Graphs of Relations

6.1

***Mathematics 10*, pages 268–278**

Suggested Timing

100–120 min

Materials

- ruler
- grid paper
- CBL interface with a motion detector
- computer or graphing calculator with appropriate software

Blackline Masters

BLM 6–3 Chapter 6 Warm-Up
BLM 6–5 Section 6.1 Extra Practice

Mathematical Processes

- ✓ Communication (C)
- ✓ Connections (CN)
- Mental Math and Estimation (ME)
- ✓ Problem Solving (PS)
- ✓ Reasoning (R)
- ✓ Technology (T)
- ✓ Visualization (V)

Specific Outcome

RF1 Interpret and explain the relationships among data, graphs and situations.
RF4 Describe and represent linear relations, using:

- words
- ordered pairs
- tables of values
- graphs
- equations.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–3, 6, 8, 15–17
Typical	#1–3, 5, 8–10
Extension/Enrichment	#3, 4, 11–15, 17

Planning Notes

Have students complete the warm-up questions on **BLM 6–3 Chapter 6 Warm-Up** to reinforce prerequisite skills needed for this section.

Students get a sense for reading and interpreting general graphs by comparing the goalie stats. Since there is no scale given on each axis, students will need to realize that, whatever the scale, it is the same for each goalie, so they should look at the "trend." Ask questions to prompt their thinking:

- Why are the points in each graph not connected? (Do not use the word *discrete*. Just get students thinking about something they will explore further in section 6.2.)
- Who lets in more goals per game?
- Who faces more shots per game?
- Can you determine which goalie has the better goals-against average? Explain.
- Can you determine which goalie has the better save percent? Explain.

Encourage a debate among students by trying to get them to decide who the better goalie is.

Investigate Describing and Sketching Graphs

You may wish to discuss the results of #1 and 2 before students proceed with the remainder of the investigation. After the pairs of students have finished #1, select a small number of students to report on their responses. Ask if other pairs of students have different responses. In a similar fashion, after pairs have finished #2, you may wish to choose a small number of students to report on and defend their results.

As the groups work on #3, monitor their progress. Watch to see that the correct quantities are placed on the x-axis and y-axis. You may want to ask the following questions:

- Why is time placed on the horizontal axis?
- Does the placement of the quantities on the axes fit with what you have done in other classes, for example, science?

You may want to ask them to review their graph and make sure that it reflects each part of the written description. Allow time for all groups of students to finish #3 before they pass their work to another group. When they examine the work of others, you may want to ask if the scenario they have been given provides

enough detail that they can tell which piece of the graph each part of the description pertains to. You can use students' scenarios to check for understanding.

In #4, it is inevitable that groups will have different graphs. As students find discrepancies, ask the following questions:

- Are the differences in the graphs evidence that one group made an error?
- Is it possible that both graphs are correct? If so, how can they be different?
- Is it all right for a different group's graph to show a different speed or length of time than yours for the same part of Connor's trip?

As students are already working in small groups, you may wish to have them remain in those groups for the Reflect and Respond in #5. After each group has had a few minutes to decide on their responses, choose a representative from each group to report to the whole class.

Meeting Student Needs

- Have students create two other scenarios that could be represented by the graphs in the opener.
- Have students complete #1 and 2, assigning one or the other to pairs of students. Each pair can make a brief presentation outlining their responses to the question assigned. Students might complete #3 in small groups and then work as an entire class on #4 and 5. Ensure that all students have a chance to write, sketch, and present.
- For #3, if your community does not have paved roads, skateboarding may not be an activity your students engage in. Have students go online to research what an ollie looks like. You may wish to have them look at other aspects of skateboarding as well. See the related Web Link that follows in this Teacher's Resource.
- If you discussed global warming in the chapter opener, relate back to it in the Reflect and Respond by asking how students would graph ice melt and temperature, or ice melt and water level.

ELL

- Some students may not be familiar with the term *goalie* or perhaps even the sport of hockey. Show them pictures or a video of the game and indicate the goalie while you describe what the goalie's role is in the game. Alternatively, have a volunteer describe the game and what a goalie does.
- Have students who are not familiar with skateboarding view a video of skateboarding and how to do an ollie. See the related Web Link that follows in this Teacher's Resource.
- For the Investigate, you may wish to have students pair up with another student who can assist them with the language and explain the steps.

Common Errors

- Some students may create graphs that do not completely agree with the scenario presented. Often one or more of the statements in the scenario is not represented on the graph.

