Linear Relations

6.2

***Mathematics 10*, pages 279–291**

Suggested Timing

100–120 min

Materials

- measuring tape
- ruler
- grid paper or graphing technology

Blackline Masters

BLM 6–3 Chapter 6 Warm-Up
BLM 6–6 Section 6.2 Extra Practice

Mathematical Processes

- ✓ Communication (C)
- ✓ Connections (CN)
- ✓ Mental Math and Estimation (ME)
- ✓ Problem Solving (PS)
- ✓ Reasoning (R)
- ✓ Technology (T)
- ✓ Visualization (V)

Specific Outcome

RF2 Demonstrate an understanding of relations and functions.
RF8 Represent a linear function, using function notation.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–3, 5–8, 13
Typical	#1–3, 5, 7, 9, 13, 14
Extension/Enrichment	#4, 10–14

Planning Notes

Have students complete the warm-up questions on **BLM 6–3 Chapter 6 Warm-Up** to reinforce prerequisite skills needed for this section.

Investigate Relationships in the Human Body

You may want to discuss with the class real-world data, which is often imperfect, versus the mathematical models that are used for making predictions.

As you monitor students' work through the investigation, ensure that they make measurements in centimetres and that in #2 they answer the question "How many of your feet does it take to make your height?" If students need to round this value, have them round to the nearest tenth of a centimetre for convenience.

Since the class needs to have access to each student's data, you may wish to set up a convenient way to share the information. For example, each student could record his or her information on the board, overhead, or interactive whiteboard after taking measurements and making calculations. You may wish to have the class pause after #3 and compare to see that there is agreement on the mean number of feet in the height of a high-school student.

After students have completed #4 to 6, you may wish to have another quick debriefing. You could have volunteers show their graphs and share their answers to #5 and 6. Alternatively, you could have each pair of students pass their graph and answers to another pair for comment.

For #6, discuss whether the graph is a straight line or a curve, and how this relates to whether or not the relationship is linear.

As students answer #7 with their partners, circulate and listen to their answers to assess understanding. Answers to part a) should include tables, graphs, equations, and written descriptions, based on this Investigate, but students may have additional methods. Part b) may be more challenging and may require a little more discussion. You may wish to have some tables of values available for some simple linear functions, so that students can test their hypotheses.

You may wish to encourage students to research Leonardo da Vinci and his study of the proportions of the human body to help them make a connection between da Vinci's work and the relationships. See the related Web Link that follows in this Teacher's Resource. You might wish to have students take the investigation further by answering these questions:

- Look at the illustration of the Vitruvian man. What relation is da Vinci suggesting by inscribing a man, with outstretched arms, in a square?

- By using a person's height only, how would you determine the area of the square the person would fit into? Write the relation as an equation: Area = _____
- Complete a table of values for the relation between a person's height and the area of the square. Start at a height of 1 ft and use increments of 0.5 ft. Use your equation to determine the corresponding area.
- Plot the results from your table of values to see a graph of this relation. Place height on the horizontal axis and area on the vertical axis.
- Should you draw a line or curve through the points on the graph? Why or why not?
- Would you describe this relation as linear or non-linear? Why?
- Determine the difference between each area value in your table. What do you notice?
- What changes would you need to make to the height values if you wanted the graph to reflect this relation for grade 10 students only?

Students may be interested in the following facts about average body proportions. The ratio of

- hand to foot is 7 : 9
- arm span to height is 1 : 1
- leg to height is 1 : 2
- torso and head to height is 1 : 2
- shoulder width to height is 1 : 4
- waist width to height is 1 : 6
- width of top of leg to height is 1 : 12
- head to height is 1 : 8
- neck to height is 1 : 12
- foot to forearm is 1 : 1

Meeting Student Needs

- Have students identify the root word in *linear* to guide them to understand that all linear relationships will form a straight *line* when graphed. Then, engage students in a discussion about the characteristics of a straight line.
- The investigation lends itself well to partner or small-group work.
- Explain to students that a *model relation* is a general relationship between two variables created from data that has been collected experimentally.
- Students may need help setting up the scale on the axes for their graph.

