7.4 Parallel and Perpendicular Lines

***Mathematics 10*, pages 383–395**

Suggested Timing

80–120 min

Materials

- grid paper
- scissors
- ruler

Blackline Masters

BLM 7–3 Chapter 7 Warm-Up
BLM 7–10 Section 7.4 Extra Practice

Mathematical Processes

✓ Communication (C)
✓ Connections (CN)
✓ Mental Math and Estimation (ME)
✓ Problem Solving (PS)
✓ Reasoning (R)
✓ Technology (T)
✓ Visualization (V)

Specific Outcomes

RF3 Demonstrate an understanding of slope with respect to:

- rise and run
- line segments and lines
- rate of change
- parallel lines
- perpendicular lines.

RF7 Determine the equation of a linear relation, given:

- a graph
- a point and the slope
- two points
- a point and the equation of a parallel or perpendicular line

to solve problems.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–5, 6a), c), e), 7a), c), e), 9–11, 25, 26
Typical	#1–5, 6a), c), e), 7a), c), e), 8, 11, 13, one of 14–16, 25, 26
Extension/Enrichment	#12, 14–19, 24–26

Planning Notes

Have students complete the warm-up questions on **BLM 7–3 Chapter 7 Warm-Up** to reinforce material learned in previous sections.

This section of the chapter concentrates on parallel and perpendicular lines. Students identify, write equations, and solve problems involving parallel and perpendicular lines. When discussing the section opener, you may wish to have students visualize that the gymnast's arms are perpendicular to the top of the balance beam. Have students identify objects in the classroom or on their person that are parallel or perpendicular to each other.

Investigate Slopes of Parallel and Perpendicular Lines

This Investigate uses the sides of squares and the diagonals of a rectangle to explore the slopes of parallel and perpendicular lines.

In #4, students will each have different rectangles. You may wish to circulate and check the appropriateness of their answers.

Step #5 enables students to see why the slopes of perpendicular lines are negative reciprocals. When the rectangle is rotated 90° about a vertex, the rise of the diagonal becomes the run, the run of the diagonal becomes the rise, and the orientation of the diagonal changes.

Meeting Student Needs

- You could lead a discussion on parallel and perpendicular lines. Have students point out locations in the room where they see parallel lines and then perpendicular lines.
- Based on previous knowledge, you may wish to determine what students know about the slope of vertical lines and the slope of horizontal lines. Discuss whether all vertical lines are parallel and whether all horizontal lines are parallel. Compare the slopes of horizontal lines and vertical lines.
- The Investigate lends itself to students who require hands-on activities. Through the investigation, students will come to a better understanding of parallel and perpendicular lines.

ELL

- Some students may need to be reminded of the meaning of *counterclockwise*.

Answers

Investigate Slopes of Parallel and Perpendicular Lines

2. C(1, 7) and D(–3, 4); $m_{AB} = \frac{3}{4}$, $m_{BC} = -\frac{4}{3}$, $m_{CD} = \frac{3}{4}$, $m_{AD} = -\frac{4}{3}$; Example: The slopes of opposite sides are equal, and the slopes of adjacent sides are negative reciprocals of each other.

3. C(7, 17) and D(–5,12); $m_{AB} = \frac{5}{12}$, $m_{BC} = -\frac{12}{5}$, $m_{CD} = \frac{5}{12}$, $m_{AD} = -\frac{12}{5}$; Example: The slopes of opposite sides are equal, and the slopes of adjacent sides are negative reciprocals of each other.

4. Example: A(3, 1), B(9, 1), C(9, 4) and D(3, 4); $m_{AB} = 0$, m_{BC} is undefined, $m_{CD} = 0$, m_{AD} is undefined; $m_{AB} = m_{CD}$ and $m_{BC} = m_{AD}$

5. Example: $m_1 = \frac{1}{2}$ and $m_2 = -2$; the two slopes are negative reciprocals of each other.

6. The slopes of parallel sides are equal. Example: $m_{AB} = m_{CD} = \frac{3}{4}$ in #2.

7. The slopes of perpendicular sides are negative reciprocals. Example: $m_{AB} \times m_{AD} = -1$.

8. a) The slopes are undefined.

