19. 51 years old

20. Let s represent the average speed of the swimmer, in metres per minute. Let c represent the average speed of the current, in metres per minute. $200 = 3(s - c)$ and $150 = 0.75(s + c)$; swimming speed: about 133.3 m/min, current speed: about 66.7 m/min

21. Let s represent the mass of sterling silver, in grams. Let p represent the mass of pure silver, in grams. $0.925s + 1p = 0.94(100)$ and $s + p = 100$

22. Let C represent the cost, in dollars. Let n represent the number of visits.

Option A: $C = 22 + 6n$, Option B: $C = 16.50\left(\frac{n}{2}\right)$;

Example: Eunji needs to decide how many times she will visit the amusement park. If she makes at least ten visits to the amusement park, the season's pass is a better deal.

23. a) Example:

Person	Time (h)	Speed (km/h)	Distance Travelled (km)
Gavin	t	110	$110t$
James	$t - 0.75$	240	$240(t - 0.75)$

b) Example: Graph $d = 110t$ and $d = 240t - 180$ and find the point of intersection. If the d-coordinate of this point is less than 300, then James will catch up to Gavin.

24. a) Example: Determine the slope of the line using points (0, 0) and (5, 70).

b) $C = 14t$ and $C = 50 + 4t$

c) Example: The point of intersection is (5, 70), so Bikes-to-Go is a better deal for more than 5 h. Spokz is a better deal if you're renting for less than 5 h.

8.3 Number of Solutions for Systems of Linear Equations, pages 454 to 459

1. a) A and B, A and C, A and D, B and D, C and D

b) B and C

2. a) infinite number of solutions; Example: Since the equations have the same slope and the same y-intercept, the graph will result in coincident lines. Therefore, this system has an infinite number of solutions.

b) one solution; Example: The equations have different slopes. So, the graph will result in two lines that intersect at one point. Therefore, this system has one solution.

c) no solution; Example: Since the equations have the same slope and different y-intercepts, the graph will result in parallel lines. Therefore, this system has no solution.

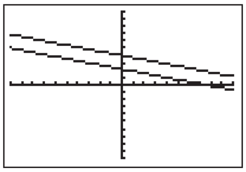
3. a) no solution; Example: Since the equations will have the same slope and different y-intercepts, the graph will result in parallel lines.

b) one solution; Example: The equations will have different slopes. So, the graph will result in two lines that intersect at one point.

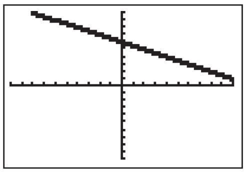
c) one solution; Example: The equations will have different slopes. So, the graph will result in two lines that intersect at one point.

4. **a)** parallel lines
b) lines with different slopes that intersect at one point
c) only one line shown on the graph

5. **a)** Example: Since the x-coefficient and y-coefficient are the same, the equations have the same slope. However, the constant values are different. Therefore, the graph will be a pair of parallel lines. The linear system has no solution.



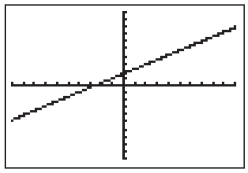
b) Example: Double the first equation creates the same equation as the second one. So, the graph of the lines will be coincident. The linear system has an infinite number of solutions.



6. Examples:
a) $2x - y + 1 = 0$
b) $3x - y + 5 = 0$
c) $4x - 2y + 10 = 0$

7. Let E represent an employee's earnings, in dollars. Let s represent the number of subscriptions sold.
a) $E = 360 + 8.25s$ and $E = 360 + 8.25s$; infinite number of solutions; Brian and Charlie will always have the same earnings.
b) $E = 472 + 7s$ and $E = 360 + 8.25s$; one solution; Brian will catch and pass Alyssa in earnings.
c) $E = 360 + 8.25s$ and $E = 413 + 8.25s$; no solution; Dena will always have earned \$53 more than Charlie.

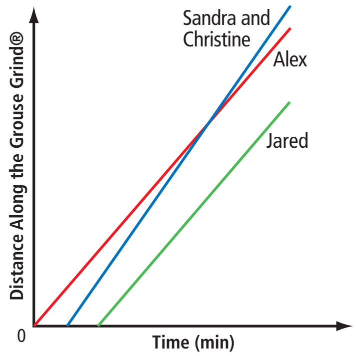
8. **a)** Compare the slopes of the two equations. Since they are different, there should be one solution.
b)



Example: Anton is correct because the equations have different slopes so there is one solution. Jeff was fooled because the graphs looked coincident on the screen.

