3.3 Solving Right Triangles

Mathematics 10, pages 125–135

Suggested Timing

100–120 min

Materials

- metre stick or measuring tape
- masking tape
- calculator

Blackline Masters

BLM 3–3 Chapter 3 Warm-Up
BLM 3–4 Chapter 3 Unit 1 Project
BLM 3–8 Section 3.3 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcomes

M4 Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1a), c), 2, 3, 4 a), b), 5a), e), 6, 7, 11, 16
Typical	#1a)–c), 2, 3, 6, 7, 11, 12, 16
Extension/Enrichment	#9, 13, 14–16

Unit Project Note that #11 is a Unit 1 project question.

Planning Notes

Have students complete the warm-up questions on **BLM 3–3 Chapter 3 Warm-Up** to reinforce prerequisite skills needed for this section. If you have posted the chapter outcomes, refer students to those that relate to this section.

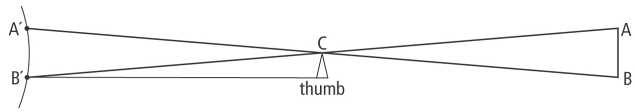
Ask students what they know about the *polar aurora*. You may need to expand on their responses by explaining that auroras are natural light displays in the sky, which are usually seen at night. Polar auroras are sometimes called the northern and southern lights. This phenomenon is more common the closer you are to the magnetic poles, so many student from lower latitudes may not have witnessed it. You may want to explain to Northern students that this experience, while common for them, is rare for most people in southern Canada.

For interest, you may want to tell students that many Aboriginal and Inuit legends attribute the northern lights to spirits at play or dancing. Some stories say that when you whistle, the lights will dance. However, the aurora was also feared, and in some Inuit traditions, children were warned never to whistle at the lights because the auroras could come closer and snatch them away.

Investigate Estimation of Distance

You could begin this investigation by discussing with students possible ways of estimating distances. You might also want to take some time to clarify the term *parallax*, since it is the basis or main idea introducing the investigation. (A parallax is an apparent change in the position of an object that is viewed along two different lines of sight.) To help students grasp this concept, draw their attention to the Did You Know? feature on page 126. Have them stretch out an arm, holding up their thumb and closing one eye. Have them line their thumb up with an object in the classroom, and then alternate their open and closed eyes. They should notice how the position of their thumb in relation to the object appears to shift. This shift is the parallax.

For most people, the angle between the lines from the eyes (A′B′) to the outstretched thumb is about 6°. That angle is the parallax of your thumb, viewed from your eyes. The triangle A′B′C is similar to the larger triangle ABC. With an angle of approximately 6°, the similarity ratio is 10:1. Therefore, if the distance B′C to the thumb is 10 times the distance A′B′ between the eyes, the distance AC to the far landmark is also 10 times the distance AB.



$$\frac{B'C}{A'B'} = \frac{AC}{AB}$$
$$= \frac{10}{1}$$

Encourage students to draw a diagram and label it with their measurements. Students should repeat their measurements to see if a pattern exists.

Meeting Student Needs

- To illustrate the concept of a parallax, you could take your class outside and ask them to focus on a distant object (a house, a tree, a soccer post, etc). Students should hold their hand out with one finger raised and close one eye. They need to line up their raised finger and try to block out the sight of their chosen object. Now without moving their arm, they should close their open eye and open their closed eye. They will find that the object, when viewed through the other eye, will no longer be blocked. Students should then move their finger to block that eye's line of sight. This change in finger position is the parallax. The distance to the chosen object can be determined by finding how far apart the finger positions are.
- Consider having students do this investigation with a partner or in groups of three, where one person could do the measuring of the distances. The same person should do the measuring to eliminate measurement error as much as possible.
- Students could complete the investigation outside in a large area and then use either a pedometer or a trundle wheel to determine the distances.
- Some student benefit from hands-on activities. If you teach in a community where there is an observatory, you could consider arranging an evening field trip to visit it. Invite an astronomer to give a presentation and to discuss what astronomers do, and how mathematics is used in their field.
- **BLM 3–4 Chapter 3 Unit 1 Project** includes all of the unit project questions for this chapter. These provide a beginning for the Unit 1 project report.