R_x Have students assign labels A, B, C, etc., to each written statement or phrase, then place that label on the corresponding segment of their graph to ensure that the graph is complete.

- Students may not obey all the conventions for making graphs.

R_x As you watch them work, remind students to give their graph a title and label the axes with titles, correct units, and variables.

WWW Web Link

To find out more about skateboarding and ollies, go to www.mhrmath10.ca and follow the links.

Answers

Investigate Describing and Sketching Graphs

1. a) Example: From 0 to A: Climbing; the distance from the base camp increases. AB: Resting; the distance from base camp does not change. BC: Climbing; the distance from the base camp increases. CD: Descending; the distance from the base camp decreases. DE: Resting; the distance from base camp does not change. EF: Climbing; the distance from the base camp increases. FG: Resting; the distance from base camp does not change.

b) Example: Yes. The climber may have been travelling across a flat plateau during the times indicated by the segments AB, DE, and FG, and the climber may have slid downward during the time indicated by the segment CD.

c) Use a steeper slope. A steeper slope indicates that more metres are climbed each minute.

d) Example: Add a line that goes from point G down to $d = 0$.

Answers

Investigate Describing and Sketching Graphs

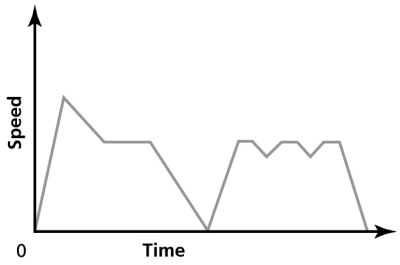
2. a) Graph C; the hot chocolate gradually cools. Vertical axis: temperature, horizontal axis: time

b) Graph B; the car speeds up until it reaches a specific speed, and then it maintains that speed. Vertical axis: speed, horizontal axis: time

c) Graph D; as time passes, the amount of distance the hiker covers also increases. Vertical axis: distance, horizontal axis: time

d) Graph A; the height increases as the ball goes up, until it reaches a maximum height, and then it goes back down. Vertical axis: height above ground, horizontal axis: time

3. Example:



5. a) a straight line with a slope

b) a straight line with a steep slope

c) a horizontal or vertical line

d) a curved line

Assessment	Supporting Learning
Assessment as Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• You may wish to complete this activity by having students work in pairs, as it may promote more discussion among students, especially when the graphs differ. • It would benefit all students to discuss #2 and 3 as a class and have students label their examples to identify what is happening in each segment. • Have students who are having difficulty with #5 use their previous labels as examples to compare to. This should make it easier for them to see the similarities between the existing graphs and what is asked for in #5. • Some students may find it difficult to identify changing quantities without a specific reference. In this case, suggest that for #5 they use a distance-versus-time graph to represent each part of #5.

Link the Ideas

You may wish to ask students to look at the graph they created in light of the information in Link the Ideas. Ask the following questions:

- Are there places on your graph that have constant rates of change?
- Which section of your graph has the highest rate of change? the lowest?
- How can you tell when your graph shows a non-constant rate of change?
- Are there sections of your graph that show no change at all? How can you tell?

Example 1

Explain to students that the letters represent where the various line segments on the graph connect. Also, ensure students note that the graph has a smooth curve. This will help them to understand the rate of change for CD, for example, which is described as being slow at first.

When students consider the example and solution, be sure that they address the questions in the margin. The second question is likely the most challenging. To help students see the answer, you could ask them to draw a point, then pick a distance and draw all the points that are that distance from the original point. They will see that the result is a circle. Ask students guiding questions:

- Why does the distance increase slowly at first?
- What kind of path would allow this?
- How do you know the rate is constant?

You may wish to have students complete the Your Turn by first writing a description, then exchanging with another student to compare. After all students have had an opportunity to complete and edit their work, you could select a number of students to present their descriptions to the whole class. Remind students that the vertical axis is the speed of the boat. To assist students in their thinking, you may wish to ask students to think of another way to describe increasing speed (acceleration).

Example 2

Students are likely to know that bacteria grow by doubling and thus increase in number quite quickly. Before students look at the solution, you may wish to ask a few questions to focus their thinking:

- How do bacteria grow?
- Is bacterial growth an example of a constant rate of change?
- Is it important to your response that the food supply is limited?

You might also wish to ask the following questions:

- If bacteria are given unlimited space and a continuous food supply, what do you think their growth would be?
- Is there a graph above that matches this scenario?