ELL

- Have students work with a partner who can help them with the language and can explain the steps.

Common Errors

- Some students may measure in inches, or may know their height in feet and inches and attempt to use that value.

R_x Remind students to read the instructions carefully, and circulate through the classroom to ensure that students measure in centimetres.

WWW Web Link

To research Leonardo da Vinci's study of human proportions, go to www.mhrmath10.ca and follow the links.

Answers

Investigate Relationships in the Human Body

1. Examples:

a) heights: 183 cm and 158 cm

b) foot lengths: 24 cm and 21 cm

2. Example: approximately 7.6 and approximately 7.5

3. Examples:

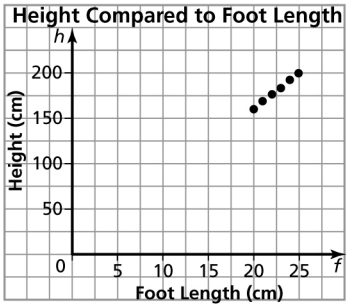
a) 7.5; 8 foot lengths to height

b) $h = 8f$, where h is height, in centimetres, and f is foot length, in centimetres

4. Example: shortest: 20 cm; longest: 25 cm

Foot Length (cm)	Estimated Height (cm)
20	160
21	168
22	176
23	184
24	192
25	200

6. a) Example:



Answers

Investigate Relationships in the Human Body

6. b) Example: Drawing a line is feasible because you can interpolate values. For example, if a foot measures 21.5 cm, the height can be estimated to be 172 cm.

c) The graph is a straight line. Example: This is a linear relation. In a linear relation, each integral increase on the x-axis results in a constant increase on the y-axis.

d) The differences are a constant of 8 cm.

7. a) Example: words, an equation, a table of values, ordered pairs, a graph

b) Example: Determine the differences between values; if the y-value changes by the same amount for each integral change in the x-value, the relation is linear. The relationship between height and shoe size may be linear since shoe size typically increases with height. However, it is more likely that various people of the same height wear different shoe sizes.

c) Height can vary by any fraction of a centimetre, whereas shoe sizes are only in whole or half sizes, so the points should not be connected.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• Students may benefit from working with a partner in answering #7a), or you may wish to encourage individual personal strategies and have them compare with a partner afterward. • For #7b), individual results in groups may not show the pattern in their table of values that will lead them to a correct solution. Use several examples and obtain suggestions regarding whether each one is linear. It may be beneficial to model some of the results on the board. Use several examples from groups to prompt a class discussion around whether the results should be linear and how students could prove it.

Link the Ideas

You may wish to take a few minutes to discuss this section. Have students decide if all of the methods for representing relations were discovered by the class during the Investigate and if the class had any representations that are not given in the student resource. Ask students if the margin definitions agree with the ones they found as they worked through the Investigate. You may wish to emphasize that there are alternative ways to express these concepts.

Many students will not be familiar with discrete functions and may even resist the concept. You may wish to ask students if they can think of situations in which a discrete relation or graph makes sense. A common example is one that involves a number of people; for example, the number of people attending a movie over a period of time.

Example 1

It may be beneficial to have students first discuss the answers to these questions without looking at the solution given. For instance, you could have students work in groups of three to five. Have students write their answers to the questions, and then have students in each group rotate their answers clockwise so that a peer can read them. Students could discuss any questions on which they disagree. You could have each group quickly report their results to the whole class.

To assess the understanding of the class, you could have each student complete the Your Turn individually, writing each answer on an index card or small piece of paper. Then, as you ask each question aloud, students could hold up the relevant response. In this way, they are able to answer anonymously, yet you can quickly judge the progress of the class. This is also easily accomplished using a student response system with an interactive whiteboard, if that technology is available.

