Slope-Intercept Form

7.1

***Mathematics 10*, pages 340–356**

Suggested Timing

100–140 min

Materials

- two metre sticks
- elastic band
- foam cup
- paper clips, string, or tape
- toothpick or straightened paper clip
- six identical marbles or other items of equal mass
- ruler
- grid paper or graphing technology

Blackline Masters

BLM 7–3 Chapter 7 Warm-Up
BLM 7–4 Chapter 7 Unit 3 Project
BLM 7–5 NATO Emblem on a Grid
BLM 7–6 Section 7.1 Extra Practice
TM 7–1 How to Do Page 343 Example 1 Using TI-Nspire™
TM 7–2 How to Do Page 343 Example 1 Using Microsoft® Excel

Mathematical Processes

- ✓ Communication (C)
- ✓ Connections (CN)
- ✓ Mental Math and Estimation (ME)
- ✓ Problem Solving (PS)
- ✓ Reasoning (R)
- ✓ Technology (T)
- ✓ Visualization (V)

Specific Outcomes

RF1 Interpret and explain the relationships among data, graphs and situations.

RF3 Demonstrate an understanding of slope with respect to:

- rise and run
- line segments and lines
- rate of change
- parallel lines
- perpendicular lines.

RF5 Determine the characteristics of the graphs of linear relations, including the:

- intercepts
- slope
- domain
- range.

RF6 Relate linear relations expressed in:

- slope-intercept form ($y = mx + b$)
- general form ($Ax + By + C = 0$)
- slope-point form ($y - y_1 = m(x - x_1)$) to their graphs.

RF7 Determine the equation of a linear relation, given:

- a graph
- a point and the slope
- two points
- a point and the equation of a parallel or perpendicular line

to solve problems.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1, 3–6, 8–10, 13, 14, 18, 24–26
Typical	#1, 2, 4–7, 9, 10, 12–14, 15 or 16, 18, 24–26
Extension/Enrichment	#17, 19–24, 26

Unit Project Note that #18 is a Unit 3 project question.

Planning Notes

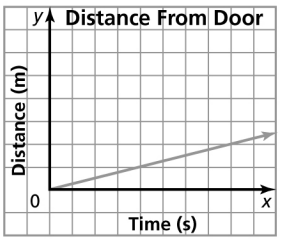
Have students complete the warm-up questions on **BLM 7–3 Chapter 7 Warm-Up** to reinforce material learned in previous sections. It is important that students be able to determine the slope of a graph. Have students discuss situations in their own life that may be modelled by the graph of a straight line. One example could be the relationship between costs on a prepaid cell phone and the calling time. You may wish to begin the discussion by asking the following questions:

- If you own a cell phone, under what conditions can a graph of a straight line model the amount you are charged?
- Besides cell phones, what other examples in your life can be modelled by the graph of a straight line?

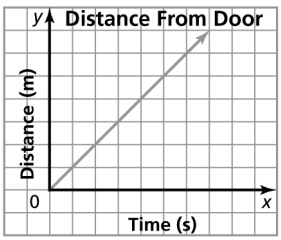
Make a list of students' examples. As you work through the chapter, add new examples to the list as they come up.

As an introduction to the chapter, you may wish to show students the following graphs. Each graph represents the distance, in metres, a student is from the door of the classroom after t seconds.

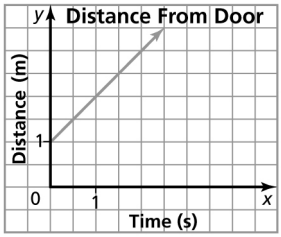
Graph A



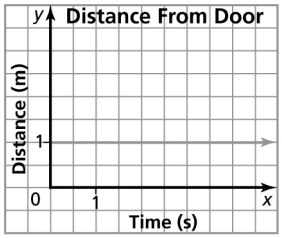
Graph B



Graph C



Graph D

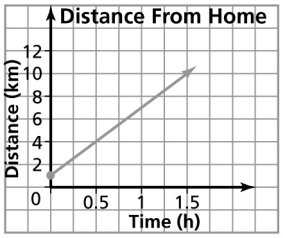


Ask students to compare and contrast the graphs. You may wish to use the following questions:

- How are the graphs similar? How do they differ?
- What are the independent and dependent variables?
- What does the slope of each line represent?
- The y-intercept in Graphs C and D is 1. What is a y-intercept? In this situation, what does the y-intercept represent?

