6.3 Domain and Range

***Mathematics 10*, pages 292–304**

Suggested Timing

120–180 minutes

Materials

- ruler
- graphing calculator or spreadsheet software
- grid paper

Blackline Masters

BLM 6–3 Chapter 6 Warm-Up
BLM 6–7 Section 6.3 Extra Practice
TM 6–1 How to Do Page 300 Example 4 Using TI-83/84
TM 6–2 How to Do Page 300 Example 4 Using Microsoft® Excel

Mathematical Processes

- ✓ Communication (C)
- ✓ Connections (CN)
- Mental Math and Estimation (ME)
- ✓ Problem Solving (PS)
- ✓ Reasoning (R)
- ✓ Technology (T)
- ✓ Visualization (V)

Specific Outcomes

RF1 Interpret and explain the relationships among data, graphs and situations.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–4, 6–8, 12
Typical	#1–6, 7 or 8, 12, 13
Extension/Enrichment	#8–13

Planning Notes

Have students complete the warm-up questions on **BLM 6–3 Chapter 6 Warm-Up** to reinforce prerequisite skills needed for this section.

Investigate Reasonable Values for the Dependent and Independent Variables

As students begin the Investigate, you may wish to have them recall prior learning. Ask the following questions concerning the two graphs given:

- Are the graphs linear or non-linear? How do you know?
- What is the independent variable?
- What is the dependent variable?
- Are the graphs discrete or continuous? How do you know?

This last question, in particular, can help serve as a connection between the prior section and this section concerning domain and range.

As students discuss the six graphs given in #3, you may want to guide their discussions with specific questions:
For the first graph in the top row:

- What does this graph say about the climate of the area?
- How would a graph of our winter climate compare?

For the second graph in the top row:

- What time frame does the story suggest this graph represents?
- Do you need to include a specific year? Explain.
- By how much does the population of caribou change?
- How could the graph be changed to allow more specific answers to these questions?

For the third graph in the first row:

- Which line represents the bear and which represents the hunters? How do you know?
- Can you tell how long a journey the bear and the hunters took? Explain.

For the first graph in the second row:

- Why are the points on this graph not connected?
- What does this graph tell you about the fight?
- What does this graph not tell you about the fight?

For the second graph in the second row:

- What units might the amount of snow be measured in?
- What can you say about the rate of change in this graph?
- Can you tell how long it took for the snow to melt? Why or why not?

- Why are the points on this graph shown as a solid line as opposed to dots?

For the third graph in the second row:
- What does the y-intercept of this graph represent?
- Can you tell the greatest and smallest points in the region? Explain.
- Why is the graph shown as a continuous curve?

For #4, since it is possible for each pair of students to choose a different set of graphs, it may be difficult for pairs to conveniently compare answers. A possible strategy to allow comparisons is to give a number of sticky notes to each pair. Have a recorder write the pair's response for each of the four chosen graphs, anonymously. Then, designate six spaces on the board, one for each of the six graphs. Have each pair place their sticky notes in the appropriate place. Then, students can circulate and read all the responses, without knowing who wrote each one.

For #4, students should focus on determining values that are appropriate for the situation. Allow some flexibility, since students may have limited experience with some situations. They should reread each section of the story before choosing their values for the dependent and independent variables, since sometimes the information is quite subtle.

The Reflect and Respond question should be answered by each pair of students. You may choose to repeat the sticky-note process described above to allow for the comparison of answers, or you may wish to have a reporter for each pair give a response. The last question, concerning axes labels, should help students understand that a graph is not complete without a domain and range, and thus should reinforce the need for students to understand these new concepts.

Meeting Student Needs

- Give students two situations in which the domain and range would be significantly different. Have them discuss the following points:
  - What size of values would need to be included?
  - What type of numbers would be useful (whole, integer, rational, real)?

