5.2 Common Factors

Mathematics 10, pages 214–223

Suggested Timing

100–150 min

Blackline Masters

BLM 5–3 Chapter 5 Warm-Up
BLM 5–6 Section 5.2 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
Technology (T)
✓ Visualization (V)

Specific Outcomes

AN1 Demonstrate an understanding of factors of whole numbers by determining the:

- prime factors
- greatest common factor
- least common multiple
- square root
- cube root.

AN5 Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1a), c), 2a), c), e), 3a), c), e), 4a), c), e), 5a), b), e), 6a), c), e), 7a), c), d), 8–10, 12, 14, 19
Typical	#1–3, 4c)–e), 5c)–e), 6c)–e), 7, 9–11, 13, 15, 18
Extension/Enrichment	#1, 6, 7, 9, 12, 14, 16–19

Planning Notes

Have students complete the warm-up questions on **BLM 5–3 Chapter 5 Warm-Up** to reinforce material learned in previous sections.

This section of the chapter is directed toward developing the skill of factoring using common factors. Students are asked to use previous skills of finding factors of given values, writing numbers as a product of prime factors, and determining common factors of numbers. They also find the greatest common factor and the least common multiple for given numbers. These skills will be extended to writing an algebraic expression in factored form using the greatest common factor.

Have students discuss the sets of formulas in the opener and answer the listed questions. This introduction should reinforce at least one reason to express polynomials in factored form.

Investigate Common Factors

The purpose of this Investigate is to help students discover how to express algebraic expressions as a sum of products with one of the factors being the greatest common factor (GCF). This is a first step in factoring polynomials using common factors.

Students may need assistance with their skills in finding the greatest common factor and least common multiple (LCM) of whole numbers. Once confident, students will be ready to use these techniques to find the greatest common factor for algebraic expressions. Avoid giving a direct solution to any of the Investigate questions. Instead, ask leading questions that involve students and allow them ownership of the concept, idea, skill, or process. Then, they will have a connection to it, and it will become part of their learning base. Encourage students to share with the class their methods for finding GCF and LCM. This will also help students to take ownership of their knowledge and to attain the new skills.

In #1, students determine the prime factors of a whole number. Then, they decide whether or not it is possible to determine the prime factors of 1 and 0. Let this question lead to a discussion of the definition of *prime factors*. This discussion sets students up for #2, in which they identify the prime factors and GCF of whole numbers.

In #2 and 3, students find the GCF and LCM. Ask questions to help lead students to gaining these skills:

- Can you express this number as a product of smaller numbers? Explain.
- Are there some special numbers that you can use to express this product?

- How can you check to see that you have the right product of prime factors for each number?

In #4, students expand on their knowledge to include writing two numbers as a product of the GCF and another factor. Allow students to explore the questions. If necessary, coach them toward an answer by using prompts such as the following:
- What number would you multiply 24 by to get 72?
- Is there a way to find that number using addition, subtraction, multiplication and/or division?

Coach students to discover that the easiest way to find the second factor is to divide the original number by the GCF. Use the following leading questions:
- What number do you multiply the GCF of 24 and 72 by to get 48?
- What operation did you use to find that number?
- Can you make a "rule" for finding the second factor if you know the original number and the GCF? (e.g., Second factor = $\frac{\text{original number}}{\text{GCF}}$)

In #5 and 6, students determine the GCF for variables with exponents. They should discover that the GCF for two exponential expressions with common bases is the expression with the smaller exponent. Students experiencing difficulty should be directed toward finding the GCF with questions like the following:
- How can you write 6^2 as a product of factors?
- How can you write 6^3 as a product of factors?
- What is the GCF of these two numbers?

Once students see the pattern, they should be encouraged to extend it by making their own "rule" for finding the GCF of algebraic expressions. Hold a class discussion for students to share their rules, and derive a class rule for finding GCFs involving expressions with variables. In #6c), students once again determine the second factor by dividing the original expression by the GCF.

In #7, students use this new skill to write a polynomial as a sum of products. They should recognize at some point that only polynomials with common factors can be written as a sum of products, and that a sum of products can include the sum of positive and negative terms. You may wish to ask more advanced students to demonstrate how they can use the reverse of the distributive property to write out the sum of products in a different way: $(4x^3)(3x) + (4x^3)(2)$ can also be written as $(4x^3)(3x + 2)$. Share with students that this is known as the factored form of the expression.

