Systems of Linear Equations and Graphs

Mathematics 10, pages 416-431

Suggested Timing

8.1

100–120 min

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Materials

- 0.5-cm grid paper
- ruler
- coloured pencils
- computer with graphing or spreadsheet software

Blackline Masters

BLM 8–3 Chapter 8 Warm-Up BLM 8–4 Chapter 8 Unit 4 Project BLM 8–5 Section 8.1 Extra Practice TM 8–1 How to Do Page 420 Example 2 Using TI-Nspire[™] TM 8–2 How to Do Page 420 Example 2 Using Microsoft® Excel

Mathematical Processes

- ✓ Communication (C)
- ✓ Connections (CN)
- ✓ Mental Math and Estimation (ME)
- ✓ Problem Solving (PS)
- ✓ Reasoning (R)
- ✓ Technology (T)
- ✓ Visualization (V)

Specific Outcomes

RF1 Interpret and explain the relationships among data, graphs and situations.

RF9 Solve problems that involve systems of linear equations in two variables, graphically and algebraically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1, 3, 5, 6, 7b), d), 8a), 10, 12, 16, 22
Typical	#1–5, 7–11, 13–16, 21, 22
Extension/Enrichment	#14, 16, 17–20, 23

Unit Project Note that #16 is a Unit 4 project question.

Planning Notes

Have students complete the warm-up questions on **BLM 8–3 Chapter 8 Warm-Up** to reinforce prerequisite skills needed for this section. Browse the Internet to find examples of graphs that represent systems of linear equations. You may wish to display the graphs using a projector. Encourage students to discuss and interpret what the point of intersection represents in each case.

Investigate Ways to Represent Linear Systems

You may wish to have students predict the outcome of the investigation before they begin. To open a discussion, ask students, "Will one plan always be better than the other?" Alternatively, you may wish to have students use technology to graph the lines. For example, students could input the tables of values into a spreadsheet and use the spreadsheet to generate the graph.

In discussing responses to #3, you may wish to lead students to the idea of slope. In this situation, the slope of each line is the cost per minute of the corresponding plan. You could ask students to discuss the lines in light of their work in previous chapters. Ask students the following questions:

- How are the graphs of the lines similar? How are they different?
- Do both lines have the same *v*-intercept?
- What does the *y*-intercept represent in this situation?
- Do both lines have the same slope? What does the slope represent in this situation?

As an extension to #5, you may wish to ask students to decide under which conditions each plan would be preferable.

Meeting Student Needs

• To make the section opener more relevant to Northern students, you may wish to discuss the gaps in cell phone coverage in the North and whether there is any possibility of this changing. You may also wish to discuss how cell phone plans work and how to choose an appropriate one.

- Invite students to share the details of their cell phone plans with the class, including base monthly cost and cost per minute for local or long distance calls. Pair students such that the students in each pair are on different plans, if possible. Then, have students create a table of values and sketch and analyse the graph. Encourage students to include a description of the plan they prefer and why.
- Some students may benefit from a discussion about how a system of two linear equations involves two variables and that there could be systems of three or more linear equations.

Common Errors

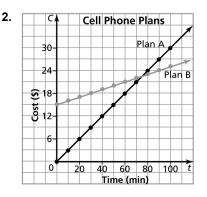
- Students may not pay careful attention to units, in this case dollars and cents.
- Rx Be sure that students have used these units properly in their tables and graphs. Encourage students to consider whether their answers make sense in the context of the problem.
- Students may not follow the conventions for making graphs.
- R_x As you watch them work, remind students to label axes, show correct units, and indicate which graph relates to each cell phone plan.

Answers

Investigate Ways to Represent Linear Systems

1.		
Pla		
Time (min)	Cost (\$)	Time (
0	0	0
10	3	10
20	6	20
30	9	30
40	12	40
50	15	50
60	18	60
70	21	70
80	24	80
90	27	90
100	30	100

Pla	Plan B	
Time (min)	Cost (\$)	
0	15	
10	16	
20	17	
30	18	
40	19	
50	20	
60	21	
70	22	
80	23	
90	24	
100	25	



- **3.** The graph of Plan A starts at zero and goes up more quickly than the graph of Plan B. The graph of Plan B starts higher but goes up more slowly. The graphs cross between 70 min and 80 min.
- **4.** The point of intersection is the number of minutes where both plans would cost the same. The point of intersection is at 75 min and a cost of \$22.50 for each plan. The table hints at these numbers being between 70 min and 80 min and between \$22 and \$23.
- **5.** Example: I prefer Plan A since I use less than 75 min per month and this plan would save money for my use.

