SI Measurement

1.1

***Mathematics 10,* pages 8–21**

Suggested Timing

100–120 min

Materials

- three items that are non-standard measuring units (e.g. coin, paperclip, and so on)
- grid paper
- SI measuring instruments, e.g., ruler, measuring tape, caliper, metre stick
- CD case
- watch
- outdoor measuring device

Blackline Masters

BLM 1–3 Chapter 1 Warm-Up
BLM 1–4 Chapter 1 Unit 1 Project
BLM 1–5 Section 1.1 Extra Practice

Mathematical Processes

- ✓ Communication (C)
- ✓ Connections (CN)
- ✓ Mental Math and Estimation (ME)
- ✓ Problem Solving (PS)
- ✓ Reasoning (R)
- ✓ Visualization (V)

Specific Outcomes

M1 Solve problems that involve linear measurement, using:

- SI and imperial units of measure
- estimation strategies
- measurement strategies.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–3, 5, 7, 9, 10, 12, 14, 18–19
Typical	#1–7, 9–14, 18–19, 21
Extension/Enrichment	#7, 8, 12–14, 16, 17–21

Unit Project Note that #14 is a Unit 1 project question.

Planning Notes

Have students complete the warm-up questions on **BLM 1–3 Chapter 1 Warm-Up** to reinforce prerequisite skills needed for this section.

Before you begin this section, you may wish to have a brief class discussion on the topic of linear measurement. In particular, students could discuss the need to measure and the units used to measure. Linear measurements are among the most likely to be made in both imperial and SI systems, so it may be beneficial to establish how familiar students are with each system and in what situations they have used each system.

One key aspect of the section is the use of personal referents, so it may be useful to discover what, if any, personal referents students have already established for themselves. Students likely have some sense of the need for standard units, so it may be productive to discuss this need and what experience students have with non-standard units, as well as any difficulties they have experienced or can imagine.

Investigate Dimensions of a Rectangle

Have students work in pairs. You may wish to have a set of non-standard items available. The student resource suggests coins, the width of a finger, and paper clips. Some other suggestions are the cap from a pen, a battery, an eraser, a pencil sharpener, a bingo chip, a USB key, and so on. To emphasize the challenges of measuring with non-standard units, you could have each pair of students use a different item to make their measurements, thus making comparing and communicating their answers more difficult.

This investigation gives students an opportunity to practise estimating. Ensure that students make the estimates required, and in their team, they should discuss their estimates and come to an agreement on them. Having students estimate their measurements in standard SI units links to the development of personal referents later in the section, and so is an important part of the investigation. At the conclusion of this lesson, students should be more comfortable with the common SI units and how to estimate with them.

For step 5 in the investigation, you may wish to emphasize that length is to be plotted on the x-axis while width is to be plotted on the y-axis. While this convention is not necessary for students to acquire the concepts in the investigation, it will help reduce

confusion when discussing in groups and as a class. For this reason, it may be helpful to agree as a class which dimension will be the length and which will be the width when beginning the investigation.

For step 6, you may wish to have students compare the graphs for the different units of measurement that they used. Encourage students to compare their results with other teams so that they can see that a constant ratio was achieved in each case. You may wish to discuss the results as a class and ask students to consider why standard measuring units are important, given the fact that the ratios are unchanged.

For step 7, this discussion serves to follow up any discussion on measurement that took place before beginning the investigation. This provides students with an opportunity to quantify what they have learned in the investigation and could be written about in their math journals. In your discussion, you may wish to compare the use of different standard measurements as a link to the work that is to come in the chapter and unit. This may be a good time to ask students to describe situations where measuring in a particular unit may make more sense than in another unit. This may lead to a discussion on how to determine the "best" unit for a particular measurement and the need for making conversions within and between measurement systems.

