Multiplying Polynomials

5.1

***Mathematics 10*, pages 204–213**

Suggested Timing

100–120 min

Materials

- algebra tiles

Blackline Masters

Master 5 Algebra Tiles (Positive Tiles)
Master 6 Algebra Tiles (Negative Tiles)
BLM 5–3 Chapter 5 Warm-Up
BLM 5–4 Chapter 5 Unit 2 Project
BLM 5–5 Section 5.1 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcomes

AN4 Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1a), b), d), 2, 3a)–d), 4a)–c), 5, 6a), 8–10, 12, 18
Typical	#1b), e), f), 3c), e), f), 4b)–d), 5, 6b), c), 7–9, 11, 12, 14, 18
Extension/Enrichment	#1f), 3f), 4e), f), 6d), e), f), 8, 9, 17, 19

Unit Project Note that #8 and 9 are Unit 2 project questions.

Planning Notes

Have students complete the warm-up questions on **BLM 5–3 Chapter 5 Warm-Up** to reinforce material learned in previous sections.

In this section, it is important that students have enough practice with models to make a connection between concrete models (such as algebra tiles) and the algebra they will now use to represent real ideas. They should use models long enough to gain an understanding of the patterns and rules for manipulating algebraic expressions. They should then make the transition to using algebraic expressions as soon as they are comfortable with these patterns and rules. Every now and then, have them revisit models to help with the connection between the concrete and the abstract.

Investigate Multiplying Polynomials

In this Investigate, students use their knowledge of rectangular area and patterns to understand the distributive property and to develop a technique for multiplying polynomials. It is important to give students an opportunity to explore with the tiles. It may be useful to have students work in small groups of two or three, depending on how many sets of algebra tiles you have available. If algebra tiles are not available, you may wish to provide students with **Master 5 Algebra Tiles (Positive Tiles)**.

Use leading questions to help students define what each tile should represent. For example, you might ask the following questions:

- Which tile do you think best represents the number 1?
- Which tiles represent a product of 1 and a number?
- Which tiles best demonstrate the product of a number and itself?
- How do you represent a negative value using the tiles?
- What happens when you add two tiles of the same size, one positive and one negative?
- Can you use algebra tiles to represent numbers? Explain.
- Which tile would you use to represent 1? 10? 100?

Now, allow students time to explore #1 to 3 of the Investigate. Remind them that their solution should form a rectangle. The following questions are some prompts you might use:

- What number does the large box represent?
- What product did you use to get each of the boxes?
- What value does each of the small boxes represent?
- What is the sum of all of the boxes?
- How does this compare with your original product?

Have students complete #4 of the investigation. They should use the remaining tiles to form a rectangle. Students need to be aware of the patterns that they see after completing a few questions using the algebra tiles. These patterns should lead them to understand the distributive property.

Have students complete #5. Then, enlarge the groups to up to five per group, and have students discuss and answer #6 in their groups and then share with the entire class.

Use prompts to lead students to a better understanding of the patterns of algebra tiles and the distributive property. For each of parts a) to c) of #5, ask the following questions:

- What product did you use to produce the large squares? Record that product.
- What product did you use to produce each of the small squares? Record those products.
- What is the sum of these products?
- How does this compare with the answer you obtained using algebra tiles?
- With the algebra tiles, which term of the answer represents the product used to get the large squares?
- Which term of the answer represents the product and sum of the rectangles?
- Which term of the answer represents the product used to get the small squares?
- Are these patterns useful for other products? Explain.

Have students create their own example of a product of two binomials and see if the pattern works with their example. Tell them to have a partner check it.

Meeting Student Needs

- Explain to students that the painting in the section opener is by Piet Mondrian. Ask students to identify ways in which elements of the painting are used in the design of the cups and building.
- Suggest that students watch for and record examples of designs in the style of Mondrian's paintings. While shopping or looking through magazines, they may find examples on clothing, furniture, art, wallpaper, etc. You might suggest that they cut out or sketch the examples, or take pictures with their cell phones or cameras. Then, you can have these pictures on display while the class works on the chapter.
- Have students research geometric designs in a culture that interests them. For example, see the Web Link that follows for sites that show how Aboriginal peoples have used geometric constructions. As an extension, have students find out about the mathematical/knowledge system that might have been used in the design of Navajo homes.
- Relate mathematical words to non-mathematical words so that students can make the connection. For example, to assist them with the term *binomial*, have them brainstorm words such as *bicycle*, *bilingual*, and *biweekly*, and discuss how *bi* means two. Similarly, to assist them with the term *trinomial*, have them brainstorm words such as *tricycle*, *tripod*, *triangle*, and *trimester*, and discuss how *tri* means three. These connections will help students to identify the new terms and their meaning more easily.

