Solving Systems of Linear Equations by Substitution

9.1

***Mathematics 10*, pages 468–479**

Suggested Timing
60–75 min

Blackline Masters
BLM 9–3 Chapter 9 Warm-Up
BLM 9–4 Chapter 9 Unit 4 Project
BLM 9–5 Section 9.1 Extra Practice

Mathematical Processes
- ✓ Communication (C)
- ✓ Connections (CN)
- Mental Math and Estimation (ME)
- ✓ Problem Solving (PS)
- ✓ Reasoning (R)
- ✓ Technology (T)
- ✓ Visualization (V)

Specific Outcome
RF9 Solve problems that involve systems of linear equations in two variables, graphically and algebraically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–3, 6–9, 12, 13, 15, 19, 26
Typical	#1–8, three of 9–13, 15–17, 19, 25, 26
Extension/Enrichment	#3–5, 7, 8, 15, 18–26

Unit Project Note that #15 is a Unit 4 project question.

Planning Notes

Have students complete the warm-up questions on **BLM 9–3 Chapter 9 Warm-Up** to reinforce prerequisite skills needed for this section.

Investigate Solving Systems of Linear Equations by Substitution

In this investigation, students get an opportunity to explore the concept of algebraic substitution through the process of substituting equivalent expressions. For students who may not be familiar with a balance scale, it may be worthwhile to borrow one from the science department or display a picture from the Internet.

Allow students to work in pairs to answer #1 to 6. As you circulate around the classroom, ask the following questions related to the diagrams:
- What does the mass of each block represent in the context of the problem?
- How is equality represented in the diagram?
- Why is it not possible to replace the blocks in Diagram 1 with cones?
- Using Diagram 2, how can you determine the cost of one light bulb? the cost of one T-shirt?

As students translate the diagrams into algebraic equations, observe whether they use appropriate letters to represent their variables (i.e., the light bulb and the T-shirt.) Check that students write correct expressions to represent the shapes in each pan of the balance scale.

Have students complete the Reflect and Respond section. Take up these questions with the class. Have a student come to the board and model the solutions to #7 and 8. Have students share their ideas for #9.

Meeting Student Needs

- Ensure that students understand what it means to *solve for y*. Explain that it means to rearrange an equation so that it has $y =$ on one side. If students have a choice between two equations, such as $4x + 5y = 23$ and $3x = y - 9$, they need to see that it is easier to solve for y in the second equation. Show students what happens when they solve for y in the first equation. The result is $y = -\frac{4}{5}x + \frac{23}{5}$. This equation complicates the substitution due to the fractional coefficients. In the second equation, the result is $y = 3x + 9$. This equation is less complicated to use for substitution due to its integer coefficients.
- Discuss the meaning of *substitution*, which is replacing one variable with an equivalent expression. If $c = 2a + 3$, the expression $2a + 3$ can be substituted for c in another equation. This must be done using brackets, since the single

variable c is being replaced with a two-term expression. For example,

$$5c + 4a = 12$$
$$5(2a + 3) + 4a = 12$$

When demonstrating to students, use a different colour for c and $2a + 3$ to emphasize the substitution.

- Introduce substitution by using coins; for example, five nickels can be substituted for one quarter.
- Bring a double-pan balance to the classroom. Allow students to place classroom objects, such as pens, pencils, erasers, markers, and planners, on the balance to determine objects that they can substitute for other objects and maintain balance. Ensure that students can replace one type of object with a certain number of another object; for example, one eraser can be replaced with three pencils.

ELL

- Have students work with a partner on the Investigate. Ensure students understand that a block represents a light bulb, and a cone represents a T-shirt.

Answers

Investigate Solving Systems of Linear Equations by Substitution

1. In diagram 2, the two cones from diagram 1 have each been replaced by three blocks.

2. Since the mass of seven blocks equals 42 g, dividing by 7 will give the mass of one block as 6 g.

3. Each cone has a mass of 18 g because the mass of three blocks equals the mass of one cone.

4. Let x represent the mass of one cone. Let y represent the mass of one block. For the first balance, $2x + y = 42$. For the second balance, $x = 3y$.

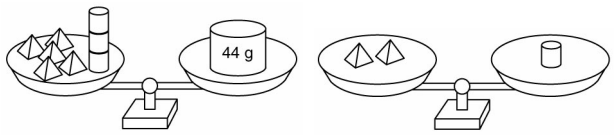
5. $7y = 42$

6. Let y represent the cost of one light bulb. Let x represent the cost of one T-shirt.

$$7y = 42$$
$$\frac{7y}{7} = \frac{42}{7}$$
$$y = 6$$

Substitute 6 for y in $x = 3y$.
$x = 3(6)$
$x = 18$
The cost of one T-shirt is \$18. The cost of one light bulb is \$6.

