5.3 Factoring Trinomials

***Mathematics 10,* pages 224–237**

Suggested Timing

120–180 min

Materials

- algebra tiles

Blackline Masters

Master 5 Algebra Tiles (Positive Tiles)
Master 6 Algebra Tiles (Negative Tiles)
BLM 5–3 Chapter 5 Warm-Up
BLM 5–4 Chapter 5 Unit 2 Project
BLM 5–7 Section 5.3 Extra Practice

Mathematical Processes

- ✓ Communication (C)
- ✓ Connections (CN)
- Mental Math and Estimation (ME)
- ✓ Problem Solving (PS)
- ✓ Reasoning (R)
- Technology (T)
- ✓ Visualization (V)

Specific Outcome

AN5 Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1a), b), 2a), b), 3a), c), 4a), c), f), 5a), b), d), f), 6a), d), f), 7a), h), 8a), 9a), 10a), 13, 14, 21, 22
Typical	#1a)–c), 2a)–c), 3a)–c), 4b), d), e), f), 5c), e), f), 6b), c), e), 7b), c), d), f), j) 10, 11b), 13–15, 21, 22
Extension/Enrichment	#5f), 7e), h), 10–12, 14, 16–22

Unit Project Note that #14 and 22 are Unit 2 project questions.

Planning Notes

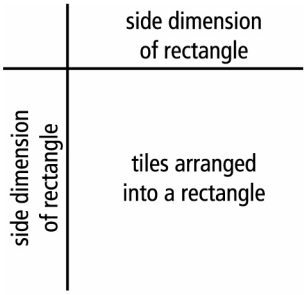
Have students complete the warm-up questions on **BLM 5–3 Chapter 5 Warm-Up** to reinforce prerequisite skills needed for this section.

In this section, students build an understanding of the concepts, skills, and processes required to factor trinomials. Have students use manipulatives to build an understanding that factoring is the opposite of multiplication. Students should use manipulatives until they have developed a full understanding of the patterns and processes involved in factoring trinomials. They will work toward developing a process to factor trinomials of the form $x^2 + bx + c$. Students' factoring knowledge will extend to incorporate trinomials of the form $ax^2 + bx + c$. Students will also be introduced to factoring bi-variable trinomials of the type $ax^2 + bxy + cy^2$. They will progress from using an algebra-tile method to an algebraic method of factoring.

Allow students time to understand this skill set fully as it is the foundation of many subsequent math skills.

Investigate Factoring Trinomials

It may be best to have students work in pairs for this activity. The algebra tiles will allow students to explore kinesthetically. If algebra tiles are not available, provide them with **Master 5 Algebra Tiles (Positive Tiles)**. You may wish to show students a template like the following to help them understand how to set up tiles.



Use leading questions to direct students' understanding regarding the length and width of the rectangle in #2b):

- What are the dimensions of the sides of your new rectangle?
- What do you think these dimensions represent?

It would be a good time to discuss the questions asked in #2 with the class. Make sure that students feel confident in determining the dimensions and have a process developed that works for them. Lead

the class, using appropriate questioning, to see the connection between the side dimensions of the rectangle and the factors of the given polynomial.

Students should start to see relationships between the product of the terms of the factors and the first and last terms of the trinomial. They should also begin to wonder what relationship exists between the factors and the middle term of the trinomial. While students are practising factoring using the tiles in #3, ask questions that may help them to make this connection. As they work either individually or as a whole class, ask students leading questions such as the following:

- What value do you get if you multiply the first terms of the factors? the last terms of the factors?
- Do these values show up in the original polynomial? Explain.
- What value do you get if you add the last terms of the factors?
- Does this value show up in the original trinomial? Explain.
- Can you make a summary statement about the product of the first terms of the factors and the first term of the original trinomial? If so, what is it?
- Can you make a summary statement about the product of the last terms of the factors and the last term of the original trinomial? Is so, what is it?
- Can you make a summary statement about the sum of the factors and the middle term of the original polynomial? If so, what is it?

Using the solutions to #3, reinforce these summary statements and have students answer #4. They should now be ready to test these observations with the questions in #5. Go over the solutions to #5, and then have students work in groups to develop their own personal method for factoring trinomials of the form $x^2 + bx + c$ as outlined in the Reflect and Respond.

