Surface Area

2.2

Mathematics 10, **pages 66–79**

Suggested Timing

100–120 min

Materials

- right cones
- spheres
- right pyramids
- right cylinders

Blackline Masters

BLM 2–3 Chapter 2 Warm-Up
BLM 2–5 Chapter 2 Unit 1 Project
BLM 2–7 Investigate Surface Area of Three-Dimensional Objects
BLM 2–8 Section 2.2 Extra Practice
TM 2–1 How to Do Page 79 #18 Using TI-Nspire™
TM 2–2 How to Do Page 79 #18 Using Microsoft® Excel

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcomes

M3 Solve problems, using SI and imperial units, that involve the surface area and volume of 3-D objects, including:

- right cones
- right cylinders
- right prisms
- right pyramids
- spheres.

AN3 Demonstrate an understanding of powers with integral and rational exponents.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–4, 6, 8, 9, 19, 20
Typical	#1–3, 5, 6, 8–10, 12–14, 18–20
Extension/Enrichment	#3, 8, 10–12, 14–20

Unit Project Note that #8 is a Unit 1 project question.

Planning Notes

Have students complete the warm-up questions on **BLM 2–3 Chapter 2 Warm-Up** to reinforce prerequisite skills needed for this section.

As a class, read and discuss the introductory text and photograph of the Blackfoot Crossing building. The first bullet in Enrichment on TR page 55 suggests how you might incorporate information about this site into classroom work.

You might discuss that architects work with other professionals including structural, mechanical, and electrical engineers to apply knowledge of surface area for calculating the amount of materials needed to construct the design. Ask how each professional applies knowledge of surface area.

You might ask what types of 3-D objects are components of the building design (e.g., truncated cones, cylinders, curved walls). Consider having students brainstorm other careers that require knowledge of surface area (e.g., furniture design, clothing design). Have students describe an application of surface area for each career.

Investigate Surface Area of Three-Dimensional Objects

In this Investigate, students work in groups and use their knowledge of surface area to develop and share strategies for determining the surface area of a right cylinder or a right rectangular prism.

When forming groups of three to four students, ensure that each group represents students with a range of abilities and learning styles. Ensure that cylinders and prisms are represented among the groups.

For #1, you might have groups use **BLM 2–7 Investigate Surface Area of Three-Dimensional Objects** to record their work. For Quadrant 3, consider having students estimate the surface area before doing the calculation.

Much of the learning for this section should take place within the Investigate; therefore, allow sufficient time for groups to circulate and complete their work.

While students are working in their groups, circulate and ask questions to help them focus on the key ideas. Consider asking the following questions:
- What do you recall about surface area?
- What ways do you know for determining the surface area of right prisms and right cylinders?
- What methods do you have for representing 3-D objects in 2-D form?
- How can representing a 3-D object in 2-D form help you determine the surface area?
- Does the diagram provide all the information you need to determine the surface area? Explain.

While groups review the work of other groups, ask the following questions:
- How do you know whether this work is correct?
- How can you differentiate between a method that is different than yours and one that is incorrect?

Debrief after the activity by having students discuss their findings. You might have students summarize the strategies used to determine surface area.

Give each group sufficient time to complete the Reflect and Respond questions. For #6, prompt students by asking the following questions:
- How are cones and pyramids similar to prisms and cylinders? How are they different?
- Some objects contain right triangles. What do you know about right triangles? How can you use what you know about right triangles to help determine surface area?

As a class, compare and contrast the group responses. Find out if there is a consensus among students about which (correct) methods are preferred. (Students might say for a cylinder: use a net to identify each surface; determine the area of one circular base ($SA = \pi r^2$); add the areas of the two circular bases; determine the area of the rectangular area (using $C = \pi d$ to find the length); and add the areas together). Honour the preferences, even if some choices are unconventional.

In your debriefing, help students make connections to their prior knowledge. In earlier math courses, they worked with 2-D nets of 3-D objects and learned how to determine the surface area of rectangular prisms and cylinders. The discussion is important for students to understand the development of the surface area formulas, rather than memorize the formulas.

