# The Tangent Ratio

# 3.1

***Mathematics 10*, pages 100–113**

**Suggested Timing**

120–140 minutes

**Materials**

- grid paper
- protractor
- ruler

**Blackline Masters**

BLM 3–3 Chapter 3 Warm-Up
BLM 3–4 Chapter 3 Unit 1 Project
BLM 3–5 Section 3.1 Extra Practice
BLM 3–6 Protractor

**Mathematical Processes**

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

**Specific Outcomes**

**M4** Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.

**Unit Project** Note that #10 and 17 are Unit 1 project questions.

| Category | Question Numbers |
|---|---|
| Essential (minimum questions to cover the outcomes) | #1a), b), 2, 3a), b), 4a), b), 5, 6, 9, 10, 17, 18, 20 |
| Typical | #1, 3a)–c), 4a), b), 5–7, 9–13, 18–21 |
| Extension/Enrichment | #10–21 |

## Planning Notes

Have students complete the warm-up questions on **BLM 3–3 Chapter 3 Warm-Up** to reinforce prerequisite skills needed for this section. If you have posted the outcomes for the chapter, encourage students to refer to those that relate to this section.

Before beginning this section, you may want to discuss with students

- that trigonometry is used to find heights and distances that might otherwise be inaccessible
- that right-angle trigonometry involves right triangles
- what a ratio is
- how to solve for a missing value in a proportion

You may want to use the following questions to engage students in a class discussion about sailing.

- Who is involved in sailing?
- How do sailors get to shore if there is an offshore wind?
- How is mathematics used in sailing?

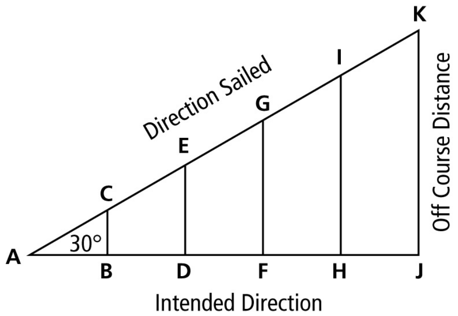
Before beginning the section, you may also want to suggest that students refer to the Key Terms list and make flashcards for any terms that are found in this section.

## Investigate the Tangent Ratio

In this investigation, students discover that the basis of trigonometric ratios comes from similar triangles. Similar triangles are scaled versions of each other and, therefore, the ratio of the sides remains the same. Since similar triangles always have equal angle measurements, the ratio of the corresponding sides is the same for any given angle. As such, each angle has a unique tangent ratio.

You could begin this investigation by having students who sail explain how tacking works and describe when they have used it. You might also want to reactivate student knowledge of similar triangles since these are the basis for connecting the data from #2 to the tangent ratio in #4.

Have students work in pairs or small groups. Ask them to complete #1 to 6. You may need to coach some students through the steps from #1c) in order to get a diagram that looks like the one below.




As students work on the investigation, consider asking such guiding questions as the following:

- What is the relationship between △ABC and △ADE?
- How are △ABC and △AHI related?
- What does the $\frac{\text{off course distance}}{\text{intended direction}}$ ratio tell you about each of these triangles?
- How is the tangent ratio of 30° related to this ratio?
- Identify the hypotenuse, opposite side, and adjacent side of your triangles.
- Using this information, how is the $\frac{\text{off course distance}}{\text{intended direction}}$ ratio related to the tangent ratio?
- How can you use this information to develop a formula for determining the tangent ratio?
- Explain how to use this formula to determine the tangent ratios for ∠A and ∠B.

You may wish to refer students to the Did You Know? since this may be the first time students have seen an angle referred to as *theta*, and labelled with the symbol θ. Review the pronunciation with students (*thay* – *ta*) and explain that this method of labelling within the space of the angle avoids having to label the angle using three letters.