Meeting Student Needs

- To write the product of binomials**,** explain that notation is very important; specifically, the placement of brackets. For example, $x + 2(x + 3)$ means that only 2 and $x + 3$ are being multiplied. The product of these two binomials should be written as $(x + 2)(x + 3)$.
- Be sure to emphasize that the method in the Investigate works for trinomials of the form $ax^2 + bx + c$, $a = 1$.
- As a class, you may wish to produce a poster that illustrates FOIL and then discuss how factoring is the reverse.
- You may wish to have students use poster paper to display their group method for factoring trinomials of the form $x^2 + bx + c$. You could hang them in class for students to refer to while doing the Practise questions.

ESL

- Before beginning the Investigate, ensure students know the difference between *binomials* and *trinomials*.

Common Errors

- Some students may make errors in determining the factors of the rectangle.

R_x Once again, a template may help students to see what they are trying to find. Also, have students start at the top of the grid and use rectangular tiles to fit over the large squares and use small squares to fit over the width of the rectangular tiles. Then, have students determine the side dimension.

- Some students may not find it easy to see the relationships between the factors and the middle terms of the polynomial.

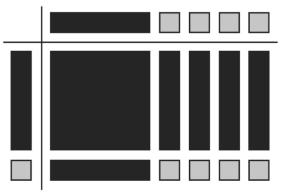
R_x Have students work in groups. By allowing students to create and draw their own group poster, all students will gain ownership for the process they develop.

Answers

Investigate Factoring Trinomials

1. a)

b)



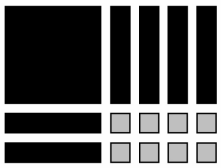
c) Example: The rectangle includes a large square that represents x^2, five rectangles that represent $5x$, and four small squares that represent 4.

Answers

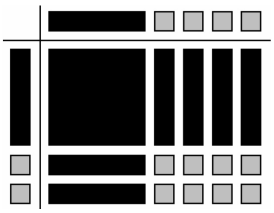
Investigate Factoring Trinomials

d) Example: The factors are shown at the top and left sides of the rectangle. At the top, the rectangle represents x and the four small squares represent 4. At the left side, the rectangle represents x and the small square represents 1.

2. a)



b)

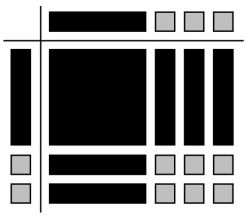


$(x + 4)$ and $(x + 2)$

c) $(x + 2)(x + 4)$. Example: The product is equivalent to the area of the rectangle.

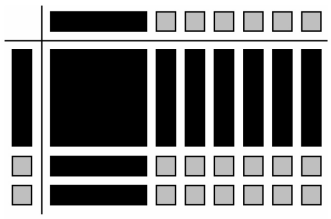
d) $x^2 + 6x + 8$; yes

3. a)



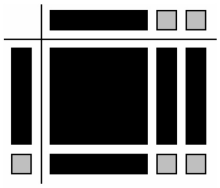
$(x + 2)$ and $(x + 3)$, $(x + 2)(x + 3)$, $x^2 + 5x + 6$, yes

b)



$(x + 2)$ and $(x + 6)$, $(x + 2)(x + 6)$, $x^2 + 8x + 12$, yes

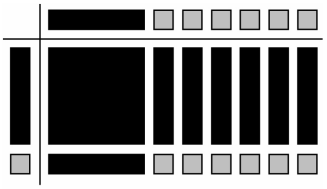
c)



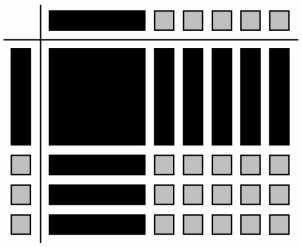
$(x + 2)$ and $(x + 1)$, $(x + 2)(x + 1)$, $x^2 + 3x + 2$, yes

4. Example: b = the sum of the integer terms of the binomial, c = the product of the integer terms of the binomial

5. a) $7 = 6 + 1$ and $6 = (6)(1)$



b) $8 = 5 + 3$ and $15 = (5)(3)$



6. Example: Find two numbers that multiply to c and add to b. If the two numbers are represented by m and n, the factors will be $(x + m)(x + n)$.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss their process for factoring trinomials.	• If students find it challenging to generate a process for #6, redirect them to what they did in #3. Ask them to look at the result of their binomial product before they added like terms. In each case, is there any relationship between the two like terms and the value of *c*? Have them add the like terms and then return to #6.