Have students act out each graph.

Alternatively, you may wish to present students with the following scenario: In the morning, Adam joins his father for a walk. The graph shows the distance, in kilometres, Adam and his father are from their house, after t hours.



Have students determine the equation of the line. Then, ask them if that is the only way to express the equation. Many students may not be able to determine an equation. In this chapter, students will learn different strategies to determine the equation of a straight-line graph. The first method is called slope-intercept form. You may wish to return to the graph as a summary activity after completion of the Investigate and Link the Ideas sections.

Investigate the Graph of a Linear Equation

This Investigate could be done as a class demonstration if necessary. You could also have students set up different stations in which the elastic band, masses, and /or foam cup size vary. Alternatively, you may wish to have different-sized materials available for students to conduct their own investigation with. When gathering the materials, long, thin elastic bands work best. Make sure the elastic band is not stretched beyond its elastic limit. It should always return to its original length when the masses are removed. The items used as masses must have enough mass to result in a measurable extension of the elastic band. Instead of marbles, bolts or washers may be used. Various types of candies, such as miniature chocolate bars, may also work well.

You may wish to discuss with the class what a scatter plot is and how it shows the relationship between two variables. Have students explain why it is or is not appropriate to connect the points on this graph. You may need to remind students of the terms *discrete* and *continuous*. Ask students to describe discrete and continuous data. Technology could be used to find the equation of the line or to check the accuracy of the equations found from students' hand-drawn lines.

This investigation is based on Hooke's law, named after the English scientist Robert Hooke (1635–1703). Students may be studying this relationship further in science. The relationship between the expansion or compression of any spring and the force acting on the spring is linear, within the elastic limits of the spring. This relationship makes springs very useful. Ask students to provide examples of how springs are used in everyday life. Possible applications include vehicle suspension springs, garage door springs, anti-vibration springs, bathroom scales, grocery produce scales, and road-side truck scales.

Meeting Student Needs

- Students may need to review how to identify independent and dependent variables and how to place them on the graph. Students may also benefit from reviewing the definition of a linear function/relation.
- Students may need to extend their graphs. You may wish to take the opportunity to discuss interpolation (estimating coordinates of points lying *between* plotted points) and extrapolation (estimating points lying *beyond* those that are plotted).

- When discussing the section opener, you may wish to have students discuss whether food for a dog team could be modelled by a linear equation.

Common Errors

- Some students determine slope as the ratio of Δx to Δy, or run over rise.

R_x Remind students of the meaning of slope: the steepness of a line. You may wish to provide students with two graphs: Graph A has a line with slope 3, and Graph B has a line with slope $\frac{1}{3}$. Students can clearly see that Graph A is steeper and therefore should have the greater slope. Graph B is less steep and therefore should have the smaller slope. Have students determine the slope both ways, i.e., rise over run and run over rise. This will help students see which formula matches what they know to be intuitively true.

Graph A

$m = \frac{\text{rise}}{\text{run}}$ or $m = \frac{\text{run}}{\text{rise}}$

$m = \frac{3}{1}$ $m = \frac{1}{3}$

Graph A should have the greater slope.

$m = \frac{\text{rise}}{\text{run}}$ gives the expected slope.

Graph B

$m = \frac{\text{rise}}{\text{run}}$ or $m = \frac{\text{run}}{\text{rise}}$

$m = \frac{1}{3}$ $m = \frac{3}{1}$

Graph B should have the smaller slope.

$m = \frac{\text{rise}}{\text{run}}$ gives the expected slope.

- Students mix up positive and negative slopes.

R_x Reinforce the concept of positive and negative slope by teaching students that graphs "run to the right." To determine the slope of a line, have students pick any point on a graph and run to the right. To reach another point on the line, they would have to go up (+ slope) or down (– slope).

WWW Web Link

To view a video showing how the extension of a spring changes when different forces are applied, go to www.mhrmath10.ca and follow the links.

Answers

Investigate The Graph of a Linear Equation

3. The independent variable is the number of marbles. The dependent variable is the length of the apparatus.

4. Example: The slope of the line is 0.5. The units are centimetres per gram. The slope of the line represents an increase of 0.5 cm in the length of the apparatus per gram.