Assessment	Supporting Learning
Assessment as Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	 Encourage students to write the coordinates of the intersection point on the graph to make a visual link to the table of values. Some students will be able to complete the table of values using patterning without really understanding what they are determining. Draw their attention to the connection between the numbers in the table and the coordinates on the graph. Use the labels on the axes to prompt students in their responses to #3 and 4. The response to #4 should be discussed as a class. To assist students in answering #5, have them choose a value for minutes that exceeds their point of intersection and ask them to describe what each plan would cost at that time. Use this response to prompt their thinking about how the point of intersection could assist students in determining which plan is the better option. Provide students with an opportunity to include the final summation in their Foldable or graphic organizer.

Link the Ideas

After discussing the different representations of systems of linear equations and how to identify the solution, you may wish to have students consider the advantages and disadvantages of each method. For instance, you could ask the following questions:

- Will the solution to a linear system always appear in the table of values?
- Is it always convenient to create a graph?
- When might it be difficult to accurately read values from a graph?

You may wish to emphasize that the solution to a system of linear equations is not a single value, but rather a coordinate pair. It is not sufficient, for example, to obtain only a value for *x* when solving a system of linear equations.

Example 1

Draw students' attention to the type of relation involved in this example by asking the following questions:

- What does it mean when points on a graph are connected by a line?
- What term describes this type of data? (continuous data)
- What does it mean when the points are not connected? What term describes this type of data? (discrete data)
- Should the data points for this example be connected? Why?

As three different solution methods are shown, you may wish to have students work in small groups. You could assign groups to each of the three methods, with the understanding that each group will explain the solution method to the whole class. If an interactive whiteboard is available, you may wish to demonstrate Method 3 to the class. Regardless of the method chosen, you should help students connect this work to their previous work graphing straight lines.

If you choose to have students work cooperatively in Example 1, then you may want to specify that students complete the Your Turn question using a different method than they did with their small group. Alternatively, you may wish to have students work independently to solve the linear system, and then discuss the advantages and disadvantages of each method with their classmates.

Example 2

Before beginning this example, you could have students brainstorm methods for graphing lines, in particular methods that do not rely on tables of values as they have used so far in this section. You may wish to have students recall the work they did in Chapter 7 and briefly summarize the different forms of writing linear equations and how to graph a line in each form. Consider allowing some time for students to explain the method they prefer.

When considering the three solution methods shown, you could ask the following questions:

- Will one method always be more efficient than the other?
- Are all methods equally precise?
- In which circumstances would you choose to use each method?
- Does every graph have an *x*-intercept and a *y*-intercept?
- Could you create the graph using points other than the intercepts?
- Does every graph have a slope?

When students are verifying their solutions, ensure that they understand they are simply checking that the solution point lies on both lines. They can check this using any representation they choose—such as a table of values, a graph, or substituting into the equations—other than the one they used to solve the linear system.

You may wish to have students use **TM 8–1 How** to **Do Page 420 Example 2 Using TI-Nspire**TM or **TM 8–2 How to Do Page 420 Example 2 Using Microsoft® Excel** as they work through part b).

Example 3

In this example, students do not determine the solution to a system of linear equations; they verify a given solution. In this case, the given point satisfies the equation of one line but not the other line. Students should note that the line containing the point corresponds to the equation that the coordinates satisfy.

You may wish to ask students, "Are there other situations where a point is not a solution to a system of linear equations?" Elicit the possibility that the point could be on neither of the lines.

Example 4

This example presents an opportunity for problem solving. Students need to visualize the travel of the trams and the fact that once during each trip, the trams pass each other. The trams are at the same altitude at the same time. Because the trams travel at different speeds, the altitude where they pass depends on which tram is going up and which is going down. The opening paragraphs provide information about the number of passengers on the trams, which may be interesting to some students but is not required to solve the problem.

