# 7.2 General Form

***Mathematics 10*, pages 357–369**

**Suggested Timing**

80–120 min

**Materials**

- bottle of water
- stopwatch
- grid paper and ruler, or graphing technology

**Blackline Masters**

BLM 7–3 Chapter 7 Warm-Up
BLM 7–7 Section 7.2 Extra Practice

**Mathematical Processes**

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
Technology (T)
✓ Visualization (V)

**Specific Outcomes**

**RF1** Interpret and explain the relationships among data, graphs and situations.

**RF3** Demonstrate an understanding of slope with respect to:

- rise and run
- line segments and lines
- rate of change
- parallel lines
- perpendicular lines.

**RF5** Determine the characteristics of the graphs of linear relations, including the:

- intercepts
- slope
- domain
- range.

**RF6** Relate linear relations expressed in:

- slope-intercept form ($y = mx + b$)
- general form ($Ax + By + C = 0$)
- slope-point form ($y – y_1 = m(x – x_1)$) to their graphs.

**RF7** Determine the equation of a linear relation, given:

- a graph
- a point and the slope
- two points
- a point and the equation of a parallel or perpendicular line

to solve problems.

| Category | Question Numbers |
|---|---|
| Essential (minimum questions to cover the outcomes) | #2, 3, 5, 6, 8, 10, 18, 19, 21 |
| Typical | #1–7, 10 or 11, 12 or 13, 15, 18, 19, 21 |
| Extension/Enrichment | #1, 4, 14–17, 20, 21 |

## Planning Notes

Have students complete the warm-up questions on **BLM 7–3 Chapter 7 Warm-Up** to reinforce prerequisite skills needed for this section.

Make sure students understand that the slope-intercept form of an equation refers to the slope and the *y*-intercept. If you get students to focus on the "$y$ =" part of the slope-intercept equation, then they will make the connection that the only equation that cannot be written in this form is an "$x$ =" equation. This will help students answer the question in the opener and understand why vertical lines cannot be written in slope-intercept form. You may wish to specifically discuss the equations of the lines shown on the Canadian flag. Use the following questions to encourage discussion:

- Does every line have a *y*-intercept?
- Does every line have a slope? If not, what type of line has a slope that cannot be expressed as a number? Why?
- What type of line is not a function? Why?

## Investigate Intercepts and General Form

Do not allow students to consume more than 1 L of water. This needs to be emphasized to students for their own health and safety, along with the fact that the activity is not a drinking race. The goal is to drink at a constant rate. Ensure that students are allowed adequate bathroom breaks following this investigation. This investigation provides a great opportunity to talk about independent and dependent variables. To help students identify the independent and dependent variables, you may wish to ask the following questions:

- Does the volume of water in the glass depend on the time that passes?
- Does the time that passes depend on volume of water in the glass?

For #1, allow students to express the domain and range in any form they choose, i.e., words, number lines, interval notation, lists, or set notation. To help students work with domain and range, you could ask the following questions:

- Are the data a set of individual points? (The domain will be the $x$-coordinates of those points, and the range will be the $y$-coordinates of those points.)
- Is there any restriction on the variable; that is, are there values the variable cannot equal? (The domain or range will be anything but those values, i.e., $x \neq \#$ or $y \neq \#$.)
- Is there a limitation on the variable; that is, must the variable be between two values? (The domain or range will be an interval, for example, $\# < x < \#$ or $\# < y < \#$.)
- Is there no limitation or restriction on the variable; that is, can the variable can be any value? (The domain or range will be all real numbers, i.e., $x \in$ R or $y \in$ R.)

In #5, reinforce to students that $x$-intercepts and $y$-intercepts are the values of the $x$-coordinate or $y$-coordinate and that they can be represented as coordinate pairs: $(x, 0)$ and $(0, y)$.

For #6, each student can drink a bottle of water or you can ask for volunteers. Use a stopwatch to measure how long it takes to drink the bottle of water at a constant rate. Have students sketch the graph using the capacity of the bottle as the vertical intercept and the time to drink the water as the horizontal intercept. Use students' graphs for a classroom discussion.

