Functions

6.4

***Mathematics 10*, pages 305–314**

Suggested Timing

180–240 min

Materials

- grid paper
- ruler

Blackline Masters

BLM 6–3 Chapter 6 Warm-Up
BLM 6–4 Chapter 6 Unit 3 Project
BLM 6–8 Section 6.4 Extra Practice

Mathematical Processes

- ✓ Communication (C)
- ✓ Connections (CN)
- Mental Math and Estimation (ME)
- ✓ Problem Solving (PS)
- ✓ Reasoning (R)
- ✓ Technology (T)
- ✓ Visualization (V)

Specific Outcomes

RF3 Demonstrate an understanding of slope with respect to:

- rise and run
- line segments and lines
- rate of change
- parallel lines
- perpendicular lines.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–4, 6–8, 9, 16, 17
Typical	#1–7, 9, 10, 16, 17
Extension/Enrichment	#9, 11–15, 18

Unit Project Note that #9 and 13 are Unit 3 project questions.

Planning Notes

Have students complete the warm-up questions on **BLM 6–3 Chapter 6 Warm-Up** to reinforce prerequisite skills needed for this section.

After you read the introduction as a class, ask students the following questions:

- Are there any items that you use daily that require an input from you and then produce one output?
- Are there other examples from music?

Investigate Functions

The Investigate takes the form of a concept-attainment exercise. You may wish to emphasize to students that the task is to develop criteria or rules that can be used to distinguish functions from non-functions. You may also want to explain that they have worked with several different representations of functions and that they will see those representations in the Investigate.

Give students a few minutes to read the instructions and orient themselves to the task. You may need to assist students in getting started. The following questions may help them:

- Why are the relations organized into two different groups?
- Does it help you to first look only at the relations that are represented in the way you are most comfortable with?
- Choose one of the functions. Can you formulate a description of it that does not apply to its corresponding relation in the other group?
- Once you have a description or rule that fits one function-relation pair, does it apply to the other pairs? If not, how can you revise your rule?
- Would changing one or more of the representations given make it easier for you to complete this task?
- Does the size of the domain and/or range have anything to do with whether a relation is a function?
- Can a function be linear?
- Can a function be non-linear?

As you circulate and monitor students' work, pay attention to misunderstandings and ask questions like those above to redirect students whose definitions of functions are not correct or complete. Students may be distracted by the representation of the relation, its domain, or its linearity, and miss the relevant criterion that each member of the domain must be related to exactly one member of the range.

You might want to ask the following questions:
- Why does the vertical line test work?
- Would a slanted line work as well?

As they complete the Reflect and Respond, you may want to encourage students to consider the expanations developed by other groups. Ask them if each expanation works for each of the eight pairs given in the student resource. After some discussion, provide time for students to revise their explanations.

Meeting Student Needs

- Open this section by displaying a variety of four-sided objects (quadrilaterals) to students, ensuring that you include several rectangles of various sizes. Ask students the following questions:
  – What do all of the objects have in common?
  – Are there objects that are unique?
  – What makes them unique?

  Lead students to the conclusion that rectangles are a special type of quadrilateral. Then, inform students that they have studied various relations but that in this section they will develop an understanding of a special type of relation called a *function*.
- *Input* and *output* are useful ways of explaining the relationship between dependent and independent variables. Use these terms interchangeably to help students understand this relationship.
- The section opener gives examples of two quantities depending on each other in a certain way. You might wish to discuss other examples that are relevant to students' experiences. For example, students in the North might discuss how the pressure on the throttle determines the speed of a snowmobile, or how the speed at which ice melts in a drinking-water tank depends on the thermostat setting in the house.
- For more assistance with the investigation, refer to notes found on the Internet about concept attainment. See the related Web Link that follows in this Teacher's Resource.

ELL

- Some students may need help with terms such as *battery*, *recharged*, *Bella Coola*, *altitude*, and *storage space*. Use visuals, descriptions, and examples to facilitate their understanding.
- Show students pictures to assist them in understanding what is meant by *above ground oval swimming pools* and *hot-air balloon*.

WWW Web Link

For more information about concept attainment, go to www.mhrmath10.ca and follow the links

Answers

Investigate Functions

1. Example: The functions all move horizontally along the page, while the non-functions have some x-values with more than one y-value.

2. a) Example: In a function, the x-value can have only one y-value, but the y-value may have one or more x-values.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• Invite each pair of students to share their answers to #1, since not each pair will see the connection between function and non-function immediately. • Ensure that the explanation students write for #2 clearly identifies an understanding of the difference between function and relation. You may wish to have a class discussion and provide several function and non-function graphs and sets of ordered pairs. Suggest to students that they may wish to come back to their explanation after completing the examples.