7.



8. For the first balance, $5p + 3c = 44$. For the second balance, $2p = c$. Substitute $2p$ for c in $5p + 3c = 44$ and solve for p.

$$5p + 3(2p) = 44$$
$$5p + 6p = 44$$
$$11p = 44$$
$$p = 4$$

Substitute 4 for p in $c = 2p$.
$c = 2(4)$
$c = 8$
The mass of one pyramid is 4 g, and the mass of one cylinder is 8 g.

9. Example: If the situation involves large numbers of objects, the diagram would be less effective than an algebraic method.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• Ask students to determine the algebraic equation for each diagram. • Have students follow the steps in the Investigate for the Reflect and Respond problem. Suggest they show each step in diagram form and follow through progressively to the end. • Some students may find it easier to translate the diagrams into algebra if the shapes are labelled with p for pyramid and c for cylinder. • For #9, listen and watch for those students who suggest the visual is a better format. These students may not be ready to move into an algebraic form and should be given additional examples to complete.

Link the Ideas

In the example provided, y is isolated in the second equation because it has a coefficient of 1. Emphasize that students should use brackets when substituting the expression equivalent to y, $3x + 9$.

Example 1

For this example, you may wish to refer students to the Did You Know? Students may be interested to learn that the Abbotsford Airshow is one of the largest of its kind in the world.

Emphasize the proper modelling of the solution, including the definition of each variable. Direct students to the questions in the margin of the solution. Some students might benefit from further elaboration of the general questions:

- What does the 2 represent in the first equation?
- What does the value $3C$ represent in the first equation?

In Method 2, point out that the entire expression $2A$ has been replaced with 50. Ensure students understand that it is not always necessary to determine the value of $1A$ before substituting.

Emphasize the process of checking the solution, as shown in the student resource.

As students solve the linear system in the Your Turn section, observe how many students directly substitute 12 for $3x$ in the first equation. For students who solve for x, ask how they could solve this problem by substituting for $3x$ instead of x. Follow up by asking these students when it might be even more beneficial to substitute for $3x$ if the situation presents itself. For example, if $3x = 17$, a direct substitution of $3x$ means avoiding working with a fractional number.

Example 2

As you define variables and set up equations, emphasize to students what the terms $9a$ and $6c$ represent. Clarify the notation for numbering an equation in a linear system with a circled number. Explain that this type of notation becomes more important in larger linear (and non-linear) systems involving three or more variables.

Ask students which variable (in which equation) they would recommend for isolation. Discuss with the class how they can reach a solution more efficiently by selecting variables with coefficients of 1 or 2. Point out that Method 2 includes improper fractions because of the decision to isolate c.

For the linear system in the Your Turn section, observe whether any students multiply the second equation by 10 in order to eliminate the decimal coefficient of y. You may wish to identify this option at this point. Focus students' attention on this topic by asking how the problem is different from the linear system in the example.

Key Ideas

This section contains a summary of the process of solving a linear system using the substitution method. If you use the example provided, check the solution at the end of the question.

Meeting Student Needs

- Emphasize that the substitution method is usually most appropriate when the value for x or y can be determined easily to be substituted into the other equation of the system.
- Some students may not be familiar with what an air show is. You may wish to have students research the Abbotsford Airshow or other air shows in Canada to gain a better understanding. See the related Web Link that follows in this Teacher's Resource.
- You may wish to model Example 1 using manipulatives, similar to what was done in the investigation. You might create magnets for the white board, which could show students in a highly visual way how this method works. Cut out squares of coloured paper and attach a small piece of magnet to each. Label each of them with a variable or number; for example, label all red papers with A, all yellow papers with C, and all green papers with numbers. These manipulatives are useful for solving all types of equations and can be stored in small plastic containers.
- For Example 2, you may wish to demonstrate the solution on a graphing calculator. Be sure to discuss the importance of selecting the correct window. Students could also determine the intersection of the two lines in order to check their work. This allows students to compare two methods of solving a system.
- Ensure that students compare the two methods demonstrated in Example 2. Ask them to write down which method they would use and why.

- Post an example of substitution. Colour code the two equations throughout the solution. This will allow students to visualize that part of one equation is substituted into another equation during the process.
- Have students create a poster stating the steps for solving linear systems using substitution. They may wish to use the Key Ideas as a guide.

ELL

- Ensure that students add the term *substitution* to their vocabulary dictionaries.
- For Example 1, students may need assistance in understanding what *admission* is. Ensure they see that it is an amount of money. Explain, using diagrams, that in this example, each admission cost mentioned in the question applies to all of the people in the car.

Common Errors

- Some students may struggle with setting up equations similar to those in Example 2, which include unit rates/prices.

R_x Coach students through additional problems where the focus is only on setting up the two linear equations.