b) The slope of a vertical line is undefined. The slope of a horizontal line is 0. The lines are perpendicular to each other.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• Before beginning #6, as a class discuss the coordinates of the vertices of the various shapes. You may wish to label them on the board. • Some students may benefit from having the slope formula placed on the board for the investigation. • Most students will determine that the slopes of parallel lines are the same; however, they may not realize that the slopes of perpendicular lines are negative reciprocals of each other. Many students will determine that they are reciprocals but miss the opposite sign. Point out to students that perpendicular slopes always have a product of –1. • Some students may need interpretation of the slopes of horizontal and vertical lines because they may have difficulty remembering which fractional value is zero and which is undefined. You may wish to use the analogy of an animal walking. A zero slope means there is no tilt, and that occurs with a horizontal line, as the *y*-values are identical. In a vertical line, it is impossible to tell whether the animal is going straight up or straight down. It can not be distinguished, therefore the slope is undefined.

Link the Ideas

The Link the Ideas section provides formal definitions for parallel lines and perpendicular lines. Reinforce that parallel lines do not intersect because they travel in the same direction (hence the equal slopes). The slopes of two non-vertical parallel lines, m_1 and m_2, are equal; that is, $m_1 = m_2$. Vertical lines are a special case. The slopes of two non-vertical perpendicular lines, m_1 and m_2, are negative reciprocals; that is, $m_1 = \frac{-1}{m_2}$ or $m_1 \times m_2 = -1$.

Vertical and horizontal lines are a special case.

Example 1

Example 1 demonstrates how to determine whether two lines are parallel, perpendicular, or neither. Prompt students' thinking with the following questions:

- When comparing two linear equations in slope-intercept form, $y = mx + b$, which parameter, m, b, or both m and b, determines whether the lines are parallel?
- When comparing two linear equations in slope-intercept form, $y = mx + b$, which parameter, m, b, or both m and b, determines whether the lines are perpendicular?
- When comparing two linear equations in slope-intercept form, $y = mx + b$, which parameter, m, b, or both m and b, determines if the lines are neither parallel nor perpendicular?

Example 2

In this example, students must determine the slope of a linear equation given in general form and then determine the equation of a line that is parallel and passes through a given point.

Make sure students understand why they are converting the original equation from general form to slope-intercept form. Once the equation is in slope-intercept form, students can find the slope of the line, thereby finding the slope of a line parallel to it.

In part a), two methods are shown for arriving at the equation in slope-intercept form. In Method 1, the slope-point form is converted to the slope-intercept form. In Method 2, the slope-intercept form is used directly. Academically strong students could be shown the following third method, which uses the general form and the point-on property.

The slope of the line $Ax + By + C = 0$ is represented by $-\frac{A}{B}$. The only values that affect the slope are A and B. Since the slopes of parallel lines are equal, an equation of a parallel line in general form can be obtained by using the same values for A and B as in the original line. The equation $2x - y + C = 0$ represents a line parallel to the original line, $2x - y + 4 = 0$.

To find C in $2x - y + C = 0$, substitute the coordinates of the point $(1, -6)$ for (x, y).

$$\begin{aligned} 2x - y + C &= 0 \\ 2(1) - 1(-6) + C &= 0 \\ 8 + C &= 0 \\ C &= -8 \end{aligned}$$

Therefore, the equation of the line in general form is $2x - y - 8 = 0$.

For part b), students could also use the slope-point form to determine the equation of the line in general form. The solution would be as follows:

$$\begin{aligned} y + 6 &= 2(x - 1) \\ y + 6 &= 2x - 2 \\ 0 &= 2x - 2 - y - 6 \\ 0 &= 2x - y - 8 \end{aligned}$$

The equation of the line in general form is $2x - y - 8 = 0$.

Part c) illustrates how to use technology to verify that the lines are parallel. Have students experiment with the graphing calculator they are using and make notes about how to perform this check.

Students can use the Try This to confirm that they understand how to use their graphing calculator to verify that lines are parallel.

Example 3

In this example, students must determine the slope of a linear equation in general form and then determine the equation of a line that is perpendicular and passes through a given point.

Students can confirm that the slopes are of perpendicular lines by multiplying the slopes to get -1.

You may wish to show students the following third method, which uses the general form and the point-on property.

The slope of the line $Ax + By + C = 0$ is represented by by $-\frac{A}{B}$. The only values that affect the slope are A and B. Since the slopes of perpendicular lines are negative reciprocals, an equation of a perpendicular line in general form can be obtained by switching the A and B coefficients and changing the sign of one of them. Hence, the equation $2x - 3y + C = 0$ represents a line perpendicular to the original line, $3x + 2y - 6 = 0$.

