Converting Between SI and Imperial Systems

1.3

Mathematics 10, **pages 36–47**

Suggested Timing

180–240 min

Materials

- compact disc (CD)
- SI measuring instrument
- imperial measuring instrument

Blackline Masters

BLM 1–3 Chapter 1 Warm-Up
BLM 1–4 Chapter 1 Unit 1 Project
BLM 1–7 Section 1.3 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcomes

M1 Solve problems that involve linear measurement, using:

- SI and imperial units of measure
- estimation strategies
- measurement strategies.

M2 Apply proportional reasoning to problems that involve conversions between SI and imperial units of measure.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–4, 6, 7, 9, 10, 14, 17, 18
Typical	#1–8, 11, 12, 14
Extension/Enrichment	#5, 14–18

Unit Project Note that #14 is a Unit 1 project question.

Planning Notes

Have students complete the warm-up questions on **BLM 1–3 Chapter 1 Warm-Up** to reinforce prerequisite skills needed for this section.

This section in the student resource begins with a suggestion to think of songs that include measurements in the title or lyrics. You may wish to prepare for the discussion by finding a few examples yourself before the class starts. In your classroom discussion, you could consider the ratio of songs that include imperial units to those that include SI units. You could classify songs that mention measurements according to genre or decade to see if there are trends. You could see which student has the highest number or highest proportion of songs mentioning measurement on his or her personal listening device. As most students identify strongly with music, this is likely to be a lively discussion, one that will take as much time as you are willing to devote to it.

The student resource suggests that students determine SI measurements comparable to the imperial measurements in the songs. This is an excellent time to practise estimation and mental mathematics, as well as to revisit the personal referents students established in the prior two sections. In addition, rewriting song titles or lyrics by replacing measurements with your students' personal referents is certain to create some humour in your classroom.

Investigate Relationships Between SI and Imperial Measurements

Unit Project

Have students complete this investigation individually or in groups of two to three students. During this investigation, which is part of the Unit 1 project, students will need to use measuring instruments, such as rulers, to measure the diameter of a CD. Some students may find it challenging measuring the diameter without knowing where the centre of the CD is located. You may wish to discuss some strategies for finding the diameter of a circle in this situation.

Students are asked to determine the diameter of several sizes of vinyl records by converting from imperial units to SI units. Explain to students the difference between proportional reasoning and unit analysis in unit conversions. Remind students that in proportional reasoning, an equation is used to solve for an unknown. In unit analysis, an expression is created to eliminate the initial unit and work toward the desired new unit.

Students also determine the circumferences and compare the sizes of the four recording devices. After students have determined the circumference of a CD, encourage them to check their calculation by measuring the circumference, to the nearest millimetre. Students may choose to use a measuring tape or a piece of string.

Students compare the amount of music stored on a CD and a vinyl record to the size of the device. Some questions to consider include the following:

- How many songs are on a typical CD? vinyl record?
- How many minutes of music can a typical CD or vinyl record play?
- What measurements could you compare on a CD and vinyl record?
- What are some advantages to having small-sized music storage devices? What are some disadvantages?

Meeting Student Needs

- Some students may benefit from tracing the outline of a CD onto paper and then measuring the diameter.
- You may wish to have students design and measure a pattern for a dance outfit using both imperial and SI measurement systems. Have students explain which system they prefer to work with and why. Can they think of a situation where they might prefer to use the other measurement system?
- Some students may benefit from asking their grandparents or elders which system they prefer to use and why. How did they feel in the 1970s when Canada transitioned to the SI system? You may wish to invite a Community Elder or one student's grandparents to share their experience.

ELL

- Prior to CDs, long-play records (LPs) were used for storing music.

Enrichment

- Have students develop methods of converting (approximately) between SI and imperial using mnemonics.

Gifted

- Ask students to research the connection between 300 000 000 m/s and 186 000 mi/s. What travels that quickly?

Answers

Investigate Relationships Between SI and Imperial Measurements

1. a) 12 cm. One small fingernail is approximately equal to 1 cm. Count the number of fingernails along the radius of a CD to be 6. Then, multiply by two. The diameter measures approximately 12 fingernails, which is approximately 12 cm.

b) The diameter of a CD measures approximately $4\frac{1}{4}$ paper clips, which is approximately $4\frac{1}{4}$ in.

2. 120 mm

3. 30.48 cm, 25.4 cm, 17.78 cm

4. a) CD: 37.7 cm, vinyl records: 95.76 cm, 79.80 cm, 55.86 cm

b) Example: The CD has the smallest circumference of the four recording devices. The LP has a circumference that is about double the circumference of the 45 record.