To help students work through the problem, you could ask questions such as the following:

- What quantity will be represented on each axis of the graph?
- How do you know that tram travel is linear?
- How will the graphs representing each tram be similar?
- How will the graphs representing each tram be different?
- Why do the trams not meet exactly halfway up the mountain?
- What properties can we expect for each graph? For example, what can we predict about the slope and the intercepts? How can you check to see if your answer is reasonable?

Encourage students to read the Mental Math and Estimation box beside the solution to part b). The reasoning described in the box allows students to double-check that their answer is consistent with information provided in the problem. Emphasize to students the importance of checking the reasonableness of their answers.

Key Ideas

The Key Ideas summarize the concepts that students need to understand in this section. Allow time for students to incorporate the Key Ideas for this section into their Foldable or their math journal. Encourage students to write the Key Ideas in their own words and to annotate them with their own notes regarding their personal preferences and strategies.

Meeting Student Needs

- You may wish to have money manipulatives available for Example 1.
- Example 1 Method 3 could be illustrated on a computer. You could have students enter data and then highlight and extend the chart. Then, you or a student could demonstrate how to create the graph from the given data. Some, but not all, students will not have experience with that step.
- Inform students that solving algebraically will be covered in Chapter 9 and remind students that they currently have the tools to verify their work algebraically once they are checking a possible solution point.
- Students could create one poster for Example 1 and one poster for Example 2 to illustrate the various methods of solving and verifying solutions to linear systems. Students should create new examples to be used on the posters.
- Draw students' attention to the table of values in the solution to Example 2b). Make sure students understand that the first and second columns represent one equation, and the first and third columns give the data for the other equation.
- If it is available, you may wish to allow students to have access to dynamic algebra/geometry software.
- Allow students to create a graph of the trams in Example 4 travelling in opposite directions. This may help students visualize that the altitude and time at which the trams pass each other depend on the directions in which the trams are travelling.
- Create an exit slip based on the Key Ideas for section 8.1. The exit slip can contain fill-in-theblank questions, true/false questions, short examples, or even pictures beside each outcome of thumbs up/thumbs down that students can circle.

ELL

- The *dimensions of the graph* on a graphing calculator refer to how much of the *x*-axis and *y*-axis can be seen on the display. Practise adjusting calculator screens with students.
- Make sure students understand that *altitude* in Example 4 refers to the vertical distance of the tram above a reference point.
- Have students use words and diagrams to distinguish between *vertical* and *horizontal* axes as referred to in Example 4 Method 1.

Enrichment

• Provide students with the following information: On the Apollo 13 mission to the moon in 1970, the astronauts dealt with a life-threatening explosion and frantic return trip to Earth. They had to use paper-and-pencil calculations in order to navigate, including the dangerous re-entry route into Earth's atmosphere. During the calculation process, engineers on Earth checked the astronauts' calculations. Challenge students to describe how being able to calculate the intersection of two simultaneous equations by hand might be valuable. Have them show their expertise by finding the intersection of the following equations using substitution.

$$2x + \frac{1}{3}y = 1$$
$$y \quad \frac{1}{2}x = 0$$

(Checking answers by hand can confirm the work of a calculator, especially if an incorrect value was

entered.
$$x = \frac{6}{13}, y = \frac{3}{13}$$
)

Common Errors

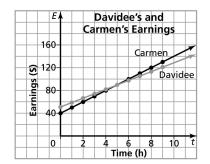
- In the verification step, students often verify the solution in only one equation, not both.
- R_x Emphasize that students need to substitute values into both equations, or look at both tables of values, depending on their choice of method.

Answers

Example 1: Your Turn a)

Davidee		
Time (min)	Cost (\$)	
0	40	
1	50	
2	60	
3	70	
4	80	
5	90	
6	100	
7	110	
8	120	
9	130	

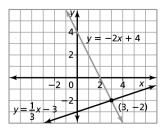
Carmen		
Time (min)	Cost (\$)	
0	50	
1	58	
2	66	
3	74	
4	82	
5	90	
6	98	
7	106	
8	114	
9	122	



b) The solution is (5, 90). Both Davidee and Carmen earn \$90 for 5 h of work.

