1.2 Imperial Measurement

***Mathematics 10,* pages 22–35**

Suggested Timing

100–120 min

Materials

- imperial measuring instrument, e.g., imperial ruler, caliper, measuring tape
- compact disc (CD)
- cassette tape case
- MP3 player
- envelopes of different sizes
- scissors

Blackline Masters

BLM 1–3 Chapter 1 Warm-Up
BLM 1–4 Chapter 1 Unit 1 Project
BLM 1–6 Section 1.2 Extra Practice

Mathematical Processes

- ✓ Communication (C)
- ✓ Connections (CN)
- ✓ Mental Math and Estimation (ME)
- ✓ Problem Solving (PS)
- ✓ Reasoning (R)
- ✓ Technology (T)
- ✓ Visualization (V)

Specific Outcomes

M1 Solve problems that involve linear measurement, using:

- SI and imperial units of measure
- estimation strategies
- measurement strategies.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1, 4–8, 12, 14, 17, 18
Typical	#1–8, 10, 12, 13, 15, 17, 18
Extension/Enrichment	#9, 12–19

Unit Project Note that #12 is a Unit 1 project question.

Planning Notes

Have students complete the warm-up questions on **BLM 1–3 Chapter 1 Warm-Up** to reinforce prerequisite skills needed for this section.

As you introduce this section, you may wish to activate students' prior knowledge of some fraction concepts. In particular, the use of mixed numbers is common in imperial measurement, as well as multiplication and addition of fractions.

Depending on your class, you may wish to discuss the intervals on an imperial ruler. Ask students the following questions:

- What measurement unit is shown on an imperial ruler?
- What do the markings represent?
- Why are the markings different lengths?
- How can you use the different markings to help you read an imperial ruler?

Investigate Referents for Imperial Measurement

Have students work in pairs, as indicated in the student resource. After each group has made a list of referents for an inch, foot, and yard, you could have each group add its referent to a master list for the class. Questions you could discuss include the following:

- Are all of these referents close (in size) to the unit they represent?
- Are all of these referents equally accessible?
- Are all of these referents equally convenient?
- Are there specific careers or situations in which some of these referents would be particularly useful?
- Which of these referents do you already use? in what context?

Meeting Student Needs

- You may wish to discuss the origin of imperial units for distance and then have students research why Canada chose to make the transition to the SI system of measurement.

- Students could be encouraged to discuss the imperial system of measurement with a parent or grandparent. They could determine whether that particular person has been able to make the transition. In the interview, students could also discuss some of the key areas where the imperial system may still be used today and determine why not all professions have changed to SI.
- Students could create posters illustrating the various units of imperial measurement and some common items that could be accurately measured using that unit. Common conversions could also be included.
- You may wish to arrange for a carpenter or other contractor to make a short presentation to the class. During the presentation, the common units of measurement could be discussed as well as the reason this trade did not switch to SI units. Alternatively, you may wish to invite an architect, mechanic, or person from another appropriate occupation to talk to the class about the tools they use to measure and whether any use both imperial and SI units.
- Some students may find it useful to keep a taped or oral summary of what they are learning. Others may benefit from having conversion charts posted in the classroom for quick reference, particularly in the imperial system, which will be new to most students.
- You may wish to have students measure objects in the classroom to determine whether they were likely built using the SI measurement system or the imperial system.
- Using tangible materials, such as measuring cups, tools, and so on, have students devise ways in which they can use the imperial measurement system to generate problems for their classmates to solve.
- Some students may benefit from conducting a survey of their friends and relatives to see who uses SI units and who uses imperial units. Do some people use both? Do more elders use imperial units? Which units do students feel most comfortable using?