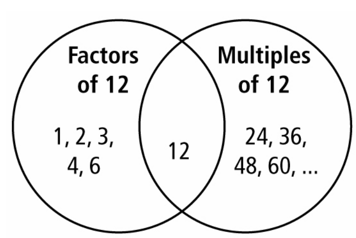
They should now be ready to discuss #8 in groups and develop their own method of factoring a polynomial using GCF.

Meeting Student Needs

- Provide to some students the three shapes in the section opener. Then, have them measure the needed dimensions and determine the surface area using both formulas. This allows students to confirm that both formulas work and assists them in developing an understanding of which formula they prefer to use. Plus, this hands-on activity may be beneficial to concrete and kinesthetic learners.
- You may have to demonstrate to some students how to write a number as a product of prime factors. Use a number, such as 72, that factors in many ways but ultimately results in the same set of prime factors. Also, demonstrate the prime factorization for an expression with variables, such as $12x^3y$, so students understand the skills they will need for the Investigate.
- Clarify to students that when a polynomial is factored, it is slightly different from when a number is factored. The number 24 could be factored as (3)(8) or (4)(6). However, when students must factor a polynomial, it means to factor out the GCF and to have a prime factor in the brackets. For discussion, write $8x - 12 = 2(4x - 6)$ on the board and ask students if $8x - 12$ has been factored.
- For the Investigate, students may need assistance in finding the GCF and LCM using the product of prime factors.

ELL

- Assist students with the term *factor*. Make sure students can differentiate it from *multiple*. One way is by using a Venn diagram:



As a class, fill in the circles and overlapping area. Discuss how not all of the multiples can be listed.

Enrichment

- Have students find and simplify the formulas for the surface areas of combinations of shapes, such as the sum of the surface areas of a right prism, cylinder, and cone.

Common Errors

- Some students may find it challenging to determine the factors of a number.

R_x Encourage students to use the Guess and Check strategy, using a calculator to check their guesses. They may need to be reminded that they are finding only the whole-number factors.

Answers

Investigate Common Factors

1. a) $30 = (2)(3)(5)$

b) No. You cannot write 1 as a product of prime factors because the only factor of 1 is 1, and the number 1 is not prime.

c) No. You cannot write 0 as a product of prime factors because a value of 0 results from the product of 0 and any number, and the number 0 is not prime.

2. a) $60 = (2)(2)(3)(5)$, $48 = (2)(2)(2)(2)(3)$, GCF: 12

b) $25 = (5)(5)$, $40 = (2)(2)(2)(5)$, GCF: 5

c) $16 = (2)(2)(2)(2)$, $24 = (2)(2)(2)(3)$, $36 = (2)(2)(3)(3)$, GCF = 4

3. a) 60 **b)** 100 **c)** 288

4. a) 24 **b)** $72 = 24(3)$, $48 = 24(2)$

c) Divide the original number by the GCF.

5. a) 6^2, 8^4, x^2

b) Example: Similarities: The GCF is the power with the smaller exponent.
Differences: The numeric examples have whole numbers as the bases of the powers whereas the algebraic expressions have variables as the bases of the powers.

6. a) x^5 **b)** $x^5(1)$ or $x^5(x^0)$, $x^5(x^2)$

c) Example: Divide the original expression by the GCF.

7. a) $4x^3$ **b)** $4x^3(3x) + 4x^3(2)$

c) Example: Divide each term in the original expression by the GCF.

8. Example:
- Find the GCF of every term in the polynomial
- Divide each term in the polynomial by the GCF to find the other factor.
- Rewrite the polynomial as a sum or difference of the products of the GCF and the second factors.
- Use the reverse of the distributive property to write the polynomial as a product of the GCF and the remaining sum or difference of the second term.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what a common factor is and how they would find it. Encourage them to try to find more than one way to identify a GCF.	• Students may need to have their skills reactivated in the various methods for finding factors. • Some visual learners may benefit from expanding the powers into repeated multiplication and circling groups of common numbers between the two given powers. This process may be very beneficial for questions with variables, such as the GCF for x^6 and x^9. You can then link this process to the exponent rules that would apply in dividing out a common factor.

Link the Ideas

The Link the Ideas statement prepares students for the skills that they will learn in the examples.