Link the Ideas

The generalization in the student resource for factoring trinomials of the form $x^2 + bx + c$ may or may not correspond with what students discovered during the Investigate. Students should use the method that they feel most comfortable with or most attached to.

Support the following statement from the student resource by reviewing the distributive property:

$(x + 2)(x + 3) = x^2 + 3x + 2x + (2)(3)$

You may wish to draw arrows to help the visual students make the connection. Some students may need you to group the like terms in brackets $(3x + 2x)$ and then discuss factoring out x from both terms to get $(3 + 2)x$.

You may also wish to use an example other than the one in the student resource for trinomials of the form $ax^2 + bx + c$, when $a \neq 1$. Students can even use their own binomials to multiply. Any product of similar binomials will work as long as all terms are positive. Using negative terms at this point may confuse students with weaker integer skills. Have students use algebra tiles to expand the product and determine the polynomial and then do the same algebraically. Use leading questions to develop the relationship for factoring trinomials of the form $ax^2 + bx + c$, when $a \neq 1$:

- What is the product of the first terms of the binomials?
- What is the product of the last terms of the binomials?
- What value do you get if you multiply these two products together?
- What is the product of the first and last terms of the trinomial?
- What is the product of the outside terms of the binomial?
- What is the product of the inside terms of the binomial?
- What is the sum of these two products?
- How does this compare with the middle term of the trinomial?
- What is the product of these two products?
- How does this compare with the product of the first and last terms of the trinomials?
- What are the factors of this product?
- Are there any factors that add together to form the middle term of the trinomial? Explain.

This is a challenging relationship for some students to grasp. You may need to spend time with some students individually to go over the process.

Example 1

Example 1 involves a progression of questions. Encourage students to make the transition from algebra tiles to an algebraic process for factoring. The example includes a table method for listing all of the possible factors of the last term of the trinomial and the sum of the factors. Coach students by asking them to determine which two numbers multiply to give a product of 4 and add together to give a sum of 5. This may help students to determine the factors in a more direct manner. It is important that all students see the relationship between terms of the trinomial and terms of the binomial factors, no matter what method is used to factor the trinomial. Note that if algebra tiles are not available for Example 1, you may wish to provide students with **Master 5 Algebra Tiles (Positive Tiles)** and **Master 6 Algebra Tiles (Negative Tiles)**.

Since the trinomial in part b) cannot be factored over the integers, hold a discussion about how many of all possible trinomials can be factored and why factorable trinomials are important. Have students share how they know that a trinomial is factorable. Some students may benefit from the following prompts:

- What two numbers multiply to 6 and add together to 4?
- Are there any two whole numbers that can do this? Explain.
- Is this trinomial factorable using whole numbers or integers? Explain.

Part c) involves a trinomial with positive and negative terms. Some students will require that you show them examples involving all of the following situations: $(x + m)(x - n)$, $(x - m)(x + n)$, and $(x - m)(x - n)$.

Some students will benefit from a discussion about the signs of the middle and last terms of the trinomial. When you discuss c as a positive value, you may wish to ask the following questions:

- What is the sign of the product of two positive numbers?
- What is the sign of the product of two negative numbers?
- What signs might the factors have if c has a positive value?
- Does the middle term, b, help you to determine if both factors are positive or negative? Explain.

When you discuss c as a negative value, you may wish to ask the following questions:

- What is the sign of the product of a positive number and a negative number?
- What sign or signs do the factors of a negative number have?
- What does the sign of the middle term, b, tell you about the values of the factors?

Have students work through a progression of factoring questions involving the same positive last term and middle terms with different signs, such as $x^2 + 5x + 6$ and $x^2 - 5x + 6$. Have students factor with algebra tiles, and then generalize the sign rules for trinomials with

- positive last terms and positive middle terms
- positive last terms and negative middle terms

Then, use examples like $x^2 + 2x - 8$ and $x^2 - 2x - 8$. Again, have students factor with algebra tiles, and then generalize the sign rules for trinomials with

- negative last terms and positive middle terms
- negative last terms and negative middle terms

Part d) introduces the concept of factoring trinomials using two variables. Since this may be confusing for some students, consider having them factor $x^2 + 3x - 18$ first. Have them compare the factors in their solution and the factors for $x^2 + 3xy - 18y^2$ as given in the student resource, and share with you similarities and differences. Coach using the following questions:

- Do the factors have the same first terms? Explain.
- Do the factors have the same last terms? Explain.
- How are they the same?
- How are they different?
- Can we use the same process to factor both trinomials? Explain.
- What do you have to do differently to factor trinomials with two variables?