Meeting Student Needs

- You may wish to invite a representative from one of the cell phone companies to give a brief presentation about how cell phone systems operate, the range of towers, and how the system is controlled, as well as the various occupations available at a cell phone company.
- Students could create posters to illustrate the various SI units and some common items that would be accurately measured with each unit.
- Students may need to be reminded about construction of rectangles. Various methods could be explored to ensure that there are four right angles and that the opposite sides are congruent and parallel.
- Some students would benefit from a chart for #2b) being posted on the whiteboard to help them organize their data.

ELL

- Some students may not be familiar with compact discs (CDs) because many students download music directly to music devices.
- You may wish to bring in a map of the school area to help reactivate the concept of map scales. Measure in SI units the distance from school to various neighbourhood landmarks and determine the actual distance to give meaning to the map scale.

Enrichment

- Suggest students create referents that define SI measurements using healthy activities, such as running, biking, and swimming. An example might be to challenge students to establish referents for the time taken to run 100 m, bike 1 km, or swim 50 m.

Gifted

- Ask students to ponder the question of SI time. What would happen if the world decided to create 10-h days, 10-day weeks, etc.? Would it be an improvement? What might go wrong?

Common Errors

- Some students may make arithmetic errors when determining the length-to-width ratio if they use fractions to express distances.

R_x Have a brief discussion of operations with fractions.

- Some students may forget which side of the rectangle they designated as the length and which as width.

R_x Suggest that students label the length and width of their rectangle on their drawing.

Answers

Investigate Dimensions of a Rectangle

1. Accept various rectangles.

2. Examples: pencil widths, eraser widths or lengths, thumb widths, pinkie finger widths. and so on. Various estimates are acceptable and depend on the size of the rectangle. Examples include 6 pencil widths wide by 3.5 pencil widths long, 3 thumb widths wide by $1\frac{1}{2}$ thumb widths long, 4.5 eraser widths wide by 2.5 eraser widths long.

Answers

3. Examples: 2.5 paper clips wide by 1.75 paper clips long, 7 pencil widths wide by $3\frac{1}{2}$ pencil widths long. 3 thumb widths wide by 1.5 thumb widths long.

4. Estimates include 50 mm wide by 30 mm long and 5 cm wide by 3 cm long. Actual measurements could be 47 mm by 26 mm and 4.7 cm by 2.6 cm.

5. Graphs will vary, but points should appear roughly on the same line. Points plotted could be (6, 3.5) for pencil widths, (3, 1.5) for thumb widths, (4.5, 2.5) for eraser widths, and (4.7, 2.6) for centimetres.

6. a) The points should fall roughly on the same line.

b) No, the length will change at the same rate as the width. So, the ratio will be constant.

7. Examples: One advantage of using standard units is that the measurements can be compared easily anywhere because the units are standard worldwide. One disadvantage is that not everyone has a measuring tape or ruler available when they need to measure something. So, using non-standard measuring instruments can still be useful.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss their graphs. Encourage them to generalize and reach a conclusion about any patterns they see in their graph or about the length-to-width ratio of their rectangle. For #6, consider having students respond in their journal to the following prompts: – What would change in your home if there were no standard measurement units for distance? – What would change at school if there were no standard measurement units for distance?	• Have students compare their graphs with those of classmates and discuss their results. • Encourage students to share their journal responses to the prompts. • Encourage discussion about the measuring process to provide struggling learners with an opportunity to verbalize the patterns that they see. • Discuss the benefits of having a standard measurement instrument and a standard unit of measurement. Ask students why they believe it is important that we have a standard unit to measure with instead of using their referent.

Link the Ideas

In this section, students are introduced to the SI system for linear measurements. Students are presented with two ways of measuring distances: a referent and a caliper. Students are talked through a strategy for estimating using a personal referent and they are given the steps involved in reading an SI caliper. A comparison is made involving the level of accuracy allowed by different measuring instruments.

You may wish to discuss the chart showing SI units and make sure that students understand how to use the multiplying factor to convert between SI units.