ELL

- Assist students with the terms *sum*, *product*, and *factor* before beginning the section by having a class brainstorming session to define these words. Write students' ideas on the board. Students can then refer to the brainstormed definitions when completing the Investigate**.**
- Students may be unfamiliar with the term *dimensions*. Explain to students that it refers to the measurements in a diagram, such as the length and width of a rectangle.

Enrichment

- As students work on the first part of the Investigate, ask them to expand the modelling of multiplication to include hundreds places; for example, (115)(100). Ask students how having three different place values affects the model. Also, ask if the model has an algebraic equivalent.

Gifted

- Encourage students to investigate polynomials that could be used to create designs similar to Mondrian's work. Have them look for patterns in polynomials that tend to produce designs that are pleasing to the eye. Ask them if they think it is possible to create great art mathematically.

Common Errors

- Some students may have difficulty determining the product if they cannot make the correct configuration with the algebra tiles.

R_x Work your way around the class giving help and asking leading questions:
- What edges of the tiles correspond?
- What shape will occur if you have a product of $(x)(1)$? $(x)(x)$? $(1)(1)$?
- What is the sum of all of the tiles in the solution?
- What do you think this sum represents?

WWW Web Link

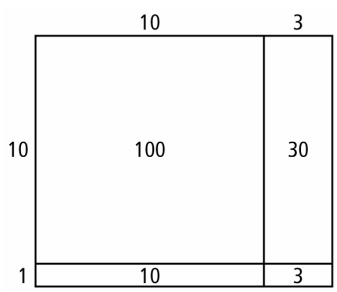
Students can research Aboriginal geometric designs by going to www.mhrmath10.ca and following the links.

Answers

Investigate Multiplying Binomials

1. 143

2. a)

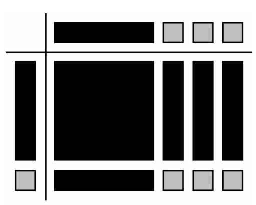


b) Example: The factor 10 + 3 is represented by the length of the rectangle, with a dividing line indicating the 10 and the 3. The factor 10 + 1 is represented by the width of the rectangle, with a dividing line indicating the 10 and the 1.

c) 143

3. Example: Differences: In #1, there are two different products, (1)(13) = 13 and (10)(13) = 130. In #2, there are four areas: (10)(10) = 100, (3)(10) = 30, (1)(10) = 10, and (1)(3) = 3. Similarities: In #1, we determine a sum: 13 + 130. In #2, we also determine a sum: 100 + 30 + 10 + 3. The answer to both is 143.

4. a)

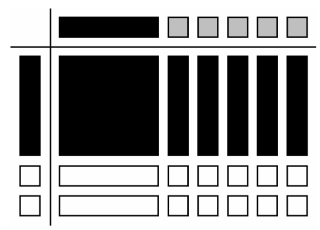


b) Example: The factor $x + 3$ is shown at the top of the diagram and represented by a rectangular x-tile and three small square 1-tiles. The factor $x + 1$ is shown at the left side of the diagram and represented by a rectangular x-tile and one small square 1-tile.

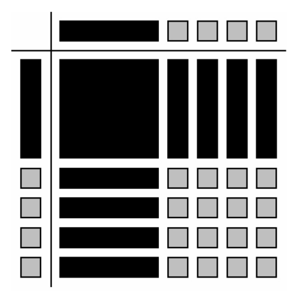
c) $x^2 + 4x + 3$

d) Example: Similarities: The models are similar in that the factors are represented by the dimensions of the rectangle and the product is represented by the area. The large square in #2 represents (10)(10), just as the large square in #3 represents $(x)(x)$.
Differences: In #2, (1)(3) and (1)(10) are each represented by just one rectangle. In #3, $(x)(1)$ is represented by one rectangle but $(3)(x)$ is represented by three rectangles, and (3)(1) is represented by three squares.