Have students connect what they discussed in #1 to 5 in their answers to #6. Consider starting with the right pyramid. Ask the following questions:
- How can you adapt techniques you used in calculating the surface area of a right prism or right cylinder to calculating the surface area of a right pyramid?
- What is the shape of each surface of a pyramid?
- How can this help you with calculating the surface area?

Have students do a hands-on activity to help them consider how to determine the surface area of a cone. Have students cut out a large circle. Use the following prompts:
- How can you make a cone using this circle? (Cut along one radius to the centre and overlap part of the circle.)
- What is the area of your circle?
- Look at the cone you have made. Is it the same area? Explain. (The cone will have a smaller area than the original circle because, to make it, you overlap parts of the circle.)

Have students experiment with making different cones from their circle. Ask the following questions:
- What do you notice about the area of the circle the cone uses? (Some cones use almost the complete circle area; others can use half or even less.)
- How can you take this into consideration when calculating the surface area of a cone?
- What else do you need to consider when calculating the surface area of a cone? (The surface area of a cone includes the surface area of the cone shape plus the surface area of the circle at the base of the cone.)

Finally, have students think about what they have done with cylinders, pyramids, and cones to consider how to determine the surface area of a sphere. Encourage them to consider what might be entailed. The purpose is to get students thinking about how such problems might be solved.

Meeting Student Needs

- Make models of right prisms, right cylinders, right cones, spheres, and right pyramids available to help students visualize the 3-D objects. The models may help them translate information between 2-D and 3-D.

Enrichment

- Before the beginning of the chapter, ask one or more students to visit the Blackfoot Crossing Historical Park web site and prepare an explanation of the importance of this site to the tribes of the Blackfoot Confederacy. Ask them to present this information when the class discusses the Blackfoot Crossing Exhibit Hall photo on page 66. Also ask them to research the cultural significance of the eagle feather for discussion with the eagle feather fan visual on page 67.
- Challenge students to answer the following questions:
  - If two objects have different shapes, but the same surface area, will they have the same volume? Justify your answer.
  - Is it possible to make a cone that uses the entire area of a circle? (Such a cone would be flat, which would not be a cone at all.)

WWW Web Link

For information about Blackfoot Crossing Historical Park, go to www.mhrmath10.ca and follow the links.

Answers

Investigate Surface Area of Three-Dimensional Objects

1., **2.** Quadrant 1: Example: The sum of the surface area of all faces of a 3-D object.
Quadrant 2: Example: Find the area of each face, and then add all the areas.
Quadrant 3: The surface area of the cylinder is 374.7 cm^2. The surface area of the prism is 348 in.^{2}.
Quadrant 4: Students should show a different method than in Quadrant 3.

6. Sample answers might be derived using formulas for the following: circle, square, sector of circle, and triangle.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss the strategies for finding surface area of 3-D objects. Consider having students use the reflection/math journal and respond to one or more of the following prompts: – How has your understanding of surface area changed? – How could formulas you already know help you with determining the surface area of a right cone, a sphere, and a right pyramid?	• For the benefit of students who may hesitate to ask questions, have other students share the journal responses to the prompts. Listen carefully and clarify any misunderstandings. • Challenge students to develop nets for a right pyramid and a sphere. Ask them how a net might help them to develop a formula. • Have students consider how to determine the surface area of a composite object. How might these ideas help them with the surface area of a right pyramid, right cone, or sphere? • Encourage students to suggest and consider many strategies. Discussing the strategies may benefit students who can use them to enhance their own understanding and possibly springboard from them to choose their personal strategy.

Link the Ideas

In this section, students are introduced to how the formulas for the surface area of a right cone and a sphere work. It is likely that students will have difficulty visualizing the cone (and the sphere) in 3-D from the 2-D presentation in the student resource. Have at least one model of a cone and a sphere available when you discuss this section.

