Factoring Special Trinomials

5.4

***Mathematics 10*, pages 238–251**

Suggested Timing

100–120 min

Materials

- centimetre grid paper
- scissors

Blackline Masters

Master 2 Centimetre Grid Paper
Master 5 Algebra Tiles (Positive Tiles)
Master 6 Algebra Tiles (Negative Tiles)
BLM 5–3 Chapter 5 Warm-Up
BLM 5–4 Chapter 5 Unit 2 Project
BLM 5–8 Section 5.4 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
Technology (T)
✓ Visualization (V)

Specific Outcome

AN5 Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–3, 5, 6, 7a)–c), 8a), b), 9, 10, 12, 13, 16, 20, 24, 25
Typical	#1–7, 8a), b), 9–13, 16, 20, 22, 24, 25
Extension/Enrichment	#8, 10, 12, 14, 15, 17–19, 21, 23, 26

Unit Project Note that #12 and 13 are Unit 2 project questions.

Planning Notes

Have students complete the warm-up questions on **BLM 5–3 Chapter 5 Warm-Up** to reinforce material learned in previous sections.

This section of the chapter helps students develop an understanding of why differences of squares and perfect square trinomials are special. It also leads them to use this understanding to develop a process of factoring these trinomials algebraically. Do not move too quickly through this section. Ensure students have sufficient time to process the concepts and develop their own models.

Direct students' attention to the photo of the patchwork quilt. This visual shows how geometric shapes (small squares) can form the area of a rectangle, just as algebra tiles form the area of a rectangle. Students should share with each other examples of items made up of geometric shapes that they have seen in their everyday life. They should be given the goal by the end of the section of connecting these examples with polynomials and factors.

Investigate Factoring Differences of Squares

This Investigate is a hands-on (kinesthetic) activity and with the right coaching will lead students to develop an intuitive proof for the difference of squares factoring relationship $a^2 - b^2 = (a + b)(a - b)$. You may wish to give students **Master 2 Centimetre Grid Paper** for the Investigate.

Allow students to work with others or to complete the Investigate by themselves, with some discussion with classmates. Have students complete #1 to 3, leading them to see that the area of a square is a product of two sides like a rectangle, $A = l \times w$, but that it is also the square of a side, $A = s^2$. It is the latter form that students need to note. If students are using only the rectangular method, prompt their thinking:

- Is there another way to find the area of a square? If so, what is it?
- What do you notice about the lengths of the sides of a square?
- Is there another way to express the product of a number and itself? If so, what is it?
- What do we call the operation when we multiply a number by itself?

Once students have made the observation that the area of a square is the square of the sides, help them to discover a method for finding the area of the new irregular shape. They can find the area of the remaining shape by using one of the following methods:

- counting the squares
- cutting it into two separate rectangles and finding the sum of those areas
- subtracting the area of the small square from the area of the large square

After students have explored one of the above methods of their choice, direct them to the last of the above methods. Remember that they will make a stronger connection through discovery than by being told the relationship. Prompt students to make a connection to the last method:

- What is the area of the large square?
- What is the area of the small square?
- What mathematical operation (addition, subtraction, multiplication, or division) do you think is represented by cutting the small square from the large square?
- Can you apply the same operation to the areas of the large and small squares to determine the area of the new shape? Explain.
- What area did you find?
- How does this compare to the area you found using another method?

In #4, students find another physical shape by cutting off part of the irregular shape and connecting the two rectangles to form a large rectangle. If students do not see how to fit the two pieces together, ask them leading questions:

- Are there any sides that have the same length?
- How can you join them together along that side?
- What dimensions will this rectangle have?
- How do we find the area of a rectangle?

Before moving on to #5, it is important that students see the area of the new shape as being determined either by taking the difference of the areas of the large square and the small area or by finding the area of the large rectangle. They must also understand that the dimensions of the rectangle can be found by the following product: (side length of large square + side length of small square) × (side length of large square – side length of small square).

Students who find this relationship challenging may need leading questions to make the discovery:

- What is the length of the new large rectangle?
- What is the sum of the sides of the large and small squares?
- How does this sum compare with the length of the large rectangle?
- Write the length as a sum of the sides of the squares.
- What is the width of the new large rectangle?
- What is the difference of the sides of the large and small squares?
- How does this difference compare with the width of the large rectangle?
- Write the width as a difference of the sides of the large and small squares.

