Integral Exponents

4.2

***Mathematics 10*, pages 162–173**

Suggested Timing

80–100 min

Materials

- ruler

Blackline Masters

BLM 4–3 Chapter 4 Warm-Up
BLM 4–4 Chapter 4 Foldable
BLM 4–7 Section 4.2 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcome

AN3 Demonstrate an understanding of powers with integral and rational exponents.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#2, 4, 5, 9, 10, 11, 14, 21, 25, 26
Typical	#1, 2, 4, 5, 7, 9–11, 13–15, 17, 19–21, 25–27
Extension/Enrichment	#1, 3, 5, 8–10, two of 11–13, 16, 18–27

Planning Notes

Have students complete the warm-up questions on **BLM 4–3 Chapter 4 Warm-Up** to reinforce prerequisite skills needed for this section.

As a class, discuss the opening paragraph and the photograph of the Rhind papyrus. You might use the following arithmetic puzzle that is recorded in the RMP to help introduce exponents: Seven houses contain seven cats. Each cat kills seven mice. Each mouse had eaten seven ears of grain. Each ear of grain would have produced seven hekats of wheat. (The hekat was an ancient Egyptian unit of volume.) Ask students how they could use exponents to express the total number of houses, mice, ears of grain, and hekats of wheat. What is the total? You might ask them to share similar puzzles they are familiar with.

Use the lead-in about the age of the RMP to read and discuss the information about carbon-14 dating. Ask students to discuss what a decreasing amount of carbon-14 means for the sign of the exponent in the formula.

You might explain that carbon-14 is formed by cosmic rays changing the nuclear structure of nitrogen-14 atoms. The ratio of carbon-14 atoms to carbon-12 atoms is relatively constant in nature. There is approximately one carbon-14 atom for every trillion carbon-12 atoms. This ratio remains constant in living organisms. When an organism dies, no new carbon-14 is incorporated into the organism and the carbon-14 that is present decays at a very slow rate. Half of a sample of carbon-14 decays in 5700 years. The ratio of carbon-14 to carbon-12 in a sample is used to date the sample. Refer students to the Did You Know? about the accuracy of carbon-14 dating for samples up to 60 000 years of age. This dating technique was devised by Willard Libby in 1949. He received the Nobel Prize for chemistry in 1960 for his discovery. Note that the formula for determining the age of a sample is beyond high school math. A formula that can be used to calculate the amount of carbon-14 remaining in a sample after t years could be approximated using the formula $N = N_0\left(\frac{1}{2}\right)^{\frac{t}{5700}}$, where N_o is the amount of carbon-14 at time 0 and N is the amount of carbon-14 after t years. By replacing $\frac{1}{2}$ with 2^{-1}, the formula can also be written as $N = N_0(2)^{\frac{-t}{5700}}$.

Investigate Negative Exponents

In this Investigate, students determine the meaning of a power with a negative exponent through a patterning exercise.

Have students work individually or with a partner to construct the number line and work through #1 to 4.

While students work, circulate and ask questions to help them focus on the key ideas about exponential form. Ensure that students halve the distance each time between 0 (left endpoint) and the previous midpoint. Some students may incorrectly halve the distance on both sides of the midpoint with the nearest endpoint.

For #2, you may need to coach students to record the value for x in base 2 after each division. After students have marked two or more midpoints, ask the following questions:

- How does the power change for each successive halving process?
- How does the power relate to the distance from 0 (the left endpoint)?

For #3, some students may record the distances less than 1 using decimal form. If so, ask them for the fractional form for each distance. Check that students have included distances up to the value for $\frac{1}{8} = 2^{-3}$.

For #4, students may organize their results differently. Consider asking a student pair to record the results on a table on the board. As a class, discuss the results and then have students compare with their own results. Have them discuss any differences in values.

Give students sufficient time to complete #5 and 6 either individually or with a partner. For #5b), some students may benefit from rewriting the denominator as a power with base 2 in order to make the connection between negative and positive exponents. For example, $\frac{1}{8} = \frac{1}{2^3}$. This may help them to develop a general form for writing any power with a negative exponent as an equivalent power with a positive exponent for #5c).