Example 1

In this example, students extend their GCF skills to factor a polynomial using common factors. Point out to students that they have just used the skills they need to do this in the Investigate. Use leading questions to coach students to make this connection.

Method 2 involves listing factors to find the GCF. Since this method may be new to some students, you may need to help them understand that the numbers listed are not products but a list of any factors of the given expression. Students should be reminded that in a list of factors, each factor appears only once.

You may wish to pose the following leading questions:
- What numbers and variables divide evenly into the expression?
- If 4 is a factor, is 2 also a factor? How do you know?
- What is the product of the circled factors? How is this product related to the GCF?

Have students complete the Your Turn questions. Discuss the solutions with the class.

Example 2

In this example, students take the next step in factoring by using prime factorization to determine the GCF. With the group, discuss the process of identifying the GCF of the numerical coefficient and then of the variables.

You may wish to discuss another approach to this question: The GCF of $7a^2b - 28ab + 14ab^2$ is $7ab$, so the expression can be written as $(7ab)(a) - (7ab)(4) + (7ab)(2b)$. By using the reverse of the distributive property, the GCF can be factored out:

$$\frac{7ab}{7ab}[(7ab)(a) - (7ab)(4) + (7ab)(2b)]$$
$$= 7ab\left[\frac{(7ab)(a)}{(7ab)} - \frac{(7ab)(4)}{(7ab)} + \frac{(7ab)(2b)}{(7ab)}\right]$$
$$= 7ab(a - 4 + 2b)$$

The above method may help students who are having difficulty seeing where the resulting factor $a - 4 + 2b$ comes from. Once again, use prompts to assist students in their understanding:

- What happens when you multiply any number by 1?
- What is $3 \div 3$ equal to? $x \div x$? $7ab \div 7ab$?
- If you multiply the original expression by 1, does its value change? Explain.
- If you multiply the original expression by $\frac{7ab}{7ab}$, does its value change? Explain.
- Is multiplying $7a^2b - 28ab + 14ab$ by $\frac{7ab}{7ab}$ the same as $7ab\left(\frac{7a^2b - 28ab + 14ab^2}{7ab}\right)$? How do you know?
- Is $\frac{7a^2b - 28ab + 14ab^2}{7ab}$ the same as $\frac{7a^2b}{7ab} - \frac{28ab}{7ab} + \frac{14ab^2}{7ab}$? How do you know?
- What do you get when you divide $7a^2b$ by $7ab$, $-28ab$ by $7ab$, and $14ab$ by $7ab$?

Leading questions such as those above can help student understanding; it is important that you adapt your questions to allow for individual students' previous knowledge.

Have students do the Your Turn questions. Then, discuss the solutions as a class. Ask students to explain what strategies they used.

Example 3

The purpose of this example is to develop a technique for factoring by grouping using the GCF. The first question should be straightforward for most students. Suggest that those having difficulty highlight the common factor in each term first: $3x(x - 4) + 5(x - 4)$. Ask leading questions:

- How can you use the reverse of the distributive property to factor out the common factor?
- What do you do with the common factor $x - 4$?
- How can you move it in front of the brackets?
- What operation would you use?

For part b) of Example 3, if students group the expression in a different order than the one given in the student resource, ask the following questions:

- Do you get the same set of factors?
- What is important in grouping the two terms being factored?

If students find this question difficult, move through the class helping students group properly before factoring.

Have students do the Your Turn questions with a partner, then discuss the solutions in groups or as a class.

Example 4

This example helps students to see an application for greatest common factors. Students need to make the connection between the concept of greatest common factor and the maximum number of groups shared by all objects.

Ask leading questions to help students understand what is meant by a group of objects:

- What does Paul mean by groups of coins?
- Is he grouping just the toonies, loonies, and quarters? Explain.
- Does each group have the same number of toonies, loonies, and quarters? Explain.
- What mathematical idea (GCF, LCM, or factoring) can you use to determine the greatest number of groups?
- What is the GCF and LCM of the number of coins?
- Which idea, GCF or LCM, best describes the maximum number of groups?
- How can you determine the number of toonies, loonies, and quarters in each group?
- How can you determine the value of the coins in each group?