For #5, give students enough time to play and explore with the grid paper and to try at least two other examples of their choosing. Have them write down the answers to the questions in #5 based on their exploration of at least two examples. Discuss as a class what relationships they discovered.

In #6, students use this same technique to find the area of similar shapes with one unknown side. Most students should be able to make the connection between areas of the rectangle and the difference of squares. Now, they learn to use x as the side length of the large square. Some students may need coaching to remember to find the side length of a square by taking the square root of the area. Use the following leading questions:

- What is the area of the small square?
- What number multiplied by itself equals 25?
- What is the side length of the small square?
- What expression multiplied by itself equals x^2?
- What is the side length of the large square?

Once students make the connection between x and the side length of the large square, have them determine the side lengths of the large rectangle.

In #7, students extend their understanding from a concrete model to an abstract relationship. Use leading questions to coach students to see that the area can be expressed as $a^2 – b^2$, which represents the difference of areas of the squares:

- What is the area of the large square if a is its side length?
- What is the area of the small square if b is its side length?
- What is the area of the shape after the small square is cut off?
- What expression, using a and b, best represents the length of the large rectangle?
- What expression, using a and b, best represents the width of the large rectangle?

In #8, students describe in their own words what patterns and relationships they observe between areas of squares.

Meeting Student Needs

- Some students may need assistance in recalling their understanding of square roots. Consider preparing a pre-assessment of students' familiarity with squares from 1 to 169. You could also discuss $\sqrt{x^2} = x$, $\sqrt{x^4} = x^2$, etc.
- Produce posters illustrating the patterns for differences of squares and perfect trinomial squares and post them in the classroom. For example, you may wish to include the following patterns:
  $a^2 - b^2 = (a - b)(a + b)$
  $(ax)^2 + 2abx + b^2 = (ax + b)^2$
  $(ax)^2 - 2abx + b^2 = (ax - b)^2$
- It may be beneficial to your class for you to split this section into two parts and teach them separately. Begin with factoring differences of squares, and have students complete the related questions. Then, move on to factoring perfect square trinomials and have students complete those related questions. Once students understand the two processes, discuss and explore the relationship between the two forms.
- Ensure students see the connection between factoring and multiplication as this will help them in their understanding of factoring.
- In #4c) of the Investigate, students must explain how one area is related to another. Before beginning the Investigate, give a few examples. For example, draw a rectangle and find its area. Draw its diagonal and find the area of one of the triangles. Write on the board a sentence that relates the area of the rectangle to the area of the triangle. Allow students to offer suggestions. Leave their examples on the board for them to refer back to.
- The Investigate presents an opportunity for students to learn through doing. Ensure that students have a chance to work through three different squares. The key to understanding differences of squares is found in #7 and 8. Students need to make the connection to $a^2 - b^2 = (a - b)(a + b)$.

Common Errors

- Some students may find it challenging to rearrange the cut sections into a larger rectangle.

R$_x$ Ask students to tape together the sides of the smaller rectangles that are the same length and then to describe the dimensions of the new large rectangle.

- Some students may not be able to find the side lengths of the squares when given the area.

R$_x$ They should be asked to describe what operation occurs when you multiply a number by itself and what operation is the opposite of multiplying a number by itself. Ask them if they can use these operations to find the area of a square, and given the area of a square, find its side length.

- Some students may not know how to use abstract variables to represent the sides of a square.

R$_x$ They should be coached to replace the length of the large square with x in #6 and replace the sides of the large and small squares with a and b for #7. Have them use centimetre grid paper to find the areas, replacing the side length of the large square with x or a and the side length of the small square with 5 or b, depending on the step that they are working on.

- Some students may find it challenging to write equations to summarize the difference of squares relationship.

R$_x$ Ask students to compare the areas of the irregular shape and the large rectangle. Use leading questions:
  - What does the term *equals* mean?
  - If you know that two expressions are equal, how can you write an equation to express this relationship?
  - How can you use the variables a and b to describe each of these areas?
  - Since the area of the irregular shape is equal to the area of the large rectangle, how can you write an equation to describe this relationship?

- Some students may not be clear on how to recognize a difference of squares.

R$_x$ Prompt students with the following questions:
  – How many terms are in a difference of squares?
  – What type of expression is the first term?
  – What type of expression is the last term?
  – Why do we call these polynomials *differences of squares*?
  – When you see a polynomial made up of two terms, how will you decide if it is a difference of squares?
  – Once you determine that the polynomial is a difference of squares, how can you factor it?