To help students find the $x$-intercepts and $y$-intercepts in #7, ask them to focus on what they are trying to find. At the point where a line intersects an axis, the coordinate of the other axis is 0. For example, to find the $x$-intercept for the equation $6x - 9y + 18 = 0$, they need to find the value of $x$ at the $x$-axis. At the $x$-axis, the value of $y$ is 0, so they can rewrite the equation as $6x + 18 = 0$ and solve for $x$.

Similarly, to find the $y$-intercept for the equation $6x - 9y + 18 = 0$, they need to find the value of $y$ at the $y$-axis. At the $y$-axis, $x = 0$, so the equation can be rewritten as $-9y + 18 = 0$ and solved for $y$.

## Meeting Student Needs

- Discuss the student learning outcomes related to this section.
- Allow students enough time to carry out the investigation and to write their strategies for finding the intercepts of a line. Alternatively, you may wish to have students work with a partner or in small groups.

### ELL

- Students may need to be reminded that the *whole numbers* are 0, 1, 2, 3, ….

## Common Errors

- Students may not follow the conventions for writing equations in general form.

$R_x$ Reinforce to students that the $x$-term is written on the left, the $y$-term is next, and then the constant comes last. The right side of the equation is 0. You may wish to discuss the benefits of having the $x$-term be a whole number.

- Students sometimes graph the dependent variable on the horizontal axis and the independent variable on the vertical axis.

$R_x$ Work closely with students, helping them properly identify and graph each variable.

## Answers

**Investigate Intercepts and General Form**

**1.** Domain: $\{x \mid 10 \le x \le 12, x \in \text{R}\}$,
Range: $\{y \mid 10 \le y \le 600, y \in \text{R}\}$

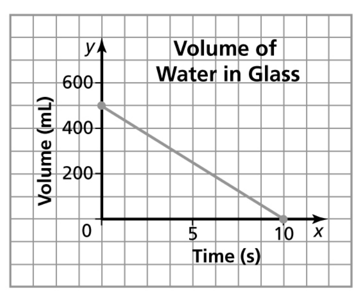
**2.** Slope = –50. The negative slope means that the volume of water decreases over time. The volume of water decreases by 50 mL every.

**3.** $y$-intercept: (0, 600). It represents how much water was in the glass before Leora started drinking.

**4. a)** $y = -50x + 600$ **b)** $50x + y - 600 = 0$

**5. a)** $x$-intercept: (12, 0).
**b)** $x$-intercept: (12, 0); $y$-intercept: (0, 600).

**6. a)** Example:




**b)** Example: Leora drank more. Leora's graph starts at 600 and mine starts at 500. I finished first in 10 s. We drank at the same rate of 50 mL/s.

**c)** Slope-intercept form: $y = -50x + 500$;
General form: $50x + y - 500 = 0$.

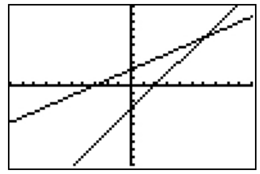
## Answers

**7. a)** yes

**b)** 0, 0. Example: I can substitute 0 for one intercept and then solve the formula to calculate the value of the other intercept.

**c)** Example: To determine the $x$-intercept, set $y = 0$ and solve for $x$. To determine the $y$-intercept, set $x = 0$ and solve for $y$.