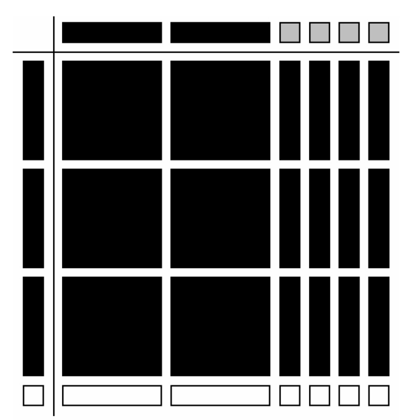
5. a)



b)



c)



Answers

Investigate Multiplying Binomials

6. a) Example: The factors are represented by the dimensions of the rectangle, and the product is represented by the area of the rectangle. An x^2-tile in the area has two corresponding x-tiles in the dimensions. An x-tile in the area has a corresponding x-tile and 1-tile in the dimensions. A 1-tile in the area has two corresponding 1-tiles in the dimensions.

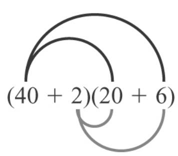
b) Example: The product of the first terms of the factors produces the first term of the answer; this is represented by the sum of the x^2-tiles. The product of the last terms of the factors produces the last term of the answer; this is represented by the sum of the 1-tiles. The sum of the products of the first term of one factor and the second term of the other factor produces the middle term of the answer; this is represented by the sum of the x^2-tiles.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• Have students work in pairs to generate possible patterns that they see. • If students are having difficulty getting started, direct them to look at the initial terms in the factors and then the first terms in the product for each question. Suggest a similar approach for the last terms in the product. • It would likely benefit the entire class to have a discussion about the patterns and how students would multiply the binomials without tiles. Their response here will help guide you to possible areas that require extra explanation as you proceed through the examples.

Link the Ideas

Use the product of 42 and 26 to demonstrate how the distributive property can be used in the multiplication of numbers.

A useful technique to demonstrate the distributive property is to use lines or arrows to indicate what expressions are being distributed.



Discuss the definition of the distributive property as outlined in the margin of the student resource.

Example 1

These questions demonstrate the basic multiplication skills required of students at this level. Have students attempt part a), using the algebra tiles and the distributive property.

Make sure that they are comfortable and competent with both methods before moving on.

Discuss the addition of positive and negative tiles. In this example, there is a sum of $-5x$ instead of $7x$, which occurs because one of the x-tiles is negative and it makes a zero pair with one of the positive x-tiles. Ensure students understand that white tiles are negative. You may wish to provide students with **Master 5 Algebra Tiles (Positive Tiles)** and **Master 6 Algebra Tiles (Negative Tiles)**.

After multiplying using algebra tiles, share with students how this same operation can be completed using the distributive property as outlined in the student resource. Ask students to determine which expression will be distributed over the other expression. You should demonstrate that the order is not important and that the same solution will occur when you multiply $(2x + 1)(x - 3)$ as when you multiply $(x - 3)(2x + 1)$.

After completing part a) of Example 1 together, you may wish to have students do Your Turn part a) using both algebra tiles and the distributive property.

Part b) of Example 1 involves an algebraic expression with two variables. Students may need some help using algebra tiles to express the second variable with tiles. Prompt them with leading questions like the following:

- Using algebra tiles, how do you express $2y$? $-2y$? $-4y$?
- How will you set up the dimensions of a rectangle to solve this product?

After going over the solution, have students attempt the same question using the distributive property. You will begin to see that some students are ready to move away from algebra tiles and use only the distributive property. Direct students to use substitution as a technique to check their answers. Have students complete Your Turn part b).

Example 2

This example extends the binomial multiplication skills to a product of a binomial and a trinomial. Once students are comfortable with binomial products and the distributive property, distributing the binomial over the trinomial should be a straightforward step. You might want to remind students of how to distribute a monomial over a trinomial first. Have them multiply each of the following: $x(x^2 + 3x - 5)$, then $3x(x^2 + 3x - 5)$, and finally $-3x(x^2 + 3x - 5)$. Beginning this way may help students to make the transition to the next level.

After completing the example, have students do Your Turn part a), share their solution, and then do Your Turn part b).

Example 3

This example involves simplifying a more complex product of polynomials. For part a), remind students that they can multiply $3(2x + 4)(6x - 2)$ in any order. Students should decide on the order that they are most comfortable with when multiplying three or more factors. Once the multiplying is done, they should then collect like terms.