For the Your Turn, you might wish to ask the following questions:

- Are there periods in a person's life when they gain height more rapidly?
- Are there times in a person's life when they do not gain height (they stop growing)?
- Why does each graph dip back down after a certain number of years?

You may wish to have students write their answer on a small piece of paper and hold it up so that only you can see it. This will enable you to check the understanding of the class quickly. If you have a student response system with your interactive whiteboard, you could use it for the same purpose. If time allows, you might ask students to write a scenario that fits each of the incorrect responses for the Your Turn. This activity should help to solidify understanding.

Example 3

Lead students to see that each part of the description of Josaphee's trip is reflected in a segment on the graph. Students should understand that each time there is a change in the situation, the graph must change as well. Ask students why line segments appear on the graph instead of curves.

You may wish to approach the Your Turn as a class. Have a volunteer sketch a graph on the board that represents the given situation. Then, have the class decide if the graph could be improved. If so, another student comes forward to make the change. The class again decides if the graph could be improved, and so on. In this activity, it is important that students understand that the mathematics is being critiqued, not the individuals presenting it. Before they begin, encourage students to think through the scenario and determine appropriate slopes for lines and appropriate time frames while applying their understanding of how to show quantities changing.

Key Ideas

The Key Ideas represent the concepts that students need to understand upon completion of this section. Allow time for students to incorporate the Key Ideas into the Foldable they created for this chapter or in their math journal. In either case, students should put the Key Ideas in their own words. As horizontal lines are a special case, you may wish to have students place some emphasis on explaining why a horizontal line represents a quantity that is not changing, or a rate of change of zero. Encourage students to give an example, either from the section or of their own, for each type of change.

Meeting Student Needs

- Depending on the makeup of your class, you may wish to change the order in which the Examples in this section are covered.
- Consider discussing with students why the endpoints of each interval in Example 1 are curves rather than points.
- Students may need to be reminded of the meaning of *constant rate of change*. Along a straight line, the rate of change is constant. The moment the line changes steepness, the rate of change also changes. The steeper the line, the greater the rate of change is.
- Explain to students that *constant distance* means the same distance. If you are moving and positioned a constant distance away from your school, you are perhaps walking around the school but staying the same distance away from the school.
- Request that students find two graphs in newspapers, magazines, or online. For each graph, have students identify and interpret what is represented. Ask them the following questions:
  - Are there any discrepancies in the graph?
  - Is any of the information distorted by the choice of units used on the axes?

  Have students mount the two graphs and their interpretation on construction paper. Then, display them in the classroom or hallway. You may choose to give students two or three days to complete the activity at home.
- Some students may not be familiar with wakeboarding. To show students what

wakeboarding looks like, have them research the sport. See the related Web Link that follows in this Teacher's Resource.
- Encourage students to study distance-versus-time graphs in detail, as this will be a concept studied again in physics. Display the graph from Example 1. Trace the graph as you discuss each segment. Some students may need another example before working through the Your Turn.
- You may want to sketch a model of the scenario in Example 1. Sketch a lake, put an X where the boarder starts, and sketch a sample path on the lake based on the distance and time information given on the graph.
- Refer to the list of definitions created at the beginning of the chapter. You may need to refresh students' understanding of terms such as *domain* and *range*.
- Discuss the step-by-step instructions for creating the distance-versus-time graph for Example 3.
- With the class, make a poster outlining the Key Ideas for this section. A small illustration for each situation would be helpful for students.

ELL

- You may need to explain such terms as *steepness*, *gradual*, *bacteria*, *limited*, *continued*, *initially*, *rapid*, *jogs*, and *curves*. Use a combination of words, visuals, and examples.
- Encourage students to add *constant* and *rate of change* to their vocabulary dictionary, Foldable, or other graphic organizer that they may be using.
- For students unfamiliar with wakeboarding, have them watch some video footage of the sport. See the related Web Link that follows.

Common Errors

- In Example 1, some students may think the horizontal line segments mean that the boat has stopped.

R_x Remind them that the quantities being compared are speed and time.

WWW Web Link

To see some online video footage of wakeboarding, go to www.mhrmath10.ca and follow the links.