**d)** Example: $3x - 2y - 6 = 0$, $x$-intercept: (2, 0), $y$-intercept: (0, –3) and $2x - 3y + 6 = 0$, $x$-intercept: (–2, 0), $y$-intercept: (0, –2)



**8.** General form. Example: No, $y$ cannot be isolated.

| Assessment | Supporting Learning |
|---|---|
| **Assessment *as* Learning** | |
| **Reflect and Respond**<br>Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings. | • Some students will choose to use the slope-intercept form to identify the *y*-intercept, as they are familiar with the format. If this is the case, reinforce the process by asking the following questions:<br>– What is the value of *x* at the *y*-intercept?<br>– What is the value of *y* when the line intersects the *x*-axis?<br>– How would you use the slope-intercept form to find both *A* and the *y*-intercept?<br>• As students improve at identifying the *x*-intercept and *y*-intercept from the slope-intercept form, point out that they could save some time by using the general form. Complete an example with them where one equation is in general form and beside it is the same equation in slope-intercept form. Solve for the intercepts in both equations and have students compare the steps required, watching for similarities. It is important that students find a strategy that is clear for them but equally important that they understand other approaches. |

## Link the Ideas

As you read through the Link the Ideas, you may wish to ask students the following questions based on the four equations.

Equation 1: $0.5x + y - 3 = 0$
Equation 2: $2y = -x + 6$
Equation 3: $x + 2y - 6 = 0$
Equation 4: $-x - 2y + 6 = 0$

- Which equations are equivalent to $y = -0.5x + 3$?
- Which equations are expressed in general form?
- Which general form equation is recommended, by convention?

You may wish to mention that *standard form* is a slight variation of general form. The *standard form* equation of a line is $Ax + By = C$, where $A$, $B$, and $C$ are real numbers, and $A$ and $B$ cannot both be zero. By convention, $A$ is a whole number. Sometimes students will be given an equation in standard form.

### Example 1

In this example, rather than using the distributive property and multiplying each side by 3, encourage students to multiply each term by three. Students may forget to multiply the constant by 6 if they use the distributive property.

As a warm-up activity, you may want to start with simpler equations like the ones below.

Express each equation in general form.

$y = 3x$
$y = 2x - 5$
$y = -6x + 7$

Then, you could challenge academically capable students with the following question:

Express the equation $y = \frac{3}{2}x - \frac{4}{5}$ in general form.

### Example 2

You may wish to encourage students to use mental mathematics by modelling the following strategy while working through parts a) and b):

- To find the $x$-intercept, put your finger over the $y$-term, and with what remains visible, solve for $x$. This works because $y = 0$ at the $x$-axis.
- To find the $y$-intercept, put your finger over the $x$-term. Solve for $x$ with what remains visible. This works because $x = 0$ at the $y$-axis.

Throughout this chapter, students solve many equations. You may wish to have students recall some strategies for isolating a variable and avoiding complicated steps, such as multiplying by a constant to eliminate fractions. In part b), the $y$-term was moved to the right side to eliminate a negative coefficient for $y$. Students could have chosen to divide each term by –3 in the third line of the solution.

Consider showing students how to use a calculator to graphically find the intercepts. Remind students to label the $x$-intercepts and $y$-intercepts on their graph.

Part c) suggests that students consider another method of graphing the equation. Discuss ways that students could check their work, perhaps by converting the equation to slope-intercept form and comparing the direction of the slope and the $y$-intercept with their graph. Students could even count the rise and run of their graph if they wish to check the slope, which is easily read from an equation in slope-intercept form.

## Example 3

This example explores the special case of the general form involving vertical and horizontal lines. You may wish to review domain and range, including the various ways of expressing domain and range, before working through this example.

Present students with the following six equations and have students sort the equations into vertical lines, horizontal lines, and oblique lines, that is, neither vertical nor horizontal.

Equation 1: $y = x + 6$ Equation 2: $x = 5$
Equation 3: $y = 0$ Equation 4: $x - y = 0$
Equation 5: $x + 6 = 0$ Equation 6: $y - 5 = 0$

Ask students to explain how they categorized the lines, giving as many different reasons as possible. For each relation, have students sketch the graph, identify the intercepts, and state the domain and range.

## Example 4

This example allows students to use the skills they have learned in this section in a real-life application. Make sure students understand how to develop the equation in part a) and how it relates to Spencer's disk space on his laptop. You may wish to point out to students that they could divide each term in the original equation $1.1T + 4.4M = 66$ by 1.1, which would provide an alternative way to reduce it to lowest terms with a whole-number coefficient for $T$.