  Discuss with students that different relationships require different scales on a graph. These values represent the domain and range.
- Post the student learning outcomes for the section as well as the key terms: *domain* and *range*.
- Extend the discussion in the opener about the necessity of well-labelled, accurate axes. Some students still may not understand that the scale on an axis must be similar to that found on a ruler or metre stick: each increment must represent the same amount.
- The six graphs in #3 could be enlarged and posted to allow students to write comments and responses directly on the poster. As a class, discuss each poster.

ELL

- Have students work in pairs or a group so that other group members can explain the vocabulary and steps for completing the Investigate.

Common Errors

- For the first graph in the second row of #3, some students may think that there should be only four dots, since there were four hunters.

R_x It may help to ask students what the two points on the axes represent.

Answers

Investigate Appropriate Values for the Dependent and Independent Variables

1. a) Example: For a number of years, the body mass for all four tyrannosaurs does not change significantly, then it begins to increase at a steady rate, and then it levels off. The body mass for the tyrannosaurus increases at a significantly faster rate and by a greater amount than the body mass for the other species.

b) The amount of time and the tyrannosaurs' age for each stage of growth is established.

2. Temperature versus Day: winter for twelve months; Population versus Year: caribou; Distance versus Time: four of the village's skilled hunters to follow the bear home; Number of Hunters versus Time: a fierce fight killed three of the hunters and mortally wounded the fourth; Amount of Snow versus Time: the snow melted; Altitude versus Distance: hills and valleys

3. a) Example: Temperature versus Day: day values from 0 to 365 and temperature values from 0 °C to –30 °C. Distance versus Time: distance values from 0 km to 2 km and time values from 0 min to 30 min. Number of Hunters versus Time: values for the number of hunters from 0 to 4 and time values from 0 h to 3 h. Amount of Snow versus Time: time values 0 days to 30 days and amount of snow from 0 cm to 200 cm

4. a) Example: Population versus Year: The story does not indicate what the population of caribou was originally and by how much it decreased. Altitude versus Distance: The story does not indicate the height and depths of the hills and valleys.

c) Example: Values define the quantity involved. For example, instead of just noting that there was a change because of the steepness of the graph, the rate of change can be determined.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• It would be beneficial to all learners to begin this Investigate in pairs and then complete it as a class. • Encourage students to compare their answers in #5 with their partner's and to agree on a final version of their work. Suggest that they give reasons or justification for their answers. For #5b), open up a class discussion and ensure students support their answers with reasons. • To answer #5c), prompt students to look back at section 6.1 #2 and 4 for graphs without units.

Link the Ideas

You may want to allow students some time to assimilate the variety of ways to represent the domain and range. For instance, if students are using Foldables or math journals, you might have them include these concepts.

Example 1

You may want to have students attempt to determine the domain and range of each graph independently before looking at the solution. They should be free to choose their preferred representation for the domain and range. You might have them write a response on an index card and hold it up so that you can assess the understanding of the class. In this way, the discussion of the examples can be directed toward any misunderstandings.

Ensure students understand that, for example, $-6 < x < 3$ means the same thing as $x > -6$ and $x < 3$. You may wish to give students some examples of combinations of inequalities and have them practise writing them as double inequalities.

After students have answered the Your Turn, you may want them to again hold up their responses on an index card. Alternatively, they could quickly exchange responses with a partner and discuss any issues that arise. If you choose the partner approach, have one of the partners report to the whole class.

Example 2

In this example, students determine the domain and range from a situation and from ordered pairs. You may wish to have them briefly refer back to the Link the Ideas to revisit how domain and range can be obtained using ordered pairs. Focus their learning by asking which coordinate they look at when analysing domain. If students prefer certain representations, you may wish to help them switch between representations. For example, if students prefer a graph, you could ask the following questions:

- Which coordinate in an ordered pair refers to the y-axis on the graph?
- Does this help us determine the domain or the range?

It may also be helpful to ask why the graph does not begin at (0, 0).