Example 2: Your Turn

Substitute x = 3 and y = -2 into both equations. In x - 3y = 9: Left Side Right Side x - 3y = 9= 3 - 3(-2)= 3 + 6



Left Side = Right Side

In 2x + y = 4: Left Side Right Side

4

- 2x + y
- = 2(3) + (-2)
- = 6 2= 4

= 9

Left Side = Right Side

Since the ordered pair (3, -2) satisfies both equations, it is the solution to the linear system.

Example 3: Your Turn

a) Substitute x = 2 and y = 5 into both equations.

In 3x - y = 2: Left Side Right Side 3x - y2 = 3(2) - 5= 6 - 5= 1 Left Side \neq Right Side In x + 4v = 32: Left Side Right Side x + 4v32 = 2 + 4(5)= 2 + 20= 22 Left Side \neq Right Side

Since both equations do not result in true statements, the point (2, 5) is not the solution to this linear system.

Answers

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b) Substitute x = -3 and y = -2 into both equations.
   In 2x + 3y = -12:
   Left Side
                     Right Side
     2x + 3y
                     -12
   = 2(-3) + 3(-2)
   = -6 - 6
   = -12
      Left Side = Right Side
   In 4x - 3y = -6:
   Left Side
                     Right Side
     4x - 3y
                     -6
   =4(-3) - 3(-2)
   = -12 + 6
   =-6
      Left Side = Right Side
```

Since the ordered pair (-3, -2) satisfies both equations, it is the solution to the linear system.

Example 4: Your Turn

The point of intersection represents when Eric and Nathan will be at the same level at the same time. This point appears to be approximately (18, 21). After about 18 s, Eric and Nathan will pass each other at about floor 21, with Eric on the stairs and Nathan in the elevator.

Assessment	Supporting Learning
Assessment for Learning	
Example 1 Have students do the Your Turn related to Example 1.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. For students having difficulty formulating their equation, ask them: Which of the dollar amounts are fixed numbers? How much does each person make per hour? after 2 h? after 3 h? Have them verbalize the equation before writing it. Draw student's attention to Example 1 and ask them: Which parts of the example and the Your Turn are similar? Which unit will go on the x-axis of the graph? Which unit goes on the y-axis? Encourage students to use patterning in creating their equation and graph. Ask students to identify what each ordered pair on their graph represents (hours, amount earned). Have students verbalize what the intersection point represents. Encourage them to visualize the labels on their axis and use them in their explanation. Have students identify the link between the table of values and the intersection point. Ask how one can be used to verify the other.
Example 2 Have students do the Your Turn related to Example 2.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. You may wish to review writing equations in slope-intercept form. Have students verbally identify the slope and <i>y</i>-intercept. Ask them to explain their meanings. You may wish to have students who are struggling use technology to assist them in graphing. Allow students choice in their method: slope-intercept form, <i>x</i>- and <i>y</i>-intercepts, technology. Encourage them to use two of the three methods. Remind students of the importance of verifying using not just one but both equations.
Example 3 Have students do the Your Turn related to Example 3.	 Encourage students to verbalize their thinking. You may wish to have students work with a partner. Review the meaning of <i>verifying</i> and <i>substituting values into an equation</i>. Ask students to identify what the ordered pair has in common with graphing the two lines. Ask them what they would expect to see if the ordered pair is a solution to the system.

Assessment	Supporting Learning
Assessment for Learning	
Example 4 Have students do the Your Turn related to Example 4.	 You may wish to have students work with a partner Encourage the use of tables for solving problems. Have students identify how the information in the table in Example 4 is translated into ordered pairs for graphing. Have them identify how they will label their graphs. Ask them what the intersection point would represent. Encourage the use of patterning between the problem and the example.

Check Your Understanding

Practise

Question #1 asks students to determine the solution to two different linear systems in two different ways. It is worth noting that the two systems are different, but have the same solution. For enrichment, students could be asked the following questions:

- How many different systems could have the same solution?
- Can you provide a third linear system that has this same solution?
- How can you prove that your system has this solution?

A natural result of #2 is to discuss situations when technology presents a clear advantage in solving linear systems. Students can be invited to verify their answer without the use of technology.

Questions #3 and 4 require students to solve linear systems using different methods and to look back at their work. As well, these questions help students make the connection between graphical and algebraic representations of linear systems.

Question #5 has students check to see if a given point is a solution to a system. You may wish to ask students to use a different method to answer each part of this question. This will help them with the metacognitive goal of deciding which method to use in a given situation, based on their own strengths and the characteristics of the system.