Make sure all students are confident factoring trinomials before they move on to Your Turn parts a) and b). You may wish to have students work in pairs on these questions and talk through their thinking.

Example 2

This example requires that students have strong number skills and be able to complete a multi-step process. Algebra tiles will allow concrete learners to make the connection between the polynomial given and the factors. Making the transition to using algebraic methods may be beyond some students and they should be encouraged to continue to use tiles.

In all questions of this type, encourage students to check for a GCF first.

Part a), Method 1, may require the use of a template. Students rearrange the tiles into a rectangle and determine the factors using the side dimensions of the rectangle.

Method 2 is similar to Method 2 in Example 1, except that the product is not the value of c but the product of a and c from the trinomial. You can introduce this idea through observation of the algebra-tile model. Ask leading questions like the following:

- How many large squares do you have?
- How many small squares do you have?
- How many rectangles do you have?
- Are there any numbers that multiply to 3 and add to 8? Explain.
- Are there any numbers that multiply to 4 and add to 8? Explain.

Discuss with students that since the last two answers are no, they will have to use a different process to factor this type of trinomial. Move on to such prompts as the following:

- When you multiply $3x$ and 2, how many rectangular tiles do you get?
- When you multiply x and 2, how many rectangular tiles do you get?
- What is the total number of rectangles?
- What is the product of the large and small squares?
- Are there two integers that multiply to 12 and add to 8?
- Do we see these numbers represented by any type of tile? If so, which tile?
- What relationship do you observe between the number of rectangular tiles and the product of the first and last terms of the trinomial?
- Which term of the trinomial do the rectangular tiles represent?
- What relationship can you see between the product of the first and last terms of the trinomial and the middle term of the trinomial?

It is important that students have an understanding of this connection between the factors of the product of the first and last terms and the sum of these factors forming the middle term. This understanding is essential for them to factor trinomials of this type.

For this example, note that a suggestion is given to replace $8x$ by the sum $2x + 6x$. The order in which these are substituted will not affect the outcome of factoring and students should play with this idea, factoring the examples using both orders of the sum.

Part b) includes a second variable in the trinomial as well as positive and negative coefficients. This type of trinomial can be challenging for some students. Some students will factor using an algebra-tile method and some using an algebraic method.

Part c) demonstrates to students that not all trinomials of this type are factorable and that the determining measure is the ability to find two factors of ac that add to b. In this case, there are no two integers that multiply to 12 and add to 2, as indicated in the student resource. Ask students how they know whether a trinomial can or cannot be factored.

Part d) introduces the greatest common factor. Suggest that students always look for the greatest common factor first when factoring. They then test the remaining trinomial to see if it can be factored.

Have students complete the Your Turn questions, and then go over these questions with them.

Example 3

This example shows how to apply factoring skills to solve word problems. It requires students to factor given polynomials in the form of an equation. They then evaluate the equation by substituting into either the original polynomial or the factored form. Have students talk through what is happening in the Example.

It may be beneficial to begin a flowchart with the class, either on the wall or in their Foldable, that outlines some of the possible routes for factoring. For example, at the top of the chart would be the GCF. The next level could break off into the number of terms in the polynomial and how each gets factored. This could be a progressive chart to work on as a class.

Have students work in pairs on the Your Turn. Encourage them to discuss the strategy they are going to use, to separately determine the answer, and then to discuss what they did.

Key Ideas

To summarize the skills learned in this section, coach students with questions such as the following:

- Given trinomials of the form $ax^2 + bx + c$, where $a = 1$, and its factors are $x + m$ and $x + n$, what relationship exists between terms m and n and terms b and c?
- Given trinomials of the form $ax^2 + bx + c$, where $a \neq 1$, and its factors are $rx + m$ and $tx + n$
  - what relationship exists between terms r and t and terms a?
  - what relationship exists between the product of m and n and the product of a and c?
  - what relationship exists between terms m and n and term b?
  - if $m + n = b$, how can you rewrite the trinomial to form a polynomial with four terms?
  - How can you factor a polynomial of four terms?
- Compare factoring the trinomials $2x^2 + 5x + 3$ and $2x^2 + 5xy + 3y^2$. How is the process of factoring these trinomials similar? different?