Enrichment

- Ask students to investigate how trigonometry could be used to find distances in space where direct measurement is impossible.

Gifted

- Suppose the first triangle of a series of right isosceles triangles has legs of 1 cm and an unknown hypotenuse. If the hypotenuse forms a leg of the next triangle in the series, have students find the dimensions of the 6th triangle in the series. (Note that you may want to have students recall the meaning of isosceles).

Answers

Investigate Estimation of Distance

4. a) Example: The distance between my partner and I was approximately six times the distance between the two locations where my partner was standing.

b) Estimate the parallax distance at your object and multiply it by six.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Have students complete the investigation. Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• The class could develop a large chart on the board, and students could compare two of their triangles with the rest of the classes' results. • Encourage students to identify whether there is a best ratio to use. • Many students would benefit from completing the investigation on grid paper.

Link the Ideas

To be able to solve trigonometric problems, students must understand the vocabulary used in the problem. Angle of elevation and angle of depression are commonly used in trigonometry problems:

- An angle of elevation is an angle made above the horizon or horizontal line.
- An angle of depression is an angle made below the horizon or horizontal line.

Because of alternate interior angles, the angle of depression is equal to the angle of elevation. You might want to review alternate angles with students. Have them recall that alternate angles are angles that are either both inside or both outside a set of parallel lines, but on opposite sides of the transversal. An angle of depression and an angle of elevation are alternate angles because they are inside a set of parallel lines (the horizontal) and on opposite sides of the transversal (the line cutting the horizontal).

Example 1

Have students talk through the strategy used to solve Example 1 and discuss how the visual assists them in solving the problem.

When students start the Your Turn, point out that a transit measures angles of elevation.

Have students draw a diagram to show the scenario in the question. You may wish to use coaching questions such as the following:

- Which side of the triangle represents the height of the silo?
- What information does the problem give you?
- Where does the reference angle go?
- How does drawing and labelling the triangle help you decide which trigonometric ratio to use?
- What trigonometric ratio is that?

Example 2

You could discuss with students what it might be like to be standing on the top of a rock and looking down. Emphasize that this is the angle of depression. Students should sketch a diagram for the problem, drawing the top horizontal line and labelling this angle 73º. Remind students that the angle of depression and the angle of elevation are congruent angles.

You may wish to have students work in pairs for the Your Turn. They can each draw and label a diagram of the scenario, then talk through how to determine the height of the balloon.

Example 3

In this example, students are asked to solve a triangle. You will need to discuss with the class what *solving* a triangle means. To *solve* a right triangle means to solve for all the missing side lengths and angles in the triangle. Encourage students, whenever possible, to use only information given in the question to find the missing values.

Students should begin by listing the quantities they know and placing them in a diagram. There are several ways to begin answering this question. You could ask the class for input on how they would start. Students will see from the varied responses that there are several ways to begin this type of question. Some students may want to use triangle sum to determine the measure of the third angle, while other students might want to use a trigonometric ratio later to determine the missing angle.

For the Your Turn question, encourage students to use a different method than the one they used in Example 3.

Example 4

Trigonometry is used to solve several types of problems in mathematics, science, and industry. Some of these problems require two right triangles to model the situation. It is often best to separate these problems into two problems.

You might want to ask students which ratio they would need to use to solve the problem. Once the two triangles are solved, ask students what they do with the answers.

You may wish to have students work in pairs on the Your Turn. They can draw a diagram representing the scenario, decide what two triangles need to be solved, and each develop that solution. Then, the students can decide what to do with their two answers in order to determine how far the bus travelled.

Key Ideas

You may wish to use coaching questions such as the following to help students talk through the Key Ideas.

- How are the angle of depression and angle of elevation similar?
- How do they differ?
- How can you remember which is which?
- What is entailed in solving a triangle?
- What three trigonometric ratios might you use in solving a triangle?
- What other strategies might you use?
- What information about the angles of a triangle might you use to help with or verify your solution?