ELL

- The imperial system can be better understood through constant exposure to imperial measuring devices. Post signs on windows or teacher boards or bulletin boards that label the sides using various imperial measurements. For example, one board could show measurements in feet and inches. Another could show yards.
- To help students recall the concept of scale factors, you may wish to demonstrate a reduction. Drawing it to scale using an imperial ruler on the board would tie the new knowledge to previous knowledge. Discuss how a reduction can also be seen as an enlargement and how to state a scale factor. Discuss strategies for using mental mathematics to determine whether a given scale indicates a reduction or an enlargement.

Enrichment

- Have students ask someone familiar with construction to explain why imperial measurement is commonly used in that industry and have students try to find out why a *two-by-four* does not measure 2 in. by 4 in.

Gifted

- Have students investigate the way in which imperial measurements have been improved in terms of consistency using science. For example, how is the measurement of a yard maintained as the temperature of the surroundings changes?

Common Errors

- Some students may try to perform fraction operations with mixed numbers.

R_x Remind students to write a mixed number as an improper fraction before performing operations.

Answers

Investigate Referents for Imperial Measurement

1. a) 12 in. **b)** 3 ft **c)** 36 in.

2. inch: eraser, foot: ruler or your own foot (two, if your feet are small), yard: one stride or big step or a metre stick.

3. 12 city blocks, or the distance travelled in 1 min by car or in 15 min by walking

4. a) Example: A referent might be 1 pencil length is approximately 6 in. long. Measure the table or desk top as 6 pencil lengths wide by 4 pencil lengths long. Then, multiply these measurements by the number of inches in 1 pencil length. An estimate for the dimensions of a desktop might be 36 in. by 24 in.

b) Examples: the referent is smaller than the object being measured; the referent is readily available and easy to use.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the investigation. Encourage them to generalize and develop a method of estimating distances.	• Some students may benefit from referring to their work on estimating in SI units in section 1.1. • As the imperial system may be very new to most students, provide ample opportunities for measuring with a referent and a measuring device. Provide objects to measure that have great variation in sizes; for example, the height of a door knob versus the length of a counter. • For part b), students may find it helpful to explain why other items may not have been appropriate referents, and this might help them verbalize why they chose their particular referent.

Link the Ideas

The Link the Ideas section shows two different methods for converting between imperial units: one uses proportions and the other substitutes inches for yards. Students should be comfortable with a variety of approaches to unit conversions, whether working with imperial or SI units.

You may wish to emphasize the conversions, because students are unlikely to be familiar with conversions among miles, yards, and feet. Draw students' attention to the symbols for feet (') and inches ("). You may want to have students examine their measuring tools to discern the differences from SI devices. In particular, you could ask the following questions:

- How many divisions is 1 in. broken into on your device?
- Does this make it practical to write fractional measurements using decimals?
- What form will measurements including parts of inches take?
- What is the smallest division on your device?
- Consider the measurement $\frac{1}{2}$ in. Using the divisions on your device, how many equivalent measurements are possible?

Use the Measuring Instruments section starting on page 23 to discuss the use of a ruler, measuring tape, and caliper. Make sure students understand how to read the particular type of caliper that you have available in the classroom. The calipers shown in the student resource measure to the nearest thousandth of an inch and give readings in decimal form. To read this type of caliper,

- Read the whole number and tenth from the fixed scale.
- Determine what fraction of a tenth is indicated by the zero on the moving scale (e.g., 0.025, 0.05, 0.075).
- Find the subdivision on the moving scale that best lines up with the subdivision on the fixed scale. Read this number as a thousandth.
- Now, add the numbers from each step.

Example 1

There is likely to be some discussion about measuring the height of the bear in the example. The fact that the bear is walking adds complexity to this discussion. Most experts agree that the best way to measure the height of a bear is to measure at the front shoulder. Similarly, biologists measure the length of a polar bear from the nose to where the tail joins the rump.