Answers

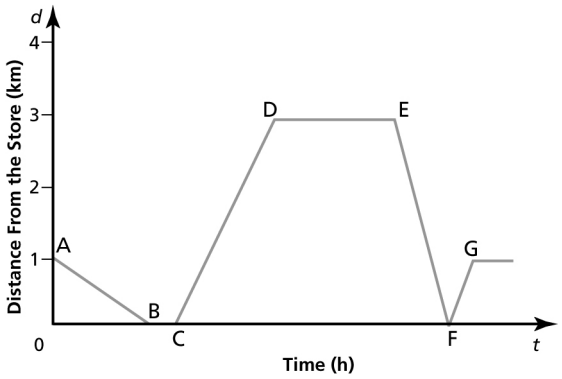
Example 1: Your Turn

The boat accelerates at a constant rate. It then travels at a constant speed. The boat quickly slows down to a stop. It accelerates, slightly slower than before, at a constant rate. The boat travels at a constant speed, the same speed at which it travelled earlier. Finally, it gradually decelerates to a stop.

Example 2: Your Turn

Example: Graph B because height increases as one gets older. Then, when one reaches physical maturity, height remains the same until the senior years when height decreases slightly.

Example 3: Your Turn



AB: Josaphee walks from her house to the store; BC: Josaphee is at the store; CD: Josaphee walks to her friend's house; DE: Josaphee is at her friend's house; EF: Josaphee runs from her friend's house to the store; FG: Josaphee continues running to her house.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner of similar ability. • Some students may need to be coached through the first three segments of the graph. It may help them to describe the segments if they are labelled individually. • Use the last three segments for students to work on alone and confirm they have grasped the concept. • Some students may benefit from a mock scenario that they can use in their description.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • You may wish to have a discussion of the physiological changes that occur with aging. • It may benefit some learners to first describe what is happening in the graph orally. This would provide an opportunity to clarify any misunderstandings in their interpretations of segments of the graph. • Encourage students to always refer back to the labels on the axes when they are describing a graph.
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner of similar ability. • Listen to the discussion that students have regarding faster/slower speeds as well as distance covered. Ensure students understand the difference between a positively sloped segment and a negatively sloped one. • If several students are having difficulty, as a class write on the board a description of what Josaphee is doing in relation to the store. Take each description individually and ask students for feedback on what that segment might look like on the graph. Work on a few together and then allow partners to complete the description.

Check Your Understanding

Practise

For #1, without using slope terminology, students state whether the slope of a segment of the graph is positive, negative, or zero. Students might refer to Example 1 if they need help with this question.

In #2, students match graphs to given scenarios, similar to their work in Example 2.

As students work on #4, you may wish to ask the following questions:

- Does your scenario contain a description for each portion of the graph?
- Does your scenario correctly explain the changes and rates of change in the graph?

Apply

In #5, students create a graph to fit a scenario. As usual, there is some ambiguity in the scenario, in particular about how long Paul must wait while he transfers trains. You may wish to ask students to stipulate any assumptions they make in sketching the graph, especially about this waiting time. This is an opportunity for students to see that many correct graphs can exist to model this situation, depending on the assumptions made.

For #8, students create a graph to show their own daily water consumption. Students will need to think about the problem before beginning the graph. You may want to direct their thinking with a couple of guiding questions:

- What will be the scale for your vertical axis?
- How many of these activities are typical for you?

These prompts should help students determine the maximum value necessary on the vertical axis. Since the topic of #8 is related to the Unit 4 project, you may want to have students refer back to this question when they work on the project.

Answers may vary for #9, as students may overlook a particular section of the ride. You may want to ask students to consider the various rates of change experienced on the roller coaster, when those changes should be represented by line segments, and when they should be represented by curves.

For #10, some students may make the mistake of sketching the graphs independently of each other. Students need to understand that for the parts of the journey where more than one person is present, those portions of the respective graphs must be identical.

For #11, ask students if they expect each graph that they sketch to be identical. Discuss that despite the times and speeds specified, there may be room for small variances in students' work, depending on the assumptions that are made about the nature of travel. For example,

- Are the changes in speed uniform?
- Does the skydiver's body position remain the same?

Extend

For #12, you may need to emphasize that these are rates of change, not numbers of births and deaths, so the changes indicated by the graph are more complicated than students may first think.

In #13, the term radioactive half-life refers to the time required for half the atomic nuclei of a radioactive sample to decay (change to a stable nuclear state).

In #14, students encounter changes in a graph with an undefined slope. Note that students are not expected to use this terminology at this point.

Create Connections

Students will need some time to complete #16. You might have them work with a partner.

Question #17 is a Mini Lab. The size of the group for the activity will depend on your access to the CBL technology. It is possible for students to complete the activity in groups as small as three and as large as the whole class. Note that students will gain most from this activity if given the opportunity to walk in front of the CBL to try to produce the desired graph.