This is a good opportunity to discuss the language and symbols of math. You could illustrate how Inuktitut and other languages have their own alphabets. You could discuss how these could easily be used as variables, and then discuss why Greek symbols are more commonly used.

## *Meeting Student Needs*

- Some students may find it easier to use **Master 8 Centimetre Grid Paper** and place the sheet horizontally to draw the triangles.
- Some students may need coaching to get the calculator in degree mode.
- **BLM 3–4 Chapter 3 Unit 1 Project** includes all of the unit project questions for this chapter. These provide a beginning for the Unit 1 project report.

### Enrichment

- Ask students to research other common Greek letters. Students may wish to use them to label the angles of their triangles.

### Gifted

Ask students to explain why the tangent ratio can have values greater than one, whereas sine and cosine are never greater than one. Ask them if the tangent ratio has an upper limit and to explain their thinking.

## *Common Errors*

- Students sometimes set up the proportion incorrectly. For example, for tangent, they will make the proportion $\frac{\text{adjacent}}{\text{opposite}}$, rather than $\frac{\text{opposite}}{\text{adjacent}}$.

**Rx** Go over labelling the triangle with opposite and adjacent sides. The tangent is the ratio of opposite side to adjacent side.

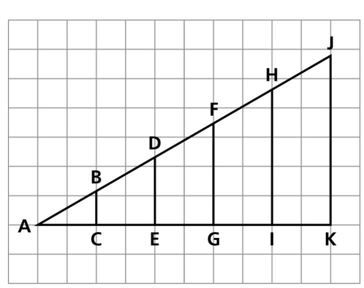
- Students solve the proportion incorrectly.

**Rx** Review how to solve a proportion by isolating the variable.

## Answers

### Investigate the Tangent Ratio

**1. c)**




**2.**

| Triangle | Intended Direction | Off Course Distance | $\frac{\text{Off Course Distance}}{\text{Intended Direction}}$ |
|---|---|---|---|
| △ABC | 2 cm | 1.15 cm | 0.5750 |
| △ADE | 4 cm | 2.3 cm | 0.5750 |
| △AFG | 6 cm | 3.4 cm | 0.5667 |
| △AHI | 8 cm | 4.6 cm | 0.5750 |
| △AJK | 10 cm | 5.75 cm | 0.5750 |

## Answers

**Investigate the Tangent Ratio**

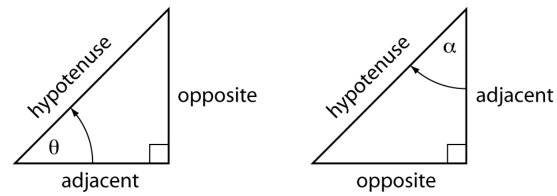
**3. a)** The triangles are similar because they have all three angles measuring the same.

**b)** The ratio $\frac{\text{Off Course Distance}}{\text{Intended Direction}}$ is constant for any side lengths with the same angle measures.

**4. a)** tangent 30° = 0.5773...

**b)** The value of tangent 30° is equal to the ratio $\frac{\text{Off Course Distance}}{\text{Intended Direction}}$.

**5.**




**6. a)** tangent of any angle = $\frac{\text{length of opposite side}}{\text{length of adjacent side}}$

**b)** tangent A = $\frac{a}{b}$; tangent B = $\frac{a}{b}$

| Assessment | Supporting Learning |
|---|---|
| **Assessment *as* Learning** | |
| **Reflect and Respond**<br>Have students complete the investigation. Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings. | • You may wish to discuss the answers to #6 with the class to assist any students who may not understand the connection between the final column in the chart in #2 and the tangent ratio, and how to use the terminology explained in #5 to develop an appropriate formula. |

## Link the Ideas

The Link the Ideas discusses the tangent ratio. You may wish to have students draw a sample right triangle in their Foldable and label the sides. Have them explain in their own words how to use the diagram to help them remember how to determine the tangent ratio.

Some students may find it beneficial to note that tan A is the short form for the tangent ratio of angle A. Discuss other mathematical short forms and how these help simplify communication.