Have students complete Your Turn part a), share their solution, and then complete part b).

Example 4

This example demonstrates how to apply the skill of multiplying polynomials to concrete problems. Students should get in the habit of communicating exactly what the variable represents at the beginning of the problem before using an algebraic expression to represent a side or part of a problem. Use prompts to set up this understanding:

- What information are you given in the problem?
- What dimension is missing?
- How can you express the side of the painting?
- How do you find the area of a square?

Once students answer part a), use leading questions to direct students in solving part b):

- What are the dimensions of the red square if the total area of the square is 3600 cm^2?
- What operation can we use to find the length of a side?

Next, have students complete the Your Turn question and check their answer.

Key Ideas

Have students summarize the relationship between multiplying polynomials and the application of the distributive property. Encourage students to use arrows to indicate how each term of the first factor will distribute multiplication over each term of the second factor. Ask students to rewrite in their notebook the product listed in the Key Ideas and draw arrows to represent the distribution of $3x$ over the factor $4x + 5$ and -2 over the factor $4x + 5$. Have students do the same for the product $(c - 3)(4c^2 - c + 6)$. Ask them to indicate what operation the arrows signify.

Meeting Student Needs

- Have students work on the Investigate in pairs or groups of three. You may need to provide extra examples to give students more practice manipulating the algebra tiles to determine specific products.
- In Example 3, students learn to *simplify*. Students often struggle with this term because a completely different set of steps might be required with a different question. Take the time to discuss why the word *simplify* is used here.
- You might want to have students look at the examples and discuss whether the questions could be simplified in fewer steps. In particular, address when it is okay to drop brackets and when it is necessary to keep the brackets.
- For Example 4, you may wish to have students create a problem based on a story or legend relevant to their community. For example, they might write an equation using animals or characters from their favourite legend(s). You may wish to invite community Elders or Knowledge Keepers to tell appropriate stories for this purpose.
- You may wish to assist students in order to reactivate their skills in simplifying polynomials. Discuss that like terms contain identical variables but may have different numerical coefficients.
- You may need to remind students to combine like terms when simplifying the product.
- It may be helpful to highlight the terms to reinforce the method of multiplication. For example, highlight +1 in one colour:
  $(x - 3)(2x + 1) = x(2x + 1) - 3(2x + 1)$
  Similarly, x, -3, and $2x$ would each be highlighted in different colours. Students could even put the terms on different-coloured construction paper and manipulate them. This visual activity may help students to better understand the method.

Common Errors

- When working with more than one variable, some students may become confused about the sum of expressions like $-4xy$ and $-2yx$.

R_x Remind students to express variables in alphabetical order.

- When working with more complicated products of binomials and trinomials, some students may be confused about which factor to distribute over the other.

R_x Have them try both ways to see if the answer is different. This will empower them to solve the question their way. Given a choice, they can decide which method they are most comfortable with.

- Some students may find it challenging to collect like terms. For example, they may confuse cubed and squared variables and may add the coefficients without realizing that the exponents of the variables are different.

R_x Remind students of the definition of like terms and suggest that they consider putting a line through terms that have already been collected.

- Some students may find problem solving challenging.

R_x Discuss with students techniques for approaching a problem. For example: First, students should make sure they understand the problem.
  - Read once for general understanding.
  - Read a second time to determine what needs to be determined.
  - Read again to gather pertinent information.

  Then, they should put together a plan. Students should ask themselves the following questions:
  - Do I need a diagram to help?
  - Do I know what I am trying to find?
  - What will I use as a variable and what will it represent?
  - Can I use an algebraic expression to help me?
  - What operation will I use?
  - Can I solve the problem now?