Link the Ideas

You may wish to ask students if they have a definition of *function* that they prefer to the two alternatives given. While the wording may be different, you can ask them if the two definitions given convey the same meaning. Students may want to record their definition of a function in their Foldable or math journal.

Function notation is mathematical shorthand for a concept students have already explored. You might ask them the following questions:
- How does this compare to the process of substituting a value into an expression?

- Why would we use this notation instead of the terminology for substituting?

Example 1

This example allows students to test their definition of a function. You may wish to give students a few minutes to decide, independently, which relations are functions. Then, you could quickly poll the class. For example, you could ask students to give a thumbs-up if the first relation is a function, and a thumbs-down if it is not. After discussing the results and achieving consensus, you could repeat the process for the second pair and third pair of relations.

You may wish to have students complete the Your Turn independently and then compare results with a partner. If desired, you could have one partner report to the class. You may wish to ask the following questions:

- Why does the vertical line test not serve as a definition for a function?
- Is it possible to rephrase the test so that it may?

Example 2

The Think-Pair-Share strategy may be an effective way to approach this example. Ask each student to answer the questions and then find a partner and come to a consensus on the correct answers. Choose a number of students to report the answers on which they agreed or to present their work to the class. Discuss the work presented to ensure that all students understand. You may wish to use the same strategy for the Your Turn.

Example 3

To help students understand the translation of the equation to function notation, you may wish to focus on the variables and refer to the Link the Ideas. Ask the following questions:

- Which variable is the independent variable? How do you know?
- Which variable is the dependent variable? How do you know?

You may also wish to point out the sidebar question and discuss that the points on the graph are not connected because the function is not discrete.

You might ask students to explain why they prefer either Method 1 or Method 2 for answering part c) or have them propose a different method that they prefer.

For the Your Turn, you may want to have students work in pairs or groups of three. Ask them to identify the domain of the function to highlight the fact that the domain is understood to be the real numbers if no restrictions are given or implied.

Key Ideas

Have students rewrite the Key Ideas in terms that are most convenient and memorable for them. You may wish to simulate a "function machine" using a calculator. There are also online function machines that may prove useful. See the related Web Link that follows in this Teacher's Resource.

Meeting Student Needs

- The new notation, $f(x)$, can be a very abstract concept for students. Be sure to take time to explain how to read the notation ("f of x" or "f at x"). To help students understand, explain that it is another way of representing y, but that it gives us more information: $f(3) = 4$ instead of $y = 4$ shows that the value of y is 4 when $x = 3$.
- You may wish to develop the idea of function notation alongside an example of evaluating an expression.

x	$3x + 1$	$f(x) = 3x + 1$
–2	$3(–2) + 1 = –5$	$f(–2) = 3(–2) + 1$ $= –5$
–1	$3(–1) + 1 = –2$	$f(–1) = 3(–1) + 1$ $= –2$
0	$3(0) + 1 = 1$	$f(0) = 3(0) + 1$ $= 1$
1	$3(1) + 1 = 4$	$f(1) = 3(1) + 1$ $= 4$

- Ensure students note that Example 1 confirms what was developed in the first part of the Link the Ideas section.
- Example 2 develops a link to a practical application of function notation. Ask students if there are other examples that they can contribute.

ELL

- Encourage students to include the term *function notation* in their vocabulary dictionary, Foldable, or other organizer that they may be using.

- Use diagrams and words to clarify the meaning of *input* and *output*.
- Students may need assistance with such terms as *corresponding*, *associated*, and *cell phone plan*.

Common Errors

- Students may want to use the vertical line test as a definition for function.

R_x Ensure students understand that this is only acceptable if the function is represented graphically. It may be possible to use the vertical line test as a definition if it is carefully phrased to indicate that the relation is first represented graphically.

WWW Web Link

To try an online function machine, go to www.mhrmath10.ca and follow the links.

Answers

Example 1: Your Turn

a) Not a function; the relation does not pass the vertical line test.

b) Function; each x-value has only one y-value.

c) Function; each x-value has only one y-value.

Example 2: Your Turn

a) $F(86) = 186.8$; this means that 86 °C is the same as 186.8 °F.

b) $F(37) = 98.6$; this means that 98.6 °F is the same as 37 °C.

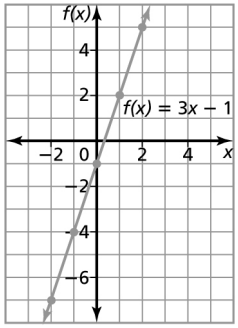
c) $K(80) = 353.15$; this means that 80 °C is the same as 353.15 K.

