6.5 Slope

Mathematics 10, pages 315–329

Suggested Timing

180–240 min

Materials

- grid paper
- plastic transparent ruler
- toothpick
- tape
- ruler
- one or more small boxes

Blackline Masters

BLM 6–3 Chapter 6 Warm-Up
BLM 6–9 Section 6.5 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
Technology (T)
✓ Visualization (V)

Specific Outcomes

RF3 Demonstrate an understanding of slope with respect to:

- rise and run
- line segments and lines
- rate of change
- parallel lines
- perpendicular lines.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1, 2, 3a)–d), 4, 5, 7, 8, 17, 19
Typical	#1–5, two of 8–12, 17–19
Extension/Enrichment	#9, 13–19

Planning Notes

Have students complete the warm-up questions on **BLM 6–3 Chapter 6 Warm-Up** to reinforce prerequisite skills needed for this section.

You may wish to connect this section to section 6.1, in which rates of change were discussed extensively. For a line or line segment, the rate of change is usually referred to as the slope. Students may be familiar with the concept and calculation of slope from science classes.

Investigate Slope

Students are likely best to work in pairs or small groups for this Investigate. If you have students that are familiar with slope from other classes, you may want to arrange the groups so that each group contains one such student.

After #1, you may wish to poll the class to ascertain that they agree on the relative steepness of the lines. One possible method is to have students write the lines, in order of increasing steepness, on the back of a sticky note. If you have them place the sticky notes on a transparency, you can quickly read the responses by turning over the transparency and placing it on an overhead.

You may wish to poll the class again after they have completed the measurement of the slopes in #3. It may be most efficient to simply ask if the measurements confirmed their initial decisions. A brief whole-group discussion may or may not be necessary, depending on the results of the groups.

In #6, if students are familiar with the concept of slope from science, you may ask if anyone recognizes the ratio or knows the usual terms for AB (rise) and AC (run).

You may want to take a few minutes for discussion after students complete each of #6 and 7. Each group of students might compare results and diagrams with one or two other groups to ensure that each of them is correctly following the directions. Students may also benefit from hearing the ways other groups express their discoveries.

The Reflect and Respond for this Investigate may be most beneficial to students if they include diagrams in their explanations and descriptions. When each group has answered one part of the Reflect and Respond, you could have them exchange with another group and come to a consensus on the answer. It would be best if students exchanged with a different group for each question.

Meeting Student Needs

- Develop a list of several places where slope can be evident to students: the incline on a treadmill, the road around a speed curve, the path of a hockey puck as it is shot toward the top corner of the net, the steepness of a ski hill or hiking trail, the edges of a framed picture or doorframe, the lines in the letter Z, etc.
- This investigation is very important for students who learn more effectively by manipulating objects. You might wish to include lines with negative slope. For these, students need to flip the ruler over horizontally to the "opposite" side. The ruler needs to be flipped because the run is now −1 not +1, as it is when the line rises from left to right. The slope, then, will be the additive inverse of the value shown on the ruler.
- Make sure grid paper is available to students.
- You may wish to have the investigation completed as an individual assignment so students have the opportunity to explore on their own.

ELL

- Have students work with a partner on the Investigate so that they have assistance with the language and in completing the steps.

Answers

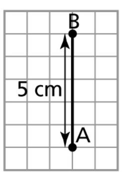
Investigate Slope

1. OC, OD, OA, and OB. Example: I compared the lines to a hill and considered which would be easiest and hardest to climb.

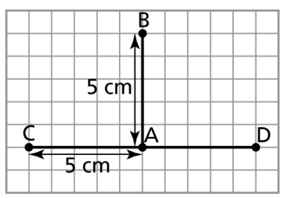
3. a) OA: 1.6 cm; OB: 2.6 cm; OC: 0.6 cm; OD: 1.2 cm

b) Example: The slope represents the number of units that the line rises for each horizontal unit.

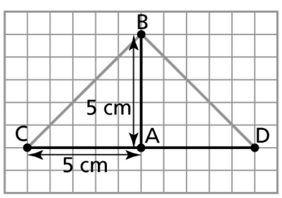
4.



5.



6. a)



$\frac{AB}{AC} = 1$

b) The ratio and the slope are equal.

7. a) Check that on one model mountain AB = 15 cm and AC = 5 cm, and on the other AB = 5 cm and AC = 10 cm.

b) The first mountain is steeper and the second mountain is less steep than the original mountain.

c) Example: The slope value for the first mountain should be greater than the slope of the original mountain, and the slope value for the second mountain should be less than the slope of the original mountain.

d) First mountain: $\frac{AB}{AC} = 3$ and the slope is 3.