Example 2

Two solution methods are given for this example. As students discuss the example, you can ask the following questions:

- Do you prefer one of these methods to the other? Why?
- Does the method you choose depend on the question? How?
- Can you think of a different method that could be used to solve the problem? How do you know that method is valid?

Students who are visual learners may prefer to draw the graph of the relation and then determine whether or not it is linear.

In the Your Turn, students have the opportunity to use any of these methods to determine the linearity of a number of relations. You may wish to challenge students by asking how many different methods they can use to answer the questions.

Example 3

Have students write on an index card or small piece of paper the letter of the representations that match each relation described in the example, and then hold up the index card when you ask. This will highlight any areas of misunderstanding and indicate the necessity to discuss those issues.

You may wish to have students answer the Your Turn in the same manner. In both cases, if there is any disagreement, students could be assigned to look back at prior work and material in the student resource in order to adjudicate. A small reward could be offered for the first student to find a way to support the correct answer.

Key Ideas

You may wish to review what students learned by asking them to create a list of all the different ways to represent a relation. Have students write each representation on the overhead, board, or interactive whiteboard. When all the ideas have been exhausted, have students look back through the section in the student resource to identify which ones have been missed and also which ones the class has identified that are not in the student resource, if any.

Meeting Student Needs

- Remind students that the *degree of an equation* is the highest exponent value found in any single term of a polynomial equation. For example, $x^2 - 5x + 7$ has a degree of 2. However, if a single term has more than one variable in it, the degree of the term is determined by adding the exponent values of the variables. For example, the term $5xy$ would have degree 2 since $1 + 1 = 2$.
- For Example 1, have students research Les Folies Grenouilles, which is held in the community of St-Pierre-Jolys, near Winnipeg. See the related Web Link that follows in this Teacher's Resource.
- For Example 1, students could make origami frogs and have their own frog-jumping competition. They could measure ten leaps for their frog, determine the mean distance per leap, and then complete the questions.
- For the Example 3 Your Turn, you may wish to change the animal in the question to one that students are more familiar with, for example, moose, deer, horses, or dogs.
- With the class, create posters illustrating the ways a relation can be represented. Use the same relationship for all ways so that students may study the connections between words, an equation, a table of values, a mapping diagram, and a graph.
- As they work through this section, students could create a chart containing information about how to identify linear relations:
  - From a table of values (or mapping diagram): Each unit increment in the domain corresponds to a constant change in the range.
  - From an equation: The equation contains one or two variables. The degree of the equation is 1.
  - From a graph: The graph is a straight line.
- Ensure that students have a chance to learn the vocabulary terms as they are presented.

ELL

- Assist students with such terms as *exceptions*, *unconnected*, *infinite*, *champion*, *consecutive*, *fireworks*, *shells* (in relation to fireworks), *duration*, *radioactive isotope*, *dance hall*, *scuba diver*, and *caribou*. Use visuals, descriptions, and examples.
- Ensure students understand the terms *relation*, *linear relation*, *non-linear relation*, *discrete data*, *continuous data*, *independent variable*, and *dependent variable*. Encourage them to add them to their vocabulary dictionary, Foldable, or other organizer they may be using.

WWW Web Link

For more information about Les Folies Grenouilles, go to www.mhrmath10.ca and follow the links.

Answers

Example 1: Your Turn

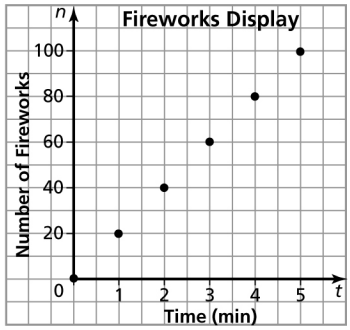
a) The relationship is linear. The total number of fireworks that have been sent off increases by 20 each time, and the time increases by one minute each time.

b) The variable n, for the number of fireworks, is the dependent variable, and the variable t, for time, is the independent variable.

c)

Time, t (min)	Number of Fireworks, n
0	0
1	20
2	40
3	60
4	80
5	100

d)



The data are discrete.