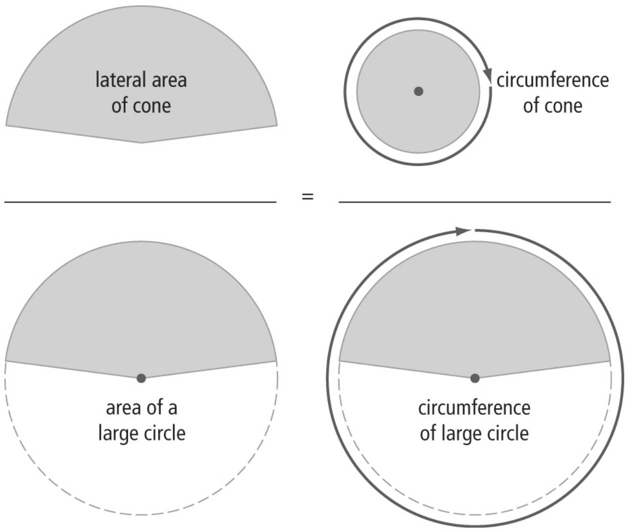
Have students make a net of a cone of their choice. To help them clarify the distance between the slant height (s) of the lateral area and the radius (r) of the base, have them label all parts on their net. Have them use this net as they work through the explanation of the surface area of a right cone.

Help them visualize that the lateral area is part of a circle by putting their net onto a piece of paper and drawing a circle with radius s. Discuss what fraction of this circle the lateral area takes up. Example: less than half in the sample on page 68.

Have them manipulate their lateral area around the base to clarify that the arc formed by the lateral area is the same length as the circumference of the base. Have them verbalize the formula for this length.

Encourage students to manipulate their net and the related circle as the class discusses the proportion developed to determine the lateral area of the cone. Use models, pieces of nets, or other visuals to make each step in this proportion concrete. It might be useful to have students sketch the visual needed for each part to clarify their understanding of the proportion.

Example:



Have students talk about how the proportion works. Ask the following questions:

- How can you make a large circle from the lateral area of the net?
- In this large circle, what is the centre? What is the radius?
- What part of this large circle does the lateral area of the cone form?
- What is the circumference of the base?
- How is the circumference of the base of the cone related to the lateral area?
- How can you determine the circumference of the large circle?
- How do you use the slant height to determine the circumference of the large circle?
- How do you use the slant height to determine the area of the large circle?

Use a model of a sphere and a strip of paper (cut to the same length as the circumference of the sphere and the same width as the diameter of the sphere) to walk through the explanation for the surface area formula of a sphere. Point out the definition of the term *sphere*. Have students consider the following questions:

- How is the diameter of the sphere related to the right cylinder?
- How is the circumference of the sphere related to the cylinder?
- Use the model and show me how the surface area of the sphere is related to the lateral area of the cylinder with similar dimensions.

Example 1

This Example is a relatively straightforward application of the formula for the surface area of a right cone. Use a model of a cone or provide students with a model to help them work through the solution.

Help students understand the solution by drawing their attention to properties of the right cone. Ask the following questions:

- What shapes make up the net of a right cone?
- What measurements do you need to determine the surface area of the base? the curved lateral surface?
- How are the diameter and the radius of a circle related?
- How would you find the total surface area?

Consider having students estimate the surface area of the cone.

$SA_{\text{cone}} = (3)(15)(15) + (3)(15)(40)$
$SA_{\text{cone}} = (3)(225) + (45)(40)$
$SA_{\text{cone}} = 675 + 1800$
$SA_{\text{cone}} = 2475 \text{ cm}^2$

Help students develop their algebra and number skills by asking why the symbol for pi is present in all lines of the solution except the last one. Ask at what point to replace pi with a numerical value. You may need to remind students to use the pi button on the calculator (not 3.14), and to round in the final step of the calculation.

Have students complete the Your Turn and then compare their solution with that of a classmate who used a different sketch. Ask the following questions:

- How did drawing a diagram help you solve the problem?
- How did drawing a net help you solve the problem?
- Which sketch did you find most helpful? Why?
- How would you find the total surface area of any right cone?

Example 2

This Example introduces the formula for the surface area of a right pyramid, which is a new concept. Direct students to the definition of *pyramid* on page 70 in the student resource. Use a model of a right pyramid or provide students with a model to help them work through the solution.

Help students understand the solution by drawing their attention to properties of the right pyramid. Ask the following questions:

- What would the net of the right pyramid look like?

- What shapes make up the pyramid?
- How can your knowledge of 2-D area help you break down the surface area problem into manageable steps?
- What measurements do you need to determine the surface area of the base? the lateral surface?
- What is the area of the base? the lateral surface?
- How do you find the total surface area?
- Why do two of the terms in the formula for surface area involve multiplying by 2?
- How would the formula change for pyramids with a square base?