Second mountain: $\frac{AB}{AC} = 0.5$ and the slope is 0.5.

8. a) Example: The lengths of AB and AC must be equal, say 2 cm. The slope is 1. Check that students drawings match their description.

b) 4 cm

c) Example: Yes. If the line intersects the slope ruler at 2.5 mm, the slope is $\frac{1}{4}$. If the slope ruler is constructed so that the length of the part of the toothpick that is exposed measures 4 cm and the line intersects the slope ruler at 1 cm, the slope is $\frac{1}{4}$.

d) Example: Yes. If a line intersects a slope ruler below the toothpick, the slope has a negative value.

e) $\frac{8}{2} = \frac{4}{1}$

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• Students might require coaching to understand that a larger scale model of the mountain drawn in #6 would have the same slope on all its sides. • Coach students in a quick reminder of scale models. You might ask how equivalent fractions relate to them. • Use the responses regarding equivalent fractions to assist them in solving #8c).

Link the Ideas

There is quite a bit of content in this Link the Ideas. You may want to have students express the main ideas in their own words, either in discussion with their group or in a whole-class discussion. The first idea is that of positive and negative slopes. While many students may find this intuitive, you may wish to have them look carefully at the four possibilities for the sign of the rise and run and ensure that they agree that this exhausts all possibilities.

You may want to allow some time for students to record a strategy of their choice. To allow for personal strategy, the student resource does not give an explicit algebraic formula for slope. Some students may want to develop one for their use, some may prefer to continue to work in terms of rise and run, and other students may have other methods. It may be productive to have students look back and make sure that their chosen strategy obtains the correct slopes for the line segments they worked with in the Investigate.

You may also want to invite students to discuss why the slope ruler they constructed works. If this is not obvious to them, you might prompt with guiding questions:

- What is the run represented by on your slope ruler when you use it?
- What is the rise represented by on your slope ruler when you use it?
- How do you get the slope from the rise and run?
- How is the slope related to the rise on your ruler? Explain why this is the case.

You may want students to think of rates of change when they consider slopes of horizontal and vertical lines. You might ask the following questions:

- Given a horizontal line, what rate of change does it show? How do you know?
- What does this rate of change mean about the rise?
- Given a vertical line, what rate of change does it show? How do you know?
- What does this rate of change mean about the run?

Students may be interested to know that in North America, *m* is the most commonly used variable to represent slope. Although there is speculation about the origins of its use, math historians have yet to determine why *m* became the popular choice. Also note that slope is not expressed with units.

Example 1

The class may complete this example quite quickly; for example, students might write *positive*, *negative*, *zero*, and *undefined* on four index cards or small pieces of paper. Then, as you call out each segment, in any arbitrary order, students hold up the card that shows the slope of that segment. Alternatively, you might use another method, such as an electronic student-response system.

Example 2

Encourage students to compare their personal strategy to the one shown in the solution. Ask students why the pitch is given as a positive value despite the fact that half of the truss has a negative slope. You may also want them to think about the following questions:

- Would it be possible to construct a truss of pitch zero? Explain.
- Would it be possible to construct a truss with undefined pitch? Explain.

Example 3

Students should be accustomed to the idea that many different strategies may be used in mathematics. Students may wish to debate the merits of each method, initially with a partner or small group and then perhaps as a class. Some of the questions they might consider in the debate are the following:

- Is each method valid? Explain.
- Is it possible to apply each method to different problems or situations? Explain.
- Will one method be more (or less) appealing to some types of learners? Explain.
- Is one method more appealing to you? Explain.
- Is each method equally efficient? Explain.
- Is each method easy to explain to another student, such as someone who missed math class?
- Is there another method that you prefer? If so, how would you answer the previously asked questions for that method?

The Your Turn is intended to help students decide on a personal strategy. If students have an alternative method that they prefer, invite them to use that strategy on the Your Turn questions to check that the strategy is correct and to confirm that it is preferable to the methods shown.

Example 4

Before beginning the example, you may want to ask students whether a point and a slope are enough information to graph a line. Allow a number of students to give their answers and rationale. Then, extend the discussion by asking if there is only one point that could be used to determine the line.

Next, make sure students have grid paper so that they can draw the line given in the example. Students can then check their work against the given solution. Then, have them complete the Your Turn. You may want to ask students to consider how they would draw a line with a slope that is an integer, such as 2 or –3. If they cannot answer readily, remind them with prompts:

- What are the numerator and denominator of an integer?
- How does this help you draw a line with that slope?