For #6a), direct students to the Did You Know? that explains that the half-life of a radioactive element is constant. Ask why this is important to know for answering part b). You might coach students to divide the total time of decay by the half-life to determine the number of half-lives. Then, calculate what fraction of the sample will remain after the calculated number of half-lives. For example, the fraction of the sample remaining after two half-lives is $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 4^{-1}$, and after three half-lives is $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 8^{-1}$.

As a class, have students discuss their findings. Invite students to discuss their conclusion about when negative exponents might be used.

Meeting Student Needs

- Consider inviting a science teacher to talk about radioactive decay and carbon-14 dating. Alternatively, you might show a video clip illustrating carbon-14 dating. If so, you may find the related Web Link at the end of this section useful.
- Students may use grid paper instead of measuring line segments with a ruler.
- Some students may benefit from working through an example of half-lives using the base 10, since this base is familiar to them. If so, draw a series of adjacent boxes and show the place value of each digit in both whole-number and power-of-10 form. When you move to the right of the decimal point, show the values as a fraction, a decimal, and a negative exponent. Using place values that students are familiar with may help them to understand the concept of negative exponents.
- Work with students to develop a table such as the one shown. Have students decide on the headings for each column and row.

Halfway Points	Value	Exponential Form
1	16	2^4
2		
3		
4		
5		
6		
7		

- For #6a), check that students understand the meaning of half-life. Ask students to imagine eating half a bag of snacks and then half of what is left and then half of that, etc. Will they ever finish all the snacks? Discuss this as a class.

ELL

- Explain that *integral exponents* refer to exponents that are integers.
- Explain the term *papyrus*. Papyrus is a thick material produced from the papyrus plant used to make paper. It was once abundant in the Nile delta of Egypt. Some students may be more familiar with birchbark. Birchbark, which is the bark of the Paper Birch tree, has been used in Aboriginal traditions to make items ranging from canoes to scrolls, art, and maps.

Enrichment

- Challenge students to write a report explaining how carbon-14 works or a discovery made with carbon-14 dating. For instance, they might research discoveries of artifacts such as arrowheads.

Common Errors

- For #3, students may struggle with determining and labelling the fractional distances between 0 and 1.

R_x Coach students to observe the pattern as the fractions are halved each time. The denominator doubles.

- Students may struggle with labelling the three powers between 0 and 1.

R_x Encourage them to expand the area between 0 and 1 on their number line by drawing a zoom-out above or below the original number line in order to label the values.

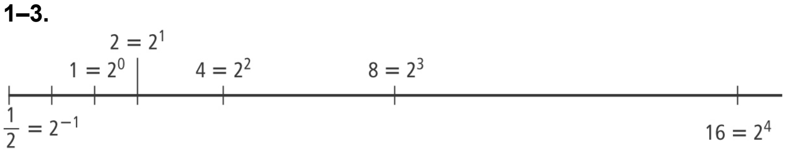
Web Link

For a video clip illustrating how carbon-14 dating works, go to www.mhrmath10.ca and follow the links.

Answers

Investigate Negative Exponents

1–3.



2. a) 4 times **b)** decreased by one each time

3. a) $\frac{1}{2}$ **b)** 2^{-1} **c)** $\frac{1}{4} = 2^{-2}; \frac{1}{8} = 2^{-3}$

4. Example:

Line Segment Lengths	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$
Exponential Form	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}

5. a) Each halving reduces the exponent by 1.

b) $\frac{1}{8} = \frac{1}{2^3}$

$= \left(\frac{1}{2}\right)^3$

$= 2^{-3}$

Example: The pattern is "1 over a power is the same as the power to a negative exponent."

c) $n^{-x} = \left(\frac{1}{n}\right)^x$

6. a) $2^{-2} = \frac{1}{4}$ remaining; $2^{-3} = \frac{1}{8}$ remaining

b) Examples: Situations involving light intensity, gravitation, or erosion.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• Have students fill in the corresponding entry on a table after each number line division. Ask how $\left(\frac{1}{2^2}\right)$, $\left(\frac{1}{3^2}\right)$, and $\left(\frac{1}{4^4}\right)$ change to 2^{-2}, 3^{-2}, and 4^{-4}. Then, have students attempt to generalize a formula. • Have students complete #6a) and check for understanding before moving on.