Answers

Example 1: Your Turn

a) $x^2 - 5x - 3x + 15 = x^2 - 8x + 15$

b) $10m^2 - 30m - 2m + 6 = 10m^2 - 32m + 6$

Example 2: Your Turn

a) $3r^3 - 4r^2 - 38r + 24$ **b)** $10x^3 - 36x^2 + 78x - 36$

Example 3: Your Turn

a) $13x^2 + 25x - 26$ **b)** $-8x^2 + 12x + 31$

Example 4: Your Turn

$x^2 - 12x + 32$

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • You may need to assist students having difficulty with the algebra-tile model by going over the significance of the colours of the tiles. Help students to understand that where different colours meet in the rectangle, you have a negative value. Where the same colours meet in the rectangle, you have a positive. • Remind students that a subtraction sign in front of a number also indicates that the number is negative. This is particularly important when students solve using the distribution method. • Encourage students to try both methods, or use one method and check with another. Have students who use different methods work in pairs. Have students talk through why they chose the method they did. • If students are having difficulty with the methods in Example 1, you could show them FOIL, the box method, or vertical multiplication. One of these approaches may be easier for them to understand.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Have students note that both binomials have a subtraction sign in them. • Alternative methods that you might show students are the box method or vertical multiplication.
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may be overwhelmed by the length of the expansion. They may benefit from dividing up the question into two parts by doing $(x + 1)(5x + 3)$ first and then $+ 3(2x + 4)(6x - 2)$, then combining like terms. • Take extra time with the class to discuss and point out the importance of brackets and how multiple brackets are used in the questions. • Have students work in pairs. Once they have done the questions, have them talk through the thinking they went through as they worked on the solutions.
Example 4 Have students do the Your Turn related to Example 4.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to draw and label their diagrams and compare them with a partner. • If students require assistance to complete the question, before moving on, have them determine the same question with new dimensions.

Check Your Understanding

Practise

For #1, students have an opportunity to practise basic binomial multiplication using algebra tiles. You may wish to have students work in pairs. Make sure that they set up the product as the dimensions of a rectangle and that they create a rectangle as the solution. Work your way around the room, making sure that the groups of students are confident in their work. Have students discuss the patterns that they see using the algebra tiles.

For #2, students must represent a given set of algebra tiles with an algebraic expression. They should be able to determine both what the product is and what the dimensions are from the given diagram.

For #3 and 4, students make the transition from multiplying polynomials using tiles to using the distributive property. Most students should be able to use the distributive property but some may have difficulty with the product of integers or the addition of integers. Provide these students with some integer remediation.

By completing #5, students will have an opportunity to see the patterns of binomial multiplication. They should start to recognize that the first term in the answer is the product of the first terms of the binomials and that the last term in the answer is the product of the last terms in the binomials.

For #6, you may wish to suggest that students use square brackets to separate the products into simpler expressions. They can then complete the product within each square bracket before collecting like terms.

Apply

For #10, encourage students to sketch a diagram with lines drawn to indicate what sections are being cut off of the picture. You may wish to inform students that the picture shows an historical elevator in McNabb, Alberta.

For #11, make sure students recognize that the area of a circle requires the radius and the question gives the diameter of the circle.

For #12, you might suggest that students complete the question themselves and then compare their answer to Bryan's answer.

Students will find #13 and 14 to be similar to #10. Again, it might be helpful to have students sketch a diagram to get a visual image of what is going on. Make sure students set a variable for the unknown dimension and then apply polynomial multiplication to simplify the area of the new object.

Question #15 provides another real-world application. Given the diagram, all students who have tried at least one of #8, 13, or 14 should be able to complete it.

Though they are similar questions, students may find #16 a little more complicated than #12; develop an appropriate solution. It is an effective question for summarizing the ideas of finding errors and checking your answer.

Extend

The Extend question is targeted at students who fully understand all of the concepts in the section. You may wish to have students discuss it in small groups before completing it.

Create Connections

All students should be able to complete part a) of #18. For part b), some students may need coaching to make the connection between representing any number by *n* and writing algebraic expressions for the next three consecutive numbers. Ask leading questions:

- What number did you start with in part a)?
- What is the second number you chose? Why?
- How much larger was the second number than the first number?
- How much larger should the second number in part b) be than *n*?
- How can you represent this new second number using *n* and the difference between your first two numbers?
- How much larger was the third number you used in part a) than the first number?
- How can you represent this new third number using *n* and the difference between the first and third numbers?
- How much larger was the fourth number you used in part a) than the first number?
- How can you represent this new fourth number using *n* and the difference between the first and fourth numbers?

Part c) of #18 will require students to complete two products using their new expressions n, $n + 1$, $n + 2$, and $n + 3$. They could be coached to develop the connection between the product of the first and last numbers and the product of the middle two numbers:

- How did you represent the first number in part b)?
- How did you represent the last number in part b)?
- Write down and multiply these two expressions. What product did you get?
- How did you represent the second number using *n*?
- How did you represent the third number using *n*?
- What is the product of these two values?
- How does the product of the first and last values compare with the product of the middle two values?