You may wish to use the Think-Pair-Share strategy to discuss this example. In this technique, each student works individually. When finished, each person works with a partner. They compare and contrast techniques, and correct any errors or misconceptions. Then, one person from each pair shares their work with the whole group. Try to discuss several different solutions

You may need to refer students to the investigation as they rewrite a number of inches as a measurement in feet and inches:

- Look back at the investigation. How many inches are in 1 ft?
- Can we find a pattern to convert inches to feet and inches? For example, how would you rewrite 18 in.? 24 in.? 32 in.? How does this apply to our example?
- How would you describe a method for rewriting inches in feet and inches?

Example 2

This situation involving the size of a television is one in which inches are the standard unit of measurement. As such, it highlights the need for fluency in both measurement systems.

To begin the discussion, you could ask students the following questions:

- What does it mean when the screen size of a TV is stated as 50 in.?
- If a TV screen is 50 in., can we determine the actual dimensions of the screen? What other information, if any, would be required?

This discussion may help students see the need to make the comparisons in the example.

This example provides a natural place for students to recall the Pythagorean relationship, although it requires a more algebraic approach than students are likely used to. Some questions to help them include

- In which situations does the Pythagorean relationship apply?
- Looking at the use of the Pythagorean relationship in this solution, how is the order of operations important?
- What process allows us to determine the value of x when we know the value of x^2?

Example 3

Ask students what referent they would use for the drum frame. Then, discuss how large they think the actual drum might be. Have them talk throught the solution.

To help students express their answers in yards and inches, ask them the following questions:

- Do you recall how many inches are in 1 yd?
- How is writing an answer in yards and inches similar to what we did in Example 1? How is it different?

Did You Know?

Point out that the *qilaut* is pronounced kee-la-oot. This drum is also called a wind drum. It is divided into two sections: the *isik*, which is the surface that is struck and the *pablu*, which is the handle of the drum. It is used for drum dances.

Key Ideas

This may be a natural place to compare aspects of the two measurement systems. For example, in SI there is a big difference between the units 1 cm and 1 m, while in imperial there is an intermediate unit: the foot exists between the inch and the yard. Students could discuss whether this poses any practical problems, and if so, how this makes each system more or less convenient in different situations.

Meeting Student Needs

- Place posters in your classroom illustrating the connections among inches, feet, yards, and miles.
- You may need to spend extra time showing students how to convert from inches to feet. Place emphasis on the idea that $3\frac{1}{2}$ ft is the same as 3 ft 6 in., or other similar conversions where students must change fractional (or decimal) values to accurate measurements.
- Spend adequate time examining Example 3; students generally have an interest in the technology of LCD televisions. You may wish to have some students investigate other aspect ratios of televisions, including 720 versus 1080 pixels, and so on. Allow students to make a presentation to the class, including the comparison of the standard 4:3 aspect ratio and the 16:9 widescreen aspect ratio.
- Students may wish to confirm that the diameter of the drum in Example 2 is just over 1″. Allow students to verify this estimate using a ruler.

Gifted

- Photographs taken with a digital camera generally use the 4:3 ratio; however, when photographs are printed, they traditionally use the ratio 3:2, as in the most common print size, 4" × 6". Have students investigate the maximum possible area that can be preserved from a 4:3 photo file if a 3:2 print is to be made.

Common Errors

- Students may incorrectly determine the power of a quotient and only determine the power of one of the factors. Students may write $\left(\frac{16}{x}\right)^2 = \frac{16}{x^2}$ or $\left(\frac{16}{x}\right)^2 = \frac{256}{x}$.

R$_x$ Remind students that when calculating the power of a quotient, such as $(ab)^m$, they need to rewrite each factor in the quotient with the same exponent and then determine the powers.

Answers

Example 1: Your Turn

5 ft, 2 ft 6 in.

Example 2: Your Turn

The standard television has a greater viewing area by about 112 in.^{2}.