Link the Ideas

Have students recall what they know about the exponent laws from previous math courses. Direct their attention to the table that summarizes the exponent laws on page 164 in the student resource. For a product of powers, you might model how to expand an example, such as $(4^5)(4^2)$ = (4)(4)(4)(4)(4)(4)(4). Explain that exponent laws can be used to simplify calculations of five 4s multiplied by two 4s, which means multiplying seven 4s together (by adding the exponents). For a quotient of powers, you might ask students to verbalize what an expression, such as $\left(\frac{5^6}{5^2}\right)$ means (six 5s divided by two 5s) and how to simplify using the related exponent law (by subtracting the exponents).

Then, have students form groups of four and verbalize the meaning of each exponent law. If necessary, coach students by using prompts such as the following:

- How can you multiply powers with the same base? (Add the exponents.)
- How can you divide powers with the same base? (Subtract the exponents.)
- How can you raise a power to an exponent? (Multiply the exponents.)
- How can you raise a product to an exponent? (Rewrite each number in the product with the same exponent.)
- How can you raise a quotient to an exponent? (Rewrite each number in the quotient with the same exponent.)
- What is the value of the power when the exponent of the power is 0? (1)

For the exponent law about zero, help students recall how they know that $a^0 = 1$ by using division to show this. Choose any power of 4, such as 3. Divide it by itself.

$$\frac{4^3}{4^3} = 4^{3-3}$$
$$= 4^0$$
$$\frac{4^3}{4^3} = 1$$

So, $4^0 = 1$.

Challenge students to develop their own example to prove that $a^0 = 1$.

You might have students use **BLM 4–4 Chapter 4 Foldable** to record the verbal explanation for each exponent law.

After time for discussion, ask the following questions:

- What did you learn by sharing your ideas with another pair?
- How did this sharing modify your ideas?
- What did you learn?
- What, if any, misconceptions did you correct?

Based on what students say, you might consider re-teaching any exponent laws that students find challenging.

Explain that some powers in this section will have *fractional bases*. Explain to students what these are and how to recognize them and differentiate between them (e.g., $\left(\frac{3}{4}\right)^2$ and $3^{\frac{2}{4}}$). Ask which power has a fractional base and which does not.

Help students recall for an integral exponent that *integers* are positive and negative whole numbers and include zero. This section features integers as the exponents in a power. The new concept is having negative whole numbers and zero as exponents.

As a class, walk through the principle that the power with a negative exponent is equal to the reciprocal of the base raised to the positive of the exponent. Show the example

$$10^{-3} = \left(\frac{10^{-3}}{1}\right)\left(\frac{10^3}{10^3}\right)$$
$$= \frac{1}{10^3}$$

Then, you might walk through the following example and show that a negative exponent in the denominator of a fraction is equal to the reciprocal of the base raised to the positive of the exponent.

If $10^{-3} = \frac{1}{10^3}$, then $(10^{-3})(10^3) = \frac{10^3}{10^3}$
$= 1$

Then, $\frac{1}{10^{-3}} = \left(\frac{1}{10^{-3}}\right)\left(\frac{10^3}{10^3}\right)$
$= 10^3$

Have students restate the meaning of the new principle in their own words. For example, a base with a negative exponent can be written as a fraction with the base and the positive of the exponent in the denominator. Also, a base with a negative exponent in the denominator can be written as a base with a positive exponent.

Example 1

In this Example, students multiply or divide powers with a common base. Work through both methods for multiplying and dividing powers as a class.

For part a) Method 1, some students may benefit from seeing the division of 5^8 and 5^3 in expanded form before applying the exponent law: $\frac{(5)(5)(5)(5)(5)(5)(5)(5)}{(5)(5)(5)}$.