Ask students for an alternative way to determine the surface area of a pyramid. For example, find the surface area by drawing the five shapes that make up the pyramid, determine the area of each shape separately, and then add them together.

Have students use more than one method for solving the Your Turn, and compare their answers.

Example 3

This Example introduces the formula for the surface area of a sphere, which is another new concept. Use a model of a sphere or provide students with a model to help them work through the solution.

Help students understand the solution by drawing their attention to properties of the sphere. Ask the following questions:

- What is the relationship between the circumference, diameter, and radius of a circle?
- Would that relationship be true for a sphere? Why or why not?
- In the calculation $\frac{95.8}{2\pi}$, what potential error could easily be made? You might have students keystroke this division to see if they arrive at the same answer. Discuss why some people got the answer in the student resource while others got 150.48. (Correct keystrokes: $95.8 \div (2 \times \pi)$. Incorrect keystrokes: $95.8 \div 2 \times \pi$.) Discuss why the last sequence is incorrect.

After students complete the Your Turn, invite them to consider questions similar to the ones above again. In particular, have students think about the relationship between radius and diameter of a sphere. You might ask students to derive a formula for surface area that is based on diameter rather than radius. If so, have them work with a partner and be prepared to discuss their findings with the class.

Example 4

This Example draws on students' prior skills with determining square roots. In this example, they need to find square roots in a problem solving context, as well as while solving an equation in which the variable is squared. To help them with the process, consider asking questions such as the following:

- How is the equation $4\pi r^2 = 459.96$ similar to the equation $4r = 459.96$? How is it different? (The equations are similar in that they require division as the first step in the solution. You can solve $4r = 459.96$ by dividing both sides by 4. Similarly, you can solve $4\pi r^2 = 459.96$ by dividing. Some students may divide by 4, and then divide the result by π, while others may divide by 4π in one step. There are several differences for $4\pi r^2 = 459.96$, including the following:
  - There are two possible divisions depending on which method is chosen.
  - To complete the solution, the square root needs to be determined.
- How do your equation solving skills come into play in this example?
- What would you do to solve the equation $4x^2 = 200$?
- What is the opposite operation of squaring a number?
- How do you determine the square root of a number that is not a perfect square?
- How do the order of operations rules help you solve an equation?

For the last line in the computation of each solution, prompt students to double the radius to get the diameter that the question asks for.

Have students complete the Your Turn. They should note that the units asked for are millimetres. Consider having students compare their solution with that of a classmate and discuss any differences. Ask the following questions:

- How do the units asked for have an impact on how you work?
- How do you prefer to handle the units in this question? Why? (Students may prefer to express 1 m^2 as 10 000 mm^2 before solving the problem.)

Example 5

This Example features a grain bin that is a composite of a right cylinder and a right cone. It may be useful to have a 3-D model of the grain bin; even one made from paper would be useful. It is likely that students will need support to visualize the bin and understand how to apply the concepts they have learned to the problem. You might use the following prompts to discuss the problem:

- What 3-D objects form the bin?
- Are all of the surfaces of each object part of the bin? Explain.
- How do you determine the surface area of each shape on the grain bin?
- What parts of each shape should not be included in the surface area?
- How are the radius and diameter of the cone and the cylinder related?
- What shape is formed by the height of a cone, the radius of a cone, and its slant height?
- How can you use this shape to determine the slant height of a cone if you know the height and radius of the cone?
- What if you just know the height and diameter?
- Why is it important to understand where each term in a surface area formula comes from?

You may wish to have students work in pairs or small groups to complete the Your Turn. As you circulate, you might guide students using prompts such as the following:

- What surface areas are included in the dumbbell?
- Are there parts of some shapes that should not be included in the surface area? Explain.
- How does where the bar meets the weight disk affect the total surface area of the dumbbell? How will this affect your calculations?