Give students time to process the terminology of the $T$-intercepts and $M$-intercepts and what the variables represent in the context.

## Key Ideas

Reinforce to students that both the general form and the slope-intercept form allow them to graph a linear relation using different methods. Make sure students feel comfortable using both methods of graphing and encourage them to check their work using each method.

The table shows examples where an equation can have two intercepts or one intercept ($x$ or $y$). Ask students whether other options are possible and why. For example, can a linear equation have no intercepts? an infinite number of intercepts?

### *Meeting Student Needs*

- Post the general form of a linear equation on the board. Emphasize that $A$, $B$, and $C$ are real numbers, $A$ is generally a whole number, and $A$ and $B$ are not both zero.
- For Example 2, allow students to use a small table of values to determine the $x$-intercept and $y$-intercept, similar to the following:

| $x$ | $y$ | |
|---|---|---|
| | 0 | (#, 0) |
| 0 | | (0, #) |

- Create an exit slip for the end of this section that has students identify three to five key ideas learned. Vocabulary may also be included. Create a spot where students can assess their own learning by circling a thumbs up/thumbs down or smiley face/sad face.

### ELL

- Students may not be familiar with the unit GB (gigabyte). Explain to them that it is roughly equal to one billion bytes, and that the byte is a unit that measures the storage capacity of a computer system.
- You may wish to discuss the term *infinite*. Some students have trouble grasping this concept. It will be revisited in section 8.3.

### Enrichment

- Ask students to predict the following outcomes:
  – Suppose the value of $B$ in the general form of an equation is doubled. How are the slope and $y$-intercept of the line affected? (They are both halved.)

– If the value of $A$ in the general form is doubled, what happens to the slope? (The slope is doubled.)
– If the value of $C$ is doubled, what happens to the slope? (no change to the slope)

### Gifted

- Challenge students to assess the circumstances in the enrichment question above, and write a generalized statement about the effect of changing the general forms affect on slope of a line.

### *Common Errors*

- Students may confuse domain and range.

**$R_x$** You may wish to remind students to think alphabetically: d (domain) comes before r (range) and $x$ comes before $y$. So, domain contains $x$-values and range contains $y$-values.

**Web Link**

For information about computer memory as referred to in Example 4, go to www.mhrmath10.ca and follow the links.

## Answers

**Example 1: Your Turn**

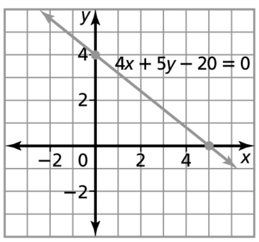
$3x - 4y - 8 = 0$

**Example 2: Your Turn**

**a)** $x$-intercept: (5, 0)

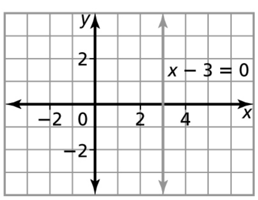
**b)** $y$-intercept: (0, 4)

**c)**

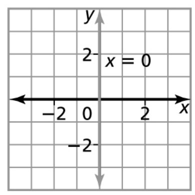



**Example 3: Your Turn**

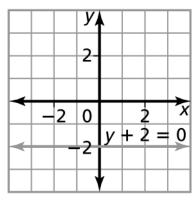
**a)** $x$-intercept: (3, 0). There is no $y$-intercept. Domain: {3}, Range: $\{y \in R\}$