For part b), Method 2, ask how they know they can convert the negative exponents to positive exponents.

For part c), you might ask students what other method could be used. Using positive exponents,

$$\begin{aligned}\frac{x^5}{x^{-3}} &= (x^5)(x^3)\\ &= x^8\end{aligned}$$

Have them try it.

For part d), you might show students an alternative way of arriving at the same solution for Method 2 using division of fractions:

$$\begin{aligned}\frac{(2x)^3}{(2x)^{-2}} &= \frac{(2x)^3}{1} \div \frac{1}{(2x)^2}\\ &= \left[\frac{(2x)^3}{1}\right]\left[\frac{(2x)^2}{1}\right]\\ &= (2x)^5\end{aligned}$$

Use the following prompts to assist students:

- Why is it important to maintain the brackets?
- Why do you take the reciprocal of the second fraction?

You might then have students work in small groups to discuss the solutions and why and how the methods shown work. Ask questions such as the following:

- What methods can be used to multiply or divide powers with the same base?
- Which method do you prefer? Why?
- For which types of questions would you use positive exponents as your preferred strategy?
- For which types of questions would you use adding or subtracting exponents as your preferred strategy?

Have students work with a partner and challenge them to use different methods to solve each Your Turn question. For part c), ask what the parentheses around the negative bases mean. Reinforce that the negative sign is part of the base and that the exponent also applies to the negative sign. Have partners discuss their answers with each other and work together to resolve any differences in the solutions.

Example 2

In this Example, students simplify and then evaluate powers of powers. Work through each problem as a class.

Before students attempt part a), you might demonstrate the law for raising a power to an exponent by using repeated multiplication.

For example, $\begin{aligned}(2^4)^3 &= 2^{4+4+4}\\ &= 2^{12}\end{aligned}$

Another way of expressing this is $2^{(4)(3)}$.

Have students discuss and try solving part b) using the exponent law about zero exponents. The solution is

$$\begin{aligned}[(a^{-2})(a^0)]^{-1} &= [(a^{-2})(1)]^{-1}\\ &= (a^{-2})^{-1}(1)^{-1}\\ &= (a^{(-2)(-1)})(1)\\ &= a^2\end{aligned}$$

For part c), ask students what other method could be used to evaluate the expression. How can they use what they know about negative exponents? Invert and power positively:

$$\begin{aligned}\left(\frac{2^4}{2^6}\right)^{-3} &= \left(\frac{2^6}{2^4}\right)^3\\ &= (2^2)^3\\ &= 64\end{aligned}$$

Ask students which method they prefer and why.

For part d), have students explain why the base is the reciprocal of the original (to divide by a fraction, take the reciprocal of the base). Draw students' attention to the Mental Math box that asks if this is true in all cases. If so, does this allow them to create a mental math shortcut for expressing fractions such as $\left(\frac{2}{3}\right)^{-4}$ as $\left(\frac{3}{2}\right)^4$?

As students analyse the methods shown in Example 2, challenge them to notice the similarities and the differences from the strategies used in Example 1. Have students discuss why certain methods are shown and the advantages and possible disadvantages of these methods. Ask students the following questions:

- What methods do you find easier to use? Why?
- What strategies do you use to help decide on a method?

- When might one method be better than another one? Explain.

Direct students to the Did You Know? on page 166 that explains John Wallis' work. Clarify that *infinity* means without end. In mathematics, it is often used in contexts as if it were a number (e.g., an infinite number of terms). Explain that the precise origin of the symbol is unknown but it is sometimes called a "lazy eight". You might have students discuss where they have seen the infinity symbol.

Did You Know?

The Métis flag features an infinity symbol. Many Métis people believe the symbol represents the joining of two peoples as a new nation into infinity or that the two nations will survive into infinity.

Have students work with a partner to complete the Your Turn questions using the methods of their choice. Have partners discuss their answers with another student pair and work together to resolve any differences in the final solutions.