Meeting Student Needs

- Make 3-D models of right prisms, right cylinders, right cones, right pyramids, and spheres available to students to help them visualize the problems.
- Before introducing the worked examples, have students who would benefit revisit how to determine the surface area of right rectangular prisms and right cylinders.
- Before introducing the surface area of a right cone, help students recall how to form a ratio.
- Have students verbalize the process for calculating the area of a right cone, a right pyramid, and a sphere.
- Have students use grid paper to draw nets and then cut out the shapes.
- Prior to starting Example 5, some students may benefit from using models, such as a right cone and a right cylinder with the same circumference, to create a composite shape to help them visualize which surfaces are eliminated.
- Have students create a poster for the classroom showing a diagram and a net of each 3-D object studied. Beside the diagram, have them record the steps for finding the surface area.

ELL

- Explain that *slant height* refers to the distance along a lateral face from the base to the highest point of a 3-D object, such as a right cone or a right pyramid. Use a model of a cone and a pyramid to show the difference in the use of this term for these two objects.

Enrichment

- For the Your Turn for Example 1, ask students how they could write a formula for surface area of a cone that uses diameter instead of radius.
- For the Your Turn for Example 2, challenge students to simplify the formula. For example, $SA = (\text{side})^2 + 2(\text{side})(\text{slant height})$.
- For Example 5, you might explain that metal fabricators build grain bins. Metal fabricators must know about blueprints, be able to use SI and imperial measures for fabrication or fitting specifications, and be knowledgeable about metals and welding procedures.

Gifted

- Give students the following challenges:
  - Develop a problem for the surface area of a right cone that provides neither the radius nor the diameter of the cone, but gives enough information to solve it.
  - Develop a problem for the surface area of a right pyramid that requires applying the Pythagorean relationship to solve it.

Common Errors

- Students may use incorrect surface area formulas in their calculations.

R_x Ensure that students use the correct formula for each object. Encourage students to refer to their list of formulas in their Foldable. For each formula, encourage students to add a diagram and a worked example that demonstrates important steps. Some students may need to make a model of each object and attach the related formula to the appropriate part. Clarify that students understand how to use their models by asking them to explain how to determine the surface area of each one. In problems where the surface area of part of a particular object may not be necessary, ask the student to pick up the model and show what parts need to be determined. For example, in the case of Example 5, it is not necessary to determine the area of the base of the cone or the ends of the cylinder.

- Students may not follow the correct order of operations when finding area.

Rx Remind students that in the formula $A = \pi r^2$, they must square the radius before multiplying by π. They could think of the formula as $A = \pi(r)(r)$ rather than $A = \pi r^2$. It may be useful for these students to record the order of operations on a sticky note and keep the sticky note in the front of their student resource.

- In Example 3, students may perform the operation $\left(\frac{95}{2}\right)(\pi)$ instead of $\frac{95.8}{2\pi}$.

Rx Have students recall the rules for the order of operations. Invite students to explore how their calculators handle the order of operations, and when brackets must be input. Have them record the correct sequence for their calculator key strokes. More importantly, remind students that the denominator of the fraction is understood to be $\frac{95.8}{(2\pi)}$, but that the brackets are usually omitted.

- When solving equations of the type $4\pi r^2 = 459.96$, students may determine the square root before dividing by the coefficient.

Rx Remind students to divide first and then determine the square root when isolating the variable, r. They may be unfamiliar with equations in which the coefficient of the variable is not a single value. Coach them through solving the equation by isolating the variable. This is done by dividing first by 4, then by π.

- Students may have difficulty distinguishing height from slant height for a right cone or right pyramid.

Rx Some students will find it beneficial to use a mnemonic. The slant part of slant height means that something is on an angle. The height is always at 90°. It may be useful to demonstrate that the slant height is the hypotenuse of a triangle formed by the radius of the base, the height of the cone or pyramid, and the slant height. Have students draw this triangle inside a cone and possibly build a triangle inside a cone to reinforce this understanding. Encourage students to recognize that the slant height is always greater than the height for any cone or pyramid.

Key Ideas

The Key Ideas summarize calculating the surface area of a right cylinder and a right prism, a right pyramid and a right cone, and a sphere. You might have students use index cards to prepare their own summary of the Key Ideas, including an example for each 3-D object. In addition, have students write their own notes on solving an unknown dimension of a right cone, a right pyramid, or a sphere when given the surface area. Beside each step, have them describe the reason for the step.

