Irrational Numbers

4.4

***Mathematics 10,* pages 184–195**

Suggested Timing

100–120 min

Materials

- ruler
- compasses
- square dot paper

Blackline Masters

Master 3 Square Dot Paper
BLM 4–3 Chapter 4 Warm-Up
BLM 4–5 Chapter 4 Unit 2 Project
BLM 4–9 Section 4.4 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcomes

AN2 Demonstrate an understanding of irrational numbers by:

- representing, identifying and simplifying irrational numbers
- ordering irrational numbers.

AN3 Demonstrate an understanding of powers with integral and rational exponents.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1a)–c), 2b)–d), 3, 4a)–c), 5a)–c), 6a)–c), 7a)–c), 8, 10, 11, 13, 21, 24
Typical	#1a)–c), 2a)–d), 3, 4a)–c), 5a)–c), 6a)–c), 7a)–c), 8, 10, 13, three of 12–17, 21, 22 *or* 23, 24
Extension/Enrichment	#1d), 2a), d), 3c), d), 4e), f), 5d), f), 6c), d), 7c)–f), 9, 13, 18, 19 *or* 20, 21, 22 *or* 23, 24

Unit Project Note that #13, 18, and 24 are Unit 2 project questions.

Planning Notes

Have students complete the warm-up questions on **BLM 4–3 Chapter 4 Warm-Up** to reinforce prerequisite skills needed for this section.

As a class, discuss the painting in the opener. Use the prompts to promote discussion:

- What is a ratio?
- What do you know about the golden ratio? (You might have students who are familiar with the golden ratio give some examples of golden ratios.)
- Why might artists and architects use the golden ratio in their designs?
- What questions do you have about the golden ratio? irrational numbers?

Direct students to the Did You Know? about Ayla Bouvette, a self-taught Métis artist. Encourage students to discuss other artists who use the golden ratio.

Investigate the Golden Rectangle

Unit Project

During this Investigate, which is part of the unit project, students

- draw a geometric representation of the golden ratio
- write an exact expression for the golden ratio
- describe the golden rectangles in a painting
- find rectangular shapes that may be in the golden ratio and determine how close the shapes are to golden rectangles

Tell students that they will construct a rectangle that expresses the golden ratio.

Have students work individually or in pairs. Have them use a blank sheet of paper, a ruler, and compasses to construct the golden ratio as outlined. Refer students to the diagrams in the student resource to help them. The non-drawing point of the compass should be placed at vertex D of the square. Emphasize the importance of accuracy in drawing and measuring.

As you circulate, use some of the following prompts:

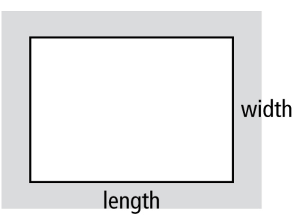
- How might you apply the Pythagorean relationship to calculate the length of DE?
- How close was your measured value of DE to the calculated value?
- In terms of computing the golden ratio, why are the dimensions of the original square irrelevant?
- Looking at your model of the golden rectangle, why do you think people find the ratio pleasing to the eye (compared to other rectangles)?

You may wish to have students use geometry software to construct the golden rectangle.

Give students enough time to answer #5 and 6. For #5a), you might ask students what values they would use to write an exact expression for the golden ratio. For #5, students should conclude that the ratio of the sides of a golden rectangle is approximately 1.618. For #6, have students compare their results with those of a classmate. You might then have students discuss as a class which items in the classroom, if any, are in the golden ratio.

In a follow-up discussion, prompt students to observe that one way to look at a number such as 1.618 is as a geometric representation, not unlike what they have done previously with square roots and cube roots.

You might display a diagram of the golden rectangle similar to the following to help students visualize the golden ratio and the ratio of the length to the width.



$$\text{golden ratio} = \frac{\text{length}}{\text{width}} \approx 1.618\ldots$$

Direct students to the Did You Know? on page 185 that introduces the term *phi* and the symbol used to represent the golden ratio. Use the diagram of the Great Pyramid of Giza to point out the proportions that form the golden ratio.