**b)** $x$-intercept: (0, 0). There are an infinite numbers of y-intercepts. Domain: {0}, Range: $\{y \in R\}$




**c)** There is no $x$-intercept. $y$-intercept: (0, –2) Domain: $\{x \in R\}$, Range: {–2}




**Example 4: Your Turn**

**a)** $12S + 16T = 336$

**b)** $S$-intercept: (28, 0). If Brooke does not work as a tutor, she needs to work 28 h as a snowboard instructor.

**c)** $T$-intercept: (0, 21). Brooke needs to work 21 h as a tutor if she does not work as a snowboard instructor.

**d)** 15 h

| Assessment | Supporting Learning |
|---|---|
| **Assessment *for* Learning** | |
| **Example 1**<br>Have students do the Your Turn related to Example 1. | • Encourage students to verbalize their thinking.<br>• You may wish to have students work with a partner.<br>• Some students have difficulty with the distributive property and multiply only the terms containing a variable, forgetting the constant term. It may benefit students to visually see each step and multiply each term. For example,<br>$y = \frac{3}{4}x - 2$<br>$\frac{y}{1} = \frac{3x}{4} - \frac{2}{1}$<br>The common denominator is still 4.<br>$4\left(\frac{y}{1}\right) = 4\left(\frac{3x}{4}\right) - 4\left(\frac{2}{1}\right)$ |

| Assessment | Supporting Learning |
|---|---|
| **Assessment *for* Learning** | |
| **Example 2**<br>Have students do the Your Turn related to Example 2. | • Encourage students to verbalize their thinking.<br>• You may wish to have students work with a partner.<br>• Success with Example 2 is directly related to students' understanding of the process in Example 1. If students are still struggling, an alternative approach would be to have students enter the equation into their calculator and teach them how to locate the intercepts from a table of values generated by the technology. Students must, however, understand and be able to relate what the meaning of the intercepts is.<br>• Most students find substituting in the value of zero for $x$ and $y$ relatively easy. They do, however, confuse what value is evaluated at zero. Many students will want to substitute the value of zero into the variable they are trying to solve for. For example, to find the $x$-intercept, they will substitute zero in for x. Remind students about working with opposites. Show them that if they substitute 0 for x when finding the $x$-intercept, they will end up with an equation and value in the form of $y$ = #, not $x$ = #. |
| **Example 3**<br>Have students do the Your Turn related to Example 3. | • Encourage students to verbalize their thinking.<br>• You may wish to have students work with a partner.<br>• Students have difficulty with vertical and horizontal lines. Remind them to locate the point on the axis in the equation. For example, if $x = 4$, locate 4 on the $x$-axis. Where else do ordered pairs have an $x$-value of 4? Plot some of the points. This process will assist in providing a visual that helps students link the $x$ = value to a vertical line. The same holds true for horizontal lines. |
| **Example 4**<br>Have students do the Your Turn related to Example 4. | • Encourage students to verbalize their thinking.<br>• You may wish to have students work with a partner.<br>• It may benefit students to discuss what variables might be appropriate for the equation. Discuss which axis each will go on.<br>• If students are having difficulty determining the intercepts because the variables are no longer $x$ and $y$, point out what the axis is labelled where it is typically "$x$". Do the same for the vertical axis. Ask what the ordered pairs look like ($S$, $T$).<br>• Some students may have difficulty with part d), not recognizing what the operation is. Prompt students by asking what the variable is for snowboarding and ask what variable they wish to solve for. |

## Check Your Understanding

### Practise

You may wish to have students complete #6 with a partner or in a group because this question involves several tasks.

### Apply

For #7, some students may need assistance working through the question. You may wish to help students with the equations of vertical and horizontal lines. Vertical lines run up and down and pass through the $x$-axis. Therefore, the equations are in the form $x = \ldots$. Horizontal lines run left and right and pass through the $y$-axis. Therefore, they are in the form $y = \ldots$.

Question #10 is similar to the Investigate activity. If students need help with this question, you may refer them to their answers to the Investigate questions.

In #12, students work with density, mass, and volume. Ensure that students have set up the equation correctly before they complete parts b) and c). Encourage students to inspect the units in their equations as a way to check their equation. If the equation is correct, the units on the right and left sides of the equal sign will be the same.

### Create Connections

These questions could be done after the Practise questions.

Question #21 is a Mini Lab and provides an opportunity for students to discover relationships among the parameters in an equation and the corresponding graph. Students can use technology to graph the lines or sketch the lines by hand using intercepts. Students should be prepared to share their conclusions.