### Example 1

In this first example, students practise writing trigonometric ratios and then solving them. Ask them how the answers to a) and b) differ from how they might usually see fractions (i.e., the fractions are not simplified).

In the Your Turn, students may wish to put their finger on the target angle before identifying the adjacent and opposite angle. Discuss whether the hypotenuse is ever an adjacent or opposite angle. (The answer is no.)

### Example 2

In this example, students are asked to use technology to determine the tangent ratio of an angle, and to find the measure of an angle given the tangent ratio. It is extremely important that students understand what to do when they have a ratio of the sides of a triangle. For example, what does the triangle look like? Drawing and labelling the triangle helps them clarify the relationship between the ratio and how this might appear on the triangle. You may wish to have students draw the diagram for b) before discussing the use of technology to determine a solution.

Review with students how to put the calculator in degree mode before working on trigonometric ratios. Some students may wish to make a note about this in their Foldable.

Have students work in pairs on the Your Turn. They can talk through the calculator key sequence needed for the tangent ratios, solve them, and then compare their answers. Then, each pair can talk through the different key sequence needed for the angles, solve them, and compare their answers.

### Example 3

This example is a practical application of the tangent ratio. Using the reference angle in the diagram, have students label the sides as opposite, adjacent, and hypotenuse in relation to the reference angle. They can then apply the tangent ratio and substitute the appropriate values.

Ask students to identify how the Your Turn involves tangent ratios. In terms of tangent ratios, what are they being asked to determine? Have them label the triangle's sides in relation to the reference angle before determining the answer.

## Example 4

Before looking at the solution to Example 4, have students sketch the scenario and label the sides in relation to the reference angle. Ask the following:
- What are you being asked to determine?
- What method can you use to do that?
- How will you state your answer?

Have your students draw a diagram showing the scenario in the Your Turn. You may wish to discuss that a clinometer is an instrument that measures angles above or below horizontal. Once students have the diagram, ask questions such as the following:
- What are you being asked to determine?
- What information do you know for sure?
- How can you verify that the other information is correct?

This question provides a good opportunity for students to try verifying different parts of their data. For example, if the measurements are correct, what should the angle be? If the angle and one measurement are correct, what should the other measurement be? Encourage students to discuss which measurement may or may not be accurate and to develop mathematical arguments supporting their point of view.

## Key Ideas

Have students discuss the Key Ideas with a partner. What information is new to them? What do they find easy to understand? What is still difficult? Have them add notes about the Key Ideas to their Foldable.

### *Meeting Student Needs*

- Have one or more students develop large visuals showing the adjacent and opposite sides related to a particular angle. Post these visuals around the classroom.
- Some student will not be familiar with the functions on their calculators. Be sure to take time to discuss the use of individual calculators, and specifically how to determine the tangent of an angle and how to find the measure of an angle.
- Have students draw a scenario that shows how to determine the measurement of an acute angle when both legs of the triangle are known, and how to determine the side length if one acute angle and the length of one leg of a right triangle are known. Enlarge and display these visuals.

#### Gifted

- A unit die has sides one unit long. Ask students to use the tangent ratio to find the minimum distance an ant would have to travel to go from one corner of a unit die to the diametrically opposite corner (you may want to discuss what is meant by *diametrically opposite*).

### *Common Errors*

- Some students may get incorrect answers because their calculator is not in degree mode.

$R_x$ You could reset the calculator or check the mode and reset it. This could be an excellent opportunity to discuss estimation skills and the reasonableness of answers, especially if the calculator is in radians.

## Answers

**Example 1: Your Turn**

**a)** $\tan L = \frac{5}{12}$ **b)** $\tan N = \frac{12}{5}$

**Example 2: Your Turn**

0.5095, 1, 1.5399; 27°, 29°, 56°

**Example 3: Your Turn**

The ladder will reach approximately 3.9252 m up the wall.