Meeting Student Needs

- Some students may be more successful if they use grid paper to construct the golden rectangle.
- For #5a), tell students to keep the length of DE in exact radical form in order to determine the exact expression of the golden ratio.
- For #6a), you might have students search for rectangular shapes at home as well as in the classroom (e.g., picture frames, playing cards, photos) and bring them in as part of the Unit 2 project.
- You might invite an artist or a craftsperson to talk to the class about how they use math concepts and skills related to the golden ratio in their work. For instance, a basket maker might use the Fibonacci sequence in creating designs.
- Invite a visual arts teacher to display and discuss some samples of the golden rectangle in student-created artwork. Alternatively, have a visual arts teacher coach students through creating a piece of art using the golden rectangle they created in the Investigate.

Common Errors

- Students may not accurately construct the arc to locate point F.

R_x Reinforce the importance of being careful in constructing the golden rectangle. Students should measure the square accurately and use the compass appropriately. Consider demonstrating how to use the compass.

- Students may not understand how they can use the Pythagorean relationship to determine DE.

R_x Coach students to identify the right triangle containing hypotenuse DE.

WWW Web Link

For an animation of the golden rectangle, go to www.mhrmath10.ca and follow the links.

Answers

Investigate the Golden Rectangle

1. Example: 2 cm

4. a) Example: 2.2361

c) Example: 3.2361

5. a) $\frac{1+\sqrt{5}}{2}$

b) 1.62

c) Example: There are two sets of golden rectangles. One set is horizontal: ground, sky, and centre. One set is vertical: centre with the birds only (not including the entire golden circle around them).

Assessment	Supporting Learning
Assessment *for* Learning	
Unit 2 Project Have students complete the Investigate.	• Consider having students work in pairs. • Listen as students discuss how to solve the problems. As you circulate, clarify any misunderstandings. • Suggest to students that they make their drawings large enough so that they can easily measure and draw using a compass. • You may wish to provide students with **BLM–5 Chapter 4 Unit 2 Project**, and have them finalize their answers. • Remind students to store all project-related materials in a portfolio for that purpose.
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• Have students discuss #5 in pairs, in large groups, and then as a class before developing their own response. • You might ask students whether they can predict the ratio for any rectangle, and if so, explain how.

Link the Ideas

In this section, students build on their knowledge of the number system to learn about real numbers and the relationships among the subsets of the real numbers. They are introduced to irrational numbers and radicals.

Use the graphic organizer in the student resource to help define the real numbers and the subsets of real numbers. Explain why the subsets of natural and whole numbers and integers are nested inside the rational numbers. Use examples to reinforce that numbers can belong to more than one subset.

Define and differentiate between rational and irrational numbers. Have students recall that a rational number is a number that can be expressed as a fraction in the form $\frac{a}{b}$, where a and b are integers, and $b \neq 0$. Ask for other examples of rational numbers (e.g., –2, $\frac{65}{4}$, $-\frac{3}{4}$, and 1.85).

Use examples and non-examples to develop the concepts. Emphasize that irrational numbers are decimals that neither terminate nor repeat. Point out the Did You Know? on page 187 in the student resource about pi as an example of a famous irrational number.

Ask students the following questions:
- Which subset do irrational numbers belong to?
- Which subsets do integers belong to? whole numbers? natural numbers?

After the discussion, use the following prompts:
- What did you learn by discussing real numbers?
- What, if any, misconceptions did you correct?

Discuss the definition of a *radical* and the terms *index* and *radicand*. Explain that a radical is a term for root. The root operation is represented by the radical sign. Use examples such as the following to show that radicals can be rational or irrational numbers.

Irrational examples include $\sqrt{2}$, $\sqrt{\frac{1}{5}}$, and $\sqrt[4]{7^3}$.

Rational examples include $\sqrt{4}$ and $\sqrt{\frac{25}{49}}$. Reinforce that where a radical that is an irrational number appears in a calculation (e.g., Pythagorean relationship, circle geometry), it has an actual value even though the radical cannot be expressed as a fraction (rational number).

As a class, walk through the examples demonstrating how powers with fractional exponents can be written as radicals. Have students discuss the meaning of the diagram showing radical form and exponential form.

Direct students to the example, $2^{\frac{3}{4}}$, written as $\sqrt[4]{2^3}$ or $\left(\sqrt[4]{2}\right)^3$. Reinforce the difference in reading these two expressions. The first one is read "the 4th root of 2 to the power of 3." The second is read "the 4th root of 2 all to the power of 3." Use a few examples (e.g., the 4th root of 2 to the power of 4, $\sqrt[5]{6^3}$) and have students practise reading and writing radicals using proper form.