Example 2: Your Turn

a) The relationship is linear. Each time the number of people attending the dance changes by 1, the cost changes by $5.

b) The degree is 2, so the relation is not linear.

c) The relation is linear. With each increase of 5 in the independent variable, the dependent variable decreases by 8.

Example 3: Your Turn

A, B, and E

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • You may see that some students find it easier to create a table of values and graph first and then respond to the linearity question. Some students may use patterning to solve the table of values. • Strongly encourage students to use the correct terminology for the question. • For students who confuse *independent* and *dependent*, ask them what an independent person is like. Ask them what a dependent person is like.
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Example 2 reinforces the different methods used to determine whether a relation is linear. Some students may prefer to join the points on a graph to obtain a visual. Although it may be a correct strategy, encourage them to look at alternative approaches and determine linearity through dependent and independent variables or a table of values. • For part b), you may wish to suggest some values for x that students could use in order to get ordered-pair values. As this graph takes the shape of a circle, students will likely need some assistance with points. You could start by suggesting x = 5, 4, 3, 0, –5. Watch that they do not end up with a rhombus. • It may benefit some of the visual learners to have grid paper available.
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Suggest to students who are having difficulty deciding which equation is representative of the caribou that they set up a table of values to determine a pattern first. Prompt them by asking them how many legs one caribou has, then two caribou, then three, and so on. Ask them to verbalize the pattern. Have them compare this pattern to the choices given.

Check Your Understanding

Practise

For #1, students might look back to Example 1 if they need to reacquaint themselves with the various types of representations.

For students to answer #2, you may wish to suggest that they look at Example 2 and ask them what their favourite method was for determining if a relation is linear. This discussion will emphasize that there are many different ways to determine the answer to this question.

In #4, students do not need to make calculations, but they may choose to change representations so that it is clear to them which graph matches the equation.

Apply

Before students attempt #5, have them read and discuss the related Did You Know? What experience do they have with bushels as a unit? Have students use a measuring stick to develop an approximation of 0.036 m^3. This could be a cube 0.4 m by 0.3 m by 0.3 m.

You may want to point out to students that the parts of #5 do not necessarily need to be completed in the order given. For example, they may wish to determine whether the relation is linear and whether it is discrete once they have drawn the graph.

In #6, students create a relation that satisfies given criteria. This is a higher-order skill, and for many students it would be best to complete this question with a partner or in a small group. If students have difficulty, you may wish to refer them to Examples 1 and 2 and have students recall their preferred methods for determining linearity.

In #7, students complete a problem-solving question that recaps the important ideas of the section. You may wish to ask students to work with a partner and alternate explaining their reasoning and justification. Students should be able to explain how they know that this is an example of a continuous relation.

Have students read the Did You Know? related to #8. The killer whale or orca is the largest member of the dolphin family. Killer whales are voracious eaters that consume a large quantity of different types of prey, including fish, squid, sharks, octopi, birds, and other marine mammals.

Did You Know?

Sgáan is pronounced suh-haan, with the accent on the second syllable. According to some sources, this word is derived from the Haida word for supernatural. A Web Link with audio of several Haida terms is referenced at the end of this section.

Question #10 presents an enrichment opportunity for students. For part b), you may wish to suggest that they begin their set of ordered pairs by choosing an arbitrary magnitude for their first ordered pair.

Extend

For #11, ask students to determine whether a relation is linear, given a table. You may wish to refer them to Example 2 in this section.

Create Connections

In #13, students' answers may require qualification. You may wish to ask the following questions:

- Does your preferred representation change depending on the context?
- In which situations would you prefer each of the most common representations: graph, equation, words, or table?