Have students attempt the Your Turn question in pairs or a small group. Students may find the list of materials daunting. Help them to organize their thinking by asking questions such as:

- How can you clarify how much of each type of wood Mr. Noyle's class has?
- Take a look at your list, and identify the GCF of the numerical coefficient.
- Take a look at your list, and identify the GCF of the variables. (There is none.)

- How can you use the GCF of the numerical coefficient to help you determine how many groups Mr. Noyle's construction class can have?
- Now that you know the number of groups in the class, how can you determine how much material each group has if the material is shared evenly?

Key Ideas

You may wish to use the following prompts to promote a discussion about what students have learned during this section of the chapter:

- When you factor a polynomial, what operation do you use?
- How can you summarize the relationship between factoring and multiplying polynomials?
- When you factor a polynomial, how do you use the GCF of the terms of the polynomial?
- What kinds of expressions can be a common factor?

To reinforce their new skills, have students work in small groups and design a poster summarizing the steps they use to factor a polynomial. The class may want to choose the best poster to hang on the wall.

Meeting Student Needs

- It may be helpful to remind students that a factor divides evenly into the given value.
- Some students may find manipulatives useful as they work on Example 4.

Gifted

- Pose the following question: When you list the factors of numerical coefficients to find the GCF, is it better to start with the greatest numbers first? Ask students to show whether this is a good strategy and to explain their thinking. Then, ask students to investigate if it is a good strategy for finding the GCF of the variables.

Answers

Example 1: Your Turn

a) $5mn$ **b)** $12ab^2c$

Example 2: Your Turn

a) $9rs^2(3r - 2r^2 - 4s)$ **b)** $2np(2p + 5n^3 - 6n^2)$

Example 3: Your Turn

a) $(x + 5)(4 - 3x)$ **b)** $(a + 2)(a + 8b)$

Example 4: Your Turn

a) 8

b) 3(10' × 1 × 4), 4(8' × 2 × 4), (4 × 8)

c) 32' of 2 by 4s and 30' of 1 by 4s

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Have students select the method they prefer to solve the problems. Some students may benefit from reviewing factor trees to find the prime factors of the lead coefficient. Some learners may benefit from circling values that are common so they see what the GCF looks like. It may also help them to expand variable powers and circle the common number of variables in each group. • You might wish to have students work in pairs. Suggest that one student use one method while the other uses a different method. Have them compare answers and discuss any differences. They may wish to talk through how each method works.
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Ensure that all students know the difference between the meaning of *factoring* and *multiplying*. You may wish to have them generate a rule for themselves to help them remember the difference. The following is a possible example: "You get rid of brackets when you multiply. You put brackets back in when you factor." • Remind students that the GCF must divide out of every term in the polynomial. • Have students check each other's work using the method shown in the example. They then can discuss any errors and how those errors occurred.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • If students are having difficulty with part a), ask them what they see that is the same in both parts of the expression. If they are able to identify quickly, show them how to factor out the common factor and how the leftover terms form the remaining binomial. Check for understanding and ask whether they are like terms and can be simplified further. • For part b), you may wish to ask students if it matters how you group the terms. Suggest that they try grouping the first and third terms and second and fourth terms. Allow them to investigate.
Example 4 Have students do the Your Turn related to Example 4.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may benefit from using manipulatives. They can choose a unique manipulative for each type of wood. Ask them to use these manipulatives to illustrate how much of each type of wood Mr. Noyle has for his construction class.

Check Your Understanding

Practise

Students complete a set of basic GCF questions in #1 and 2, listing all the factors of each number and circling the greatest numeric factors.

For #3, students practise their skill in finding the LCM. Once again, student should use methods that they feel comfortable with to do this question.

For #4, encourage students to find the GCF for the coefficients and then for each variable. The final answer will then be a product of these GCFs. They can also use the rules developed by the class as suggested previously in the Investigate.

For #5, students complete a set of basic common factor questions. Students should first find the GCF for all terms and then use either division or the reverse of the distributive property to factor each expression.

In #6, students are given one of the factors and need to find the remaining factor. Students having difficulty with this type of question should factor each expression and then compare their results with the given factor.

In #7, students get practice in factoring by grouping. All students should be able to factor parts a) and b). Parts c) to e) require students first to isolate two groups of two terms with common factors, and then factor out the common factor into a form similar to parts a) and b). Help students to choose their groups wisely by asking if both groups have common factors. Then, help them to manipulate the expression into a simpler form like in parts a) and b).