Example 3: Your Turn

a) $f(x) = 3x - 1$

b) Example:

x	f(x)
–2	–7
–1	–4
0	–1
1	2
2	5



c) $x = 18$

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Initially, some students may wish to plot the points and then use the vertical line test. Coach students to identify the characteristics of a function or non-function from looking at ordered pairs and tables of values. Have them write the characteristics into their Foldable for future reference.
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may benefit from reviewing how function notation is different from an equation in terms of y. • Ensure students are not confused by the meaning of $f(3)$. Some students will want to multiply 3 and f. Provide several examples before students move forward.
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may benefit from reviewing how functional notation is different from an equation in terms of y. • Ensure students are not confused by the meaning of $f(3)$. Some students will want to multiply 3 and f. Provide several examples before students move forward. • You may wish to set up two tables of values initially, one with function notation and the other in terms of y. Help students to see the parallels between the two. • Have students verbalize the function-writing process and identify the ordered pairs that are generated. Ask students to identify how they determine the "order" of the ordered pairs. This skill is important for graphing the given equations manually or using technology.

Check Your Understanding

Practise

For #1, ask students whether their explanations make reference to concepts such as domain, range, and independent and/or dependent variables.

For #2, ask students to explain their choice of variables and/or to identify the dependent and independent variables.

Conceptually, #4 and 5 are equivalent; however, the fraction in #5 may provide more challenge to students.

To help students with #6, you might ask the following questions:

- On which axis does the independent variable appear?
- Would it be helpful to express the question in another way to help you understand what is being asked?

In #7, you may wish to draw students' attention to the suggestion that they make a table of values in order to draw the graphs. You may want to ask students how they know whether the points on the graph should be connected.

Apply

In #8, students may be tempted not to write the explanations for part b). Encourage them to complete this part carefully, as this should help with their skills in creating functions to model problems.

In part d) of #9, student's answers for an appropriate domain may vary. The emphasis is on their ability to justify their choice and to draw a graph that reflects their stated domain.

While completing #9 is not required, doing so will assist students in understanding the unit project. If students are going to complete the unit project in groups, you may want them to work with those groups to complete this question. In particular, students may find the final task of measuring the length of their radius bone and their height to be more easily completed with help from a peer.

Most students will likely be able to begin question #11, but the discussion of Mach numbers presents an enrichment opportunity and may be more challenging for some students.

Extend

Students will likely need to experiment to complete #12. This question gives a glimpse of systems of equations, which is the topic of Chapters 8 and 9.

In #13, a unit project question, function notation is used to refer to points on the graph. If students completed #6, refer them to that question as a starting point.

You may wish to discuss the importance of bison to Blackfoot culture, including the Siksika, Kainai, and Piikani. Much of tribal life revolved around preparing for the bison hunt and utilizing the results of that hunt. Nothing was wasted. Bison meat was roasted for immediate use or made into pemmican and stored. The hides provided winter clothing, blankets, ceremonial costumes, and tipi covers. Bison sinew was used as thread, bowstrings, and snowshoe webbing. The bones made useful garden tools, arrowheads, scrapers, and pipes. Bison horns provided cups, spoons, bowls, toys, and headdresses. The tails were used as fly swatters and whips. Bison dung fueled the campfires. Their teeth were made into necklaces. Bladders became medicine bags, water containers, and pouches. Bison skulls were used for ceremonial purposes. For additional information on Head-Smashed-In Buffalo Jump, see the Web Link at the end of this section.

For #14, you might guide student's thinking by asking one of the following questions:

- What is the difference between a point that is closed and a point that is open on a graph?
- In what context have we previously seen open and closed dots? What did they mean then, and do you think they mean something similar now?

Students may think that #15 appears intimidating, but ask students to determine a pattern to help their understanding of the concept. For example, you could ask a short series of questions:

- What do you do to determine $h(1)$?
- What do you do to determine $h(-7)$?
- What do you do to determine $h(142)$?
- What do you do to determine $h(t)$?

These questions should help students to recognize that the underlying idea is that of substituting a value into an expression.

Create Connections

Invite students to discuss their answer to #16 with a partner or in a small group.

In #17, students need to understand the role of the independent and dependent variables in function notation. If students have difficulty with the abstraction of the unspecified function $f(x)$, you may want to invite them to choose a specific function to use in its place.

In #18, students encounter a situation in which their intuition often leads them astray. To help students see Jean-Marie's error, ask them to compare the function $f(x + 2)$ with the polynomial $y(x + 2)$ to highlight how the two are different, though they appear to be the same. This concept will prove to be important in the study of functions and relations in the future.

Unit Project

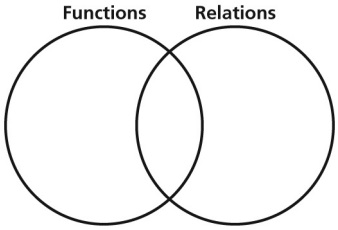
The Unit 3 project focuses on forensics and how mathematics might be useful in forensic science. You might take the opportunity to discuss the Unit 3 project described in the Unit 3 opener if you have not done so already. Students should enjoy this project, as it ties into many of the current television shows and scientific studies that they have likely been exposed to.