Example 1

This example allows students to choose referents for given lengths. Encourage students to develop their own referents. Discuss how different parts of the hand might be used for the same referent. This discussion will tie in to section 1.2 and the discussion involving the development of the imperial system. Encourage students to use referents that are different from each other's. As students choose referents, you may wish to ask them to consider the practicality of their referent by asking questions such as the following:

- Is your referent readily available?
- How many copies of your referent are needed to measure that object?
- Can you use the same referent for more than one object?
- Would you use that referent in a real-life situation?
- In what sort of situation have you needed or used a referent in the past?

You could also have students compare their choice of referents, and see if a consensus develops as to which ones are most useful or practical.

Example 2

The solutions for this example show two methods for converting between SI units. The methods involve unit analysis, discussed in the Did You Know? on page 12, and proportional reasoning. Help students discuss the two methods by asking questions such as the following:

- Do you prefer one method to another?

- Would your choice of method depend on the situation or units? Explain why.
- Is there another method that you would prefer to the ones shown? Explain why you prefer it.

Emphasize that although students may prefer one method to another, they need to be able to use both methods. Prompt students to look for opportunities to use each method in the unit. This could lead to a metacognitive goal for students: as they progress, they can decide for which types of questions they prefer each method, and why.

Help students understand the solutions by drawing their attention to the relationships between SI units. Ask students the following questions:

- How do you convert from millimetres to centimetres? from centimetres to millimetres?
- How do you convert from kilometres to metres? from metres to kilometres?

It is important that students carefully consider the units when converting. Point out to students that in the second solution, both numerators have the same units and both denominators have the same units. You may wish to emphasize to students that keeping track of units is as important in mathematics as it is in other areas, such as science.

Example 3

In this example, students consider the length of band needed to hold together a planter made from a wooden half-barrel. Encourage students to sketch and label the planter and each band. You may wish to review the formula $C = 2\pi r$. Challenge students to consider how to solve part d) using this formula.

Discuss with the class the generalization at the end of part d). You may wish to have them test the validity of this generalization using the measurements of other cylinders before and after increasing the circumference by 1 m.

Have students do the Your Turn question and then discuss their solution with a classmate and explain how they determined the diameter of the hub of a Red River cart.

Key Ideas

The Key Ideas summarize the SI system of measurement and some of the common units used for measuring distances, as well as the process of using a referent to estimate a distance. You might have students use index cards to prepare their own summary of the Key Ideas, including a linear example for each SI unit along with a referent for this unit.

If students have not been asked to work through the solutions to Example 2 using the other method, you may wish to develop the unit analysis solution for the proportional reasoning example provided on the board. Then, develop the proportional reasoning solution for the unit analysis example provided.

Meeting Student Needs

- You may wish to have students create a mnemonic to remember the basic units of the SI system.
- Encourage students to brainstorm occupations where using a referent would be helpful, such as a real estate agent estimating the length of a room.
- To help students who may not be familiar with kilometres, create a personal referent for 1 km; suggest that they ask someone with a car to drive 1 km from a point outside the school. Have them identify a landmark at the other end and then walk the distance back.
- The concept of a standard measurement system may be new to students who have not seen or used measuring tools, because standard tools were not traditionally used. Allow students the opportunity to explore the units on an SI ruler and to practise making measurements and reading the values off the ruler.
- Have a class discussion about current examples of the use of the SI system.
- For the Example 2 Your Turn, if students are having difficulty determining an appropriate unit, suggest that they think of the conversion in one of two ways: What unit would enable the given measurement to be a more reasonable number to work with? What unit would seem appropriate to measure the distance required?

Common Errors

- When converting between SI units, students may confuse multiplying and dividing. For example, 3.5 m may become 0.035 cm.

R_x Remind students of the importance of mental mathematics and considering whether an answer is reasonable. You may wish to suggest that students estimate a measurement first, using a referent, and then perform any calculations. This will allow them an extra check to see whether their solution seems reasonable.