To answer the Your Turn, students may prefer to work with a partner. You might want to remind them that the representations for domain and range need not be given in the order specified but that they may begin with the representation they find most accessible.

Example 3

When students consider the list, you may want to ask the following questions:

- Why does this method include set notation, but not the other notation, like inequality symbols?
- Would it be convenient to write a statement using set notation that is more like the previous examples? Explain.

Example 4

Many students will see a natural connection between domain and range and calculator settings. You may wish to ask students to graph the function given, without further instruction, and then poll them to see how many changed the window settings as a matter of course. Even if students do not automatically change the window, they will see that the default window is not suitable and they will also see the need for understanding domain and range in order to complete the question.

There are many ways for students to determine the answer to the margin question, "Which variable is the independent variable?" For example, they may

refer to the fact that age is the independent variable, or they may observe that since the equation is written with height isolated, it is the dependent variable.

After students have completed the Your Turn, you may want to have them hold up their calculators so that you can quickly check their understanding. This will help you decide if further class discussion is needed and will give a jumping-off point for that discussion.

Key Ideas

As students consider the Key Ideas, they should state whether there are representations for domain and range that they feel more comfortable with. Alternatively, they may identify situations in which one representation seems more natural than the others. They should be able to use all of the representations, but if they are cognizant of their own preferences, they may use that knowledge as a starting point when solving problems.

Meeting Student Needs

- Take time to reaquaint students with the difference between an open and a closed circle when used on a number line. Spend time discussing interval notation and set notation. Emphasize when to use square brackets or parentheses within the notation. Also, have students discuss what they remember about the various sets of numbers, such as integers and real, rational, irrational, natural, and whole numbers.
- As a class, create a poster listing the five methods of displaying domain and range: words, number line, interval notation, set notation, and list.
- You may wish to give students different representations of the same domain or range and have them group the like ones. Also, you might set up a matching game in which there are two cards with the same domain or range but they are given in different forms.
- Example 3 discusses music written in 4/4 time, where there are four beats per measure (shown by vertical lines in the music). Divide the class into two groups. Play eight bars of 4/4 music. Have one group clap on the first beat of each measure and count how many times they clapped. Have the other group tap their foot with each beat and determine how many times they tapped. Students could then compare their data with the table of values provided. Ask students how this data would change if they counted the number of steps taken while dancing a two-step for the eight bars of music. Invite students to demonstrate.
- Explain to students that a *restricted domain* means the dependent variables are limited to certain values. In this section, the domain is restricted when using technology to graph a situation. Another way to help students understand the meaning of restricting the domain is to describe it as "limiting the values of the domain." This way, they will see that other values are permitted when graphing; however, to see the particular part of the graph that needs to be looked at closely, the domain can be restricted so only that part is displayed.
- For Example 4, have students enter the equation into their graphing calculator without a discussion about the window. Ensure that the graph is not visible in the standard window. Students will soon see that an understanding of domain and range is necessary to accurately represent mathematical relationships.
- As a class or in small groups, have students create a summary of the content of this section. Post their responses.

ELL

- Students may need assistance with terms such as *permitted*, *element*, *leftmost, rightmost*, *Ferris wheel*, *revolution*, *inclusive*, *motorized*, and *bar* (in relation to music).
- You may need to show a number of examples of *interval notation* and *set notation* to reinforce the name for this way of representing domain and range.
- Ensure students add the terms *domain*, *range*, *interval notation*, and *set notation* to their vocabulary dictionary, Foldable, or other organizer that they might be using.
- For students unfamiliar with *corn* and *cornstalks*, show pictures of each while repeating each word.

Enrichment

- Encourage students to make a list of real-life situations where the domain and range of a relation must be integers, whole numbers, and rational numbers. Challenge them to create a Venn diagram that displays their response. Ensure that they include areas of overlap. (Students might choose counting livestock for whole numbers, loans and savings for integers, and ratios of males to females for rational numbers. An overlap might be a historical situation where livestock was counted as payment for debt.)