In #9, students have an opportunity to apply their knowledge of solving for an unknown variable or evaluating a function to solve for the missing height of a man. The question has a direct connection to their project, so students would benefit from completing this question for the final assignment. A good understanding of functions is necessary for this question. Further, students will apply their knowledge of domain and range to a similar function representing the length of female bones.

Students may find #13 more difficult. This question requires a good understanding of functions from a graphing perspective. In addition, students must be able to demonstrate an ability to interpret information from a graph and apply it to a context to solve a problem.

Meeting Student Needs

- Provide **BLM 6–8 Section 6.4 Extra Practice** to students who would benefit from more practice.
- Post examples and non-examples of functions in each of the forms: ordered pairs, mapping diagram, table of values, and graph.
- For #7, students could graph the functions on their graphing calculator and then transfer the graph onto grid paper. Emphasize that the graphs cannot have arrows.
- Encourage all students to complete #9. Some students will be able to complete the question with paper and pencil. Allow others to use a graphing calculator to determine the values for parts a), b), and c). They may require some instruction as to how to change the table of values or to determine the information from the graph.
- In relation to #13, students may be interested to learn the following: The Métis relationship with the buffalo was central to the cultural way of life of the early Métis. The buffalo hunt was an event that happened twice a year (spring and autumn) and dictated most other Métis community events. The organization of the buffalo hunt was rigorous and regimented, creating a chain of command that was respected outside of the hunt as well. St. Albert, AB, was established by Métis, and the first laws that governed the community were based upon the rules of the buffalo hunt.
- You may need to display a blank Venn diagram suitable for answering #16.



ELL

- You may need to explain to students what is meant by *saving pattern*, *force of gravity*, *elevation*, and *initial*.
- For #9, 11, and 13, have students work with a partner who can explain the language and can assist them through the steps of the questions.
- Note that if students experience difficulty with #12, their difficulty may be caused by the language in the question rather than lack of mathematical understanding.

Enrichment

- Challenge students to draw and explain a function that shows the temperature of water in a bath from when a person first draws the bath to when the tub is completely empty at the end of the bath. Ensure that they consider the effect of the person entering the tub and any other factors they think of that may affect the water temperature, such as bubbles. (Example: The graph may show the temperature of the water as initially being hot, then decreasing as the coolness of the tub affects it, then rising again

as the water volume increases. A person entering the tub cools the water as long as the person's temperature is less than the water temperature. Bubbles insulate the warmth of the water.)

Gifted

- Challenge students to examine the following situation: Suppose an office worker buys a coffee from the building's cafeteria. She wants her drink to be at the maximum temperature possible when she returns to her desk. Should she put the cream in the coffee before she leaves the cafeteria or when she arrives back at her desk? Have them determine their answer by graphing time versus temperature. Ensure that they explain their answer. (Cooling is more rapid the greater the temperature difference. Adding the cream decreases the temperature difference between the coffee and the air. Therefore, the office worker should add the cream before leaving the cafeteria. The negative slope of this graph is greater initially, but has a lesser negative slope than the graph for the coffee that has the cream added later.)

Web Link

For additional information on buffalo jumps and the importance of bison to Blackfoot culture, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1–4 and 6–8. Students who have no problems with these questions can go on to the remaining questions.	• For #1, you may wish to review the rules for determining a function. Encourage students to refer to their Foldable or Example 1 if further scaffolding is needed. • For #2 to 6, students need a good understanding of both function notation and evaluating functions. For #4c), ask students to verbalize what 42 represents. A reminder of solving basic equations may be helpful. • For #7, ask students to demonstrate another way the domain could be used in function notation. Coach them through the first question and have them complete the rest independently. • In #8, students are given the formula. They should be confident in answering #8b). Ensure that they are not simply learning to substitute values in for a function without understanding what the variables represent.
Unit 3 Project If students complete #9 and 13, which are related to the Unit 3 project, take the opportunity to assess how their understanding of chapter outcomes is progressing.	• You may wish to provide students with **BLM 6-4 Chapter 6 Unit 3 Project**, and have them finalize their answers. • Since #9 is an entry-level project question, all students should be able to complete it. It links directly to #8 in that they must understand what the variable represents. • You may consider having students work in pairs to complete #9. • Assign #13 only to students who are confident and strong in their ability to interpret graphs.
Assessment *as* Learning	
Create Connections Have all students complete #16 and 17.	• You may wish to use #16 as an Assessment *as* Learning or Assessment *of* Learning. It provides an opportunity for students to explain their understanding of functions. Similarly, #17 provides students an opportunity to determine the difference between evaluating a function at a value and the final value of the function. • Assign #18 to students who require more challenge and have a good understanding of functions.