Apply

Point out to students that #8, 13, and 15 are similar to Example 4 in the student resource. Discuss strategies for solving these problems.

In #9, students summarize the difference between listing factors and multiples of numbers. Students should also be able to explain which of these lists is useful for finding GCF and which for finding LCM.

Question #10 provides another opportunity for students to move from concrete to abstract models for polynomials.

The open-ended question in #11 has many different correct answers. Prompt students with the following questions:
- If this is the GCF, what polynomials can you produce?
- If you have a polynomial and a GCF, what are possible remaining terms in the other factor?

This question may be quite abstract for some students and is a great opportunity to build understanding of factoring using common factors.

The higher-level questions in #12 require students to recognize errors in factoring and make corrections. Suggest that students unable to see the errors do the factoring and compare their answers with the given examples.

Question #14 requires students to recognize that the length and width of the rectangle will involve the variable r from the circles. Prompts might include the following:
- What is the width of the rectangle in terms of r?
- What is the length of the rectangle in terms of r?

- What is the total area of the large rectangle in terms of r?
- How many circles are there?
- What is the total area of all four circles in terms of r?
- How can you find the area of the plate around the circles?
- What algebraic expression represents this area?
- Can this expression be factored? How do you know?
- Is there a common factor? If so, what is it?
- Write the expression in factored form.

In #16, students use their new skill of factoring to solve an area problem algebraically.

Extend

Solving #17 requires a good understanding of the relationship between GCF and multiples of the GCF. Students may need to solve this question using Guess and Check; others will list multiples of 871 until they find the smallest two numbers that satisfy the given constraints.

The skills and processes learned to this point in the chapter are covered in #18. In addition, this question challenges students to go beyond the concepts of this topic.

Create Connections

Question #19 requires students to use the skills of factoring and substitution, and then to consider their thinking process and state a preference for a particular formula.

Meeting Student Needs

- Provide **BLM 5–6 Section 5.2 Extra Practice** to students who would benefit from more practice.
- Students who find these concepts challenging should be encouraged to complete all of the questions in #1 to 7. It may be helpful for them to draw diagrams or work with manipulatives.
- You may wish to pair students to work together on the more difficult problems. Encourage students to write how they determined their answers.
- You may wish to conclude the section by having students work in pairs or groups to create a PowerPoint presentation that will demonstrate their understanding of factoring polynomials.

ELL

- Make sure students understand the meaning of *surface area* as the total area of the surfaces that can be touched.

Common Errors

- Some students may find it challenging to determine all of the factors for each value given.

R_x Suggest that students use another method that they have learned, such as writing out the prime factors and circling the common factors for each group.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1a), c), 2a), c), e), 3a), c), e), 4a), c), e), 5a), b), e), 6a), c), e), 7a), c), d), 8–10, 12, and 14. Students who have no problems with these questions can go on to the remaining questions.	• For #1 and 2, students may find it easier to draw a factor tree to determine the GCF. • Some learners may benefit from writing out the multiples for #3 until they find one in common. Others may benefit from having a multiples chart in their Foldable for quick reference for both multiples and factors. • For #5, make sure students understand that the common factor must divide out from each term. Further, ensure students know that we always factor out the greatest common factor. You could use part d) as an example and factor out 2. Ask students if this is factored completely. This exercise would help with #6 if students are having difficulty finding the missing factor. For part a), ask them questions like, "What could you multiply $2ac$ by to get $6a^2bc$?" For some students, focusing on the numerical coefficient first will assist them in starting to find the missing factors. • It may be beneficial for you to have learners explain what #8 is asking for. Ensure they understand that they are finding the minimum common height. Encourage them to draw multiple heights until they are the same. Ask students how they could have found the answer without drawing. Suggest that they list the factors in numerical order. • Encourage students to share their response to #9 with a partner and then write their response in their Foldable. Suggest that they include a diagram or model to help explain their thinking.
Assessment *as* Learning	
Create Connections Have all students complete #19.	• This question allows students to think about whether factoring does or does not make calculating easier. Discuss with students which formula is easier to calculate. • Students should be able to determine whether or not the answers using both formulas should be the same. Listen to any discussion that occurs around this issue and clarify any misunderstandings.