Example 3

In this Example, students solve a population density problem involving powers with integral exponents using two different methods.

As a class, read the problem about grasshopper population density. Use the Did You Know? to help provide the context. Explain that grasshopper surveys and forecast maps help determine the need for control measures. You might point out the table in the related Your Turn to reinforce that control measures would be needed for moderate to severe infestations.

Before having students consider the solution, you may have them use their own strategies to solve the problem. Ask the following questions:

- How did you determine the approximate population per square kilometre?
- What answer did you get?

As a class, work through the solution. Note that the first method uses arithmetic while the second method uses exponent rules. You might ask students which method they prefer and why.

Have students do the Your Turn using the method of their choice and then explain their method to a classmate.

Key Ideas

The Key Ideas reinforce that the pattern $2^0 = 1$, $2^{-1} = \frac{1}{2}$, $2^{-2} = \frac{1}{2^2}$, $2^{-3} = \frac{1}{2^3}$, and $2^{-4} = \frac{1}{2^4}$ allows us to define the expression $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$, when a does not equal 0 and n is a positive integer. Have students develop their own example and record it in section 4.2 of their Foldable.

Direct students to the table summarizing the exponent laws. You might have students verify that the equivalent expressions are equal using technology. Have students use the back of the insert for section 4.2 to provide their own example of an integral exponent for each exponent law. You might encourage students to exchange their examples with those of a partner and check the solutions.

Meeting Student Needs

- For the Link the Ideas, consider modelling an example of each exponent law to help reinforce students' understanding. Alternatively, have students work with a partner to verify each example.
  - Product of Powers: $(4^5)(4^2) = 4^7$
  - Quotient of Powers: $\frac{5^6}{5^2} = 5^4$
  - Power of a Power: $(4^3)^5 = 4^{15}$
  - Power of a Product: $(4y)^2 = (4^2)(y^2)$
  - Power of a Quotient: $\left(\frac{4}{5}\right)^3 = \frac{4^3}{5^3}$
  - Zero Exponent: $(-5)^0 = 1$
- Consider having students work in pairs to create posters that illustrate the exponent laws. Each student pair could develop a poster illustrating one exponent law. Have students use the posters displayed in the classroom as a reference tool.
- Some students may benefit from recalling the order of operations. You might have students record the acronym BEDMAS into their Foldable and using it as a reference in solving problems involving order of operations. Coach them to recall the correct order of operations: brackets, exponents, divide and multiply in order from left to right, and add and subtract in order from left to right. Alternatively, some students may prefer the phrase **P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally (parentheses, exponents, multiplication and division, addition and subtraction).

- For Example 1, some students may benefit from practising multiplying powers by expanding first before applying the exponent law.
- Some students may need to recall how to divide fractions (multiply by the reciprocal).
- Some students may find it helpful to think of rewriting expressions in positive exponent form as moving factors from a denominator to a numerator or vice versa. For example, $\frac{2x^{-3}}{5}$ becomes $\frac{2}{5x^3}$ and $\frac{2}{5x^{-3}}$ becomes $\frac{2x^3}{5}$. The example shown for the power of a quotient in the Key Ideas clearly demonstrates this shortcut.
- The Your Turn for Example 2 asks students to simplify and evaluate where possible. You might explain that *simplify* means to make more simple. For example, you might use the order of operations to simplify an expression. Explain that *evaluate* in this case means to determine the value of an expression. For example, 10 – 3 = 7.

ELL

- Explain that an *integral exponent* is an exponent that is an integer, such as $2a^3$ and 9^{-4}.

Enrichment

- For Example 2, challenge students to solve a problem in which they need to convert the powers to the same base first. For example, $\left(\frac{2^4}{8^2}\right)^{-3}$.
- Challenge students to research the origins of the infinity symbol and how it is used. For example, the Métis flag features an infinity symbol. Students may find the related Web Link at the end of the section useful.

Gifted

- Challenge students to explain the reasoning that produces negative exponents.

Common Errors

- Some students may be confused about the exponent rules for multiplying and dividing powers.