Gifted

- Challenge students to create questions to test for understanding of the restrictions of domain and range. Suggest that they create expressions in which, without restrictions, the denominator could be zero or in which there could be a negative sign under a square root. Have them write the question and explain the answer. (Example: What is the domain for the expression $\frac{1}{1-x}$? Answer: The domain is all numbers greater than or less than 1, since there cannot be a denominator of zero.)

Common Errors

- Students may develop the misconception that graphs always begin at (0, 0).

R$_x$ Direct them to the side bar in Example 2 that asks, "Why does the graph not begin at (0, 0)?" To help them see the answer to the question, you might ask the following questions:

- How much time has passed when a person starts a ride on the Ferris wheel?
- When a person boards a Ferris wheel, do they do so at ground level?

Answers

Example 1: Your Turn

Graph A: Domain: all real numbers greater than –2

$(-2, \infty)$, $\{x \mid x > -2, x \in R\}$;

Range: all real numbers greater than –3

$(-3, \infty)$, $\{y \mid y > -3, x \in R\}$;

Graph B: Domain: all real numbers from 0 to 5, inclusive

$[0, 5]$, $\{x \mid 10 \le x \le 5, x \in R\}$;

Range: all real numbers between 2 and 7, including 2 but not including 7

$[2, 7)$, $\{y \mid 12 \le y < 7, y \in R\}$

Example 2: Your Turn

a) Point A represents the largest point on the Ferris wheel, 47 cm. Point B represents the starting point of the chair at 3 cm above the base. Point C is the origin and represents base level on the vertical axis and the starting time on the horizontal axis. Point D represents the time it takes for two complete revolutions. Its value is 20 s.

b) Domain: all times between 0 s and 20 s, inclusive

$[0, 20]$, $\{x \mid 10 \le x \le 20, x \in R\}$;

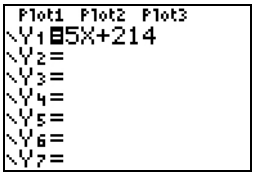
Range: the Ferris wheel's height above the base is between 3 cm and 47 cm, inclusive

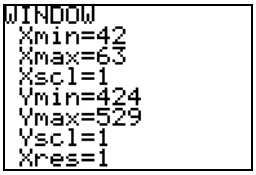
$[3, 47]$, $\{y \mid 13 \le y \le 47, y \in R\}$

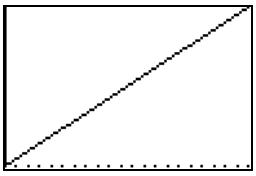
Example 3: Your Turn

Domain: $\{x \mid -3 \le x \le 2, x \in I\}$, $\{-3, -2, -1, 0, 1, 2\}$;
Range: $\{y \mid 15 \le y \le 10, y \in I\}$, $\{5, 6, 7, 8, 9, 10\}$

Example 4: Your Turn







Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Have students identify which of the four representations they feel least comfortable with and coach them through it. Check for understanding by providing another graph. • Some students have difficulty remembering domain and range and which variable in the ordered pair each represents. Coach students to think alphabetically: *d* (domain) comes before *r* (range) and *x* comes before *y*. • Check that students are clear on how to do set notation. Provide additional examples, if needed.
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • You may wish to start with value B if students are having difficulty. • Encourage students to copy the graph into their books, label the distance B and have them verbalize how far up from the ground the wheel is. • From the information given, ask what role the 22-cm radius plays. • Encourage students to label ordered pairs for values on their graph, for example, (0, 3). Ask how this might assist in determining the domain and range. • Alternatively, you could have students act out the motion of the Ferris wheel.
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Example 3 reinforces the notion of domain and range. Some students might find it easier to complete the Example 3 Your Turn and then go back and do the example. • Remind students of the link between the domain and values of *x*, and between the range and values of *y*. • Coach students to explain what the values mean beyond the link with values of *x* and *y*.
Example 4 Have students do the Your Turn related to Example 4.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • It is important for students to understand how the domain can help with determining the values for the range, which provide the window settings in a graphing calculator. You may wish to complete the example by first putting the equation on the board and asking students to graph it. For many, they will go back and change the window setting until it provides a solution. Guide students to see the efficiency in using the domain.