Answers

Example 1: Your Turn

$SA = 502.65$ cm^2

Example 2: Your Turn

$SA = 270$ cm^2

Example 3: Your Turn

1787 cm^2

Example 4: Your Turn

282 mm

Example 5: Your Turn

1041.3 cm^2

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• You may wish to have students work with a partner. • Have students use mental math to do a quick estimate of the surface area of the cone. They can use this quick calculation to check the reasonableness of their answer. (SA = (3)(8)(8) + (3)(8)(12) Round the 8 and 12 to numbers that are easy to work with. $SA \approx (3)(10)(10) + (3)(10)(10)$ $SA \approx 600$ cm^2) • Have students explain how to determine the surface area if they know the diameter instead of the radius. • Encourage visual learners to use grid paper and draw and label a diagram and a net of the cone. • Have students verbalize the method they prefer to solve the surface area of any right cone. • Consider having students check each others' answer. • Provide students with a similar problem to solve.
Example 2 Have students do the Your Turn related to Example 2.	• You may wish to have students work with a partner. • Encourage visual learners to use grid paper and draw and label a diagram and a net of the pyramid. • Have students verbalize the method they prefer to solve the surface area of any right pyramid. • Provide students with a similar problem to solve.
Example 3 Have students do the Your Turn related to Example 3.	• You may wish to have students work with a partner. • Have students verbalize the method they prefer to solve the surface area of any sphere. • Provide students with a similar problem to solve.
Example 4 Have students do the Your Turn related to Example 4.	• You may wish to have students work with a partner. • Have students draw and label a diagram to help visualize the problem. Use the following prompts: – What measurements are needed to solve the problem? – What measurements are given? – What measurement is missing? – How can you use what you know to find the missing measurement? • Provide students with practice solving for a variable to gain proficiency in the algorithm of solving for a variable and in order to master the necessary calculator keystrokes for their particular calculator. • Provide two similar problems: one that has students determine square root, and another where determining a square root is not required. Discuss how to tell the difference.
Example 5 Have students do the Your Turn related to Example 5.	• You may wish to have students work with a partner. • Use a 3-D model of the dumbbell using three cylinders that disconnect so students can see what surfaces need to be subtracted from their calculations. (Four bases need to be subtracted: two bases on the bar and a fraction of a base on one side of each disk.) Have students identify the components of the object and the surfaces of each component (three cylinders; three surfaces per cylinder), and the areas to be subtracted. Ask students how they could determine the part of this surface area (pointing at the inside disk where the bar meets it) that needs to be subtracted. • Have students use the formula and estimate the surface area of the composite object to check the reasonableness of their answer. • This is a multi-step problem. Prompt students to verbalize the steps needed to solve the problem. They may wish to check off each part of the diagram as they determine its surface area. • Have students verbalize their thinking related to their calculations and the number of surfaces used in them. • Provide students with a 3-D composite object in the classroom and its measurements, and have them determine its surface area. • Provide students with an additional question that allows them to determine slant height using the Pythagorean relationship.

Check Your Understanding

Practise

Question 1 provides an opportunity for students to demonstrate their understanding of calculating the surface area of right prisms, right cylinders, right pyramids, right cones, and spheres. The diagrams are provided and no unit conversions are required.

For #2, students are asked to make their own sketch before solving the problem. This is intended to help students develop the habit of drawing a diagram or a net as part of the problem solving process.

For #3 and 4, students are required to find a missing dimension, given the surface area.

The sculpture in #5 is produced by Linda Tanaka, an artist from Lethbridge, Alberta. Students solve a surface area problem involving a composite object composed of a sphere and a right pyramid with a rectangular base. Before students begin working on this question, ask what assumptions they need to make to answer this question. Also ask if everyone's assumptions will be the same.

Consider pairing students with different assumptions. Have them each solve the problem and then talk through their thinking with their partner. They can check each other's answers for reasonableness and discuss how differing assumptions may result in different answers.

Apply

The Apply questions provide a variety of contexts for students to solve problems involving surface area. In some cases, the real work simulations are very realistic. You may wish to engage students in a discussion of which simulations are more accurate than others, and discuss the idea that although mathematical models are not always exact representations of real life, they are useful.

