7.3 Slope-Point Form

***Mathematics 10*, pages 370–382**

Suggested Timing

80–120 min

Materials

- grid paper
- ruler
- SI measuring tape

Blackline Masters

BLM 7–3 Chapter 7 Warm-Up
BLM 7–4 Chapter 7 Unit 3 Project
BLM 7–8 7.3 Investigate: Figure 1
BLM 7–9 Section 7.3 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcomes

RF6 Relate linear relations expressed in:

- slope-intercept form ($y = mx + b$)
- general form ($Ax + By + C = 0$)
- slope-point form ($y - y_1 = m(x - x_1)$) to their graphs.

RF7 Determine the equation of a linear relation, given:

- a graph
- a point and the slope
- two points
- a point and the equation of a parallel or perpendicular line

to solve problems.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–3, 5a), b), 6a), c), d), 7–9, 11, 21–24
Typical	#1–5, 7, 8, 10, 11, 14, 24
Extension/Enrichment	#12, 13, 15–20, 22, 24

Unit Project Note that #24 is a Unit 3 project Mini Lab.

Planning Notes

Have students complete the warm-up questions on **BLM 7–3 Chapter 7 Warm-Up** to reinforce material learned in previous sections.

In this section of the chapter, students are introduced to a third form for writing an equation of a line. They will discover that this new form can only be used for non-vertical lines.

Present students with the following three equations: $y - 3 = 2(x - 4)$, $2x - y - 5 = 0$, and $y = 2x - 5$. Ask students to identify the equation that is written in slope-intercept form. Next, have them identify the equation that is in general form. The third equation is in slope-point form. Ask students to show that all three equations are equivalent.

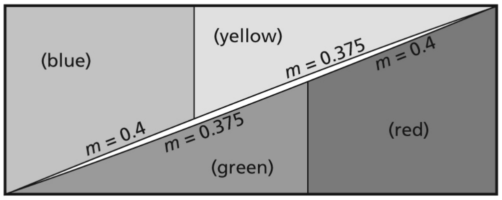
Investigate Equations in Slope-Point Form

In this Investigate, students discover that any two points on a line can be used to write the equation of the line. This fact is then generalized and written as the slope-point form of a linear equation.

In #3, students may need assistance with determining the discrepancy between the areas. As a hint, suggest that students determine if the path from point E to G in Figure 2 is a straight line. They should justify their answer.

It may be useful for students to use **BLM 7–8 7.3 Investigate: Figure 1** to explore what is happening here. This BLM provides an enlarged version of Figure 1 on page 370 in the textbook. If students cut out this figure and carefully put it together, they may see that there is an empty area between the two sets of polygons.

The figure below demonstrates why this occurs. The yellow and green shapes from Figure 1 have a rise of 3 and a run of 8. This gives a slope of $\frac{3}{8}$ or 0.375. The triangles on the blue and red shapes have a rise of 2 and a run of 5. This gives a slope of $\frac{2}{5}$ or 0.4. Since these two slopes are very close, the difference will not be evident on a small version of the visual. On a larger version, however, the difference will be clear. The larger you make the visual, the clearer the difference will be.



In #4, students should start with points that produce a line with an integral y-intercept and slope. For academically capable students, ask them how they could find the equation of the line when the y-intercept is not integral. For example, find the equation of a line with a slope of $\frac{1}{2}$ .passing through (3, 5).

Students may need assistance with #5. Using their graphs as a guide, ask students how they could numerically find the y-intercept of a line passing through a given point with a given slope. For example, state the equation of a line with a slope of 2 passing through (3, 10). Then, have a discussion similar to the following:

- To move from point (3, 10) to the y-intercept, the run would be –3. Using the slope relationship $2 = \frac{\text{rise}}{-3}$, the corresponding rise would be 2(–3).

Hence, the y-intercept is 10 + 2(–3), or 4. This leads to the generalization that a line with slope m passing through point (x_1, y_1) has a y-intercept of $y_1 + m(-x_1)$. Hence, the equation of the line in slope-intercept form is $y = mx + (y_1 + m(-x_1))$. This simplifies into slope-point form:

$$y = mx + (y_1 + m(-x_1))$$
$$y = mx - mx_1 + y_1$$
$$y - y_1 = m(x - x_1)$$

For #7, have students develop this explanation in their own words. You may wish to have partners explain this to each other.