Walk through the example of rewriting a power with a fractional exponent in decimal form and then evaluating. You might walk through a few additional examples of numerical radicands and have students practise converting and evaluating them.

Reinforce that in the example $\sqrt{5^3}$, the index, and therefore denominator of the rational exponent, is 2 ($5^{\frac{3}{2}}$).

Example 1

In this Example, students rewrite powers with fractional exponents in radical form.

As a class, work through the solutions. Encourage students to explain their thinking for each.

Have students work with a partner to complete the Your Turn questions. Have partners discuss their answers with another student pair and work together to resolve any differences in the solutions.

Example 2

In this Example, students rewrite radicals as powers with fractional exponents.

As a class, work through the solutions. Remind students that if no index is indicated for a radical, the index is understood to be 2. Ask students: Which part of a radical corresponds to the denominator of the exponent? the numerator of the exponent?

Have students work with a partner to complete the Your Turn questions. Have partners discuss their answers with another student pair and work together to resolve any differences in the solutions.

Example 3

In this Example, students convert mixed radicals into entire radical form.

As a class, walk through the definitions of mixed radicals and entire radicals. For the examples of mixed radicals, have students practise reading them: $3\sqrt{2}$ as "3 times the square root of 2"; $\frac{1}{3}\sqrt{5}$ as "$\frac{1}{3}$ times the square root of 5", and $4\sqrt[3]{6}$ as "4 times the cube root of 6".

As a class, work through the solutions. Alternatively, consider having students work in pairs or small groups to answer the questions using their own strategies before working through the given solutions as a class. Point out that every mixed radical can be converted to entire radical form but not every entire radical will simplify to mixed radical form.

For each question, ask students to discuss any other methods they used. Ask which methods they prefer and why.

Have students work with a partner to complete the Your Turn questions. Have partners discuss their methods and answers with another student pair and work together to resolve any differences in the solutions.

Example 4

In this Example, students convert entire radicals into mixed radical form.

As a class, work through the solutions.

For part a), ask students what two numbers multiply to 27 so that you can take the square root of one of the factors. Think of all the numbers that multiply to 27. The answer is (1)(27) and (9)(3). Nine is a perfect square. Think $\sqrt{(9)(3)} = 3\sqrt{3}$. The perfect square 9 can be simplified so the final answer is $3\sqrt{3}$. This may help students understand multiplying radicals later on and reinforces that $3\sqrt{3}$ means we are multiplying.

Some students may benefit from a reminder about how to determine the prime factorization of a number. Make the connection between the index of the radical and the number of factors that must be present in order to simplify the radical. Emphasize that the index of the radical must be recorded in the mixed radical form.

Direct students to the Did You Know? About pentagrams. Point out the features of the pentagram and ask students how the pentagram represents a golden ratio.

Have students work with a partner to complete the Your Turn questions. Have partners discuss their answers with another student pair and work together to resolve any differences in the solutions.

Example 5

In this Example, students compare and order a set of radicals.

As a class, work through both methods shown in the student resource. Ask students what other method might be used. Ask them what they notice about the index of each radical. (The index is the same.) Since the index of each radicand is the same, you might have students express each radical as a mixed radical with the same radicand, and then compare.

$$\begin{aligned} 2\sqrt{18} &= 2\sqrt{(2)(9)} \\ &= 2\sqrt{(2)(3^2)} \\ &= (2)(3)\sqrt{2} \\ &= 6\sqrt{2} \end{aligned} \qquad \begin{aligned} \sqrt{8} &= \sqrt{(4)(2)} \\ &= \sqrt{(2^2)(2)} \\ &= 2\sqrt{2} \end{aligned}$$

$$3\sqrt{2} \qquad \begin{aligned} \sqrt{32} &= \sqrt{(16)(2)} \\ &= \sqrt{(4^2)(2)} \\ &= 4\sqrt{2} \end{aligned}$$

Ask students why it is not always possible to use this method.

Have students work in pairs to complete the Your Turn question. Have each partner use a different method to solve, and then discuss their method and answer with each other.

Example 6

In this Example, students solve a problem involving irrational numbers.