Answers

Investigate Factoring Differences of Squares

1. a) 100 cm^2

b) Example: Determine the product of the side lengths (10 × 10) or the square of the side lengths (10^2).

2. a) 16 cm^2

b) Example: Determine the product of the side lengths (4 × 4) or the square of the side lengths (4^2).

3. a) 84 cm^2

b) Example: Area of large square – area of small square

c) Example: Yes. Count the number of centimetre squares.

4. a) 6 cm × 14 cm **b)** 84 cm^2 **c)** The areas are the same.

6. a) Example: $x^2 - 25$

b) $x + 5$ and $x - 5$

c) Example: The product of the dimensions $x + 5$ and $x - 5$ is equal to the area $x^2 - 25$.

d) $x^2 - 25 = (x + 5)(x - 5)$

7. a) $a^2 - b^2$ **b)** $a + b$ and $a - b$

c) $(a + b)(a - b$ **d)** $a^2 - b^2 = (a + b)(a - b)$

8. a) Example: The rectangle has dimensions of (sum of the side lengths the large square and small square) × (difference of the side lengths of the large square and small square). The area of the rectangle is the same as the area of the irregular shapes. The area can be found by finding the side lengths of the rectangle.

b) Example: The area you get from subtracting the area of the smaller square from the area of the larger square is equal to the area of the large rectangle with side lengths listed in a) above.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond It may be beneficial to complete the responses to #7 and 8 as a class.	• Have students work in pairs to complete the Investigate. Have the pairs share their answers to #7 and 8 with the class. • As a class, generate a pattern and then ask students to create their own pattern to put into their Foldable. This activity sets a strong foundation for students' conceptual understanding of the difference of squares.

Link the Ideas

The Link the Ideas section summarizes the discovery from the Investigate and introduces students to another special product that they will be factoring later, perfect square trinomials. Discuss with the class if *u* and *v* represent a relationship different from or similar to the one in the Investigate involving *a* and *b*. Have students describe both special products. Ask students what pattern or connection they see between the factors and the product.

Most students should understand the difference of squares example but may have difficulty with the development of the perfect square trinomials example. Prompt them with the following questions:

- What relationship exists between the first terms of the factors and the first term?
- What relationship exists between the last terms of the factors and the last term?
- What special relationship exists between the square roots of the first and last terms and the middle term?
- Is this relationship always true? Explain.
- Do the equations $(ax)^2 + 2abx + b^2 = (ax + b)^2$ and $(ax)^2 - 2abx + b^2 = (ax - b)^2$ describe the relationship that exists between the square of a binomial and the resulting trinomials? Explain.
- What do we call trinomials formed by squaring binomial expressions?
- How can you use this relationship to factor trinomials of this type?

Encourage students to write in their Foldable anything from the Link the Ideas section that may assist them.

Example 1

Example 1 demonstrates various forms of trinomials involving differences of squares that students may see. It also shows students that they can factor these types of polynomials in more than one way. You may wish to provide students with **Master 5 Algebra Tiles (Positive Tiles)** and **Master 6 Algebra Tiles (Negative Tiles)**.

In Method 1, students may not recognize the reasoning behind adding the positive and negative rectangles. Remind them that the product of binomials will always form a rectangle and ask what parts of the rectangle are missing in the first diagram. Prompt their thinking:

- What happens when you add a positive and negative rectangular tile?
- How many positive tiles are needed to form a rectangle?
- How many negative tiles are required if the sum is zero?
- What binomial represents the top of the large rectangle?
- What binomial represents the side of the large rectangle?
- What relationship exists between the binomials $x - 3$ and $x + 3$ and the product $x^2 - 9$?
- How can you check that the factors are correct?

In Method 2, students may wonder why the terms $-3x$ and $+3x$ are inserted as middle terms in the first step. Prompt them with questions like the following:
- What is the sum of $-3x$ and $+3x$?
- Where did the value 3 come from?

Students should see Method 3 as similar to the method developed in the Investigate. If they do not, refer them back to #6 of the Investigate and ask if they see similarities in the methods.

Part b) requires students to factor a difference of squares involving two variables.
- When have you factored polynomials with two variables before?
- How did you change your factoring method from factoring single-variable polynomials?
- Can you use the same technique with differences of squares? Explain.