Meeting Student Needs

- Provide **BLM 6–6 Section 6.2 Extra Practice** to students who would benefit from more practice.
- Encourage students to refer to the graphics placed in the classroom during the section and to the list of Key Terms they developed. Allow students time to discuss the terms with a partner. Have students consult with another pair of students when there is a discrepancy in the information shared within a pair.
- Post large sheets of paper around the classroom, each labelled with one question from #5 to 10. Each pair or small group of students can start at one question. In 5 min, they should answer as much as possible on the sheet before moving to the next sheet. If there is information on the sheet that the group does not agree with, they should add their comments or suggestions to the sheet. By the time each group has rotated through all of the questions, the final solution should be available. Students have the advantage of looking through information suggested by other groups along with seeing the final solution. Manipulatives and graphing calculators may be placed at each station, and students may request to use a manipulative other than what is provided.
- For #5, some students may not know what a bushel is. Explain and make comparisons to mass in SI

units. Note that one bushel is approximately equal to 0.02722 tonne (t).
- For #8, you may wish to read to students a couple of Aboriginal legends related to the killer whale and discuss killer whales in Aboriginal art. See the related Web Link that follows in this Teacher's Resource.
- For #10, encourage students to find out more about earthquakes in Canada. See the related Web Link that follows in this Teacher's Resource.

ELL

- Students may need assistance with terms such as *convert*, *current*, and *characteristic*.
- For some questions that are more language based, such as #5, 7, 9, 10, and 12, you may wish to have students pair up with another student who could assist with the vocabulary.

Enrichment

- Tell students to suppose $a = \frac{b}{c}$ where b and c are positive integers. Ask them to predict what happens to a if b is doubled and c is tripled. Have them explain their thinking. (Example: When b and c are positive integers, a decreases because the change in c, the denominator, is greater than the change in b, the numerator.)

Gifted

- Give students the following information: The International Space Station orbits Earth, travelling in a curve with its distance from Earth remaining constant. The force of gravity pulls the station toward Earth. Ask students what stops the station from crashing to Earth. Also, ask why the station's time-versus-speed graph remains linear. (The centripetal force acting on the International Space Station corresponds to the Earth's gravitational force. The station's inertia keeps it in orbit.)

WWW Web Link

To find Aboriginal legends about killer whales and information about killer whales in Aboriginal art, go to to www.mhrmath10.ca and follow the links.

For a recording of how to pronounce words in the Haida language, go to www.mhrmath10.ca and follow the links.

To research earthquakes in Canada, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1–3 and 5–7. Students who have no problems with these questions can go on to the remaining questions.	• Students having difficulty with #1 should refer back to Example 1 or their Foldable for the representative models they learned. Encourage them to use as many of the methods as possible. • In #2, students can use their own personal strategy. Again, they can refer back to Example 2 and the Your Turn, but encourage them to link the strategies from #1 to solving #2. Challenge students to be comfortable using three strategies to solve this question. • Remind students that the variable for which they can select any value to substitute into is the independent variable and the answer that results is the dependent variable. Alternatively, you may wish to discuss independent and dependent variables in terms of horizontal and vertical axes. • Some students may prefer to complete #3c) first and then return to parts a) and b). Remind students that variables should be appropriate for the question so they are easy to remember. • If students have difficulty with #6, encourage them to graph the given context first and then generate a table of values. Using their table of values, prompt them to verbalize how they could have achieved the answer without graphing first. • For #7, students are given the equation. If they do not recognize a linear graph, ask them to create a table of values and then use their own strategy to explain whether it is a linear relation or not.
Assessment *as* Learning	
Create Connections Have all students complete #13.	• In #13, students have an opportunity to assess the methods they find easiest when representing a relation. Encourage them to identify as many methods as they can confidently complete. They could also provide an example of their own to model each method. This would make an excellent Assessment *as* Learning question for the Foldable or to collect from each student. • Encourage students who complete #13 quickly to try #14.