In #8, you could ask students to explain why the equation is for a non-vertical line. Why is it not possible to represent a vertical line in this way?

Meeting Student Needs

- Discuss the student learning outcomes for this section. Post them in the classroom.
- Post the three forms that can be used to represent a linear equation. Explain that students will focus on the third form, slope-point form. It is to be used when given the slope of the line and a point on the line.
- Give students an enlarged photocopy of Figure 1. Have them cut the square into the four polygons and assemble the parts into a rectangle by placing them on grid paper. What do they notice?
- Allow students to work in pairs on the Investigate.

Answers

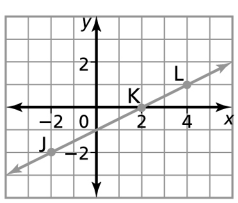
Investigate Equations in Slope-Point Form

1. 64 square units

2. 65 square units

3. Example: The second figure is not a rectangle. “Diagonal” EG is not a straight line.

4. Example:



slope = $\frac{1}{2}$

a) $y = \frac{1}{2}x - 1$

b) $y = \frac{1}{2}x - 1$, $y = \frac{1}{2}x - 1$; The equations are the same.

c) Example: If you know the slope and the coordinates of one point, you can use those values in $y = mx + b$ to find the value of b and the equation of the line.

5. $y - y_1 = m(x - x_1)$

7. Encourage students to develop and share their own strategies. Discuss these strategies as a class before proceeding to #8.

8. Example: Substitute x for x_2 and y for y_2 and multiply both sides by $(x - x_1)$.

9. Vertical line; the slope is undefined. $x_2 - x_1 = 0$

10. Example: Always true. The slope of a line can be determined by any two points on the line.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• If students are having difficulty getting started with #7, write the slope-intercept form on the board and ask how the look of it might be changed using the slope formula. • For #8, suggest students sketch a vertical and a horizontal line anywhere on a coordinate grid. Ask them to determine the equation for each of their lines. This should reactivate previous learning and provide the support to answer the question. • You may wish to complete #9 as a class. Perhaps sketch a parabola on the board and label the points (0, 0), (1, 1), and (2, 4). Ask students to use these to respond to the question.

Link the Ideas

Slope-point form is best expressed as

$y - y_1 = m(x - x_1)$ and not as $\frac{y - y_1}{x - x_1} = m$. That way all points on the line, including the point (x_1, y_1), satisfy the equation of the line, $y - y_1 = m(x - x_1)$.

Students need to realize that x_1 and y_1 represent the coordinates of a specific point on the line, whereas x and y represent the coordinates of any point on the line. You may wish to ask the following questions:

- In the formula $y - y_1 = m(x - x_1)$, what do the coordinates (x_1, y_1) represent?
- What do the coordinates (x, y) represent?

Example 1

In this example, students will be asked for the equation in slope-point form and slope-intercept form. Then, students use technology to graph their equation. Encourage students to consider looking for ways to verify that their answer is correct. Refer students to the Mental Math and Estimation box beside the solution to part b) for one way to check part of an answer.

After completing the example, have students work through the Your Turn.

Example 2

Once the slope is determined, the coordinates of any point can be used to find the equation of a line in slope-point form. Calculations are sometimes easier for one point than the other. In part a), you may wish to ask the following questions:

- Do you prefer using one point over the other?
- How could you check your equation?

Have students use the coordinates of the point they did not use to check their equation. Students could also use technology to check their equation.

When going over the procedure to determine the equation in general form for part c), you could show students that multiplying each side by 2 is equivalent to multiplying each term by 2. This might eliminate some errors.

Example 3

This example allows students to use the skills they have learned in a real-life application. Ask students who have done mountain climbing to share their experience.

Note that the sample solution uses the y-intercept as the point to substitute. This often results in the most efficient solutions.

In part a), emphasize that d is the distance from the *top* of the first pitch. The slope is negative since the distance to the *top* of the pitch is decreasing. A decrease suggests a negative value. Similarly, an increase would suggest a positive value.