Meeting Student Needs

- Provide **BLM 6–5 Section 6.1 Extra Practice** to students who would benefit from more practice.
- Ask students for at least five main ideas from the section. Encourage them to use the terminology and concepts from the section. Allow them to include diagrams to illustrate specific information.
- Some students may benefit from a discussion about the distances described in #10. One student could create a table of values while another student could move manipulatives from one location to the next on a diagram similar to the one given.
- For #14, explain to students that for them to describe the *rate scheme*, they need to determine how much a particular item or service costs for a specific time (e.g., 1 h, 1 day). Also, discuss the meaning of open and closed circles on the graph.
- For #15a), discuss "negative time" with students. Many will not realize that the graph shows time moving backward.

ELL

- Some students may not be familiar with terms such as *scenario*, *profit*, *distributing*, *recorded*, *complete*, *daycare*, *skydiver*, *parachute*, *demography*, *mortality, half life*, *substance*, *decay*, *vehicle*, *rate scheme*, and *impossible*. Assist them to understand by using descriptions, examples, and visuals.
- For #2, it may be helpful to provide diagrams or photos to students in addition to the descriptions of the different scenarios in part a).
- For #5, have students work with a partner who can guide them through the question by indicating Paul's journey on the diagram and explaining what happens during the journey.
- For #6, make available pictures that will show the meaning of *vinyl albums*, *cassette tapes*, *compact discs*, and *digital downloads*.
- For #7, some students may not be familiar with *snowmobiling*. Show pictures while you describe this activity.
- For #8, students may not be familiar with some of the activities listed under Water Use in the table. Invite them to pair up with another student and search for each of these activities on the Internet.
- For #9, invite students to share their experiences with roller coasters. This will help English language learners to learn the term and may encourage them to share their own stories of riding roller coasters or other similar rides, if they have done so. Also, have students watch a video of a roller coaster ride. See the Web Link in the student resource.
- For #17, have students work in a group in which others can help them with the language and can explain the steps.

Enrichment

- Give students the following scenario: When people consume food and drink that contain sugars, their blood sugar level rises. When they exercise, their muscles burn sugar, and so their blood sugar falls. Encourage students to track for 24 h their consumption of food and drink that may increase their blood sugar and also their physical activities that may burn sugar. Have them create a time-versus–blood sugar level graph. Ask students to comment on the slope of the blood sugar level line. (Graphs should show a positive slope soon after consuming sugars, and a negative slope as the result of physical activity.)

Gifted

- Present students with the following information: A golf ball follows a curve as it travels through the air. The graph of height versus distance for the ball is the same as the curve. Suppose the ball travelled 20 yd horizontally every second and reached a maximum height of 50 yd. Have students sketch a

graph of height versus distance for a golf ball that hits the ground 100 yd from where it was struck. Ask students to place discrete points every second of travel and then join those points with straight lines. Have them comment on the slopes of those lines. Then, ask them to speculate on the meaning of those slopes relative to the way the ball travels. (Example: The slope of the travel up to maximum height is positive. At the maximum, the slope is zero. The slope becomes negative after that until the ball hits the ground.)

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1–3, 6, and 8. Students who have no problems with these questions can go on to the remaining questions.	• Students having difficulty with #1 should be encouraged to review Example 1 as well as their response to the Your Turn for that example. • Students having difficulty with #2 should be encouraged to review Example 2 as well as their response to the Your Turn for that question. The description and diagram for part b) would be useful to include in students' Foldable. If students are struggling with matching the graph, ask them to verbally describe what is happening in each graph. You will find that they will substitute their own label for the vertical axis. Based on their description, coach them to find one from the graphs given. • For #3, you may wish to discuss what each of the choices means and possible pairings. Based on the suggestions, have students see if any of the pairings are found in the graphs given. • You may need to review the different recorded music formats for #6, as not all students will know what they are. • To prompt students for #8, as a class, write out a brief description of water use in a 24-h period. Without using exact values of litres, identify where more or less or no water use at all might be likely. Then, pair students up to complete the question.
Assessment *as* Learning	
Create Connections Have all students complete #15–17.	• Both #15 and 16 are useful Assessment *as* Learning questions that students should include in their Foldable. Encourage students to use unique stories for #16. Have a partner review the story for accuracy. • You may wish to have students work in groups of three or four to complete the Mini Lab, #17. Have students share their results on the board so they can compare and determine whether their graphs are similar. Suggest that students identify each graph with a quick description of the action that resulted in the graph.