5. a) Example: CD

b) Example: Determine the ratio of the number of minutes of music stored on a device to the area of one side of the device. Example: a CD might store 0.6 min per cm^2 or 3.9 min per $in.^2$.

c) Example: Laser technology enables tremendous amounts of music to be stored on very small devices and to be played with high quality sound.

Assessment	Supporting Learning
Assessment *for* Learning	
Unit 1 Project Have students complete the investigation.	• Consider having students work in pairs. • Help students recall what they learned in earlier courses about determining diameter and circumference and making conversions between SI and imperial units for length. Provide coaching for students who need it. • You may wish to provide students with **BLM 1–4 Chapter 1 Unit 1 Project** and have them finalize their answers.
Assessment *as* Learning	
Reflect and Respond Listen as students discuss the amount of music held on a CD versus a vinyl record.	• Encourage students to suggest and consider many methods for comparing the amount of music and size of recording devices. • This reflection allows students to bring together the measurements they have completed in sections 1.1 and 1.2 and to draw the conclusion that bigger is not necessarily better. Some students will need to have their attention drawn to the number of songs stored on each device. Encourage them to write ratios for each to help them visualize the size of the device versus the capacity.

Link the Ideas

This section provides an opportunity to discuss the difference between exact and approximate values. The conversion given, 1 yd = 0.9144 m, is an exact value. Students can compare this, for example, with the times in this unit when they have made calculations using pi and have had to give approximate answers. You may want to have the class brainstorm some reasons that an answer would be approximate and instances where exact values exist. You could also discuss situations in which an exact value is required and those in which an approximation is expected.

This section also provides a reference for exact and approximate conversions between SI and imperial measurements.

Example 1

Encourage students to use mental mathematics to estimate the answer to this question before beginning the calculations or looking at the solution. You may want to allow different students to show their method and defend their choice of unit.

For access to more information about the research project referred to in this example, see the Web Links at the end of this section.

Example 2

Visualization is an integral part of this example. As students determine the dimensions of the mats or read the solution in the student resource, ask them, "Why is the factor 30.5 being used in the calculations for each measurement system?" This will direct their attention to the scale being used and may help explain why the length-to-width ratio is the same for the mats regardless of the measurement units used. This relationship was explored previously in the section 1.1 Investigate.

To determine the number of mats needed, the relevant information is not the area of the mats or the area of the gymnasium floor, but rather the (linear) dimensions of the gym floor and the mats. It is quite common for students to solve problems like these by comparing the area of the gym to the area of one mat and obtaining the answer by dividing. A common situation that models this problem is the tiling of a floor. Encourage students who may have encountered a similar situation to share their experiences.

To illustrate this, you could present a similar but smaller problem. Once the size of the mats has been determined to be 8' by 4', suppose that a floor measuring 20' by 10' needs to be covered. Ask students to find the relevant areas and determine the number of mats needed according to the area calculation. Then, have the students make a diagram of how they could physically lay out the mats, and count them this way. This experiment should help them see that it is the length and width of the mats and the room dimensions that matter in this problem, not the areas.

The Your Turn questions allow students to explore this concept in a different context. You may want to remind students that drawing a diagram is an important part of the problem solving process.

Example 3

Many students in grade 10 are learning to drive. A short discussion of reaction time would be appropriate. Have students brainstorm factors that affect reaction time, as well as braking time, before reading those listed in the question. You may wish to have students predict whether reaction time or braking time is affected more as speed increases and whether students expect total braking time to be directly proportional to speed. For example, do students think that braking from 100 km/h takes twice as long as braking from 50 km/h.

It will take some practice for students to learn to read the graph given in this example. Before working through the example, you may wish to have students work with a partner and determine the answers to questions like the following:

- What does the 16.7 m label indicate?
- The dot farthest to the left is labelled with 135.6 m and 6.76 s. What do each of these values represent?
- How could you determine the reaction time for a given speed?
- Why is the graph curved?

The SI calculations are quite straightforward in this example. The imperial calculations allow for students to make conversions by their preferred method.

This example also provides an opportunity to revisit personal referents. Given that driving is (or will be) a reality for much of the class, students should recognize that this is a situation in which it is important to have a way to estimate distances in the 50–150 m range.

When students have completed the Your Turn section, you may want to complete your discussion of the non-linearity of this relationship. That is, they should note that the increase in stopping distance from 110 km/h to 120 km/h is proportionally larger than the increase in speed.