For #19, have students represent any multiple of 10, using *t* as a replacement for 10. Have them use a coefficient and *t* to represent numbers like 60, 70, and 90. Then, using *t*, a coefficient, and a sum, they can represent numbers like 63, 72, and 95. When they are comfortable with using algebraic expressions to represent other numbers, have them tell you how they will represent the numbers 45 and 34 using t, a coefficient, and a sum. Then, have them write the product of the new factors to represent the multiplication algebraically. Ask them if they can use the distributive property to multiply these factors. Have them draw lines or arrows to show which terms are being multiplied.

Unit Project

The Unit 2 project questions, #8 and 9, provide an opportunity for students to solve problems involving combining like polynomial terms as well as to generate their own polynomials to model an area formula.

Students will approach #8 in a variety of ways. Have students clarify that they understand the meaning of each polynomial and how many terms each contains. If students are having difficulty generating their own polynomial, have them work with a partner before developing an individual response.

For #9, students need to demonstrate combining like polynomial terms, arrange them in an artistic design, and write the algebraic equation. Encourage students to check that each other's algebraic equation correctly summarizes the addition or subtraction.

You might have students use **BLM 5–4 Chapter 5 Unit 2 Project**.

Meeting Student Needs

- Provide **BLM 5–5 Section 5.1 Extra Practice** to students who would benefit from more practice.
- Allow students to continue to use algebra tiles as long as they need them.
- In connection with #7, you may wish to have students research Aboriginal art online. See the Web Link that follows in this resource.

ELL

- For #11, clarify that the word *inset* is another word for within or inside of.

Gifted

- As soon as students begin to see the connection between the middle term of the answer and the sum of the products of the remaining terms of the binomials, challenge these students to look for these patterns and to make generalizations about them.

Common Errors

- For #2, some students may not understand what each tile represents.

R$_x$ Assist students by asking leading questions to elicit this information from them.

- For #6, some students may have difficulty with the subtraction of two products.

R$_x$ Remind students to multiply by –1 each term in the polynomial that follows the subtraction sign.

- For #11, some students may have difficulty finding the product of π and the square of $3x + 2$.

R$_x$ It may help students to write $(3x + 2)^2$ as the product $(3x + 2)(3x + 2)$ first.

WWW Web Link

To view prints by Inuit artists, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1a), b), d), 2, 3a)–d), 4a)–c), 5, 6a), 8–10, and 12. Students who have no problems with these questions can go on to the remaining questions.	• Students may need help recalling the values of the tiles and how to model the question. Remind them that the product of a positive and a negative will result in a white tile and the same signs will result in a shaded tile. • If students are having difficulty starting #3, have them verbalize how they divide the first binomial into two parts. Ensure they take the operational sign with the second term. • If students find #4 challenging, review the distribution of a monomial over a polynomial, and have students draw lines or arrows from the first factor (monomial) to each term in the second factor (polynomial). • Encourage students to use one of the methods that they feel most comfortable with to solve #5. It could be a method other than the distributive property or algebra tiles. Point out to students that answers are generally written in descending order in terms of the exponent of the variable. • Asking students to explain the order of operations may assist some learners to begin #6. • Encourage all students to draw and label a diagram for #10. Encourage visual learners to verify the multiplication with tiles. • Ask students who do not see an error in #12 to explain like terms. Refer them back to the solved steps to check whether like terms were used consistently.
Unit 2 Project If students complete #8 and 9, which are related to the Unit 2 project, take the opportunity to assess how their understanding of the chapter outcomes is progressing.	• You may wish to provide students with **BLM 5–4 Chapter 5 Unit 2 Project**, and have them finalize their answers.
Assessment *as* Learning	
Create Connections Have all students complete #18.	• Encourage students to verbalize their thinking. • Allow students to work with a partner to discuss the questions, and then have them provide individual responses orally or in written form. • Students may require clarification and an example of *consecutive numbers*. You could have students generate sets of consecutive numbers to clarify understanding. • Have students compare their conclusions regarding the pattern before they go on to part b). • If students are unable to see a pattern, complete a sample problem using the numbers 1, 2, 3, and 4. Determine the pattern and an algebraic expression for the multiplication. Once students understand the process, ensure they select a different set of four numbers.