Answers

Example 1: Your Turn

Example for the marker tray on a whiteboard: 5 hand lengths (wrist to fingertips) with an actual measurement of 80 cm

Example 2: Your Turn

a) 20.5 cm. The centimetre is a larger unit than the millimetre and the centimetre is used more commonly.

b) 6 cm. The centimetre is a smaller unit than the metre, and then no decimals are needed.

c) 590 km. Using the kilometre gives a smaller value; this unit is more commonly used for road distances than the metre.

d) 211 cm or 2.11 m. The millimetre is not commonly used when referring to buildings.

Example 3: Your Turn

0.26 m or 26 cm

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• You may wish to have students work with a partner. • Encourage students to verbalize their thinking. • Assist students having difficulty deciding on a personal referent by suggesting a readily available tool. Have them measure their desk and demonstrate their ability to use the referent effectively before assigning any further questions. • Some students might benefit from a discussion about the usefulness of their referent.
Example 2 Have students do the Your Turn related to Example 2.	• You may wish to have students work with a partner. • Encourage students to verbalize their thinking. • Encourage visual learners to physically measure each distance if they are having difficulty determining an appropriate unit. Alternatively, you may suggest that students convert the given measurement to several other SI units until they determine one that seems most appropriate. • Although students can choose which method they prefer, they need to be able to solve questions using both methods. Have students identify their method of choice and solve several questions with their preferred process. • Have students use the other method and solve some of the same questions. Have students verbally describe how the methods differ, how they are similar, and why students chose the method they did.
Example 3 Have students do the Your Turn related to Example 3.	• You may wish to have students work with a partner. • Provide students with a similar problem to solve. • Have students draw a diagram of a Red River cart and label the measurements they know and any that they can determine using circle properties.

Check Your Understanding

Practise

Question #1 provides an opportunity for students to select a referent to use to estimate the perimeter of a shape.

For #2, students practise drawing objects of specified sizes, in specified units.

For #3, students read measurements from rulers and calipers. Students often read calipers incorrectly, so it may be necessary to discuss this question and reinforce that students must choose the divisions that line up most closely with the sliding (vernier) scale.

Emphasize that readings must be made by looking directly at the caliper and that reading it from an angle will cause errors.

Question #4 gives students an opportunity to work with ratios as they use the photograph.

Question #5 has students identify the most appropriate unit for a measurement and make a conversion. This question may provoke some discussion about whether there is only one correct answer for the most appropriate unit. Encourage students to contribute their experiences in life or in other classes to this discussion.

Apply

For #6, students relate the circumference of a circle to its radius. They make a conversion from centimetres to millimetres. This question requires students to use their estimation skills.

Question #8 may be challenging for some students. In order to make the question more accessible, you may want to have several small wheels available so that students can model the problem.

Question #9 relates to the investigation for this section. You may want to suggest to students having difficulty that they revisit that investigation. You can ask questions such as the following:

- How is the task in this question related to the investigation?
- What property of rectangles did you discover in the investigation?

Did You Know?

For #10, point out that in Inuktitut, the word *Inukshuk* means likeness of a person. The pronunciation of *Inukshuk* varies slightly in different communities. In Nunavik, it is pronounced i-nuk-suk. In other communities, it is pronounced inn-uuk-shook.

For #11, students need to measure and determine the ratios among the sizes of Canadian coins. This allows for several solution methods and could provide an opportunity for students to explain the method they have chosen and why they chose it.

For #13, students extend the idea of radius and circumference to the situation of a geostationary satellite. This may be a good opportunity for students to work with one or two partners.

Extend

Question #15 relates to photography. While students are working with scale, in this case the units are different, so students are required to make unit conversions as part of their solution. This question also links to the career of photogrammetrist as profiled in the chapter opener.

For #16, students may model the situation and add up the distances travelled, or they may find a pattern in the distances formed as the lawn is mowed. Whatever the method they use, students need to make unit conversions and decide which units to convert and when in the process to make the conversions.

Create Connections

For #17, you may wish to refer students to Example 3.

Students will need to work in teams of two for #18. This problem refers to the technique of wrapping an ankle and foot in a figure 8 pattern. Some students may not be familiar with this technique and may need help visualizing that the 8 is formed by 'circling' the ankle and then 'circling' the foot.