Example 5

Before they look at the graph, ask students to predict characteristics of the line based on the written information:

- In which quadrant would you expect to find the graph? Why?
- Can you predict the sign of the slope? Explain.
- What would a slope of zero look like? What would that mean in the context of a race?

To emphasize that the graph is a model, or approximation, of the boat's travel, ask students the following questions:

- What does the graph tell us about the way the boat travels? Is this realistic?
- Why would it be a good idea to use a simplified model?

Students should be able to quickly complete the Your Turn individually. You may want to have them compare responses with a peer. Alternatively, take a quick poll of the class to check understanding.

Key Ideas

Suggest that students put the Key Ideas in terms that are most convenient and memorable for them. They may wish to record the Key Ideas in their Foldable or math journal.

Meeting Student Needs

- When explaining rise and run, it is important students realize that the direction they move in has either a positive or negative value. For the rise, a movement up will have a positive value and a movement down will have a negative value. For the run, a movement right will have a positive value and a movement left will have a negative value. Emphasize that they should begin at point A and describe the rise and the run until they get to point B. A common mistake is to start at A, describe the rise, then start at B and describe the run. This mistake results in the slope having the wrong sign.
- Point out to students that the slope of the line is the same between any two points on the line. Take the time to plot a line with eight or more points and have pairs of students determine the slope between various points on the line. Record the results for all students to see.
- Once students have seen the formula for slope, $m = \frac{\text{rise}}{\text{run}}$, have them return to the investigation and determine the slope of each line.
- When the two-point slope formula is introduced, discuss the use of subscripts such as x_1 and x_2, and y_1 and y_2. You may wish to illustrate that the naming of the two points as 1 and 2 will make no difference to the outcome: both will produce the same slope. Also, ensure that students leave the signs of the formula intact and correctly substitute both positive and negative values into the formula. You may wish to show a colour-coded example of substitution.
- Example 3 develops the concept that horizontal lines have a slope of zero and vertical lines have an undefined slope. Have students create separate graphs for all three parts, to visualize the lines. Encourage them to connect the graphs to something in real life.
- Example 5 develops a concept that will also be used in physics. You may wish to provide one or two more examples for students.
- Some students may benefit from exploring slope online. See the related Web Link that follows in this Teacher's Resource.

ELL

- Suggest that students add *slope*, *rise*, and *run* to their vocabulary dictionary, Foldable, or other organizer.
- You may need to have English language learners work alongside another student for Examples 1 and 5, since there are a number of terms in these questions that they may not be familiar with.
- For Example 2, ensure students understand the connection between *width* and *span*, and between *slope* and *pitch*.
- You may need to help students with the terms *roof truss* and *undefined*.

Common Errors

- Some students may confuse slopes of zero with those that are undefined.

R_x You might help them by asking the following questions:
  - For which type of line is the run equal to zero?
  - For which type of line is the rise equal to zero?
  - Which type of line shows no change in the quantity?

- In Example 3, when using the slope formula, students may substitute the *x*-coordinates and *y*-coordinates in the wrong order: $m = \frac{6-2}{5-(-3)}$.

R_x One way to assist students with this potential error is to guide them with prompts:
  - Did you make a small sketch of the points and line segment?
  - Does your answer seem appropriate given your sketch?

Web Link

For online interactive activities related to slope, go to www.mhrmath10.ca and follow the links.

Answers

Example 1: Your Turn

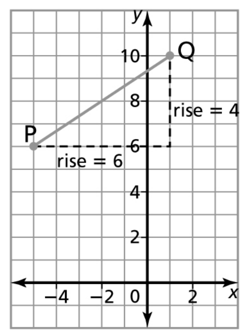
AB: positive; CD: neither; EF: negative; GH: negative; IJ: positive; KL: neither

Example 2: Your Turn

$\frac{1}{4}$. This means that the roof rises 1 unit for every 4 units of horizontal distance.

Example 3: Your Turn

a)



$m = \frac{2}{3}$

b) $m = -1$

Example 4: Your Turn

Example: (–3, 2), (0, 3), and (3, 4). Check that (–6, 1) and students' three other points are plotted on the graph.