Consider having students form groups to design three posters that summarize the process of factoring trinomials: one poster for trinomials of the form $ax^2 + bx + c$, where $a = 1$, one for trinomials of the form $ax^2 + bx + c$, where $a \neq 1$, and one to summarize the difference between factoring trinomials with one variable and factoring trinomials with two variables. Hang the posters in the room as a reminder of how to factor trinomials.

Meeting Student Needs

- It may help some students if you break down the process and have visual steps for them to follow. For example they can perform the following steps:
  - Factor out the GCF.
  - Look at the number of terms:
    Two Terms: Look for the difference of squares.
    Three Terms: Factor the trinomial.
    Four Terms: Factor by grouping
  - Factor completely and check by multiplying.

  Explain and have visuals for the rules for determining the signs in each factor.
- It may be helpful for you to provide additional polynomials for students to solve in pairs or groups.
- Some students will find it difficult to distinguish between the two types of trinomials, $ax^2 + bx + c$, $a = 1$ and $a \neq 1$. Ensure they are aware that the method used for trinomials with $a \neq 1$ will work for all trinomials in the form $ax^2 + bx + c$.
- A visual can be a helpful tool for students finding it challenging to determine the integers they need to factor a trinomial. For example, when factoring $x^2 - 8x - 20$, set up the following for students:
  $\square \times \triangle = -20$
  $\square + \triangle = -8$
  This visual shows them that the same two integers must multiply to –20 and add to –8.
- You may wish to discuss what is meant when you cannot factor a trinomial *over the integers*. Inform students that this means that no integers can be used to factor a given trinomial. Refer to and discuss the Did You Know! on page 228.
- For Example 3, be prepared to discuss why the formulas are factored first. Make sure students see that they get the same answer if the time is substituted into the original formula or into the factored form.

WWW Web Link

For more information and practice with factoring trinomials, have students go to www.mhrmath10.ca and follow the links.

Answers

Example 1: Your Turn

1. a) $(x + 2)(x + 5)$ **b)** $(r - 9s)(r - s)$

Example 2: Your Turn

2. a) $(2x - 1)(x + 4)$ **b)** $-3(s^2 + 17s + 10)$ **c)** $(2y + x)(y + 3x)$

Example 3: Your Turn

3. a) $h = -16(t - 10)(t + 1)$ **b)** 464.64 ft

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Allow students who need it extra time for practice and exploration. • Some students may need to use the algebra tiles longer to ensure that they see the patterns of factoring. • If students are using the table, encourage them to identify the *b* value and *c* value for each question and the pairs of factors they will use. Developing this habit will assist them in quickly reviewing for incorrect signs on factors in later problems. • If students do not have strong number skills, provide a multiplication table that could be inserted into their Foldable for quick reference.
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Remind students that the operation sign in front of a term also identifies whether it is positive or negative. This understanding is especially important when students find the value for *c*. • For students who may get confused about whether or not to multiply *a* and *c*, show them that all factorable polynomials can be factored using the method in this example; therefore, they need learn only one method. • Identify for students that when using grouping, the signs of each factor must be enclosed within the brackets. Initially, the operation sign between the brackets will be positive. • Encourage students to experiment with different methods as they work on these questions. They may factor with one method and check their answer using a second method.
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Remind students to check polynomials for a GCF before proceeding to finding the factors of *c*. • Discuss with students whether or not it matters which formula they use to evaluate. Their response will serve as a good assessment of their understanding of factoring. • You may wish to have students work in pairs to solve this problem, then join with a second pair and share what they did. Encourage them to talk through the different methods they used, and why.

Check Your Understanding

Practise

For #1, students are given algebra-tile models. If students have difficulty with this question, coach them on the use of algebra tiles to represent trinomials. For #2, students do basic factoring questions using tiles. For #3, students practise number skills required for factoring trinomials.

For #4, students complete basic $x^2 + bx + c$ factoring questions that involve trinomials with only positive terms. It is useful to reinforce the understanding of the relationship between the last term of the trinomial and the last terms of the binomial factors.

In #5, positive and negative terms are introduced. This question will help you to identify which students need coaching on integer skills. These students may need extra practice on this type of question.

In #6 and 7, students build from basic trinomials with positive terms, to trinomials with two variables, to trinomials involving positive and negative terms. Note that some trinomials may not be factorable.

Apply

For #8, students make a connection between algebra tiles and a geometric area with algebraic dimensions.

In #9 to 12, students have an opportunity to use generalizations about factoring to develop factorable trinomials. Some students will need encouragement to apply the concept of factoring trinomials and then use their number skills to decide on the possible solutions.