Question #19 is another real-life application. Students need to work within a context and make unit conversions as they solve a problem. In this question, a variety of answers are possible, so students will need to be prepared to justify their answers.

Question #20 requires students to make a judgment about the accuracy of the formula and to justify their response.

Question #21 provides an opportunity for students to work outside the classroom and develop a referent for a distance that most students are unlikely to have a personal referent for.

Unit Project

The Unit 1 project question, #14, provides students with an opportunity to use a personal referent to estimate the dimensions of a cassette case. Students can choose which instrument to use for their measurements.

Meeting Student Needs

- Provide **BLM 1–5 Section 1.1 Extra Practice** to students who would benefit from more practice.
- The last question could be linked to an "in-motion" walk for your class. You could actually measure out a route prior to class of either 1 km or 2 km. Then, have students walk the route. Each student should stop at the point where they believe they have walked 1 km. You could then indicate the 1-km marker you put out the day before.
- Allow students to use manipulatives as necessary to work through the problems. For example, in #9, students may need to physically cut out rectangles that represent the photograph and the given rectangle.
- When completing #11, allow students to work with a partner to make sure they have access to the required coins and in case there are not enough calipers for each student.
- Concrete and kinesthetic learners may benefit from using items to help them visualize and work through some of the problems. For example, students may use a tennis ball to model Earth in #13, drinking straws for #19, or a frying pan for #20.

- For #16, you may wish to allow students to use a wide highlighter or paintbrush to model the lawn mower's path around a scale diagram of the lawn.

ELL

- Remind students of the meaning of reduction factor (and enlargement factor), so that they can apply their knowledge as they work through the scale problems in the Check Your Understanding.
- Explain the meaning of a trundle wheel and what it is used for.

Common Errors

- Some students may forget how to read a caliper.

R_x Refer students to the Link the Ideas section on measuring instruments and have them review the steps involved in reading an SI caliper. Alternatively, you may wish to have students watch a video showing how to read a caliper by going to www.mhrmath10.ca and following the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1–3, 5, 7, 9, 10, 12, 14, and 18. Students who are able to complete these questions could move on to the other Apply and Extend questions.	• For #1, make sure that students have a good understanding of the use of their referent before beginning. Have them measure an item on their desk with their referent before proceeding. Refer students who have difficulty converting between SI units to the relevant worked example. Then, provide a similar problem to solve by changing the dimensions. • Encourage estimation and have students verbalize why they feel their estimates may be reasonable. • Students experiencing difficulty with #3 should be prompted in the use of the measuring device and provided with additional items to measure to demonstrate their learning. • You may wish to suggest to students completing #5 and 7 that they use their solutions as samples in their Foldable. • Question #10 provides students an opportunity to use a variety of measuring tools. Students may benefit from working with a partner for the activity. • Question #12 allows students to estimate distances. Students could write their response to the comparison between their estimate and determined distance, providing work for the problem. This could be used as an Assessment as Learning grade if needed.
Unit 1 Project If students complete #14, which is related to the Unit 1 project, take the opportunity to assess how their understanding of the chapter outcomes is progressing.	• You may wish to provide students with **BLM 1–4 Chapter 1 Unit 1 Project** and have them finalize their answers. • Students may benefit from handling and viewing an actual cassette case to help them visualize the perimeters required. • Make sure students can identify which record in the photo is the LP and which record is the 45. • Bringing in actual records for students to see and hear may further engage them in the project.
Assessment *as* Learning	
Create Connections Have all students complete #18 and #19	• Encourage students to verbalize their thinking with a partner. • The hands-on activity in #18 provides students a link between their estimation skills and actual product use. When they have completed that activity, have them write responses to questions such as the following: – Why is estimating important? – Where in your daily routine do you estimate? – Can you think of any careers that would use estimation? The responses could be entered into their Foldable or used as an Assessment as Learning piece.