Example 5: Your Turn

$\frac{500}{92}$, or approximately 5.43 m/s

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • An alternative approach for students having difficulty could be to have them focus on the top of each line (the largest point) and ask whether the line would point to the right of the *y*-axis (positive slope) or to the left of the *y*-axis (negative slope). Vertical lines do not point left or right and therefore are undefined; horizontal lines, if extended, go left and right at the same height and therefore have a slope of zero.
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Ensure students understand that slope is $\frac{\Delta y}{\Delta x}$ and not $\frac{\Delta x}{\Delta y}$. • Remind students to reduce the fractional values of slopes where possible. • Ensure students know how to write an integer value for slope as a fraction to represent rise and run.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • When substituting into the slope formula, it is common for students to confuse the values of *x* and *y*. Having them label each ordered pair as (x_1, y_1) and (x_2, y_2) will assist them. • Caution students to watch for when they need to subtract negatives. • Coach students to realize that the placement of a single negative can appear in the numerator or denominator without changing the slope of the line. You may wish to model this case for students.
Example 4 Have students do the Your Turn related to Example 4.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Remind students that slope is $\frac{\text{rise}}{\text{run}}$. Review with them the counting method that provides another approach to graphing a line. Count units downward for the negative rise value, and count upward for the positive rise value. Count units to the right for the positive run value, and count to the left for the negative run value. A slope of $\frac{-2}{5}$ would be down 2 and right 5. Ask students what would happen if it was $\frac{2}{-5}$.
Example 5 Have students do the Your Turn related to Example 5.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Review with students that rate of change can be thought of as a move from one ordered pair or location to another. • Coach them to see how the slope formula links to the calculation of rate. • Having them label each ordered pair as (x_1, y_1) and (x_2, y_2) will assist them.

Check Your Understanding

Practise

For #1, students may wish to refer to the Key Ideas, and for #2, they may wish to refer to Example 1.

Point out to students that #4 is related to Example 4. You may wish to refer students to the Key Ideas if they need assistance. Since the question includes lines with slopes that are integers, you may want to ask how students can rewrite a number like –4 as a ratio between the rise and the run.

For #5, to help students interpret the physical meaning of the slope in the situation, ask the following questions:
- What does the slope mean in words?
- Who would be concerned about this slope? Explain.

Apply

The task in #7 is similar to Example 4, so encourage students to look back at this example if necessary. Have students compare their explanation with at least one peer. To help them understand the meaning of the points on the graph, you could ask the following questions:
- What does each of the coordinates represent?
- What are the independent and dependent variables in the situation?
- The rise describes a change in which quantity?
- The run describes a change in which quantity?

For #8, guide students' thinking by asking the following questions:
- If you think of the slope as a ratio, what have we previously done in math to compare ratios?
- Why does $\frac{1}{16}$ represent a gentler slope?
- How can you use a diagram to show that $\frac{1}{16}$ is a gentler slope?

For #9, prompt students to recall previous learning by asking the following questions:
- What does it mean when a number is expressed as a percent?
- How can you write a percent in a different way so that it looks more like what we expect of a slope?

To help students with #10, you might ask the following questions:
- Which is the independent variable in this question?
- Which of rise or run refers to the independent variable?
- Which is the dependent variable in this question?
- Does *rise* or *run* refer to the dependent variable?

- How could a graph help you solve this problem?
- What units would you use to express the answer to this problem?
- How does that help you determine the correct slope?

In #11, students will likely need to sketch a graph representing the situation. You might ask the following questions:
- Which quantity should be graphed on the x-axis? Why?
- Which quantity should be graphed on the y-axis? Why?

When students have obtained answers, have them look at the signs of their slopes and ask them how they know whether the slopes are appropriate.

Students need to recognize that the numbers in #12 are written in two different forms. As they sketch a graph for the situation, you might ask the following questions:
- How would you label the values on the axes?
- What is another way to write a number like 2.8 million?

A real-life application of slope is presented in #13. Assist students' understanding by asking the following questions:
- What other representation or representations of this function can you use to make determining slope easier?
- How can you tell which quantity is changing in the run?
- How can you tell which quantity is changing in the rise?

In part c), students solve a proportion. Ask them what strategy they have used previously when they know two ratios are the same but do not know one part of one ratio.

Extend

For #14, have a number of boxes available to help students visualize the situation. Students will need to use the Pythagorean Theorem to determine the run along the diagonal of the base of the box. Guide them by asking the following questions:
- What type of triangle is formed by the three vertices indicated in the diagram?
- Are other triangles of this type part of this situation?
- What strategy or strategies do we have for working with this type of triangle?