Part c) involves a polynomial that cannot be factored over the integers. Students should be able to describe in their own words why $m^2 + 16$ cannot be factored using a difference of squares method. Ask students what is meant by *the binomial cannot be factored over the integers*. Discuss as a class having only integer coefficients in the factors.

Part d) introduces a common factor and requires students to recognize that the expression $7g^3h^2 - 28g^5$ is not a difference of squares but that one of its factors is a difference of squares. Prompt their thinking with the following questions:
- Do the terms $7g^3h^2$ and $28g^5$ have a common factor? What is it?
- What expression do you get when you factor out the common factor?
- Is this expression a difference of squares? How do you know?
- What are the factors of $h^2 - 4g^2$?
- What three factors need to be multiplied together to produce the polynomial $7g^3h^2 - 28g^5$?

Have students complete the Your Turn questions. Suggest that they use the following strategy:
- Is this polynomial a difference of squares?

↓ YES ↓ Factor it.

↓ NO ↓ Is there a common factor?

↓ YES ↓ Factor it out. Then, ask: Is the remaining factor a difference of squares?

↓ YES ↓ Factor it.

↓ NO ↓ It is not factorable.

↓ NO ↓ It is not factorable.

Example 2

This example demonstrates three methods of factoring perfect square trinomials and then shows examples of types of polynomials that can be factored using one of these methods.

In Method 1, have students look for patterns in the tiles to determine what is special about perfect square trinomials. Ask questions like the following:
- What do you notice about the first term?
- What do you notice about the last term?
- What do you notice about the number of rectangles to the right of the large square and the number of rectangles below the large square?
- What relationship exists between the number of rectangles to the right of the large square and the middle term of the trinomial?
- What relationship exists between the number of rectangles below the large square and the middle term of the trinomial?
- How can you determine if a polynomial is a perfect square trinomial?

In Method 2, if students are unsure of how to factor by grouping, prompt them with leading questions:
- What two numbers have a product of 9 and a sum of 6?
- Why is $6x$ replaced by the sum of $3x$ and $3x$?

- Why is the trinomial written as $(x^2 + 3x) + (3x + 9)$?
- How is it possible to write $(x^2 + 3x)$ as $x(x + 3)$?
- What is the common factor in the expression $x(x + 3) + 3(x + 3)$?

For Method 3, prompt students with the following questions:

- How do we know that $x^2 + 6x + 9$ is a perfect square?
- What is $\sqrt{x^2}$?
- What is $\sqrt{9}$?
- How can you express 9 as a square?
- Why is it important that the middle term is twice the product of the square root of the first term and the square root of the last term?

In part b), the trinomial includes a common factor. Some students may not see that the polynomial can be factored using a perfect square trinomial method. Assist them with the following prompts:

- Is $2x^2 – 44x + 242$ a perfect square trinomial? How do you know?
- Does the polynomial $2x^2 – 44x + 242$ have a common factor? What is it?
- What expression do you get after factoring out the common factor 2?
- Is the remaining polynomial a perfect square?
- Can you factor $x^2 – 22x + 121$ using a perfect square trinomial method?
- How can you check that $2(x – 11)(x – 11)$ is the factored form of $2x^2 – 44x + 242$?

Part c) challenges students to determine if a given polynomial is a perfect square trinomial. Ask students how they know that $c^2 – 12c – 36$ is not a perfect square trinomial. If students cannot answer, ask the following questions:

- What is the square of +6?
- What is the square of –6?
- What do you notice about the signs of the squares of each number?
- Can a perfect square term have a negative value?

Have students complete the Your Turn questions. Suggest that they ask themselves the following questions:

- Is the polynomial a perfect square trinomial?
- If no, does the polynomial have a common factor?
- If yes, is the remaining factor a perfect square trinomial?

These questions should help them determine if they can use the perfect square trinomial method to factor the polynomial.

Key Ideas

The Key Ideas points summarize the methods used to factor differences of squares and perfect square trinomials. Have students write these summaries in their own words in their notebook and refer to them when factoring polynomials. Suggest that they include in their notes how to determine if a polynomial is a special product as well as how to determine if a polynomial is factorable.