## *Meeting Student Needs*

- Provide **BLM 7–7 Section 7.2 Extra Practice** to students who would benefit from more practice.
- Post both forms of linear equations studied so far: slope-intercept form and general form. You may wish to identify equations of horizontal lines ($y = \#$) and vertical lines ($x = \#$).
- Allow students to work in pairs or small groups to complete #7–14.
- Have each student complete #18; then, discuss as an entire class.
- Have students complete #21 in small groups and compare answers.
- Allow students to determine the percent in #12c) using any method they wish.
- Some students may benefit from using money manipulatives for #13, as well as manipulatives representing the tickets.
- Encourage students to use coloured pencils when drawing the triangles in #17 and to clearly label their graphs.
- In #19b), students may come up with one method of determining the $y$-intercept, but they may not be able to think of another method. Encourage students to look back on their work in this chapter and to review what they have learned about finding $y$-intercepts.

### ELL

- You may wish to explain the two swimming styles mentioned in the table in #11, *backstroke* and *butterfly*.
- In #12, some students may not be familiar with the terms *traction*, *density*, *volume*, and *mass*. Explain these terms. Students will likely encounter the three measurements in science class, and they may even discuss *traction*, but it will be called *friction*.
- For #20, students may need to be reminded that *oblique* means slanting.

## *Common Errors*

- For #3g), students may recognize that the line $y = 0$ intercepts the $y$-axis at 0, but they may neglect to consider where this line intercepts the $x$-axis.

$R_x$ Encourage students to think of each axis separately when determining the intercepts.

- For #5 and 8, students may confuse whether a linear equation has an intercept of zero or no intercept with a particular axis.

$R_x$ Review and clarify these two situations with students. Have them come up with an example of each type of line. Ask them to graph the line and determine an equation of the line.

### Web Link

For information on different types of exercise and the calories burned as discussed in #11, go to www.mhrmath10.ca and follow the links.

| Assessment | Supporting Learning |
|---|---|
| **Assessment *for* Learning** | |
| **Practise and Apply**<br>Have students do #2, 3, 5, 6, 8, and 10. Students who have no problems with these questions can go on to the remaining questions. | • Encourage students to work with a partner.<br>• Remind students that writing the general form in #2 requires them to work with opposite operations. You may need to review the characteristics of general form and how to remove fractions from an equation. Suggest that students complete one of each section on each question and have a partner check their work before completing all the assigned problems.<br>• Some students may be confused by the many lines in #6. Have them make a list in their books of the equation numbers 1 to 8 and identify the intercepts for each equation. Suggest that below the list, they determine the $x$-intercepts and $y$-intercepts of the given equations. Use information from both to match the graphs.<br>• Students having difficulty with #8 should be prompted to describe a line that does not intersect the $y$-axis. Ask what type of line it is. Encourage students to sketch a graph with the point (3, 6) on it. Ask students how they could sketch the line so it did not touch the $y$-axis.<br>• For #10, encourage students to write the intercepts as ordered pairs, as this could then be used to find the slope in part c). Prompt students to identify what their preferred form is for writing the equation of a line. Ask them to identify what information they have that will be used (slope and intercepts). |

| Assessment | Supporting Learning |
|---|---|
| **Assessment *as* Learning** | |
| **Create Connections**<br>Have all students complete #18, 19, and 21. | • Students capable of finishing the recommended questions with few supports should attempt #20.<br>• Question #18 provides an excellent Assessment *as* Learning question that could be placed into their Foldable.<br>• For #19, prompt students to think of opposites. If we are looking for the *x*-intercept, what must be zero? Some students may prefer to graph the equation and read off values from the graph. Encourage whatever strategy is easier for them to remember.<br>• Question #21 is an extension of the idea of families of lines that was started in the previous section. Encourage students to label information for each grouping before they graph the lines. A suggested list of questions to refer to might include the following:<br>– When the equations are written in slope-intercept form, what is the same? What is different?<br>– When the equations are written in general form, what is the same? Different?<br>– When the equations are graphed, what is the same and what is different? |