For #15, to emphasize that volume, not length, is the quantity under consideration, you may wish to ask students what they would place on the y-axis to sketch a graph.

At first, #16 may look very different to students. Ask them to think of the rule they use to determine the slope and apply it without regard to the appearance of the coordinates. For example, to show a pattern, ask the following questions:
- How do you determine the slope if your coordinates are (1, 3) and (4, 15)?
- How do you determine the slope if your coordinates are (–2, 1) and (5, –13)?
- How do you determine the slope if your coordinates are (0, 0) and (–3, –12)?

Then, ask students to apply the pattern to the coordinates given.

Create Connections

You may wish to make #17 more accessible to students by prompting their thinking:
- What does the word *constant* mean in English?
- What does the word *constant* mean in math?
- For any line, does it matter which two points you use to determine the slope?

For #18, you may need to assist students by using guiding questions:
- What type of triangles are you able to analyse using trigonometry?
- How can you form that sort of triangle if you know a slope?

For the Mini Lab, #19, have students work in small groups. After groups have completed Step 1, you may want them to compare answers to ensure that they understand how to read a topographic map. As you monitor students' work, watch for correct units in Step 3. When groups have finished the question, you may want to have a whole-class discussion for students to compare answers and hear alternative explanations and rationales.

Meeting Student Needs

- Provide **BLM 6–9 Section 6.5 Extra Practice** to students who would benefit from more practice.
- For #2, remind students to use the formula $m = \frac{\text{rise}}{\text{run}}$.

- When students graph lines given a point and a negative slope, encourage them to put the negative sign on the "rise" and always "run" to the right. Some students may be confused unless directly informed of how to deal with a negative value.
- When first viewed, the graph in #5 may be confusing to some students. Ask them to describe why this graph appears in quadrant IV.
- For #7, students interested in pursuing carpentry might be encouraged to determine the regulations for ramps in their own province.

ELL

- You may wish to have students work with a partner on #8, 10, 11, and 19 due to the challenging language.
- Students may need assistance in understanding the terms *mountain pine beetle*, *infested*, *lightning*, and *thunderclap*. Use descriptions and visuals to clarify.

Enrichment

- Have students list potential problems when graphing a relation to show real-life information. For example, they might mention the zero-denominator problem, the vertical-line issue, or the difference between discrete points and lines. (Example: When showing time versus the population of an endangered species, for small numbers of the species, it is incorrect to use a line rather than discrete points.)

Gifted

- Have students create a list of ordered pairs for a linear graph that includes fractions, square roots, and negative numbers. Have them draw the graph of the ordered pairs. (Check that students create a list of values that can be graphed, yet meet the criteria. One strategy is to use such numbers as $\sqrt{4}$ for the square root and $\frac{4}{1}$ for the fraction.)

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1, 2, 3a)–d), 4, 5, 7, 8, 17, and 19. Students who have no problems with these questions can go on to the remaining questions.	• Students having difficulty with #1 and 2 may wish to review the Link the Ideas, Key Ideas, and Example 1. • For #2 and 3, review the meaning of *slope* and the terms *rise* and *run*. Have students link rise and run to the parts of the slope formula related to each. A common mistake for students is to confuse the rise and run in the calculation. Coach them through one of the parts in each question. Some students may find "counting" the vertical changes and then the horizontal changes an easier way to determine the slope. • For #4, clarify that students know how to interpret the negative sign. Show them that it will not alter the final answer whether the negative is applied to the numerator or to the denominator. • Students who prefer to count the slope rather than use the formula must remember that a downward count is interpreted as a negative when solving #5. Have them identify the rise units and the run units so that they use the correct terminology. • For #7, ask students to provide an equivalent value for –3 in fraction form. This will assist those looking for the run value in the slope. • For #8, encourage students to draw a diagram and coach them through the labelling of it. You may wish to go over a few equivalent fraction questions before asking students to solve the shortest length of the ramp.
Assessment *as* Learning	
Create Connections Have all students complete #17 and 19.	• For #17, ask students to explain their thinking behind the meaning of *constant* (a number that does not change). Have them verbalize why the meaning is important when they draw a line. Have them draw a line with a slope of their own choice and ask them to select any two points on their line. Ask what the rise and run are. Encourage them to generalize their thinking to answer the question. • All students should work through #19. Put students in pairs or small groups. The information and processes used in this Mini Lab will assist them with their project. Coach students through the contour line map if they have never worked with one before. Select one of the mountains and review the indicated elevations, how they were determined, and what they mean. Some students may have a difficult time visualizing the map.