Key Ideas

As you discuss the Key Ideas, you may wish to ask students to describe the consequences of using standard units in different measurement systems when solving a problem. Some real-life examples include a 767 aircraft that made an emergency landing in Gimli, MB, when it ran out of fuel, and the Mars Climate Orbiter lost by NASA because one team working on it measured in SI units while another measured in imperial units.

This may be a good time to further discuss situations in which estimating values is acceptable (for example, theoretical braking distance) compared to situations where estimating is not appropriate (installing baseboards).

Meeting Student Needs

- You may wish to have students create bookmarks from recipe cards. The bookmarks could summarize the various units of measurement in both the SI and imperial systems. This would enable students to have an easy reference chart when working through problems in this section. You would need to determine whether students will be allowed to use this bookmark for the summative assessment for the chapter.
- Example 2 will be of interest to students currently participating in a driver training program. You may wish to have an accident investigator visit your classroom to discuss some of the methods used to determine the speed at which an accident might have taken place. The presentation should include pictures of skid marks and a discussion on how skid marks are used to analyse and determine which person is at fault.
- Allow students to use a manipulative to represent the mats in Example 3 and lay them out in various patterns to fill a designated space.

ELL

- For Example 2, review the labels on the chart and make sure that students understand what is being indicated. Encourage students to make the connection between the description of the label and the measurement units in the chart.
- In Example 3, the terms *rows* and *columns* may be new to some students, although most students should be familiar with these terms from using tables and spreadsheets. Explain to students that columns run vertically side by side and rows run horizontally above and below each other. Students may find it helpful to use a diagram with labelled columns and rows.

Answers

Example 1: Your Turn

400 m = 0.4 km and 1 mi ≈ 1.609 km. So $\frac{1}{4}$ mi ≈ 0.40225 km.

Therefore, the two measurements are not equivalent. Example: The SI measurement is likely more accurate because the SI system is used internationally and might be the preferred system at an international competition, such as the Olympics.

Example 2: Your Turn

a) 16 paving stones **b)** 88 tiles

Example 3: Your Turn

a) 6 ft **b)** 55.923 mph, about 77 yd

Web Link

For more information about the research done on "frozen light," go to www.mhrmath10.ca and follow the links.

To learn more about the first SI aircraft in Canada making an emergency landing in Gimli, MB, including specific details about the measurements and subsequent calculations, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• You may wish to have students work with a partner. • Suggest that students write the conversions presented in section 1.3 into their Foldable for future reference. • Multiple approaches are possible in solving the problem. Ask students to verbalize what they see as being reasonable units to compare for miles and metres. Encourage students to predict which is likely more accurate and why. • Allow students to write the units in their ratios to help visualize the conversion process.
Example 2 Have students do the Your Turn related to Example 2.	• You may wish to have students work with a partner. • This is a multi-step problem. Prompt students to verbalize the steps needed to solve the problem. They may wish to check off each step as they work through the problem. • Allow students who are having difficulty to measure and cut out a representative tile that they can then use their referent on to make predictions. They may also wish to mark off an area of 1 square yard to help them visualize the placement. Although it would not be practical to do this for every real-life scenario, it is important that students be provided opportunities initially to make these comparisons and engage in these hands-on approaches. They are converting between two systems of linear measurement, one of which is new to them. Measuring and cutting out the two areas allows students to strengthen their visual images of what is being compared. • Allow students to work through the example by laying the mats in the other direction, i.e., horizontally. Encourage students to discuss their results with a classmate.
Example 3 Have students do the Your Turn related to Example 3.	• You may wish to have students work with a partner. • This is a multi-step problem. Prompt students to verbalize the steps needed to solve the problem. They may wish to check off each step as they work through the problem. • You may wish to work through Example 3 as a group and then have individuals complete the Your Turn. • Encourage students to write out all the values with the units from the diagram provided in stopping distances before beginning their conversions. • You may wish to provide the class with a scenario such as, "You are travelling at 70 mph and a deer crosses the road. You have 4 feet to stop. Will you be able to stop in time?" This would provide students an additional opportunity to demonstrate their understanding of the chart as well as conversions.

Check Your Understanding

Practise

Students must use an SI ruler to complete #4.

Question #6 involves the use of calipers. Some students may benefit from a quick discussion of this measuring device.

Apply

For #7, students are given data in imperial units and required to solve the problem and give final answers in SI units.

Did You Know?

Point out that in Iniktitut, *komatik* means dog sled. It is pronounced ko-'ma-tik. A Web Link with audio of several Inuit terms is referenced at the end of this section.