Meeting Student Needs

- You may wish to use whole numbers to show students the pattern for differences of squares. For example:
$$\begin{aligned}(5+3)(5-3) &= (5)(5)+(5)(-3)+(3)(5)+(3)(-3)\\ &= 25-15+15-9\\ &= 25-9\end{aligned}$$
Ask students to discuss the relationship between the numbers found in the original question and the numbers in the bottom line.
- For perfect square trinomials, ensure that students make the connection to the middle term. Show several examples and highlight the two middle terms so that students know why the middle term is twice the product of the square root of the first term and the square root of the last term.
- Encourage students to do more questions similar to Example 1, Method 1. The visual representation will assist many students.
- Ensure students know that if the last term is negative, the trinomial is not a perfect square trinomial and that any number squared is always positive. Also, ensure that they understand the reasoning behind these concepts.
- The perfect square trinomial may be a challenge for some students to recognize. Have them do the multiplication of perfect squares until they start to see a pattern. Then, have them write this pattern on an index card to refer to as they do the Check Your Understanding questions.
- For some students, it will be very important that they experience a good deal of modelling using diagrams or manipulatives before they begin the Check Your Understanding questions.

Gifted

- For Example 1, have students explore whether or not $m^2 + 16$ can be factored using a set of numbers other than integers, such as rational numbers, irrational numbers, or real numbers.

Common Errors

- Some students may not recognize that a given polynomial is a difference of squares or a perfect square trinomial.

R_x Coach students to remember what is special about each of these two types of products, and as a class, brainstorm a lists of characteristics that make them special. Students could work in groups and design posters to describe the special relationship between terms of differences of squares and perfect square trinomials. The posters could also summarize the skills of factoring each type of special product. Display the posters in class.

Web Link

To give students an opportunity to practise factoring special products, go to www.mhrmath10.ca and follow the links.

Answers

Example 1: Your Turn

a) $(7a + 5)(7a - 5)$ **b)** $5(25x^2 - 8y^2)$ **c)** $(3pq + 5)(3pq - 5)$

Example 2: Your Turn

a) $(x - 12)(x - 12)$ or $(x - 12)^2$

b) not factorable

c) $3(b + 4)(b + 4)$ or $3(b + 4)^2$

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Review to ensure students can explain what *difference of squares* means. Ask students how they would be able to distinguish a difference of squares question from a trinomial question. Ask them to consider whether the answer would be the same if the question were $x^2 + 25$. • Initially, when students are factoring a difference of squares, allow them to use a method of their choice. However, encourage them to find a second method by which they can factor. This will help them to identify different presentations of the differences of squares. • Some students may still be at the concrete stage. Encourage them to use manipulatives, as needed.
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Students often confuse the difference of squares and perfect squares because they focus solely on the constants and do not recognize the significance of the different signs. Point out how the factors differ between the two types of questions. • Allow students to use a method of their choice for factoring. Point out that the methods used in the previous lessons will be used here. Encourage students to review their factoring method of choice from the previous section.

Check Your Understanding

Practise

For #1, students complete basic algebra tiles factoring questions. Most students should be able to complete #2, which is a review of multiplying polynomials. If they are not confident, coach them on multiplying polynomials and give some remedial questions as practice before they complete #2. For #3, students also multiply polynomials, this time determining the squares of binomials. Ask students if they can write the question out as a product of two binomials instead of binomials squared.

Since #4 is a higher-level question, students should work with a partner if they find it challenging. You may need to work individually with some students, asking them to identify the type of special product each question represents and then leading them to use the appropriate method of factoring to find the missing expressions.

For #5, students factor differences of squares. Note that some are not factorable. Students should indicate which questions cannot be factored and state why.

For #6, students factor trinomials. Again, some are not factorable. The questions require students first to decide if the trinomial is factorable and then to factor it as a perfect square trinomial. Have students explain why a polynomial cannot be factored.

For #7, students factor special product polynomials with common factors. Parts a) and c) involve factoring by using only common factors. All other questions require students to factor out the common factor and then use a second step to factor using a method for differences of squares or perfect square trinomials.

Apply

In #8, students have a chance to reinforce their understanding of the relationship between the middle term of a perfect square trinomial and the first and last terms of the trinomial. Discuss that perfect square trinomials can be either in the form $(ax)^2 + 2abx + b^2$ or in the form $(ax)^2 - 2abx + b^2$. To help them in their thinking, ask the following questions:

- Using either $2abx$ or $-2abx$ from the definition, what expression is equal to n in the question?
- How can you find the value of a?
- How can you find the value of b?
- What value of n will satisfy the relationship if the middle term is $2abx$ or $-2abx$?