5. a) Example: The line intersects the y-axis at (0, 14). This point represents the length of the apparatus before any marble is placed in the cup.

b) Example: Multiply the slope by the number of marbles and add 14.

6. Example: 19 cm; Yes.

7. a) Example: $y = 0.5x + 14$

b) Example: 21.5 cm

8. a) Example: All measurements will be 4 cm more than the initial measurements.

b) Example: The graph will be vertically translated 4 units up.

c) Example: $y = 0.5x + 18$

9. a) Example: Measurements in #2 will be twice the apparatus length compared to the initial measurements.

b) The slope will be steeper, 2(0.5) or 1.

c) Example: $y = x + 14$

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• Some students may benefit from having extra time to try the suggested change in #8. Without completely redoing the Investigate, suggest that they physically hold the larger cup in the place of the original. This may prompt their thinking about the effects on the *y*-intercept. • For #9, provide sufficient time for students to try one or two objects of different masses. It may benefit some students to physically try to visualize the effects of the change before predicting how the graph will change. • You may wish to divide the class into two groups and have one group complete and respond to #8 while the other group completes #9. If students completed trials, have representatives present their results of #8 and #9 to the rest of the class.

Link the Ideas

To help students remember the equation $y = mx + b$, they can think of b as the beginning y-value when x is zero, and m as the rate or slope needed to move to another point on the line. Discussing the information in the table will reinforce these concepts. You may wish to have students extend the table. You could fill in the first column with a table of values and have students complete the other four columns. You may wish to have a class discussion about the following questions:

- When calculating slope, does it matter which coordinates are chosen? Why?
- For a particular example, why is Δy positive or negative? Why is Δx positive or negative?

This will help students understand the meaning of slope rather than merely applying a formula.

Example 1

Students should be able to do this without difficulty. They could also find the slope of the line by picking two "nice" points and counting the rise and run between the two points. Encourage students to determine the sign of the slope by the direction of the line in the graph. This will help them confirm that their answer has the correct sign. Remind students to reduce any fractions for slope to lowest terms. You may wish to draw students' attention to coefficients of 1 and 0. Remind students that $x = 1x$, $-x = -1x$, and $0x = 0$. Students may benefit from making these distinctions.

You may wish to draw students' attention to the order of the solution for part a). The question asks for the slope and y-intercept of the line. The first answer provided is the y-intercept, not the slope, because the y-intercept is easily read off the graph without performing any calculations. Encourage students to always try to answer the parts of a question that they are most confident with and can answer quickly first.

For part c), students need to use graphing technology to check their equation. You may wish to have students use **TM 7–1 How to Do Page 343 Example 1 Using TI-Nspire™** or **TM 7–2 How to Do Page 343 Example 1 Using Microsoft® Excel** to work through this solution.

Example 2

You may wish to discuss the positive and negative values of y within the context of this question (making money, losing money) and what a value of y of 0 means. Students may have difficulty grasping the concept of breaking even. You may want to discuss other situations that could involve a break-even point.

Emphasize to students that there are several methods for isolating a variable in an equation. You may wish to have a discussion about some strategies for reducing the number of steps and thereby reducing the possibility of introducing an error. One such strategy is shown in the Mental Math and Estimation box beside the solution.

Example 3

Work closely with students when they are defining their variables. Ensure that students choose variables meaningfully and with the proper units. Ask students to indentify the independent and dependent variables and to justify their choices.

In part c), students are asked to identify and interpret the meaning of the y-intercept. You may wish to have sudents do the same for the x-intercept. Students have likely worked with parameters before, but they may

not be familiar with the term. You may wish to have students provide some examples of parameters they have used, perhaps from science class.

Although it is virtually unknown, Canada has a distinguished heritage of submarine service that dates back to 1914. As part of the British fleet during World War II, Canadian submariners were involved in every type of naval operation. Since 1961, the Canadian Navy has had at least one submarine in its fleet. You may wish to discuss the following questions as a class or have students interested in this topic research it further and report back to the class.

- Why is it important for Canada to have operational submarines?
- What are the advantages to having non-nuclear submarines?