R*x* Coach students through solving additional problems using expansion and then simplification. Help them discover that when powers are multiplied, the exponent in the final power is the sum of the initial exponents. Similarly, when powers are divided, the exponent in the final power is the difference between the initial exponents.

- Some students may multiply and divide positive and negative integers incorrectly.

R*x* Coach students through additional problems to help them recall the rules for multiplying and dividing positive and negative integers.

- Some students may add exponents when a power is raised to another exponent.

R*x* Coach students through additional problems using expansion and simplification to help them recall the rule for multiplying the exponents when a power is raised to another exponent.

- Some students may add the bases and the exponents. For example, $(2^3)(2^4) = 4^7$.

R*x* Remind students to keep the bases the same and add only the exponents.

- When working with negative exponents, some students may change the base to a negative number.

R*x* Have students talk through their thinking as they work with a negative exponent.

WWW Web Link

For information about the Métis flag, which includes an infinity symbol, go to www.mhrmath10.ca and follow the links.

Answers

Example 1: Your Turn

a) $\frac{1}{8}(32) = 4$ or $2^{(-3+5)} = 2^2$ **b)** $7^{(-5-3)} = 7^{-8}$ or $\frac{1}{7^8}$

c) $(-3.5)^{[4-(-3)]} = (-3.5)^7$ **d)** $(3y)^{[2-(-6)]} = (3y)^8$

Example 2: Your Turn

a) $(0.6^0)^{-5} = 1$ **b)** $(t^{-1})^{-3} = t^3$ **c)** $(x^2)^{-2} = x^{-4}$ **d)** $\left(\frac{1}{y^3}\right)^{-3} = \frac{1}{y^9}$

Example 3: Your Turn

There are 16.04 grasshoppers per square metre. This is a severe infestation.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Remind students that when they are multiplying or dividing powers with the same base, the exponents change but the base does not. • Encourage students to apply the rules of their choice. • Provide similar questions before having students try the Your Turn. For example, $(3x)^4(3x)^3$ and $\frac{(4x)^5}{(4x)^{-3}}$. • For part d), reinforce the importance of maintaining the brackets by asking what the difference is between $(2x)^2$ and $2x^2$.
Example 2 Have students do the Your Turn related to Example 2.	• Provide similar questions before having students try the Your Turn. • Encourage students to verbalize the process for simplifying each question. • You may wish to have students work with a partner. • Remind students to evaluate only after simplifying.
Example 3 Have students do the Your Turn related to Example 3.	• Some students may prefer to solve using arithmetic. You might point out the possibility of making an error when entering a number with multiple zeros. Encourage multiple approaches. Encourage students to use a second method and check that the solution is correct. • Provide a similar problem to students who would benefit from more practice. Allow them to work with a partner and talk through their thinking.

Check Your Understanding

Practise

For #1, students need to understand the meaning of a negative or a positive exponent used to represent a situation. You might prompt students to recognize that a positive exponent means increasing in value and a negative exponent means decreasing in value.

For #3, have students correct the error and then provide an explanation of the error and correction to a classmate.

For #4 and 5, remind students that for #5c), $t \neq 0$ and for part e), $n \neq 0$. The introduction of coefficients in #5c) and d) requires special consideration. For #5d), check that students apply the exponent 3 to the coefficient 2.

In #6, the bases are rational numbers. Encourage students to simplify using the exponent laws before applying technology to determine the answer.

For #7, you may need to remind students that in time, *four years ago* can be represented by –4. The word *ago* refers to the past, which requires using a negative exponent.

For #8, you may need to help students understand that the base in an exponential expression that simulates growth must be larger than 1 (or 100%). In this case where the growth rate is 1.05 or 5%, the base is 1.05 or 105%.

Apply

It may not be necessary to assign all questions to all students. Allow students some choice in the questions they need to do. Encourage students to solve the problems using a method of their choice.

You might have students compare their method for some problems with that of a classmate who solved the problems in a different way.