Meeting Student Needs

- For the Link the Ideas, consider having students create posters representing examples of "Angle of Elevation" and "Angle of Depression" as they relate to something familiar to them. For example, if students are skiers, they could sketch a diagram showing the angle to the top of the ski slope.
- In Example 1, you may have to remind some students that the height of the transit has to be added to the height calculation of the totem pole.
- In Example 4, you might want to review compass directions.

Enrichment

- To give students the opportunity to solve triangles with and without technology, consider having them work in pairs to construct their own right triangles. They can use a protractor and ruler to measure and provide either two sides lengths, or one side and one internal angle. They can then use what they have learned in sections 3.2 and 3.3 to solve the triangle. They can check their calculations by measuring with a ruler and protractor. (You might extend this exercise by having students mark the unknown sides and angles as variables. They can then solve for these, writing the corresponding values on the back of the triangle. Pairs can then exchange triangles with other pairs, solve the triangles, and check their answers with the ones provided on the back.)

Common Errors

- Some students will make a mistake on one value and use this incorrect value throughout the question.

R_x Encourage students, whenever possible, to use only the original values given in the question. If students make a mistake, it will not affect the remaining measurements if they always go back to the original question.

Answers

Example 1: Your Turn

85 m

Example 2: Your Turn

150 yd

Example 3: Your Turn

52 m, 54°, 36°

Example 4 Your Turn

151 m

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Have students draw and label a diagram for all angles. • Students commonly have difficulty with deciding where to draw shadows and typically draw them on the hypotenuse. Have students go outside on a sunny day to visually see where a shadow would appear (on the ground). • You may need to coach students through the meaning of *elevation*, and how it applies to angle of elevation or the angle measured from the base. • Encourage students to write the meanings of elevation and depression into their Foldable, along with a definition and example in their own words.
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Have students draw and label a diagram for all angles. • Students commonly have difficulty with deciding where to draw and how to measure, the angle of depression. Have students go to a window or higher place and look down at an object. Help them relate their line of sight to the triangle needed. • You may need to coach students through the meaning of *depression* and how it applies to angle of depression or the angle measured from the imaginary horizon line.
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to draw and label a diagram for each question. Label the corresponding sides and determine which angle is being referenced. • Have students verbalize which method is easier for them to understand when solving. • Have students check that their calculator is in degree mode.
Example 4 Have students do the Your Turn related to Example 4.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Students may find multiple right triangles confusing. Encourage them to redraw and label separate triangles when solving. Encourage them to label the sides and highlight the common sides.

Check Your Understanding

Practise

In #1, students should label the sides of each triangle and then decide which trigonometric ratio to use.

Questions #2 and 3 are multiple-triangle questions: one triangle is dependent upon the other. Students will decide from the diagram which elements are missing and what ratio they need to solve them with.

Question #5 is based on the fact that many towns and cities across Canada have roadside attractions that represent an aspect of the community. You could ask students if anyone has seen an attraction like this. If not, ask students about other attractions they have come across when travelling through communities.

Apply

Ask students where the given angle for #6 should be placed and how to draw the diagram.

For #7, students should sketch and label a diagram. There is extra information provided in this problem; students will have to decide where it fits in. What will they need to do to justify their answer?

Question #9 may provide material for an interesting discussion. You could ask students the following questions:

- How are cranes raised as the height of the building increases?
- Why don't the cranes tip over?

Students will need to be aware of how the arm moves and the different angles that the operator would see as he looks down to negotiate the boom.

In #10, you might want to ask students what kind of work would be needed to arrange people into the design shown in the Arctic Wisdom picture. Arctic Wisdom was a special two-day briefing on the scientific, cultural, and political issues related to climate change in the Arctic.

Students should sketch a diagram for #12. They may need to understand where the helicopter is in relation to the boats. Remind them that this is a two-triangle problem.