Example 3: Your Turn

$13\frac{3}{4}$ ft

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• You may wish to have students work with a partner. • This is a multi-step problem. Prompt students to verbalize the steps needed to solve the problem. They may wish to check off each step as they work through the problem. • Some students may need prompting and reminding of the meaning of scale. Coach students through the questions and provide an additional question from the problem set for them to demonstrate their learning. • Provide students with a similar problem to solve.
Example 2 Have students do the Your Turn related to Example 2.	• Provide students with a problem similar to Example 2 before they work on the Your Turn. Allow them to work with a partner and talk through their thinking. • Allow students to select one method for determining the scale factor and to use only this method when solving the problem. • Encourage students to describe the steps they will follow as they work through this problem. • Some students may need coaching and review work on the Pythagorean relationship. Provide students with several sample questions to determine the hypotenuse to one decimal place.
Example 3 Have students do the Your Turn related to Example 3.	• You may wish to have students work with a partner. • Have students draw and label a diagram to visualize the problem. • Some students may benefit from cutting one piece of string to represent the perimeter of the frame and another piece of string to represent the length of sinew. Then, have students explain the relationship between intervals on each piece of string.

Check Your Understanding

Practise

For #1 and 3, students read measurements from rulers and calipers. It may benefit some students to review the Link the Ideas starting on page 23.

Question #2 requires students to round imperial measurements. Many students may benefit from some discussion of their results and methods with a partner, a small group, or the class.

For #4, students will need access to imperial rulers, calipers, and/or micrometers. You may wish to talk through measuring an item to the nearest sixteenth of an inch before having students complete this question.

For #5, you may wish to have students assess the accuracy of their referent compared with the actual measurement.

For #6, it may be beneficial for students to work with a partner. Encourage students to justify their choice of a personal referent.

For #7, in order to give students a sense of the significance of the feat, you could invite them to compare Billy Loutit's achievement to that of a contemporary marathon or long-distance runner.

Apply

The window in #8 is a composite figure, and allows students to activate prior knowledge of perimeter and circumference.

When students have completed #9, you might ask them to consider these questions:

- What ratio do you get between the drive wheels and caster wheels if you use the circumferences?
- How does this compare to the ratios between the diameters? Why?

Have students discuss the strategies they used to solve #10. They may use the idea of a constant length-to-width ratio or they may approach it by using proportional reasoning.

Question #11 addresses an error often made with imperial measurements. While students are quite used to equating the fraction $\frac{1}{2}$ to the decimal value 0.5, this is not as straight-forward when measuring in feet and inches.

In question #13, discuss with students the difference between distances according to a GPS reading and what they might find if they walked the same distance.

Extend

Consider having students work in pairs to complete the Extend questions.

When working on #14, encourage students to discuss the advantages and disadvantages of using each length of material. They may extend this discussion to consider how the different lengths of material could affect both the amount of waste and the circumference of the retaining wall around the pool, and how the distance between the retaining wall and the pool wall might affect the pool's ability to retain heat. Remind students that they need to justify their ideas for part c); there is no one correct answer.

In #15, students use proportional reasoning to make conversions between miles and astronomical units, a unit that is likely new to most students. This is a more abstract problem.

Create Connections

For #17, ask students to consider, "What would be some implications of purchasing the bed without first doing the mathematical analysis?"

Question #18 refers to a unit that students might not be very familiar with, the megawatt (MW). You may wish to mention that mega is an SI prefix meaning million. Some students may recognize that the watt is a unit for measuring electricity as indicated on light bulbs.

You may wish to have students work in pairs for #19, then compare and discuss their solutions with another group.

Unit Project

You might have students use **BLM 1–4 Chapter 1 Unit 1 Project** and finalize their answers to #12 on the master.

You may wish to have a class discussion, before students work on #12, on how the sizes of music devices have changed over time and whether students think that music devices could be getting too small and what some disadvantages or advantages might be.

Students measure the dimensions of a CD, cassette case, and MP3 player using imperial units.

Meeting Student Needs

- Provide **BLM 1–6 Section 1.2 Extra Practice** to students who would benefit from more practice.
- Ensure that students spend an adequate amount of time measuring various objects in the classroom. Encourage students to record their results and then have a classmate check the measurement. If they do not have the same measurement recorded, they will need to discuss which answer is correct and why.
- Students may benefit from using linking cubes when working on #12.
- When working on #9, concrete and kinesthetic learners may benefit from cutting out a circle to represent each wheel. This may help students visualize the required ratio as they solve the problem.