As a class, read the problem and the Did You Know? features about gold production. Then, have students work in pairs to solve the problem.

As a class, have students discuss the method they used and the solution before walking through the given solution. Ask the following questions:
- What method did you use?
- Which method do you prefer? Why?

Have students do the Your Turn and then explain their method and solution to a classmate.

Key Ideas

The Key Ideas summarize the set of real numbers. As a class, discuss the following points as you review the organizer about real numbers:
- Real numbers include the rational numbers and irrational numbers.
- Rational numbers include the integers, whole numbers, natural numbers, fractions, and decimals that terminate or repeat. For example, $\frac{15}{4} = 3.75$.
  - Integers include the whole numbers and natural numbers (i.e., 0, –1, –2 … and 1, 2, 3 …).
  - Whole numbers include the natural numbers (i.e., 0, 1, 2 …).
  - Natural numbers are 1, 2, 3 … .
- Irrational numbers cannot be expressed in the form $\frac{a}{b}$, where a and b are integers, or as decimals that terminate or repeat. For example, $\sqrt{\frac{4}{5}}$ and $\sqrt[4]{35}$. Irrational numbers can be represented using decimal approximations. For example, $\pi \approx 3.142$ and $\varphi \approx 1.618$.

Remind students that radicals can be rational or irrational numbers. Ask for examples of each.

Review expressing radicals as powers with fractional exponents. You might have students use an example to show how a power with a fractional exponent can be written in decimal form, such as $\sqrt[5]{2^4} = 2^{\frac{4}{5}}$
$= 2^{0.8}$

Use the following prompts to discuss converting between radical and exponential forms:
- Do you prefer working with rational exponents that are in fractional or decimal form? Why?
- Is there ever a situation where you should use one form over the other? Explain.
- Explain how to convert from radical to exponential form.

Have students explain how to convert between entire radicals and mixed radicals. Have them demonstrate using examples such as the following:
- Using prime factorization:
$$\begin{aligned} \sqrt{72} &= \sqrt{(6^2)(2)} \\ &= 6\sqrt{2} \end{aligned}$$
- Multiplying:
$$\begin{aligned} 2\sqrt[5]{3} &= \sqrt[5]{(2^5)(3)} \\ &= \sqrt[5]{(32)(3)} \\ &= \sqrt[5]{96} \end{aligned}$$

Have students explain how they would order radicals that are irrational numbers.

Have students write their own summary of the Key Ideas and record it in section 4.4 of their Foldable. Also, have students use their Foldable to define the Key Terms in this section. Have them develop examples for each term. You might have students exchange their examples with those of a classmate and check that they are appropriate.

Meeting Student Needs

- Display a poster that illustrates a radical and labels the radical sign, radicand, and index. Encourage students to draw their own diagram and store it in their Foldable.
- Reinforce the idea that in order to remove a number from a square root, it needs to appear twice in the radicand. In order to remove a number from a cube root, it needs to appear three times in the radicand. Have students extend the pattern for other root indexes.
- For Example 4, discuss that $\sqrt{4} = 2$ and

  $$\begin{aligned}\sqrt{4} &= \sqrt{(2)(2)} \\ &= 2\end{aligned}$$

  Students can use these big ideas of multiplying or dividing radicals to help convert radicals.

Show students the general form $\left(\sqrt{a}\right)\left(\sqrt{b}\right) = \sqrt{ab}$ and $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$. Have them apply the general form using examples.

Enrichment

- Encourage students to use the Web Link in the student resource on page 189 about pentagrams to help them draw one.

Common Errors

- Some students may struggle with converting between mixed and entire radical forms when the index of the radical is larger than 2.

Rx Give students extra practice to determine the prime factorization of numbers. Also, stress the connection between the index of the radical and the number of identical factors that must be present in the radicand in order for a number to become a coefficient of the radical.

WWW Web Link

For a video about expressing roots with fractional exponents, go to www.mhrmath10.ca and follow the links.