**Example 4: Your Turn**

The guy wire forms a 65° angle with the ground.

| Assessment | Supporting Learning |
|---|---|
| **Assessment *for* Learning** | |
| **Example 1**<br>Have students do the Your Turn related to Example 1. | • Encourage students to verbalize their thinking.<br>• You may wish to have students work with a partner. |
| **Example 2**<br>Have students do the Your Turn related to Example 2. | • Encourage students to verbalize their thinking.<br>• You may wish to have students work with a partner.<br>• Provide additional angle measures and ask students to write the tangent values to four decimal places.<br>• Provide additional tan values and ask students to determine the degree measure to four decimal places. |
| **Example 3**<br>Have students do the Your Turn related to Example 3. | • Encourage students to verbalize their thinking.<br>• You may wish to have students work with a partner.<br>• Some students may benefit from labelling the sides of the triangle formed by the ladder, wall, and ground according to the reference angle (opposite side and adjacent side). Ask the following:<br>– What is the missing value?<br>– How does this scenario relate to tangent ratios?<br>– How can you use tangent ratios to determine the missing value? |
| **Example 4**<br>Have students do the Your Turn related to Example 4. | • Encourage students to verbalize their thinking.<br>• You may wish to have students work with a partner.<br>• Have students draw the scenario and label the measurements given. Challenge them to consider how to verify any one measurement using the other two. Allow different methods for answering this question. |

## Check Your Understanding

### Practise

Questions #1 to 5 give students practice labelling the sides of a triangle, working with similar triangles, and using their calculator.

Question #6 gives students diagrams and contexts for using the tangent ratio. Have students identify the reference angle, adjacent side, and opposite side in each triangle.

Students need to develop their own diagram for Question #7. Considering that this question deals with an Olympic athlete, you could ask if any students have ever been to an Olympic event. What was their favorite event? Have they ever watched a floor exercise? Are any of them involved in gymnastics? Ask about taking the diagonal run for their routines.

### Apply

Students might need help identifying the rise and run in the diagram for #8.

For #11, students should make a diagram with the information given in the problem. Again, remind them to label the sides of the triangle in relation to the reference angle.

For #13, you could link the idea of steepness of a line to the slope of a line. Either give students general guidelines as to what is considered steep, or have students decide amongst themselves.

### Extend

For #14, students will have to work with the given diagram, solving the problem using two triangles.

In #15, students will have to be able to draw in the right triangles and appropriately label them in order to determine how the tangent ratio is used. This question is based on a relevant environmental theme, since green building practices are a growing concern in building homes and places of employment. Energy-conscious designers are always trying to come up with ways to help reduce the amount of energy being used. Using the energy from natural resources such as the sun is just one such step in reducing energy consumption.

In #16, students will need to read very carefully and add in the dimensions as they pertain to the problem. Remind students that when they label an angle, the vertex is always in the middle. For example, the vertex of $\angle ACD$ is at $\angle C$. The second part to this question will use the first calculation for the distance across the water. It should be noted that this is how surveyors find inaccessible distances.

## Create Connections

Question #18 provides a good section summary. It is recommended that all students answer this question.

For #19, students will have to be able to understand what a ratio of 1 means in terms of the tangent ratio. Have them sketch a diagram and mark a reference angle and give a tangent ratio of 1 for the opposite and adjacent sides. Ask them what this angle measure is.

Question #20 demonstrates how trigonometry may be used to find heights. Devin used material around him to determine the height of the silo. Ask students how he was able to do this. Using things that are around them, could they come up with ways to measure heights of objects in the schoolyard?

For #21, have students work with a partner to measure angles. They would be doing similar things that a surveyor would do to measure distances that are inaccessible or difficult to measure.

## Unit Project

The Unit 1 project questions give students opportunities to solve problems involving the use of the primary trigonometric ratios and the Pythagorean relationship to explore how wireless systems have impacted music distribution.