For #9, students explain why a given polynomial is not a difference of squares or not a perfect square trinomial. Ask students the following questions:

- What do you notice about the first term in a product involving a difference of squares and a perfect square trinomial?
- What do you notice about the last term in a product involving a difference of squares and a perfect square trinomial?
- What do you notice about the number of terms in a difference of squares?
- Why are they called differences of squares?
- What relationship exists between the first and last terms and the middle term of a perfect square trinomial?

In #11, students apply the patterns they have observed using differences of squares to understand how some number tricks might work. This question can be used as an optional activity to challenge and entertain interested students.

For #13, students apply the concept of difference of squares to the area of a property. It allows students to make the connection between a concrete problem and an algebraic expression to model the situation.

Students may find #14 to be a higher-level question since it involves a formula with difference of squares and binomial perfect square terms.

Before students begin #15, you may wish to inform them that Gena LaCoste, the artist who created the original watercolour painting, is a southern Alberta native and has studied and painted across Western Canada and in parts of the southwestern United States. See the Web Link that follows in this resource for more information on Gena LaCoste.

For #15, students must recognize that if a print is enlarged by a factor of 3, all sides increase by a factor of 3. They may need some coaching to make this connection:

- How can you express the area of the original print if its side is $(2x - 3)$ cm long?
- If the original print is $(2x - 3)$ cm long, how can you express the side if it is 3 times longer?
- If you express the length of a side as $3(2x - 3)$ cm, how would you express the area of the large print?
- How can you express the difference of these areas using a difference of squares?

Once students understand these expressions, they should be able to expand and simplify, and then check using substitution. Make sure that they use a small enough value for substitution to keep the check reasonable.

For #17 and 18, students use the concept of perfect squares to solve area problems. In #17, students use this concept to determine the side length of a square when given its area, and then use the side length to find the perimeter of the square.

In #18, students use the concept to find the radius of a drum given its area. Some students may need coaching to recognize that $r^2 = 9x^2 + 30x + 25$. They should then be able to factor this perfect square trinomial to find the radius and then the diameter.

Question #19 requires students to verify statements using their understanding of differences of squares and perfect square trinomials. Part b) may provide a challenge, since if $b = 0$, the statement is only sometimes true. This question is useful to develop students' ability to think in the abstract.

In #20, students must find the error. This type of question is useful for students to self-assess their work and be aware of common errors made by students. This is a good class discussion topic and should lead to a better recognition of common errors in factoring special products.

Since #21 is a volume question, to determine the answer, students must express the product as three factors. The most challenging step in this question is recognizing that the original polynomial can be grouped and then factored using common factors. Coach students to see this relationship using the following prompts:

- How many terms do you see in the volume expression?
- How can we factor polynomials with four terms?
- How can you group the terms into two sets of binomials with common factors?
- Is there more than one way to group these terms into two sets of binomials with common factors? Explain.
- Factor these two groups.
- Do you see another common factor in your solution? If so, what is it?
- How can you factor it out of the expression?
- Can one of the factors be factored further? Explain.
- How many factors do you have in total?
- How many dimensions do you require to describe the volume of a rectangular prism?
- How can you check that you have found the correct dimensions?

Extend

In #22, students apply the concepts developed in the Investigate to solve the problem. Some students may need some coaching to recognize that one of the factors of the perfect square trinomial represents the side of the square. If they can make that connection, they should be able to determine the dimensions of the rectangle and then its area.

For #23, students apply to integers their understanding of the relationship between the terms of a difference of squares and the sum of the square roots of the terms. For students who are not sure how to begin, tell them to use values between 1 and 5 and use Guess and Check to find one set of integers whose difference of squares equals the sum of the integers. Once they have found one set, have them find two more. They should then be able to make a generalization about integers that satisfy these conditions. Part b) will require the application of the generalization made in part a).

Create Connections

For #24, students summarize their understanding of the relationship between the terms a, b, and c of a perfect square trinomial.

For #25, students summarize their understanding of the relationship $a^2 - b^2 = (a + b)(a - b)$, using two methods, likely a concrete visual model and an algebraic model, to prove the equality between a difference of squares and its factors.

In #26, students show their understanding of the patterns they have discovered when factoring different trinomials.

In #27, students apply their understanding of how a difference of squares relationship can be used to simplify the product of any two numbers that differ by 2. Then, they extend their understanding to develop a technique for multiplying any two numbers with an even number difference.