In #6 to 8, students solve linear systems graphically. Note that in #6, students are instructed to make a graph without the use of technology, while in #7 and 8, no such stipulations are made. Depending on the technology available, you may wish to have students use different technology tools in #7 and 8, and discuss the advantages and disadvantages of each tool. In #9, students are asked to check a potential solution to a system of linear equations. However, they are asked to go a step further and describe the graph of the system and the relationship between the point specified and the graphs in the system. Students need to be able to state whether the point is a solution or not a solution. In this case, students should be able to decide if the point is on one of the lines or neither of the lines in the system. You may wish to refer students to Example 3 if they need help with this question.

For #10, some students may benefit from working with a partner and comparing their methods and the systems of equations they develop.

Apply

Question #11 is a problem-solving question. The equations in the system are given. You may want to have students make sure that they understand why those equations are the correct ones for the context given. Part c) asks students how to use the graph to determine profit—a skill that is not specific to systems of equations, but a review of their previous work with functions.

In #12, students solve a linear system graphically, by hand and with technology. This presents another opportunity to address the appropriate use of technology and situations in which the use of technology offers a clear advantage.

Question #13 has students address a shortcoming of tables of values. The tables of values given for the system do not show the solution to the system. Students need to explain why the tables do not yield a solution. You may wish to use the Think-Pair-Share strategy to have students answer the following questions:

- Is it possible to use tables of values to solve this system?
- Is another method preferable for solving this system?

The slopes and *y*-intercepts in #14 are very similar, requiring care in obtaining a solution. If students graph the system by hand, it may be difficult to work

precisely. If students use technology, they will likely not be able to easily read the solution from the graph, but will need to use the functions of the technology to find the intersection point.

Question #15 gives students an opportunity to explain how a graph relates to a given situation as part of the solution to the system.

Question #16 is related to the unit project. This problem should be accessible to all students; the equations are given.

Question #17 requires students to construct a system with a small amount of information and very little scaffolding. You might suggest that students look back to Example 4 if they experience difficulty with this question.

Question #18 is an enrichment opportunity. Students need to translate the information given into a graph of the linear system. If students require help you could suggest that they consider the time and distance at the beginning and end of each trip for each rider. This gives the endpoints of the line segment representing each rider's travel.

Extend

Question #19 involves a system of three linear equations graphed on the same coordinate grid. Some students may not recognize this. You may want to suggest that students consider the equations two at a time as they answer the question.

To help students answer #20, you may want to ask them, "Does it matter how long the truck travels before reaching the car?" If students need help answering this question, you may want to suggest that they perform a few experimental calculations with several values for the time the truck travels before reaching the car.

Create Connections

Questions #21 and 22 give students opportunities to create their own systems of linear equations and applications. You may want to provide time for students to work in small groups to discuss their answers. Alternatively, #21 provides an opportunity for a gallery walk. Students can (anonymously) post their systems throughout the classroom and then take a few minutes to tour the class and see the linear systems and situations created by their peers. For #23, if students are having difficulty beginning the question, you may wish to suggest that they rewrite the equations in slope-intercept form. This might enable students to analyse and compare the slopes and *y*-intercepts.

(Unit Project)

The Unit 4 project questions give students an opportunity to solve problems involving graphs and systems of equations. Students will be asked to prepare a presentation that convinces people to make changes to help reduce water use. You may wish to discuss why we are concerned about our water use and what its importance is in our future. It may give you a better sense of students' understanding of the issues.

Question #16 is a Unit 4 project question. The question provides the system of equations that represents the movement of two different types of ducks to water. It is important for students to be able to identify which equation represents each type of duck and how to evaluate an equation for a specific value, which, in this case, is 25 min. Students have several methods available to them for graphing, including using the slope-intercept form, table of vales, *x*- and *y*-intercepts, and technology. Ask students to identify which method is easiest for them to understand; however, encourage them to use more than one method in solving the question. Ask how they could check their work. Point out how two methods can be used to verify their solution.

Meeting Student Needs

- Provide **BLM 8–5 Section 8.1 Extra Practice** to students who would benefit from more practice.
- Allow students to brainstorm possible examples of situations that could be represented with a system of linear equations. You may wish to compile a list ahead of time.