For #10, students use the information about a satellite radio cell tower and three substations to determine the distance of each substation from the intersection of two roads.

Question #17 compares the storage capacities of wax cylinders and today's digital storage systems. Most students will be able to relate to the number of songs that they are able to store in their MP3 players. It is interesting to note that early recording devices contained 2 to 4 min of songs. The standard for a 3-min song came about because of the storage capacity of the early wax cylinders. In this question, students will be able to see how technology has changed in how the music industry is able to bring their product to their fans. This might be a time to discuss what future technology might bring, and how things might change over the next generation of recording devices.

### *Meeting Student Needs*

- Provide **BLM 3–5 Section 3.1 Extra Practice** to students who would benefit from more practice.
- For #21, you may wish to give kinesthetic/tactile learners a separate assignment. They could determine the height of a building in your community, or the rise : run ratio of a ramp (see #9).
- You could arrange for a surveyor to make a presentation in your classroom. The surveyor could demonstrate how to use the equipment as well as discuss the career.

### Enrichment

- You can create further discussion for #19 by leading students to find the length of the hypotenuse ( $\sqrt{2}$ ). This triangle allows us to find exact values of trigonometric ratios. You may wish to discuss what an exact value is.

| Assessment | Supporting Learning |
|---|---|
| **Assessment *for* Learning** | |
| **Practise and Apply**<br>Have students do questions #1a), b), 2, 3a), b), 4a), b), 5, 6, 9. Students who have no problems with these questions can go on to the remaining questions. | • Have students who are having difficulty with #1 and 2, review Example 1. Coach students who are experiencing difficulty, and assign the balance of #1 to check for understanding.<br>• For students who are having difficulty with #3, 4, and 5, coach them through the appropriate parts of Example 2.<br>• Ensure students have a good working knowledge of their calculator, and ensure that they have their calculator in degree mode. You may wish to give students a quick check to perform, so that they always know that they are in the correct mode. Teach them that tan 45° = 1. If, when they enter this in their calculator, they do not get 1, then their calculator is in an incorrect mode. Assign some of the unused parts of questions to check for understanding.<br>• Encourage students having difficulty with #6 and 9 to draw a diagram first and label it with the information given in the question. Have them verbally identify what they will solve for. Reviewing Examples 3 and 4 may assist them in starting their questions. |

<table>
<tr><th>Assessment</th><th>Supporting Learning</th></tr>
<tr><td colspan="2">Assessment for Learning</td></tr>
<tr><td>Unit 1 Project<br>If students complete #10 and 17, which are related to the Unit 1 project, take the opportunity to assess how their understanding of the chapter outcomes is progressing.</td><td>• Have students use different colours to highlight the right triangle for each cell tower in #10. Have them identify the angle and opposite side, then explain how to determine the length of the missing side. Some students may need to talk through how to solve the first triangle before doing the others independently.<br>• Question #17 requires students to sketch a diagram first. Encourage them to compare their diagram with a partner before beginning to solve.<br>• Encourage students to label the sides of the given angles to help determine which ratio will be used to solve.<br>• Remind students to place their solution and all supporting documents into their project portfolio.</td></tr>
<tr><td colspan="2">Assessment as Learning</td></tr>
<tr><td>Create Connections<br>Have all students do questions #18, 19, and 20.</td><td>• Have students check each other's notes for #18. This organizer should be added to the Foldable.<br>• Students having difficulty with #19 should be encouraged to start with labelling a right triangle of their choosing and then identify the opposite and adjacent sides. Prompt them with questions, such as the following:<br>– What is the tangent ratio, expressed as a ratio of sides?<br>– How could you use this ratio to solve the missing side lengths? Hint: if the ratio has to equal 1, what numbers are in your ratio?<br>– If you write a fraction and reduce it to 1, what types of numbers were in the fraction?<br>• Encourage students to draw two triangles separately for #20. Have them verbalize whether they will need to add or subtract values.</td></tr>
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