For #8, you may wish to have students work in pairs, with one student converting the distances to imperial units and the other student converting to SI units. Then, have students compare and discuss their answers.

Question #9 has students discuss their methods of conversion with a classmate. Students should justify their preferences. Also, they should identify whether they prefer the same method for each given conversion or if their preferences vary. This may also be a good time to discuss exact versus approximate values again.

For #11 students need to choose a unit in order to make a comparison.

In #13, students need to determine the perimeter of a composite object with curved sides and discuss their steps with a classmate. Then, students identify and describe what they believe to be the easiest way to solve a similar problem.

Extend

Question #15 gives students additional experience in determining the number of small objects needed to complete a particular task.

For #16, students work with an unfamiliar formula and the measuring and estimating skills they have acquired in this chapter to find the area under a curved arch. You may wish to have students discuss the pros and cons of using the formula for imperial units

Create Connections

Question #17 provides an opportunity for students to look back and summarize their learning.

You may want to allow classmates to compare their answers to #18 to see which units were chosen and whether the factors are consistent.

Unit Project

For #14, students compare the storage on an MP3 player to that on an LP. The size of this ratio is likely to make students question the answers they obtain.

You may wish to have a class discussion about why it might be practical to have small devices with great storage capacities.

Meeting Student Needs

- Provide **BLM 1–7 Section 1.3 Extra Practice** to students who would benefit from more practice.
- For #4, you may wish to have floor plans of cottages, homes, bedrooms, etc., available. Students could be encouraged to visualize the placement of various pieces of furniture, and they could discuss whether the rooms would be adequate in size. This could serve as a project for those students who prefer to work with a partner.
- Students sometimes have difficulty attempting to use unit analysis when converting between or among measurement systems. Allow students to convert units in intermediate steps to get them closer to the required conversion. You may wish to have students review the examples in the student resource that explain unit analysis.

Common Errors

- Some students may use diameter or circumference rather than radius when solving #12 because the question is about Earth.

R_x Emphasize to students the importance of reading a problem carefully and making sure they understand what they are being asked to find.

- Some students may convert the harness length in #7 to an inappropriate unit in SI.

R_x Once they have completed their conversion, ask students to explain their rationale for the unit they used. Provide students with a similar scenario, such as a tug of war rope, the length of their classroom, and ask them what unit would be most appropriate.

WWW Web Link

For audio files that provide pronunciation of common Inuit words, including *quimmiq*, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1–4, 6, 7, 9, and 10. Students who have no problems with these questions can go on to the remaining questions.	• Questions #1 to 4 provide students the opportunities to convert between measurement systems. • Prompt students to determine which of the measurements given in the floor plan for #4 will assist them in determining the scale. Encourage students to write the units in their ratios when completing the conversion so it is clear what they are solving for. • For #6, ask students to identify whether the ruler represents SI or imperial units before reading the values. Some students may need prompting to help them determine the fractional divisions. • It is important that all students be provided with ample opportunities to visualize equivalent linear measurements in SI and imperial systems to assist students in making mental connections while learning the imperial system. • Question #10 provides students a real-world calculation of distances. Prompt students through the conversion. You may wish to provide an additional question to check for understanding such as 15 km versus 9 miles. If students are struggling with number calculations, keep the values smaller because you are checking for understanding in conversions and not for the students' ability to manipulate larger numbers.
Unit 1 Project If students complete #14, which is related to the Unit 1 project, take the opportunity to assess how students' understanding of the chapter outcomes is progressing.	• You may wish to provide students with **BLM 1–4 Chapter 1 Unit 1 Project** and have them finalize their answers. • Question #14 provides students with a link to the measurements they have been making and the reality of how storage devices for music have changed over time. • Assist struggling learners by asking them to verbalize and demonstrate how they will determine the number of LPs needed to store 20 000 songs. Prompt students through how this number will be used to find the height of the stack. • Ask the class, in general, whether they expect a smaller number for the number of required LPs or a larger number. To assist in putting their answers in perspective, challenge the class to generate some examples of easily visible heights that would model their LP stack.
Assessment *as* Learning	
Create Connections Have students complete #17 and 18.	• Both questions #17 and 18 allow students to generate their own examples and explain their thinking related to measurement and conversion. You may wish to suggest that students write their responses into their Foldable. Encourage them to use examples in #17 that are common or often used measurements as they will likely be using these types of measurements in subsequent chapters. Encourage students to write out as many examples as they feel are necessary to help prompt their own thinking in other chapters.