Check Your Understanding

Practise

For #2, students may refer to Example 2 if they need help getting started or if they experience difficulty.

For #3, students can revisit Example 3 for help with this question if needed.

In #4, remind students that the parts need not be answered in the order given. You may wish to have students explain their method for answering the question.

Apply

As students work on #5, you may want to ask them if the points on the graph should be joined. While the answer may seem obvious, students may note that graphs of this type frequently have points joined when presented in the media. If you have access to several days' newspapers, you may be able to locate an example and have students discuss why this is mathematically incorrect.

For #6, you may want to ask students if they notice any connection between interval and set notation.

In #7, students encounter a problem-solving situation from which they obtain the domain and range. You may wish to ask the following questions:

- To make your graph, would it be more convenient to write the relation in a different representation first?
- If so, which representation would you choose?

Since #8 is similar to Example 2, you might refer students there if necessary.

For the problem-solving context in #9, you may want to ask students which representation they could use to express the relation that would make it more convenient to graph.

Extend

In #10, students graph a discrete function to determine missing elements of the domain and range. You may want to ask students if graphing the points that they know both coordinates for makes a pattern appear.

In #11, some students may be able to write the domain and range from the description given, while others may wish to first write an equation or draw a graph.

Create Connections

Students will be more likely to remember the definitions for domain and range if they write them in their own words. They are given this opportunity in #14. You may wish to have students include these definitions in their math journal or Foldable.

In #15, assist students by asking them if it is necessary to have an equation or graph in order to know that a domain or range is restricted. Refer them to their work on #9 to see that it is not possible to use many representations of the relation to answer this question.

Meeting Student Needs

- Provide **BLM 6–7 Section 6.3 Extra Practice** to students who would benefit from more practice.
- When students are looking at the graphs, they will need to interpret the value represented by each line of the grid. In most instances, the value is one unit. Some students may need labels in increments of one unit. Also, discuss the symbols on the end of a graph to ensure that all students understand their meaning and how these symbols affect the domain and range.
- Students could have teacher-prepared integer number lines to view while deciding on the domain and range for each graph in #2 and 3.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1–4 and 6–8. Students who have no problems with these questions can go on to the remaining questions.	• If students find #1 and 2 challenging, have them review Examples 1 and 2. Ask them the following questions: – Can you explain how you know when a number is and is not included on a number line? – What type of inequality signs can you identify? Ensure they are not confusing greater than and less than with interval notation. • For #3 and 4, coach students to verbalize what they know about domain and range. Prompt them by asking what you look for in the domain and range of ordered pairs, of a graph, and of a table of values. Responses to these questions should help clarify the work in #4. • Some students may find it easier to use a ruler to find the exact values for domain and range. • Students must be familiar with set and interval notations for this question. Coach students as needed. • Students may wish to use the same technique of graphing and labelling segments as was completed in Example 2 and the Your Turn. Prompt students to label the coordinates of points A and B. This will help to determine the domain and range. Ensure students do not simply state values for the domain and range without being able to interpret what the values represent.
Assessment *as* Learning	
Create Connections Have all students complete #12. If students successfully answer this question, have them complete #13.	• In #12, students articulate their own personal understanding of domain and range. Encourage students to write their response in their Foldable along with their examples and explanations. This is an excellent Assessment *as* Learning question. • For students who have little difficulty, you may wish them to combine #12 and 13 into one question and base it on a topic of their own choice.