To find C in $2x - 3y + C = 0$, substitute the coordinates of the point $(9, 0)$ for (x, y).

$$\begin{aligned} 2x - 3y + C &= 0 \\ 2(9) - 3(0) + C &= 0 \\ 18 + C &= 0 \\ C &= -18 \end{aligned}$$

Therefore, the equation of the line in general form is $2x - 3y - 18 = 0$.

The equation of the line in slope-intercept form is $y = \frac{2}{3}x - 6$.

To answer the blue type near the bottom of the page, students could use the slope-point form of the equation and convert that equation to general form. The advantage to working with equations developed earlier in the solution is that there is less chance of incorporating an error than when working with a solution that has been manipulated many times.

Key Ideas

Have students summarize what they have learned. Ensure that students understand the properties of parallel and perpendicular lines.

Meeting Student Needs

- You may wish to post the information from the beginning of this section for students to use as a reference for the section. This information can be placed on a bookmark with other key information from the chapter, or it may be enlarged on a photocopier and posted at the front of the room for continual reference.
- Be sure students understand the concept of negative reciprocals (reciprocals that are opposite in sign; they have a product of –1).
- Allow students to use a graphing calculator for Example 1.
- You may wish to demonstrate that a line parallel to $2x + y + 1 = 0$ will be of the form $2x + y + C = 0$ (based on the fact that the slope $= -\frac{A}{B}$, so the values of A and B do not change). An extension for perpendicular lines can show that the slope of a line perpendicular to $Ax + By + C = 0$ will be $\frac{B}{A}$. Hence, the equation of a perpendicular line will be of the form $Bx - Ay + C = 0$.

Enrichment

- An old-fashioned analog clock has three hands, for hours, minutes, and seconds. How often in a 24-h day will the second hand be parallel to either of the other hands?

Gifted

- Challenge students to explain mathematically why parallel railway tracks appear to touch in images of such tracks as on the prairies. (Example: Suppose a big triangle represents the relationship between the actual image and the image that appears on the retina. One side is the distance to the tracks and a right angle between the two tracks. The farther the tracks are away, the lesser the angle from the eyes' perspective. The change in angle lessens the distance between far-away objects, such as the two sides of the tracks in the distance. Since the change in angle is linear, the lines still appear straight, but they converge.)

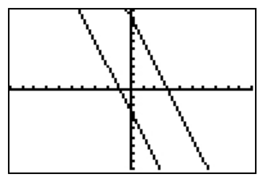
Answers

Example 1: Your Turn

a) neither **b)** parallel **c)** perpendicular

Example 2: Your Turn

$y = -3x + 9$, $3x + y - 9 = 0$



Example: The graphs are parallel lines with different *x*- and *y*-intercepts.

Example 3: Your Turn

$y = \frac{1}{4}x - 8$ or $x - 4y - 32 = 0$

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• You may wish to have students work in pairs. • Encourage students who are having difficulty to develop a chart that allows them to fill in the characteristics of parallel and perpendicular lines. Specifically, the column for slope and *y*-intercept would serve as a quick reference. • Remind students that the negative reciprocals' product is –1. • Ensure that students understand that the comparisons can only be made when the forms of the equations are identical.
Example 2 Have students do the Your Turn related to Example 2.	• You may wish to have students work in pairs. • Review with students the characteristics of parallel lines. Remind them that knowing the slope is the key as the tilts of the lines must be the same. • Encourage students to use their favourite personal strategy for determining the equation of a parallel line. Where possible, encourage them to try more than one method of explaining how a second approach could be completed.
Example 3 Have students do the Your Turn related to Example 3.	• You may wish to have students work in pairs. • Review with students the characteristics of perpendicular lines. Remind them that knowing the slope of the original allows them to determine the negative reciprocal of the value and find a perpendicular line. • Encourage students to use their favourite personal strategy for determining the equation of a perpendicular line. Where possible, encourage students to try more than one method of explaining how a second approach could be completed.

Check Your Understanding

Practise

For #1g), you may wish to suggest that students begin by writing 0 as a fraction, for example, $\frac{0}{1}$.