Extend

Question #14 involves another step. Once students have determined both distances, they must determine the truck's speed. Another twist to the problem is that the initial measurement is given in metres, but the speed limit is given in kilometres per hour. Students will have to convert these values to the same units. Also, since the speed of the truck is given in kilometres per hour, and the time span is given in seconds, students will have to compute the equivalent speed.

Question #15 is the first three-dimensional trigonometric problem students have seen. A possible first step to solving this problem is to draw the right triangle that has the rod as its hypotenuse, and then decide what pieces are missing and what needs to be done to find those missing pieces.

Create Connections

In #16, students will need to sketch the problem from both Jennie's and Mike's perspectives. They need to do both sets of calculations to establish whether Richard is correct.

Unit Project

The Unit 1 project questions give students opportunities to solve problems involving the use of the primary trigonometric ratios and the Pythagorean relationship to explore how wireless systems have impacted music distribution.

Question #11 is a multi-triangle question in which students have to find a length in one triangle to find the length in another triangle. You might want to discuss with students the fact that cell phones are now equipped with a GPS that will allow the user to be located. At least three towers will pick up the signal, transferring your call to the tower that is least busy or that gives the strongest signal. It is possible to triangulate a cell phone as the radio waves from the cell phone are sent out. As a result of the triangulation, cell phone users are able to be located when they initiate any kind of call with the cell phone.

Meeting Student Needs

- Provide **BLM 3–8 Section 3.3 Extra Practice** to students who would benefit from more practice.
- Some questions in the Check Your Understanding rely on the idea of complementary angles adding to 90º. You might want to review this concept with students who may not recall it from earlier math courses.
- Some students may benefit from building physical models of some of the questions. These models can be very simple, built from items such as a cardboard box and a rope or string.

- For #8, you might want to explain what an alidade is a device, originally used in astronomy, that allows someone to sight a distant object and use the line of sight to measure the angle.
- Some students may have difficulty with the conversions in #14. You may want to provide guidance or have students work in pairs.
- Some students may have difficulty visualizing and drawing the situation in #15. They may benefit from using manipulatives to model the problem and then draw it.

Enrichment

- One of the many uses of trigonometry is to aid in navigation. Encourage students to create questions that involve airplanes traveling with winds at right angles blowing them off course.

Gifted

- Ask students to explore how triangles with angles greater than 90º could be solved using combinations of right triangles.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students complete questions #1a), c), 2, 3, 4a), b), 5a), e), 6, 7. Students who have no problems with these questions can go on to the remaining questions.	• By labelling the sides of the triangles for #1, students should be able to make choices as to which trigonometric ratio to use. Problems in this section will combine all the trigonometric ratios that the students have worked with. • Questions #2 and 3 use multiple right triangles. Encourage students to redraw the triangles separately, and highlight the side common to them. Labelling the sides using their reference angle should assist in deciding which trigonometric ratio to use. • For question #4, have students review Example 2. They should remember that the elevation angle is measured from the ground, and angle of depression is measured from the line of sight. Have students label the sides of the triangle. • Question #5 provides an opportunity to solve triangles in real world scenarios, using tourism as an example. Encourage students to draw and label a diagram before beginning. • For any questions from #1 to 5 that required extra coaching, use the unassigned questions to check for understanding. • Questions #6 and 7 require students to model a scenario with a diagram and solve it. Remind students that they need to support their answer with work and calculations.
Unit 1 Project If students complete #11, which is related to the Unit 1 project, take the opportunity to assess how their understanding of the chapter outcomes is progressing.	• For question #11, remind students that, as part of their submission for the Unit 1 project, they need to draw and label their diagram and show their thinking, including all calculations. • Remind students to store all project-related materials in their project portfolio.
Assessment *as* Learning	
Create Connections Have all students do #16.	• Encourage students to verbalize their thinking. • Allow students to work with a partner to discuss the question, and then have them provide individual responses orally or in written form. • Encourage students to offer a good rationale for angles of elevation and depression. Remind students that they are to explain their thinking in both parts.