For #9, encourage students to express the diameter as a power with base 10 in order to use exponent laws for solving it. Consider extending the question by asking students to estimate how many bacteria could fit on the head of the pin.

For #10b), encourage students who are stuck to solve the exponential equation for $t = 0$.

For #11, you might mention that the Great Galaxy in Andromeda, also called M131, is the nearest spiral galaxy to our own galaxy, the Milky Way. M131 is one of the farthest objects visible to the naked eye.

For #13, refer students to the Did You Know? about whooping cranes to help set the context.

For #16, refer students to the Did You Know? about algae. You might have students discuss problems with algae that they are familiar with.

For #18, refer students to the Did You Know? about oil spills to help explain the context. Emphasize the importance of fast and effective action to contain an oil spill. For instance, a spot of oil no larger than a quarter may be enough to kill a seabird. Ask students what strategies they could use to determine the time needed for the bacteria to reach the required concentration. Encourage them to solve the problem using mental math and estimation.

Extend

For #20a), you might suggest that students raise each side of the equation to the exponent –1. Students will find that the right side of the equation will convert to the reciprocal. For part b), ask students how the power can be rewritten with a base of 3. For part c), encourage students to rewrite the fraction on the right side as powers with base 3 or 4.

For #21, you might provide a clue about rewriting 8 in base 2.

For #22a), some students may need help to realize that $n = 0$ in this case.

Create Connections

For #25, consider having students work in pairs to develop their own pattern to demonstrate the relationship between negative and positive exponents. Have students verbally describe the relationship to their partner before recording a response.

For #26, consider having a class discussion and then have students use the ideas as a springboard to develop their own response.

For #27, students determine how to compare the sizes of different powers. The questions are set up to help students conclude that it is possible to compare different powers when either the bases are the same or the exponents are the same.

For part c), encourage students to try evaluating the powers using technology. They will note the limitation of the calculator in that it cannot display such large values. Allowing students to see the limitations with technology is important in the math classroom. In order to successfully evaluate the powers, students must be prepared to rewrite the exponents in factored form and identify the greatest common factor of the exponents. Subsequently, students can apply the exponent law of powers to rewrite each power with a base that can then be compared. In this case, the greatest common factor of the exponents is 111. The first power could then be rewritten in the following equivalent form:

$$2^{666} = (2^6)^{111}$$
$$= 64^{111}$$

Have students rewrite each power in an equivalent form with an exponent of 111 and a base that can be evaluated and then compared with the other powers.

As a class, have students discuss their results and conclusions.

Meeting Student Needs

- Allow students to work in pairs.
- For #16, coach students to explain the percent of the pond covered today and last week.
- For #27, some students may benefit from coaching before developing their response. Use prompts and examples to help students realize it is possible to compare powers when
  - the bases are the same. (2^5 is larger than 2^4 since the exponent 5 is larger than 4, and the bases are both 2.)
  - the exponents are the same. (4^5 is greater than 3^5 since the base 4 is larger than the base of 3, and the exponents are both 5.)
- Some students may benefit from doing the tutorial about exponent laws described in the Web Link at the end of this section.
- Students need to build a solid understanding of the exponent laws using integral exponents in order to be successful with rational exponents in the next section. Some students may benefit from extra practice questions to reinforce understanding.
- Provide **BLM 4–7 Section 4.2 Extra Practice** to students who would benefit from more practice.

ELL

- Teach the following terms in context: *radioactive*, (bacterial) *culture*, *French-language publishing*, *bacterium*, *red blood cell*, *endangered species*, *atoms*, *rechargeable batteries, nickel-metal hydride battery, voltage, pledges, oil spill*, *crude oil*, *degrading*, *concentration*, *intensity*, *coloured gels,* and *radium.*
- For #11, clarify that a galaxy is a massive system of stars, gas, and dust. A spiral galaxy is one of the three main classes of galaxies. Use the photo to help explain that M131 consists of a flat disk that looks like spirals with long arms winding toward a central concentration of stars called a bulge.