It is useful for students to assess an effort to solve a problem, as in #6. Before looking at the solution, ask students to consider the problem and decide how they would solve it. Do they agree with Austin's method and solution? Discuss why or why not.

You might have students extend #7 by having them research the number of tipis set up at Blackfoot Crossing Historical Park and the total amount of canvas needed for these structures.

Did You Know?

Some students may question the spelling of tipi. Point out that tipi is the Canadian spelling; teepee is the American spelling. You may wish to brainstorm math terms that have different spellings, such as centre (Canadian) and center (American) and metre (Canadian) and meter (American).

For #9, ask students how their answer would change if they decided to use a diameter of 60 cm.

For #10, students use the concept of percent. You may wish to mention that this problem involves proportional reasoning.

For #11 to 13, challenge students to convert their answer to the other measurement system.

In #11, ask students familiar with hand drums to share their knowledge of the cultural significance and use of these drums.

Refer students who are interested in the Muttart Conservatory in #12 to the Web Link at the end of this section.

For #14, students solve a surface area problem involving a composite object. You may wish to refer students to look at how they solved #5, if they encounter difficulty.

If students are unfamiliar with the use of a light tent such as that shown in #14, you may wish to explain that the lights shining on the outside walls of the tent provide diffused and reflected lighting for the items inside. Tents such as these are commonly used by professional photographers because the diffused and reflected lighting is ideal for capturing the beauty of small shiny objects. See the related Web Link at the end of this section.

For #15, explain that 40 mm ± 0.5 mm is a short-hand way of writing: 40 mm + 0.5 mm and 40 mm – 0.5 mm. Explain that in this question the diameters of the squash balls can fall between the values of 40.5 mm and 39.5 mm. For part b), encourage students to draw a diagram on grid paper to help understand the problem.

Extend

These questions require students to perform several steps to solve the problems.

For #16, students determine the surface area of a truncated cone. If students encounter difficulty, prompt them to identify the shape of the piece of the cone that is cut out, and determine its surface area. Encourage students to model the problem to help visualize the cones (one cut out and one whole). Assume that there is no paper overlap or rim.

For #17, students are challenged to calculate the surface area of a composite block that has holes in it. Since the dimensions are fairly small, have students draw a scale model or create a 10:1 scale using centimetre grid paper and shade parts to be painted. Prompt students to think about how the holes in the block reduce the surface area in one way, but increase it in another way. Students should also think about the cylindrical shape of the holes, and which parts of the cylinder add to and subtract from the surface area.

For #18, students need to use spreadsheet software to analyse how changing the radius of a sphere changes its surface area. You may wish to have students use **TM 2–1 How to Do Page 79 #18 Using TI-Nspire™** or **TM 2–2 How to Do Page 79 #18 Using Microsoft® Excel** to do this question.

The result is counterintuitive; caution students that seem to answer the question very quickly. If you have access to some 3-D spheres, you may wish to make them available to students to assist them in visualizing the relationship.

Have students discuss the meaning of *stretch ratio* in the given context.

Create Connections

These questions allow students to communicate their understanding about surface area and why it is expressed in square units.

For #19, consider inviting students to present their response in a class discussion.

For #20, allow students to use a communication method of their choice; i.e., a written, taped, or graphic explanation. The visual shows students using American Sign Language.

Unit Project

The Unit 1 project question, #8, provides students with an opportunity to solve problems involving the surface area of a cylindrical CD case and a rectangular CD jewel case. Students can use multiple strategies including using grid paper and drawing and labelling a net, or making a table and listing the shapes for which calculations are needed.

Meeting Student Needs

- For #6, students find the error in a solution, which is a higher-order skill. Consider having students complete their own solution to the problem before attempting to identify the error in the given solution.
- For #11, help students recall how to change $14\frac{7}{8}$ to a decimal.
- For #14, refer students who have difficulty to Example 5.
- If you wish to make the spreadsheet work in #18 accessible to all students, use the Web Link on the next page.
- Provide **BLM 2–8 Section 2.2 Extra Practice** to students who would benefit from more practice.

Enrichment

- For #12, mention that although architects designed the Muttart Conservatory, likely ironworkers and glaziers were involved to create the buildings and install the glass. Ask how these trades apply knowledge of surface area.