Answers

Example 1: Your Turn

a) $\sqrt[4]{10}$ **b)** $\sqrt[3]{1024}$ **c)** $x^{\frac{3}{2}} = \sqrt{x^3}$

Example 2: Your Turn

a) $125^{\frac{1}{2}}$ or $5^{\frac{3}{2}}$ **b)** $y^{\frac{5}{3}}$ **c)** $27^{\frac{2}{n}}$ or $3^{\frac{6}{n}}$

Example 3: Your Turn

a) $\sqrt[3]{2916}$ **b)** $\sqrt{317.52}$ **c)** $\sqrt{\frac{5}{2}}$ or $\sqrt{2.5}$

Example 4: Your Turn

a) $2\sqrt{10}$ **b)** $6\sqrt{3}$ **c)** $2\sqrt[3]{4}$

Example 5: Your Turn

Look for two methods. Example:

- Compare entire radicals: $2\sqrt{54} = \sqrt{216}$; $\sqrt{192}$; $5\sqrt{10} = \sqrt{250}$.

  From greatest to least: $5\sqrt{10}, 2\sqrt{54}$, and $\sqrt{192}$.
- Use a calculator to determine decimal equivalents and compare:

  $2\sqrt{54} = 14.696938...$; $\sqrt{192} = 13.856406...$; $5\sqrt{10} = 15.811388...$.

  From greatest to least: $5\sqrt{10}, 2\sqrt{54}$, and $\sqrt{192}$.

Example 6: Your Turn

$\sqrt[3]{(360)(5)} = 12.164403...$. The edge length is of the cube is approximately 12.2 cm.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Provide similar questions before having students try the Your Turn. • You may wish to have students work with a partner. • Some students may benefit from reviewing exponents and indexes and how they relate to a power. You might have students work in small groups to create a poster that can be displayed in the classroom and serve as a reference tool. • Remind students that when converting powers to radicals, it is the exponents that change and not the base. • Remind students to use brackets around a radicand that has more than one element.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 2 Have students do the Your Turn related to Example 2.	• Provide similar questions before having students try the Your Turn. • Encourage students to verbalize their thinking. • You may wish to have students work with a partner.
Example 3 Have students do the Your Turn related to Example 3.	• Provide similar questions before having students try the Your Turn. • Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Reinforce that the index becomes the exponent of the coefficient. For example: $4\sqrt{8} = \sqrt{(4^2)(8)}$
Example 4 Have students do the Your Turn related to Example 4.	• Provide similar questions before having students try the Your Turn. • Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may benefit from an alternative approach to finding factors of the radicand that multiply together where one factor is a perfect square or cube. For example: $\sqrt{27} = \left(\sqrt{9}\right)\left(\sqrt{3}\right)$ $= 3\sqrt{3}$
Example 5 Have students do the Your Turn related to Example 5.	• Provide a similar question before having students try the Your Turn. • Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to estimate their answers first. • Allow students to use technology as appropriate and as needed.
Example 6 Have students do the Your Turn related to Example 6.	• It may be beneficial to review the rules for isolating a variable in a linear equation. Emphasize the use of opposite operations. Ask students to verbalize the opposite of squaring and cubing. • Provide a similar problem to students who would benefit from more practice. Allow them to work with a partner and talk through their thinking.

Check Your Understanding

Practise

For #1, students convert powers to equivalent radical form. Except for #1a), students need to simplify using the exponent laws before converting to radical form. Since the emphasis is on the process of expressing powers as radicals, students are not required to evaluate the expressions. However, some students may wish to evaluate the powers with a numerical base.

For #2, encourage students to simplify the final expression.

For #3, check that students use brackets around the exponent in order to ensure that the calculator is performing the proper order of operations.

For #4 and 5, you might have students observe how the index of the radical becomes the exponent for the coefficient in order to bring it into the radicand.

For #6 and 7, students need to use prime factorization in order to convert the larger radicands into mixed radicals.

For #8 and 9, the indexes of the radicals are not all the same. Therefore, students may need to use a calculator to evaluate these numbers.

Students will need to solve #10 using a calculator.

For #11, direct students to the Did You Know? about Pacific halibut to help set the context for the problem.

Apply

Allow students some choice in the questions they need to do. Encourage students to solve the problems using a method of their choice. You might have students compare their method for some problems with that of a classmate who solved the problems in a different way.

For #14, students may mistakenly place the denominator under the radical sign. For part b), refer students to the Did You Know? for an explanation of the term *geosynchronous orbit*. Check that students make the connection that 1 orbit of Earth is equivalent to 1 day (24 h).

For #15, tell students to use the pi button on their calculator to maintain reasonable accuracy.