To help students visualize the situation and show that they understand the problem, you may wish to have them sketch and label a diagram showing the heights of the pitches and Brad's height at each of the times given in the problem.

In part a) of the solution, you may wish to discuss how the slope formula is related to the given formula, $m = \frac{d_2 - d_1}{t_2 - t_1}$. Encourage students to discuss how they need to consider how far the climber has moved from the second distance to the first distance. Some students may find this confusing because Brad is moving up the slope; however, the numbers are given from the top of the slope so are getting smaller. So, when Brad moves from 60 m down to 40 m down, he has actually moved 20+ m. That move took him ten minutes, which is a negative number because it refers to time elapsed.

Have students complete the Your Turn question.

Key Ideas

Have students summarize what they have learned. Ensure that students understand why a vertical line cannot be written in slope-point form. Emphasize that any point on a line can be used to determine the equation of a line in slope-point form if you know the slope. A second point on the line can be used to check the equation. Reinforce to students that they only need two points to graph a linear equation. They can use the points to find the slope and then use the slope-point form with either of the points given.

Meeting Student Needs

- Establish which form you want students to use when writing an equation, if it is not stated in the question. Students should also be given opportunities to choose the form they write the equation in.
- Create two posters on chart paper illustrating Examples 1 and 2. Ask students to compare and contrast the two examples.
- To help students understand the benefit of writing an equation in slope-point form, explain that this form requires less work and fewer calculations. That is because students will only need to determine the slope of the line and any point on the line, not a specific point, such as the y-intercept, as required for the slope-intercept form.

Enrichment

- Ask students to write in slope-intercept form four lines whose internal area is 100. For example,
  $y = \frac{4}{3}x + 0$, $y = \frac{4}{3}x + 8$, $y = -\frac{4}{3}x + 8$,
  $y = -\frac{4}{3}x + 33$

Gifted

- The enrichment question above cannot be done for a number of real shapes created by four lines and having an internal area of 100. Challenge students to investigate what the common characteristic of these shapes is. (Any shape with a vertical line on the x-y coordinate grid cannot be written in slope-intercept form because the slope of a vertical line is undefined.)

Common Errors

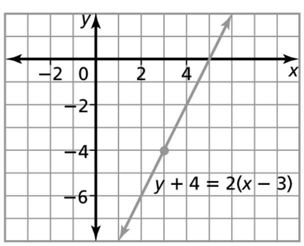
- Students may not apply the distributive property correctly. For example, to expand $y + 4 = \frac{3}{2}x - 3$, some students will write
  $2(y + 4) = 2\left(\frac{3}{2}x - 3\right)$
  $2y + 4 \neq 3x - 3$

R$_x$ Show students that what you do to one *term* you do to all the others.

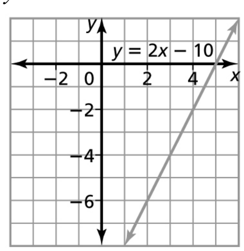
Answers

Example 1: Your Turn

a) $y + 4 = 2(x - 3)$



b) $y = 2x - 10$



c) The graphs are identical.

Example 2: Your Turn

Example: First, find the slope, m. Then, substitute the value of m and the coordinates of one point into the slope-point form of the equation. $x + 3y - 1 = 0$

Example 3: Your Turn

a) $d = -90t + 540$ **b)** 3 p.m.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Students may benefit from starting a list of formulas in their Foldable or notes with the title beside them. Have students verbalize which value in the ordered pair represents *x* and which represents *y*, and where they will substitute each in. • Remind students of the distributive property that requires that they multiply each term in the brackets. • You may wish to have students graph their line by hand first and then verify it using technology.
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to write out the ordered pairs and label them below each coordinate as x_1, y_1 and x_2, y_2. It may help organize their thinking when substituting into the slope formula. • Some students might be challenged by suggesting that there is a two-point formula, and it appears as $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$. Ask students to determine whether it is correct and whether it will work each time they encounter two points.
Example 3 Have students do the Your Turn related to Example 3.	• Students who have difficulty determining the equation for the context may need prompting to clarify the meaning of certain values. You may wish to review Example 3 and ask what the initial distance was (60 m) at time 0 min. Ask what the initial distance is in the Your Turn (540 km). Go back to the example and ask what the finishing distance was (40 km in the example and 315 km in the Your Turn). Help students to identify where these fit into the slope formula. Repeat the same process for time and then allow students to find the slope.