Gifted

- The palm of the hand is approximately 1 percent of the total surface area of a human body. This is sometimes used by medical staff to estimate the percentage of an individual's body damaged by a burn. This is useful in determining treatment.

  Ask students to trace their palm onto 1-cm grid paper and count squares to find its surface area. Have them approximate their total body surface area from this information. Ask students to predict how changing the grid paper to 0.5 cm might affect the accuracy of their findings. Have them test the prediction and look for a pattern. Then, have them test with even smaller grid paper. How does decreasing the size of the grid paper squares affect the accuracy of the approximation of surface area of irregularly shaped objects? Note that this line of thinking is related to calculus reasoning.

Common Errors

- Students may incorrectly round their answers.

Rx Review any rules for rounding. Remind students that giving an answer to the nearest tenth of a unit requires one place after the decimal point. To the nearest hundredth is the same as two places after the decimal. When using a calculator, students should maintain exact values in all computation steps, rounding only at the end.

Students who still have problems with rounding might use the following process:

1. Identify the part of the number that might be rounded. Circle that number.

2. Underline the numeral that immediately follows the numeral that might be rounded.

3. Ask the following question:
Is the underlined numeral 5 or more?

YES	NO
↓	↓
Leave the circled number as is.	Round the circled number up one.

- Students may neglect to include the area of one side of a 3-D object when finding surface area.

Rx Remind students to add the areas of all sides of the object unless the question states otherwise or it is impractical to do so. Have a model of a right prism, a right cylinder, a right cone, a right pyramid, and a sphere on display in the classroom. Encourage students to examine the models and to consciously ask themselves whether it makes sense to include the base of the solid. For example, would you include the surface area of the base for a conical drinking cup?

Some students may need to make a model of the item they are calculating the surface area for. Have them put a checkmark on each surface as they determine its surface area. You may wish to have them number the surfaces and put matching numbers on the parts of their solution. This will help some students identify what surfaces they might have missed and what surfaces they may have used that they should not have.

WWW Web Link

For information about the Muttart Conservatory, go to www.mhrmath10.ca and follow the links.

For more information on the use of light tents such as that discussed in #14, go to www.mhrmath10.ca and follow the links.

To make #18 available to more students, have students go online to www.mhrmath10.ca and download one of the prepared spreadsheets. By entering the stretch ratio and radius, students can get the calculations, determine a pattern, and make predictions.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1 to 4, 6, and 9. Use students' responses to #1 to assess their understanding of calculating the surface area of prisms, cylinders, pyramids, cones, and spheres. Then, have students attempt #3, which asks students to solve for the missing dimension. Students who have no problems with these questions can go on to the remaining Apply questions.	• For #1, have models of all shapes available for students to examine and manipulate. Make sure that students have a good understanding before proceeding. • Have students note any 3-D objects that they have difficulty with. Coach them through the corrections and clarify any misunderstandings. Provide a problem similar to #1 before having students move on to 3. • Provide grid paper to students who would benefit from drawing and labelling a diagram and/or a net for each object, particularly for #2 and 9. • Provide coaching to students who have difficulty with #3. Coach them through solving equations and calculating square roots, if needed. • Refer students who have difficulty with recalling the formulas to their Foldable. • It may benefit some students to verbalize the dimensions of each object to assist in linking them to the variables in a formula. Have students write the dimension that corresponds to each variable in the formula. • Encourage students to use the method they prefer for solving surface area problems.
Unit 1 Project If students complete #8, which is related to the Unit 1 project, take the opportunity to assess how their understanding of the chapter outcomes is progressing.	• Encourage students to use the actual items, manipulate a model of the same shape, and/or draw and label a diagram and a net for each situation. • You may wish to assign only #8a) to students who are having difficulty with surface area problems. • You may wish to provide students with **BLM 2–5 Chapter 2 Unit 1 Project**, and have them finalize their answers. Have them store the work in their project portfolio.
Assessment *as* Learning	
Create Connections Have all students complete #19 and 20.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner and then prepare an individual response. • Some students may wish to answer #19 on the back of their Foldable, where they can refer to it for review purposes.