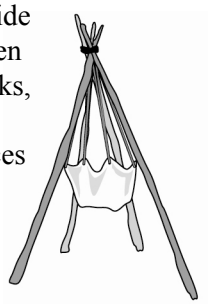
Example 4

This example uses a real-life situation to reinforce the concepts in this section. Make sure students have a solid understanding of the previous examples before working through this one. Emphasize to students the importance of using brackets to show a number being substituted. Some students may need to be reminded of the order of operations.

You may wish to write the slope-intercept form of a linear equation on the board with (x, y) beside it. Then, write the equation for boiling water with (t, W) below it to help students understand the link between the equations. Colour-coding the similar variables might also be helpful.

Draw students' attention to the Did You Know? box beside the solution. The property described can be referred to as the point-on property (POP). Make sure students understand this concept, because it will be useful when they work through many problems involving equations.

In addition to the method described in the student resource, First Nations people used another method of boiling water. They used to hang a sac made from a bison stomach or piece of hide on a wooden stand. The sac was then filled with water. Using forked sticks, red-hot rocks were scooped from a campfire and added to the sac. Pieces of meat and vegetables were then added to make a soup or stew.



Key Ideas

Remind students to always include the following when graphing:

- labels for the axes, equation of the line, and scale on the vertical and horizontal axes
- labels for the coordinates of the x-intercept and y-intercept

Meeting Student Needs

- Allow students to use manipulatives, such as algebra tiles, when isolating y in an equation.
- You may wish to create an overhead displaying the table from the Link the Ideas and then use markers to highlight the slope and y-intercept in each graph and corresponding equation.
- Colour-coding may be useful when working through Example 1.
- Allow students to work through Example 4 in pairs or small groups. Encourage discussion and a written explanation of each group's findings.

Enrichment

- Challenge students to compare the lines $x = 0$ and $y = 0$ in terms of slope-intercept form. ($y = 0x + 0$ is appropriate, but $x = 0$ is a vertical line.)

Gifted

- Ask students to create a poster that shows three applications of slope-intercept form that meet the following criteria: they are real-life applications, they show a step-by-step approach to using the slope-intercept form, and both slope and y-intercept are shown.
  (Examples might include slope of a distance-time graph representing speed and the y-intercept representing initial starting point.)

Common Errors

- Many students have difficulty converting equations to slope-intercept form.

R_x Remind students that there are many strategies they can use to isolate y in an equation. Encourage students to use the methods they are comfortable with. You may wish to have different students show their methods and discuss some of the strategies they use and why they prefer a certain method.

- Students may have difficulty interpreting the meaning of slope and y-intercept in a context.

R_x To help students interpret the meaning of slope in a particular situation, direct them to the units being used. To interpret the meaning of the y-intercept, ask students what happens when the independent variable is zero.

WWW Web Link

To learn how to make a dunk tank, go to www.mhrmath10.ca and follow the links.

For a graphical representation of loss, break-even, and profit, go to www.mhrmath10.ca and follow the links.

For information on how submarines work, go to www.mhrmath10.ca and follow the links.

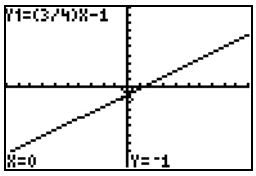
Answers

Example 1: Your Turn

a) The slope is $\frac{3}{4}$ and the y-intercept is -1.

b) $y = \frac{3}{4}x - 1$

c)



Example 2: Your Turn

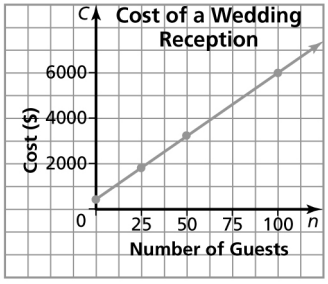
a) The slope of the line is 12. It represents income of \$12 per ticket.

b) The y-intercept is -840. It represents the cost of \$840 to rent the hall.

c) 70 tickets

Example 3: Your Turn

a)



b) The slope of the line is 55. This means that the price increases by \$55 per guest. The y-intercept is 425, and it represents the cost of renting the ballroom.