Unit Project

As Unit 2 project questions, #10 and 12 allow students to apply their skills from the section and create some artwork. In particular, students with a more concrete approach to learning will benefit from these activities.

Meeting Student Needs

- Provide **BLM 5–8 Section 5.4 Extra Practice** to students who would benefit from more practice.
- Before assigning questions, refer students to the student learning outcomes for this section.
- It may benefit your students for you to assign one part from each Practise question and have students work on them in small groups.
- Encourage students to look for the patterns as they complete the questions, but also allow some students to use either algebra tiles or multiplication to determine each product.
- For #7, explain to students that *factor completely* likely means the question involves a common factor plus another type of factoring. For example, $5x^2 + 25x + 30 = 5(x^2 + 5x + 6)$, which factors further to become $5(x + 2)(x + 3)$.
- For #11, you may wish to challenge students: While they complete the questions using a calculator, you complete the questions mentally. Afterward, demonstrate how you were able to find the answers so quickly. Encourage them to challenge someone at home.
- For #14, ensure students understand that concentric circles are circles that share the same centre, with one circle lying within the other.

- Associated with #18, you may wish to consult Elders and Knowledge Keepers about someone who could visit the class to talk about the traditional drums used or created locally.
- Remind students that when substituting a value for x, they may choose whatever value they wish for x. In this section, they check that the factored form matches the expanded form. Ensure that students understand to substitute their chosen value into each form and that the objective is to end up with the same result from both forms.

Common Errors

- Some students may not be able to recognize that a trinomial is a special product.

R_x Coach students to see that they can factor perfect square trinomials using other methods developed for factoring trinomials, but that it is much easier to factor if they recognize the special product. Assist students in seeing that a difference of squares has a form unique from other polynomials and that binomials can only be factored using common factors or a difference of squares.

Web Link

For students to see other examples of Gena Lacoste's work and to learn more about the artist, have them go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1–3, 5, 6, 7a)–c), 8a), b), 9, 10, 12, 13, 16, and 20. Students who have no problems with these questions can go on to the remaining questions.	• In #2 and 3, students have an opportunity to demonstrate their multiplying ability. Encourage them to look back in section 5.1 and review possible methods of finding products. Ask them to identify which are the factors of a difference of squares and which are factors of a perfect square. Have them explain their reasoning. You could also ask students how many terms they expect in their product. • In #5, 6, and 7, students are required to factor. Review what the first steps are in factoring. Some students may have developed their own form of inspection to factor. Encourage them to use whatever method is easiest for them. If students still find a question challenging, ask them to identify whether the question is a difference of squares or perfect square trinomial. This will help narrow the possibilities for the solution. • Students requiring help with #8 should be coached in the meaning of b and c in the trinomial. You could prompt them by asking the following questions: – What is a perfect square? – Are 25 and 100 perfect squares? How do you know? – How could the square roots help you find n? • Since #9 is an excellent question for assessment *for* learning, use this question to identify any weaknesses in students' understanding of the concepts. • Use the discussion from #10 to assist in prompting for #16. Students should be able to see the visual link between the two questions. If not, reviewing the Investigate may be beneficial.

<table>
<tr><th>Assessment</th><th>Supporting Learning</th></tr>
<tr><td colspan="2">Assessment for Learning</td></tr>
<tr><td>Unit 2 Project
If students complete #10 and 12, which are related to the Unit 2 project, take the opportunity to assess how their understanding of the chapter outcomes is progressing.</td><td>• You may wish to provide students with BLM 5–4 Chapter 5 Unit 2 Project and have them finalize their answers.
• For #10, encourage students to choose a difference of squares that is not too large for modelling. Visual learners will likely benefit from using algebra tiles. Some students may prefer grid paper.
• In #12, students model the squaring of a binomial. Some students may find it easier to square a binomial algebraically first and then design their model based on their answer. If this is the case, ask them how they know their product is correct. Have students do several examples before generating their rule for squaring. Discuss this rule with the class. Have them write their rule, with examples, into their Foldable for future reference.</td></tr>
<tr><td colspan="2">Assessment as Learning</td></tr>
<tr><td>Create Connections
Have all students complete #24 and 25.</td><td>• Both #24 and 25 allow students to summarize, in their own words, the key concepts presented in this lesson. Again, encourage students to include their own specific examples, models, and diagrams to help in the completion of their explanation.</td></tr>
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