Enrichment

- Students with an interest in the Paleolithic caves and rock art discovered in the Pyrenees, in France, may enjoy researching dating methods such as carbon dating to determine the age of the rock paintings. Have students present a report of their findings with a particular focus on the problems encountered in using carbon dating. Students may find the related Web Link at the end of this section helpful.
- Discuss some examples of exponential growth and decay (e.g., calculating the growth of a caribou population of 1400 using the expression $1400(1.05)^n$; calculating the growth of bacteria using the expression 0.85^{-n}; calculating the decline in value of a mutual fund using the expression $500(1.5)^{-n}$; calculating the amount of a radioactive substance remaining using the expression 0.5^n). Have students observe the base and the exponent of the power in each example and identify when a positive or a negative exponent is used to model a situation. Then, prompt them to draw conclusions. Ask students how they can use the base and the exponent of a power to determine if the power represents a growth or a decay situation.

  You might have students develop a table similar to the one shown to organize their findings.

Growth Model	Decay Model
• a base larger than 1 and an exponent that is positive	• a base larger than 1 and an exponent that is negative
• a base between 0 and 1 and an exponent that is negative	• a base between 0 and 1 and an exponent that is positive

  (These conditions are due to the fact that raising a base to –1 results in the reciprocal of the base.)

- Have students who are interested in bioremediation research how scientists use nutrient enrichment to clean up oil-contaminated shorelines. They may find the related Web Link at the end of this section helpful.

Gifted

- Challenge students to develop and solve their own carbon-14 dating problem: Research a painting from a cultural heritage of your choice. The related Web Link at the end of this section may be helpful. For example, you might choose a painting titled Hero and Heroine by French painter Jacques Iverny (active from 1411–1435). Create a question that requires using the carbon-14 formula to date the painting *or* determine if it is fake based on the amount of carbon-14 remaining on a sample taken from the painting today. Hint: Choosing an older painting is preferable to illustrate carbon-14 dating.
- Challenge students to show the exponential growth of a circular oil spill and resulting environmental damage for every hour or day of delay in containing the spill. Encourage students to graph the results and present their findings to the class.
- Challenge students to solve a problem that illustrates exponential growth: Ms. Lee has a class rule that students who show up late for class must contribute 1¢ toward a class pizza party and every day that they show up late afterward, they must double the amount contributed. Write the formula. How much would a student owe after showing up late 4 days? 8 days? 20 days? 30 days?

Web Link

For a tutorial about the exponent laws, go to www.mhrmath10.ca and follow the links.

For information about cave paintings and the dating methods used to determine the age of the artifacts, go to www.mhrmath10.ca and follow the links.

For a database of European painting and sculpture and a web gallery of artists' works, go to www.mhrmath10.ca and follow the links.

For a video about how bioremediation was used to clean up an oil-contaminated shoreline in Nova Scotia, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise, Apply, and Extend Have students do #2, 4, 5, 9, 10, 11, 14, and 21. Students who have no problems with these questions can go on to the remaining questions.	• Provide additional coaching to students who need support with #2, 4, and 5. These questions provide students with practice writing expressions with positive exponents, and in the case of #4 and 5, practice simplifying expressions. Work with students to correct their errors and then have them try #6 on their own. • Some students may benefit from reviewing the exponent laws using integral values before attempting questions with variables. Refer students to the notes in their Foldable or a class poster of the exponent laws to help them. • For #9, 10, 11, and 14, students apply their understanding of exponents to real-world situations. It may be beneficial to discuss situations when a negative or a positive exponent should be used. Encourage students to add any additional notes to their Foldable. • For #21, students apply the exponent laws in different ways to solve for a missing exponent or base. For each case, encourage students to identify the exponent law that is being used and then attempt to solve the problem.
Assessment *as* Learning	
Create Connections Have all students complete #25 and 26.	• Encourage students to verbalize their thinking. • Allow students to work with a partner to discuss the questions, and then have them provide individual responses orally or in written form. • For #25, students could record their response in their Foldable and use it as a study tool. • For #26, consider allowing students to adapt a real-life situation from the student resource.