c) Let C represent the cost, in dollars. Let n represent the number of guest. $C = 55n + 425$

d) \$8125

e) 265 guests

Example 4: Your Turn

a) \$150 **b)** 11 h

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may need coaching and a review of the terms *slope* and *y-intercept*. • Remind students to select two ordered pairs that are "nice" or fall exactly on grid values that make them easier to read when calculating slope. • Some students may benefit from using the counting method to determine the slope from the y-intercept. Have students start at the y-intercept and locate the closest ordered pair that is on the line and made up of integer values. Then, have students count from the y-intercept the number of units up (+) or down (–) and then left (–) or right (+) to the ordered pair. The up or down count represents the rise, and the left or right count represents the run. • Once students have completed the count, ensure they can correctly place the slope and y-intercept into the form $y = mx + b$.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students will have difficulty with the interpretation of the slope and/or *y*-intercept. They may become proficient at identifying the values and the process to graph them, but are unable to understand their meaning. Coach students through rewriting the general form into slope-intercept form. Have them verbally identify the slope and the *y*-intercept. Ask students to identify the dependent and independent variables from the question. Then, ask students how the slope and *y*-intercept are related to the problem. • Remind students they are not asked to graph this question, but if they choose to do so, they may need assistance in choosing appropriate scales for the axis. • Help students to understand that at the break-even point, revenue equals expenses. If necessary, provide an example with smaller numbers. For example, you buy ten pencils at 15¢ each. How many pencils do you have to sell at 25¢ each to break even?
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Remind students that graphing from a table means to write ordered pairs from the data. Ask them to identify what the ordered pair represents. • Remind students that changes in *x* and *y* can be determined from the table: $\left(\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1800 - 425}{25 - 0}\right)$ • Have students verbally explain what is happening in the graph before they answer part c).
Example 4 Have students do the Your Turn related to Example 4.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • When students write their own equations, remind them to select variables that are easy to remember and make sense in the context of the problem. For this problem, ask students to identify the meaning of the variables. • You may wish to review some simple equations and solving for a variable; then, use the same equations and use substitution to solve for a missing value.

Check Your Understanding

Practise

For #1c), reinforce that the slope is not 0. The number 1 is implied in front of x, so the slope is 1. In part d), you may wish to explain that the x-intercept is 0. Since the equation has no constant, the constant is 0. This equation could be rewritten as $y = -4x + 0$. Make sure students understand that there is a y-intercept and that it is 0. They might mistakenly assume this equation has no y-intercept.

For #3, students may need to be reminded to place brackets around fractions if they are entering these equations into a graphing calculator.

For #8, you may wish to show students a quick way to determine slope given a graph. On the graph, identify the y-intercept and one other nice point (a point with whole number coordinates). Start at the point on the left and count up or down and then to the right to the second point. Place the number of times you count up or down with the sign (+ for up, – for down) in the numerator and the number of times you count right in the denominator. Reduce the fraction to lowest terms.

For #9 and 10, students may need help seeing the connection between the x and y in the equation and the x and y in the coordinate pair. You may wish to emphasize the point-on property mentioned earlier.

Apply

Question #14 will take some time. Allow students to complete this question either with a partner or in a group. Encourage students to look for patterns to make the work easier.

For #15, students need to realize that springs can expand or compress. In either case, the relationship between the length of the spring and the force applied is linear.

Question #18 is similar to Example 3. It is an important question because it ties together many of the concepts in the section.

Extend

For #20, encourage students to use more than one approach to solve this problem. Students can start by letting $x = 0$ or by isolating the variable y in both equations.

Create Connections

Question #24 could be done after Example 1.

Have students complete #25 in conjunction with #3 and 6.

For #26, you may wish to create a wall of family portraits. Students can post the families of lines they created, showing the equations of the lines, their corresponding graphs, and a description of how the lines are related. Posted families could be grouped along common themes. Encourage students to use patterning as they work through this Mini Lab.

Unit Project

The Unit 3 project questions give students an opportunity to solve problems that will assist them in solving an archaeological mystery in the final project assignment.

Question #18 is a Unit 3 project question. It is an application of determining whether data in a table of values represent a straight line, the calculation of the slope and what it represents, and writing an equation and interpreting the results. The topic of how weather affects humans is appropriate practice for the final project, in which students will set out to solve the mystery of discovered human bones and the disappearance of three Klondike gold miners.