9. **a)** Example: Stephanie doesn't see a point of intersection on the screen.
b) She is incorrect. The lines have different slopes and will intersect somewhere to the left of the current screen.

10. **a)** Example:



b) Alex and Jared: no solution; Sandra and Christine: an infinite number of solutions; Sandra or Christine and Alex: one solution; Sandra or Christine and Jared: no solution; Sandra and Christine will catch and pass Alex, but they will always be ahead of Jared and will steadily increase their lead. Jared is always the same distance behind Alex.

11. False. Example: Any point on one line is also on the other line. Not all points in the coordinate plane are on the lines.

12. **a)** No. The lines could be parallel with no solution or coincident with an infinite number of solutions.
b) No. The lines could be coincident or have one solution (at the y-intercept).
c) Yes. The lines will be coincident with an infinite number of solutions.

13. a) Let P represent the napkins manufactured, in kilograms. Let w represent the number of weeks. $P = 5000 + 350w$ and $P = 28\ 000 + 350w$

b) Example: Since the graphs of the lines are parallel, there is no solution to the system. Northern Paper will always have produced 23 000 kg more napkins than PaperWest.

14. a) $C = 24$

b) Any value except 24 will yield no solution ($C \neq 24$).

15. a) $y = 20x + 6$ and $y = 20x$

b) There are no solutions, because both taxis have the same fuel economy but stated in different ways. The second taxi will never catch the other taxi based on fuel used.

16. a) With the domain restricted, there is no solution. The lines do not intersect within these limits.

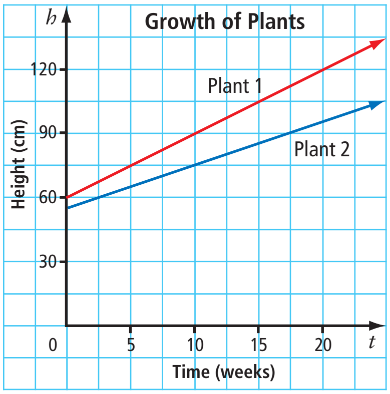
b) one solution

c) Example: Only parallel and coincident situations will be predictable. Graphs of lines with different slopes might not intersect in the window defined by the restricted domain and range.

17. a) $W = 6$ or $W = -6$

b) Another number will yield one solution. Example: $5x + 3y = 10$ and $12x + 5y = 24$ have two different slopes, $-\frac{5}{3}$ and $-\frac{12}{5}$, so the system will have one solution.

18. Wendy. Example: The shorter plant is growing slower so it will never catch up. The two equations have different slopes, but the intersection would be before the time starts.



19. Example: No. The graph of the *straight* lines would have to curve. On a globe, a line of latitude will meet a line of longitude at two points, one on either side of Earth.

20. Example: Yes, as long as there are no restrictions on the domain. With restrictions, you cannot be sure (like the plants in #18).

21. Examples:

a) In a race, a head start is given to one runner. If the two runners are running at the same speed, there will be no solution. $d = 5t$ and $d = 5t - 15$

b) In a race, a head start is given to one runner, and the second runner is faster. There will be one solution. $d = 5t$ and $d = 7t - 14$

c) In a race, the runners start at the same time and run at the same speed. They will always be beside one another (infinite number of solutions). $d = 5t$ and $d = 5t$

22. Step 1: Example: A: $2x + y = 4$ and $6x + 3y = 12$, B: $2x + y = 4$ and $2x + y = 10$, C: $2x + y = 4$ and $2x + y + 5 = 9$, D: $2x + y = 4$ and $4x + 2y = 4$

Step 2: Example: A: an infinite number of solutions, B: no solution, C: an infinite number of solutions, D: no solution

Step 3: One equation as a multiple of another produces coincident equations. If two equations have identical x-coefficients and y-coefficients but different constants, then they are parallel lines and there will be no solution.

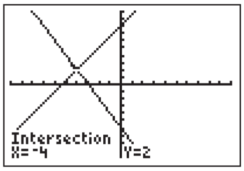
Step 4: Example: Any equation of the form $Ax + By = C$ has a slope of $-\frac{A}{B}$. Any other equation whose value of $-\frac{A}{B}$ is not the same will have one solution with the first equation, since the slopes will be different.

Chapter 8 Review, pages 460 to 462

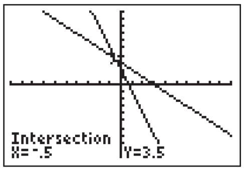
1. a) Example: Substitute $x = -7$ and $y = 4$ into each equation. Evaluate each side to determine whether the values satisfy both equations. The point $(-7, 4)$ is the solution to the linear system.

b) Example: Substitute $x = 3$ and $y = -5$ into each equation. Evaluate each side to determine whether the values satisfy both equations. The point $(3, -5)$ is not the solution to the linear system.