So, the reciprocal becomes $\frac{1}{0}$. Since division by 0 is not allowed, the slope must be undefined. A similar approach can be used for #1h).

For #2, you may wish to show students how to find the slope of an equation in general form. Please refer to the notes for Example 2.

Before students complete #4, you may want to present a simpler example, such as the following: The slopes of two lines are $\frac{2}{5}$ and $-\frac{6}{n}$. Find the value of n if the lines are parallel. Find the value of n if the lines are perpendicular.

Apply

In #8, students should learn that drawing conclusions from a sketch may be misleading. Slope is the best way to prove two lines are parallel. You may wish to reinforce this concept by asking students to prove two lines are parallel using the distance formula.

For #10, students may need to be reminded that $y = 0$ is the equation for the x-axis.

For #11, suggest that students equate expressions for the slopes of the lines and then solve for n.

For #12, students can prove ΔABC is a right triangle either by using slopes or by using the Pythagorean relationship.

Question #13 requires students to use skills from sections 7.1 and 7.2.

For #16, direct students to the definition of a tangent to a circle shown beside #16. A tangent to a circle was introduced in grade 9.

For #18, using the three end points given, (20, 14), (23, 14) and (23, 12), will give equal slopes only if decimals are used for calculating the slope. Students working with fractions will have slopes that are very close, but not equal.

Extend

If you have shown students how to find the slope of an equation in general form, questions #19, 20, 23, and 24 should be straightforward.

For #21, students must realize that the shortest distance between two lines is a perpendicular distance. This question has many steps to it. Students should pick a point on one line and find the equation of the line passing through that point and perpendicular to the second line. Students then need to find the intersection point of the perpendicular line and the second line. Finally, the perpendicular distance between the lines is the distance from the original point and the intersection point.

For #23, you may wish to inform students that there are two values for n that will make the lines parallel.

Create Connections

Encourage students to experiment with perpendicular lines in varying orientations as they work on #25. They need to determine the slopes of vertical, horizontal, and oblique perpendicular lines in order to fully answer the question.

You may wish to have students complete #26 after the Link the Ideas section.

Meeting Student Needs

- Provide **BLM 7–10 Section 7.4 Extra Practice** to students who would benefit from more practice.
- Questions #1 to 5 should be completed to fully develop the concept of slope as related to parallel and perpendicular lines.
- Remind students of the three forms of expressing linear equations. Students may need to use different forms to assist them to complete part of the assignment. For example, #6 requires the slope-point form.
- For the Apply questions, allow students to work in either pairs or small groups. These questions are abstract and students will need assistance. You may wish to have students complete the questions in a station approach, where the questions are written on chart paper and each group adds at least one key comment/response on the chart paper.
- For #12 and 22, students could also plot the points on grid paper to visualize which lines need to be perpendicular in order for the triangle to be a right triangle.

Enrichment

- Related to #17, ask students to play with the formula to answer the following questions:
  - Will the lines get closer together or farther apart as you age?
  - When will the lines intersect? What would this mean? (Answer: You would be 220 years old and your heart rate would be 0.)
  - Is the above answer valid?

Common Errors

- Some students may not determine a negative reciprocal correctly.

R_x Reiterate that the slopes of perpendicular lines are related in two ways: they have opposite signs and are reciprocals. Having opposite signs means if m_1 is positive, then m_2 is negative, and vice versa. Being reciprocals means their numerators and denominators are interchanged.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1–5, 6a), c), e), 7a), c), e), and 9–11. Students who have no problems with these questions can go on to the remaining questions.	• You may wish to have students work in pairs to discuss and compare responses. • You may wish to review how an integer can be written as a fraction when finding the perpendicular slopes in #1, 2, and 7. • Encourage students to refer to their Foldable or any summary page they have created for solving problems relating to formulas. • Some students may benefit from reactivating what an equivalent fraction is. Provide them with several questions that they can practise with before starting #4. • For #9, struggling learners may benefit from recalling the definition of a parallelogram. • If students have difficulty with #10, have them sketch a horizontal line and perpendicular line of their own choosing. Have them determine the slope. Ask them which is parallel to the *x*-axis and which to the *y*-axis.
Assessment *as* Learning	
Create Connections Have all students complete #25 and 26.	• Both questions are excellent Assessment *as* Learning questions and should be considered for the students' Foldable. Encourage students to include specific examples in their explanation.