For #16, a partial solution is shown. Check that students isolate the radical. Then, to undo the cube root, students need to cube both sides of the equation.

For #17, students should make the connection that the expression on the right is equivalent to the square root of the division of the power bythe resistance.

Extend

For #19, you may need to coach students to convert each radical sign into the corresponding exponential form. In part a), the power in the radicand needs to be raised 2 times successively to the exponent $\frac{1}{2}$.

For #20, students can begin by evaluating the expression with different types of numbers: positive and negative integers and rational numbers. Then, encourage students to develop an explanation that would extend to any number in general.

Create Connections

Question #24 is a Unit 2 project question. Refer to the Unit 2 project notes for this question.

For #21a), make **Master 3 Square Dot Paper** available. Have students exchange and compare their solution with that of a classmate.

For #22, have students work individually to describe the relationship between a radical and its equivalent power with a rational exponent and then describe the relationship to a classmate or small group.

For #23, students research the history of algebraic or mathematical symbols. They may find the related Web Link at the end of this section helpful.

Unit Project

The Unit 2 project questions, #13, 18, and 24, provide opportunities for students to apply their understanding of irrational numbers.

For #13, you might extend the question by providing the shorter dimension of a painting and asking students to determine the perimeter or the area.

For #18, direct students to the Web Link and the image of a pine cone showing the Fibonacci sequence. Explain this is an illustration of the spiral patterns of seeds. Have students try to locate two sets of spirals. Going clockwise, there are 8 spirals radiating from the centre and going counterclockwise, there are 13 spirals. Ask how the spirals are related to the Fibonacci sequence.

Have students try to simplify the expression in #18 using technology.

For #24, ensure that students describe a different example of golden geometry than what has already been presented during the chapter. You might direct them to the related Web Link at the end of this section to help them research examples of the golden ratio. You might have students present their visuals in a whole-class activity.

Meeting Student Needs

- Allow students to work in pairs.
- When expressing a radical as a power, some students may find it helpful to remember that the number in the notch goes in the denominator.
- Some students may rely heavily on using technology. Ensure they know the order in which to solve problems using technology.
- For #10, show students some Rubik's Cubes. Consider allowing students to use the actual measurements of the cube to answer the question. Invite students to solve the cube.
- Provide **BLM 4–9 Section 4.4 Extra Practice** to students who would benefit from more practice.

ELL

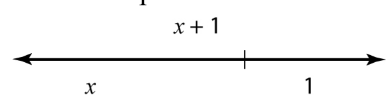
- For #14, clarify that an orbit is the path that an object, such as a satellite, takes as it travels around another object.
- Teach the following terms in context: *skid marks*, *coefficient of friction*, *weaver*, *tapestry*, *telecommunications satellite*, *swing of a pendulum*, *limited-time sale*, *sales discount*, *appliance*, *watts*, *resistance*, *ohms*, and *spiral patterns*.

Enrichment

- For #6, challenge students to express an entire radical such as $\sqrt[3]{\frac{16}{27}}$ as a mixed radical. Tell them to write the numerator and denominator in prime factorization form and simplify each independently.
- For #12, you may wish to invite a police officer to talk to the class about speed and braking distance and the importance of keeping a safe distance between moving vehicles. This may be a timely presentation for students who are learning to drive.
- Challenge students to create a geometric representation of an irrational number of their choice. Students may benefit from some guidance to represent an irrational number geometrically. You might suggest drawing a right triangle with short sides of 1 and 1 and a hypotenuse of $\sqrt{2}$, or a right triangle with short sides of 1 and 2 and a hypotenuse of $\sqrt{5}$. An interesting question would be to create a right triangle with a hypotenuse of $\sqrt{3}$. Alternatively, students might use a diagram that proves the Pythagorean theorem, such as placing three squares along the sides of a right triangle that models $a^2 + b^2 = c^2$, for a triangle with dimensions of 3, 4, and 5 units. Ask them how this model might be used to generate an irrational number for the hypotenuse (e.g., $9 + 4 = 13$).
- Have students research examples of the golden ratio in art and architecture. They might consider Canadian architect Douglas Cardinal and identify golden ratios in his designs.
- Have students explore the connection between the Fibonacci sequence and the golden ratio. Explain that in the Fibonacci sequence, each term is the sum of the two previous terms: $a_n = a_{n-1} + a_{n-2}$. For example, the 12th and 13th terms (144 and 233 respectively) are added to form the 14th term in the sequence (377). Dividing consecutive pairs of numbers in the sequence gives results that get closer and closer to the approximation of the golden ratio, which is $\frac{1+\sqrt{5}}{2}$ = 1.6180339887….
Consider $\frac{233}{144}$ = 1.6180556… and $\frac{377}{233}$ = 1.6180258… . Challenge students to try using a different pair of consecutive terms and compare their results with those of other students.
- Have students research the Fibonacci sequence in art, architecture, or nature. Allow them to present their findings in a format of their choice.
  - For the Fibonacci sequence in nature, have students access the related Web Link on page 195 in the student resource.
  - For the Fibonacci sequence in art and architecture, have students access the related Web Link at the end of this section.
- For #23, some students may be interested in researching the origin of the term *algebra* and al-Khwarizmi, who is considered by some writers to be the father of algebra. They may find the related Web Link at the end of this section helpful.