Meeting Student Needs

- Allow students to use a graphing calculator if available.
- Keep the equation $y = mx + b$ on display in the classroom with the slope and y-intercept identified.
- Provide **BLM 7–6 Section 7.1 Extra Practice** to students who would benefit from more practice.
- Question #9 can be made more visual by using a color-coded example illustrating the substitution of x and y into the equation to find b. Question #10 can also be illustrated this way.
- For #14, provide students with **BLM 7–5 NATO Emblem on a Grid** to allow students to extend the lines and determine each equation.
- For #15, some students may need to graph the data shown to determine the slope and y-intercept. If students must graph first, encourage them to write a response demonstrating knowledge that the y-intercept corresponded to where x had a value of zero, and that the slope is determined by studying the y-values.

Common Errors

- Students often confuse no intercept with an intercept of 0.

R_x Have students sketch a line that has no y-intercept and have them sketch a line that has a y-intercept of 0. Explain to students that a line with no y-intercept does not intersect the y-axis. It is a vertical line. A line with a y-intercept of 0 intersects the y-axis at the origin, or point (0, 0). Encourage students to determine the equation of each line they drew, and to justify their equations.

Web Link

For information about NATO, go to www.mhrmath10.ca and follow the links.

To learn more about how scientists use bones to find the age of dinosaurs, go to www.mhrmath10.ca and follow the links.

For information about extinct animals, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1, 3–6, 8–10, 13, and 14. Students who have no problems with these questions can go on to the remaining questions.	• You may wish to partner students of like ability to work together. • Review with students how to identify the slope and *y*-intercept. Specifically, ensure that they can identify that given $y = x + 2$, the slope is 1, and given $y = 2$, the slope is 0. Also review that the *y*-intercept is zero when the equation does not contain a constant term. For example, $y = 3x$ could be written as $y = 3x + 0$. This will assist students who might have difficulty with #1–3. • Questions #4 and 5 require students to rewrite equations in slope-intercept form. Students who are having difficulty should be coached through solving for *y* in simpler forms. Encourage students to verbalize the steps they perform and why they are doing so. Coach and remind students that they are working on opposite operations. Try examples such as solving for *y* in the following: $y + 1 = 2x$ $y - 3 = 5x$ $y + 2x = 4$ $2y = 4x + 6$ $3y = x + 9$ $-2y = 4x - 5$ • For #8, encourage students to use their own personal strategy to find a slope. Some will count, some will find the difference, and some may wish to sketch a triangle created by the line. • For #9 and 10, students may require coaching on evaluating and solving for a missing variable. Review the meaning of ordered pair (x, y) and ensure students are clear on which value to substitute for *x* and which for *y*. • Students having difficulty with #13 should review Example #3 and their response to the Your Turn. • Some students may find the diagram overwhelming in #14. Assure them that the diagram shows multiple straight lines. Have students use a ruler to help them focus on one line at a time. Remind them that it is easier to determine the slope when they select ordered pairs that do not fall between two integer values. Choose "nice" points.
Unit 3 Project If students complete #18, which is related to the Unit 3 project, take the opportunity to assess how their understanding of chapter outcomes is progressing.	• Question #18 is modelled after Example 4. Students could refer to the example and pattern their work after it. In part b), encourage and listen for the approaches students use. Encourage them to describe the one that works best for them. • Part c) can be approached in several ways. Encourage students to explain or describe their strategy and why. • Part f) will require students to interpret a graph. Before they respond, have them verbally estimate from the values in the table; for example, what altitude is likely to have a temperature of –20 °C? Visual learners should verify their estimates from their graph. Challenge students to answer this using more than one strategy. • You may wish to provide students with **BLM 7–4 Chapter 7 Unit 3 Project**, and have them finalize their answers.
Assessment *as* Learning	
Create Connections Have all students complete #25 and 26.	• Questions #24 and 25 will provide important feedback for both you and students in determining their understanding of solving for slope and *y*-intercepts using more than one method. These questions would be an excellent summary to include in the student Foldable. • For #26, partner students of like ability. Encourage students to write out all the similarities and differences they see in each group of equations before they complete their graphs. Have students do the same after they graph a family. Ask how the answers and graphs compare. For students who need more prompting, ask them to create a chart for each family and identify the slope and *y*-intercept for a family before they graph it. Ask how they can use these patterns to explain the graphs.