ELL

- Explain to students that a Norman window is a composite figure with a semicircle above a rectangle, as illustrated in #8.
- Some students may not be familiar with the term *geocache* referred to in #13. Explain to students that it is a hiding place (or cache) in the ground that can be used for storage.

Enrichment

- You may wish to have students use imperial calipers that measure in 32nds or 64ths of an inch. Students will be required to add the fractions from each step of the reading. For most students, this is a more difficult task and will require additional time and practice.
- You may wish to ask students to estimate the length of magnetic tape in a 90-min audio cassette tape and then research to find out the actual measurement.

Common Errors

- Some students may confuse the markings on an imperial ruler.

R_x Remind students to identify the whole numbers that indicate inches on the ruler and then look at the markings that represent fractions of an inch.

- In #2, students may not give their answer to the required fraction of a unit.

R_x Make sure students understand what units the question is asking for; that is, for part a), students need to understand that their answer will be given in either whole inches or whole inches plus one half of an inch. You may wish to suggest that students write down some sample measurements that indicate the possible fractions of a unit that their measurement could be written in.

- In #11, students may think that 37.5 ft is the same as 37 ft 5 in.

R_x Remind students that 0.5 is a decimal which equals $\frac{1}{2}$. So, 37.5 ft = $37\frac{1}{2}$ ft, which is the same as 37 ft 6 in., not 37 ft 5 in.

WWW Web Link

Some students may find a visual representation helpful when working on #7. For an illustration of the journey completed by William Loutit, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1–8. Students who have no problems with these questions can go on to the remaining questions.	• Have students draw part of an imperial ruler and highlight the smallest subdivision. • Some students may benefit from a review and coaching on how to identify the fraction of a segment being represented on the ruler. • Provide additional coaching to students who have difficulty reading imperial measuring instruments. Talk them through corrections and clarify any misunderstandings before allowing them to proceed. • Questions #1 to 6 are directly related to measuring. You may wish to provide additional classroom examples for students to practise their measurements. Some students may require additional coaching to work with the scale values given. Reviewing Example 1 may assist students in their attempts to begin. • Some students may need prompting for #7. To assist them in setting up their ratio, ask them what fraction of an hour 15 min represents. • Students having difficulty with #8 should be asked to verbalize their thinking. Some students may attempt to measure the trim on the window, including the semi-circle at the top. Ask these students what formulas they have used in the past that allowed them to find the distance around a circle. Prompt them to explain how they could use this to solve the problem. • If students are having difficulty using scale factors, you may wish to use an overhead and measure or estimate the dimensions of a picture, paragraph, or something "projectable." Enlarging and measuring it may benefit the entire class.
Unit 1 Project If students complete #12, which is related to the Unit 1 project, take the opportunity to access how their understanding of the chapter outcomes is progressing.	• You may wish to provide students with **BLM 1–4 Chapter 1 Unit 1 Project** and have them finalize their answers. • You may wish to suggest that students store their work on #12 in their project portfolio. • Encourage students to compare units used with a partner. Some students may require prompting relating to the unit of size in the imperial system. Ask questions such as the following: – Which is the largest unit? – What units can we use to measure small distances? • If students have placed the conversion charts in their Foldable, encourage students to use these throughout their work. The emphasis is not on how much each device stores, but on the comparison of size by calculating diameter and perimeter. • Where possible, bring in samples that the students can handle and accurately measure.
Assessment *as* Learning	
Create Connections Have all students complete #17 and 18.	• Encourage students to work with the dimensions they know and to cut out 2-D scale diagrams to represent the furniture, so that they can move each piece around to see how things might fit in the bedroom. • You may wish to have students share and compare their answers to #18 with a partner.