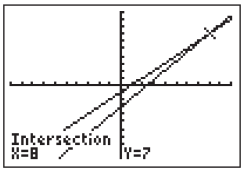
2. a) (−4, 2)



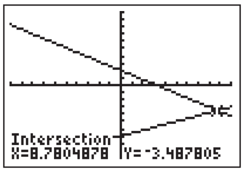
b) (−0.5, 3.5)



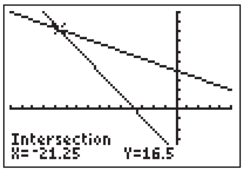
c) (8, 7)



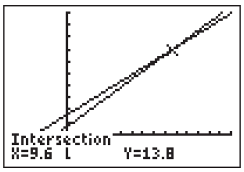
3. a) approximately (8.8, −3.5)



b) (−21.25, 16.5)

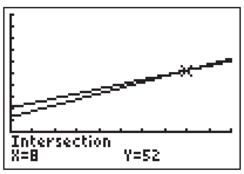


c) (9.6, 13.8)



4. a) Example:

t	$d = 4t + 20$	$d = 5t + 12$
0	20	12
1	24	17
2	28	22
3	32	27
4	36	32
5	40	37
6	44	42
7	48	47
8	52	52



b) Example: In the table of values, the solution occurs when the values of d are equal for the same value of t. On the graph, the solution is the point of intersection.

c) (8, 52); At 8 s, both boats are at a distance of 52 m.

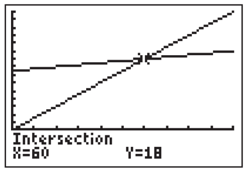
5. a) Let C represent the cost of a gym membership. Let m represent the number of months as a member. $C = 85 + 30m$ and $C = 35m$

b) Let G represent the amount of grain, in cubic metres. Let t represent time, in minutes. $G = 5 + 2.5t$ and $G = 5t$

c) Let D represent the length of road left to resurface, in metres. Let t represent time, in hours. $D = 3000 - 200t$ and $D = 4000 - 250t$

6. a) Let P represent the cost of the cell phone plan. Let m represent the number of minutes used. $P = 15 + 0.05m$ and $P = 0.3m$

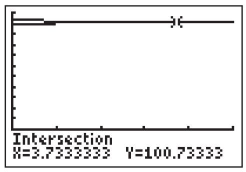
b)



c) Example: The point of intersection shows that 60 min costs $18 on both plans. If a person would use more than 60 min per month, then the better choice is Plan #1. If a person would use less than 60 min per month, then the better choice is Plan #2.

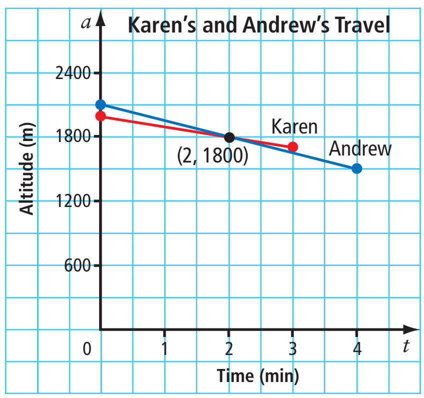
7. a) Let P represent the atmospheric pressure, in kilopascals. Let t represent time, in hours. $P = 102.6 - 0.5t$ and $P = 99.8 + 0.25t$

b)



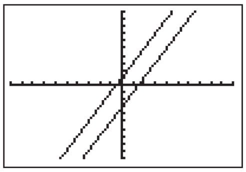
In 3 h 44 min, both cities will have the same pressure of about 100.7 kPa.

8. Andrew will pass Karen 2 min into the skiing, at 1800 m.

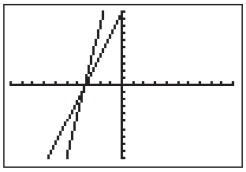


9. a) parallel lines
b) intersecting lines
c) coincident lines

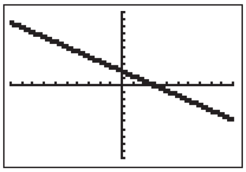
10. a) no solution; Example: Since the equations have the same slope and different y-intercepts, the graph will result in parallel lines.



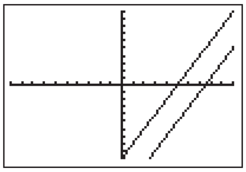
b) one solution; Example: The equations have different slopes. So, the graph will result in two lines that intersect at one point.