Check Your Understanding

Practise

To save time, you may wish to ask students to express equations in slope-intercept form or general form, but not necessarily both. You can also let students choose the form of the equation.

Completing #4 will help students work through #8.

Question #5 prepares students for #6. In #5, students express the answer in slope-point form only. There are two possible answers for each graph. In #6, students express the equation in slope-intercept form and general form.

Question #7 gives students the opportunity to communicate and apply their understanding of linear equations. You may wish to discuss part a) as a class.

Apply

The strategies from #9 for determining the *y*-intercept of a line can be applied in #10.

In #11 and 12, students have the opportunity to apply skills from sections 7.1 and 7.2.

For #17 and 18, students must determine the equation of a line and then determine a particular characteristic of that line. Question #18 has students read information from a label on a bag of potatoes. You may wish to have a class discussion about the information provided on the label.

Create Connections

These questions could be done after the Practise questions.

For #22, students summarize their understanding of what information is needed to determine the equation of a line.

In #23, students use their creativity to illustrate their thought process for deciding on a form of a linear equation.

Unit Project

The Unit 3 project question, #24, provides an opportunity for students to apply their ability to determine the equation of a line to real data. Discuss with students the conditions under which the equation of their line would be useful in predicting a person's height from their bone remains. The equation would be useful if the person was of the same gender and similar ethnic background. It would not be useful for the remains of a young child nor of an elderly person. You may wish to discuss and compare the results for males and females.

Meeting Student Needs

- Provide **BLM 7–9 Section 7.3 Extra Practice** to students who would benefit from more practice.
- Before students begin, illustrate the three forms of a linear equation and emphasize the key components for each equation: m, b, A, B, C, y, y_1, x, and x_1.
- Students are given the opportunity to practise converting from one form to another. They should be aware of whether fractions or decimals are acceptable for certain situations, and when a value can be negative (e.g., in general form, A can never be negative by convention).
- Encourage students to write two equations for each graph in #5. Can they prove why each equation is correct?
- Allow students to use graphing calculators for all questions in this section.
- Questions #13, 14, 17, and 18 may be difficult for some students. Ensure they have a partner who will be able to assist them. Encourage a discussion of the question prior to writing the response.

WWW Web Link

To learn more about paleontology in Canada, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1–3, 5a), b), 6a), c), d), 7–9, and 11. Students who have no problems with these questions can go on to the remaining questions.	• Remind students about multiplying through brackets when using the distributive property. This is important for #1 and for subsequent questions. To help organize and compare, you may wish to suggest that students include columns on their page. • Coach students to verbalize how the labelled ordered pairs in #2 can be used and identified in the formula. Encourage them to use the dotted triangle to determine the vertical change first and then the horizontal. For #5, suggest students could either use the slope formula or dot in a triangle of their own to mark the moves from one point to another. • For #6, suggest that students label the points so they are not confused when substituting the values into the slope-point form. Visual students may prefer to graph the points to determine the slope and *y*-intercept first. • Students who are having difficulty with the slope-point form might benefit from completing the first two questions by writing the equations first in slope-intercept form. Coach students to see the link between the process in #2 and 8. • Once students have verified their answer for #9, encourage them to include it in their Foldable, as it identifies two personal strategies that students could use.
Unit 3 Project If students complete #24, which is related to the Unit 3 project, take the opportunity to assess how their understanding of chapter outcomes is progressing.	• You may wish to provide students with **BLM 7–4 Chapter 7 Unit 3 Project** and have them finalize their answers. • Students will enjoy the activity in the Mini Lab. Remind them that the information here can be useful for their final project. You may wish to make a large chart on the board and have student pairs indicate their results as they work through the Mini Lab.
Assessment *as* Learning	
Create Connections Have all students complete #21 and 22.	• Some students may benefit from working with a partner of like ability in completing #21 and 22. Have them discuss their responses. Both questions are excellent Assessment *as* Learning pieces and would benefit the students to include in their notes or Foldable.