- For #11, you may wish to allow students to create a beaded necklace as an example of the activity.
- Suggest that students develop a question similar to #11. Encourage students to research a fundraising project of their own and use the data. For example, they could sell cases of oranges. There may be a minimum order requirement plus a shipping cost. Students would need to determine the retail price. They could graph the two relations to determine the break-even point.

- For #16, when discussing the Unit 4 project on water conservation, you may wish to invite an Elder or Knowledge Keeper to speak about the Aboriginal teaching of water. Have students discuss in groups the importance of water. For part b), encourage students to explain how the equations relate to the situation.
- Provide time for students to work through at least one part of each of #1 to 8 before continuing with #9 to 22.
- Some students have trouble solving linear systems graphically by hand. Encourage students to develop their skills using technology—graphing calculators or spreadsheet software. You may wish to have students work in pairs or small groups.

ELL

• For #11, students may not be familiar with the terms *revenue*, *break even*, or *profit*. Explain these terms using an example.

Common Errors

- Some students forget what their variables represent and then cannot state their answers accurately. For example, when specifying the time of an event, students may not pay attention to what time = 0 represents and then are not able to interpret what time a number actually means in the context.
- R_x Encourage students to clearly identify what each variable represents, including any specific details they need to remember.
- Some students will have difficulty interpreting how the point of intersection relates to the situation.
- R_x Suggest that students carefully consider the variables for each axis and visualize the relationship between them. Encourage students to think about what it means in the situation if one point satisfies both equations and if that point contains a coordinate for each variable.
- In #8, students may recognize that the equations in part a) are in slope-intercept form for graphing, but they may have difficulty determining the slope on the graph and finding appropriate points to plot.
- R_x Have students convert each decimal slope to a fraction and then count the $\frac{\text{rise}}{\text{run}}$ squares to determine the slope.

Assessment	Supporting Learning
Assessment for Learning	
Practise and Apply Have students do #1, 3, 5, 6, 7b), d), 8a), 10, 12, and 22. Students who have no problems with these questions can go on to the remaining questions.	 For #1, students may need clarification regarding what is being asked. Prompt students to explain what the intersection point on the graph means. Ask them to explain how a table of values might be the same as a graph. Ask them what they would expect to see in the graph and the table of vales if the solutions were the same. Reinforce the process for graphing an equation in slope-intercept form. Have students verbalize the process and then have them complete one question that models their understanding. This will help reinforce their confidence for #3, 6, and 7. Ensure that students are able to change the forms of the equations into slope-intercept form. Again, have them verbalize and model one of the questions before completing # 6 and 7. Students having difficulty with #10 should review Example 1 and the Your Turn they completed. Ask students to identify what the fixed amount is (\$35) and what can change (\$5/day and \$12/day). Some students will find it easier to generate a table of values by repeated addition and then use these points to graph the system. Although correct, encourage them to write a system that represents the problem.

Assessment	Supporting Learning
Assessment for Learning	
Unit 4 Project If students complete #16, which is related to the Unit 4 project, take the opportunity to assess how their understanding of the chapter outcomes is progressing.	 Students may benefit form working with a partner to solve the problem. Most students are able to complete a table of values from the given information. Encourage them to complete the table and use the points to generate the graph, if necessary. Encourage students to graph from the given system in slope-intercept form. Point out that the graph will verify their solution. Some students may need prompting to describe the trip the ducks take to the water. Have students verbally describe what is immediately different about the two ducks' journey. Ask students: If they both start at the same time, what would an early start mean for one breed of duck? Which duck flies faster? How would we know if the canvasbacks catch up to the green-winged teals? Where might this show up in a table of values or a graph? You may wish to provide students with BLM 8-4 Chapter 8 Unit 4 Project, and have them finalize their answers.
Assessment as Learning	
Create Connections Have all students complete selected questions from #21 to 23 depending on their level.	 Question #22 serves as an excellent assessment question. All students should be able to explain the difference between solving and verifying. Prompt students to consider where each occurs in the process of finding a solution to a system of equations. This question is appropriate for all levels of learners. Some students may be assigned #21. Prompt them for ideas for #21, or have them refer back to Example 1. For #23, students need to have a good knowledge of parameters and their effects on the graphs created.