Gifted

- Have students prove that the square root of 2 is irrational. Have them find a proof and work through it.
- Have students try the animation illustrating the connection between the Fibonacci sequence and the golden ratio described in the related Web Link at the end of this section. Have them summarize the relationship between the Fibonacci sequence and the golden ratio.
- For #24, some students may use the following idea to describe an example of the golden ratio. Divide a line segment of any length into two pieces so that the ratio of the entire segment to the longer piece equals the ratio of the longer piece to the shorter piece.



The ratios produce $\frac{x+1}{x} = \frac{x}{1}$.
So $x^2 = x + 1$ and $x^2 - x - 1 = 0$.
Then, by the quadratic formula,
$x = \frac{1 \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$ and the positive value is
$x = \frac{1+\sqrt{5}}{2}$, which is the golden ratio. Even though students are not familiar with the quadratic formula in grade 10, they could experiment with different lengths of line segments and determine where to divide the segment in order to come as close as possible to the golden ratio.

Common Errors

- Students may struggle with correct calculator sequencing for formulas.

R_x Ensure that students use brackets around fractional exponents. Also, make sure that students use a closing bracket at the end of a radicand.

- Some students may struggle with solving for a variable inside a radicand in a formula.

R_x Coach students through some worked examples that involve isolating a variable inside a radicand.

WWW Web Link

For information about the Fibonacci sequence in art and architecture, go to www.mhrmath10.ca and follow the links.

For an animation of the connection between the Fibonacci sequence and the golden ratio, go to www.mhrmath10.ca and follow the links.

For information about several famous irrational numbers, go to www.mhrmath10.ca and follow the links.

For information about the origin of the term *algebra*, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1a) to c), 2b) to d), 3, 4a) to c), 5a) to c), 6a) to c), 7a) to c), 8, 10, 11, and 13. Students who have no problems with these questions can go on to the remaining questions.	• Provide additional coaching with Example 1 to students who need support with #1, with Example 2 for #2, with Example 3 for #4 and 5, with Example 4 for #6 and 7, and with Example 5 for #8. Coach students through correcting their errors before having them try some of the questions that were not assigned. • Provide additional coaching with Example 6 for #10. Have students talk through their thinking and help them correct the errors. Check for understanding before assigning #11 and 13. • For #8, review the meaning of *irrational* and the bar over the number in part a). • For the word problems, have students record the meaning of each variable before beginning to solve.
Unit 2 Project If students complete #13, 18, and 24, which are related to the Unit 2 project, take the opportunity to assess how their understanding of the chapter outcomes is progressing.	• For #13, you may need to help students recall what they learned in the investigation about the golden ratio. • For #18, you might provide scaffolding to help students evaluate the brackets first before completing the solution on their own. • For #24, encourage students to use the guidelines to help organize their research plan. • You may wish to provide students with **BLM 4–5 Chapter 4 Unit 2 Project**, and have them finalize their answers. • Remind them to store all project-related materials in their project portfolio.
Assessment *as* Learning	
Create Connections Have all students complete #21.	• Encourage students to verbalize their thinking. • Allow students to work with a partner to discuss the questions and then have them provide individual responses orally or in written form. • Allow students to make revisions before handing in their response.