For #13, students demonstrate much of the same understanding as #9 to 12 but are using their knowledge to create a trinomial expression that cannot be factored.

For #15 and 16, students apply their factoring skills to real-life contexts. All students who can factor trinomials of the form $ax^2 + bx + c$, where $a \neq 1$, should be able to complete #15. For #16, students go beyond factoring to making a connection between the factors and an expression of the number of jackets sold and a price per jacket.

Extend

Question #17 is open-ended. Challenge students to try to come up with different answers for this question, and then to compare answers.

You may wish to have students draw a diagram for #18, discuss the strategies they can use, and then have them solve the question and compare their answers. Challenge students to verify that their side length is correct by multiplying to determine the area.

You may wish to have students work together on #19.

Have students compare the polynomial for #20 to the polynomial for #18 and discuss whether or not they are working with a square. Ask them to verify their thinking mathematically. Challenge them to consider whether more than one shape could provide this area.

Create Connections

For #21, students summarize their understanding of the connection between multiplying binomials and factoring trinomials.

Unit Project

The Unit 2 project questions, #14 and 22, give students an opportunity to model their understanding of the link between multiplying and factoring as well as their ability to model a trinomial. They then incorporate these models into an abstract piece of art of their choice. This activity relates to the overall goals of the unit project.

Meeting Student Needs

- Provide **BLM 5–7 Section 5.3 Extra Practice** to students who would benefit from more practice.
- Remind students that the GCF should always be removed first before they factor any trinomial.
- Before students begin #8 to 12, remind them of the importance of sum and product when factoring trinomials.

Enrichment

- Ask students to use "reverse engineering" to create trinomials that can be factored by first creating binomials and then multiplying. Encourage the use of challenging coefficients. Have them switch trinomials with a classmate to practise factoring.

Gifted

- The formula for finding the distance travelled by a falling object is $d = \frac{1}{2}at^2 + vt$, where d is distance, in metres, a is acceleration due to gravity, t is time, in seconds, and v is the starting velocity of the object that is falling. Ask students to explore the formula, including factoring it and entering various values for time, starting velocity, etc., in order to get a sense of the effect of gravity on objects. Students might consider graphing the formula, using technology. Ask them how this type of formula might be used to create video games such as a golf game.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1a), b), 2a), b), 3a), c), 4a)–c), f), 5a), b), d), f), 6a), d), f), 7a), h), 8a), 9a), 10a), 13, and 14. Students who have no problems with these questions can go on to the remaining questions.	• In #1 and 2, students have an opportunity to demonstrate that they can determine an algebra-tile model given a polynomial and determine the factors from an algebra-tile model. Allow students to use algebra tiles, if necessary. Although the requested approach may vary in each question, students can use the tiles to verify their thinking. • Caution students to watch for the signs on numbers in #3 as they will affect the signs on their factors. This activity relates directly to the questions in #4 and 5, in which they find factors. • Have students verbalize the factoring process to be used in #6. It may be beneficial to have them complete the first question as they verbalize the steps. Ensure students understand the process and are successful in #6 where all values are positive, before moving to #7 where factors will have different signs. • If students are confused about the role of 15 cm in #8, have them explain the dimensions of a rectangle and how to determine area. Then, ask how this links 15 cm. • In #9 and 10, students must demonstrate that they understand the role of b and c in the factoring process. If students are unsure, use $x^2 + bx + 16$ as an example. Have students generate as many possible factors for 16 as they can. Encourage students to use two negative numbers as well.
Unit 2 Project If students complete #14 and 22, which are related to the Unit 2 project, take the opportunity to assess how their understanding of the chapter outcomes is progressing.	• You may wish to provide students with **BLM 5–4 Chapter 5 Unit 2 Project** and have them finalize their answers. • For students who are unsure of how to begin their designs for #22, encourage them to make several models, lay them out on their desk, and slide them around until they create a design.
Assessment *as* Learning	
Create Connections Have students complete #21 and 22.	• In #21, students demonstrate their understanding that binomial multiplication can result in a trinomial. Encourage students to use whatever method makes most sense for them in their description. • Make sure algebra tiles are readily available. • Encourage students to generate their own examples. They may wish to review Examples 1 or 2 for ideas. Show students that they can choose their own factors, e.g., 4 and 5, and write their own binomials. From there, they can generate a trinomial. This will help them to personalize their responses.