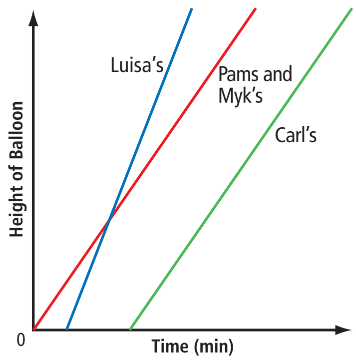
c) infinite number of solutions; Example: Since 7 times the first equation equals the second equation, the graph will result in coincident lines.



d) no solution; Example: Since the equations will have the same slope and different y-intercepts, the graph will result in parallel lines.



11. a)



b) Example: Zero solutions would be for Pam's and Carl's or Myk's and Carl's, as the graphs of their balloons are parallel. Luisa's and Carl's balloons will also never cross, because Luisa's had a head start and rises faster. One solution for Pam's and Luisa's or for Myk's and Luisa's, because their balloons have different slopes and Luisa's balloon will pass the other two. There are an infinite number of solutions for Pam's and Myk's, because their balloons are always side by side.

12. a) Let P represent the production of sports beverages, in litres. Let h represent the number of production hours. $P = 150 + 300h$ and $P = 600 + 300h$

b) There are no solutions to this system. Both companies have the same rate of production. Company B will always have 450 L more in production.

13. a) one solution

b) Example: One solution is not possible, as they are earning interest at the same rate (slope). They either started with the same amount of money (principal) and always have the same amount, or they began with different principals and will always be the same amount apart.

Chapter 8 Practice Test, pages 463 to 465

1. D
2. C
3. D
4. B
5. D

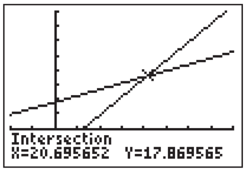
6. a) No. Example: The point (5, 2) does not lie on the graph of $x + y = 10$ or the graph of $2x - y = 3$.

b) Yes. Example: The point $(-1, 4)$ lies on the graphs of both lines as the point of intersection.

c) No. Example: The point $(8, -3)$ lies on the graph of $x + 4y + 4 = 0$ but not on the graph of $2x - 3y = 27$.

7. Example: Graph the equations on the same grid and look for the point of intersection.

8. a)



b) (20.70, 17.87)

c) Example: Store the x-value and y-value after finding the point of intersection on your graphing calculator. Check that $5x - 4y$ is 32 and $3x - 7y$ is -63. Use the full decimal strings and not the rounded off values for best results.

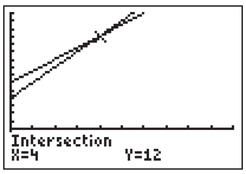
9. a) Example: Since the x-coefficients and y-coefficients are the same, the equations have the same slope. However, the constant values are different. Therefore, the graph will be a pair of parallel lines. The linear system has no solution.

b) Example: Since the equations have the same slope and the same y-intercept, the graph will result in coincident lines. Therefore, this system has an infinite number of solutions.

10. $B = 10$

11. a) Let L represent the height of the grass, in centimetres. Let w represent the number of weeks. $L = 6 + 1.5w$ and $L = 4 + 2w$

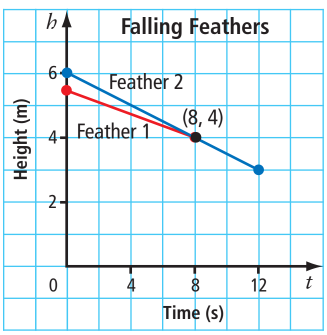
b)



One grass starts higher but grows more slowly. The shorter grass will grow quickly and pass the first grass in 4 weeks, when they are both 12 cm high.

12. Example: Paige had to walk 5 km to get to school, while Quinn had to walk only 3.5 km. Even though Paige had further to walk to get to school, she walked faster (0.125 km/min) than Quinn did (0.05 km/min) and arrived earlier. Paige passed Quinn 2.5 km from school and 20 min into the walk.

13. Yes. After falling for 8 s, they are both at 4 m.



14. Let d represent distance, in miles. Let t represent time, in hours.

a) $d = 11.8 + 7.2t$ and $d = 11.8 + 7.2t$; Donna and Marcus are always beside each other and thus have an infinite number of times that they are at the same point in the river.

b) $d = 12.6 + 7.0t$ and $d = 11.8 + 7.2t$; There is one solution. Donna catches Taj at 4 h and 40.6 km into the race.

c) $d = 11.8 + 7.2t$ and $d = 11.2 + 7.2t$; There is no solution. Marcus stays ahead of Rose a consistent 0.6 km throughout the race.