- Some students will not necessarily choose the easier variable to isolate. For example, the easier variable may appear on the right side of the equation and the student is more comfortable isolating on the left side.

R_x Provide students with additional systems where they are encouraged to isolate a variable on the right side of the equation.

WWW Web Link

For more information about the Abbotsford Airshow, go to www.mhrmath10.ca and follow the links.

Answers

Example 1: Your Turn

$x = 4$ and $y = 3$

Example 2: Your Turn

$x = -6$ and $y = 25$

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. Some students may question why there is a need to solve for A if the two equations both contain $2A$. Have students recognize that they can substitute 50 in immediately, as indicated in the blue sidebar. Prompt discussion by asking students to identify when it is appropriate to make this substitution and when solving for the variable is necessary. Note that the Your Turn provides an opportunity for students to first solve for the single variable and then substitute. • Remind students about the importance of verifying their solution.
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Students may need to refresh their skills in simplifying equations that contain fractions or decimals. Model several examples. • As students work on the Your Turn, ask them to identify which variable they believe would be easier to solve for and why. • The skill of recognizing by inspection which variable is the easier to isolate will not come immediately to all students. Provide sufficient examples where one variable would clearly be a better choice and several where either variable could be used. Try to use variables other than x and y in example equations, so students become accustomed to identifying any variable.

Check Your Understanding

Practise

In the first three questions, there is at least one variable that has a coefficient of 1.

In #4, some students may need assistance with eliminating fractional coefficients. You may wish to have a student model solving this type of linear system for the class.

In #7, students discover one of the disadvantages of a graphical solution for a linear system: determining non-integer coordinates of the point of intersection.

Apply

In #5, students are shown two approaches to solving a linear system. Using Helen's method results in fractions that cannot be represented with terminating decimals. However, students should feel comfortable working with fractions as they solve linear systems.

For #13, encourage students to check their answer graphically as a review of Chapter 8.

For #16, some students may start part a) by using the value of a nickel and of a dime. Remind them that the question has to do with the number of coins rather than their values.

Extend

For #22, it may be necessary to remind students to work with one unit of time (hours or minutes).

In #23, students work with some physics formulas for simple circuits, which they may have seen in science class.

Encourage all students to try #24.

Create Connections

For #26, you may wish to guide students to #13 for a question that lends itself to a graphical solution.

Unit Project

The Unit 4 project question, #15, provides an opportunity for students to assign variables to a bush and a tree and set up a system of equations to solve for the cost of each. Students may wish to review either Example 1 or 2 and model their system using the same format of identifying variables and writing equations. Reinforce that students must solve the system. Ask them what it is that they are determining in this problem. Ask what units their answer will require.

Encourage students to sketch a diagram to help visualize what the problem is asking. For example, they may wish to sketch and label a scale.

Meeting Student Needs

- Provide **BLM 9–5 Section 9.1 Extra Practice** to students who would benefit from more practice.
- Algebra tiles or other manipulatives may be useful in helping students to work through #1 to 3. The movement of the manipulatives and actual physical substitution will solidify the concept for some students. Not all students need to use this method, but it may be good to demonstrate one question for the entire class.
- For verbal learners, #3 and 5 are useful questions. Ensure that these students have the opportunity to share their writing.
- Have all students complete #5 individually, as it is an effective question for Assessment *as* Learning. A discussion should follow this question, so that students may learn from errors before they complete the rest of the Check Your Understanding questions.
- For #10 and 12, you may wish to have students research data about snowfall and TV-watching habits in their own community. They might then create questions based on the information they find.
- Some students will benefit from completing the unit project question, #15.
- You may wish to organize students into groups and vary the assigned questions for each group. Have students produce an answer key once all questions have been answered.

ELL

- For #9, show students visuals of cable so they understand it is a long piece of material that can be cut into pieces.
- Since #15, 18, and 23 include challenging language, you may wish to have students work with a partner on these questions.
- For #16 and 19, you may wish to provide actual coins for students to handle if Canadian money is new to them.

Enrichment

- Present the following scenario: A fraction's value in lowest terms is $\frac{5}{6}$. Adding 11 to the numerator of the fraction creates a fraction whose value is $\frac{6}{5}$. Challenge students to use a system of linear equations to determine the fraction. $\left(\frac{25}{30}\right)$

Gifted

- Have students solve the following problem using a system of linear equations and substitution as the method: A fraction has a two-digit numerator and denominator whose place values are reversed (such as $\frac{13}{31}$). If the value of the fraction is $\frac{7}{4}$, determine at least two fractions that meet the criteria. (Examples: $\frac{21}{12}, \frac{42}{24}, \frac{63}{36}, \frac{84}{48}$)

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1–3, 6–9, 12, 13, and 19. Students who have no problems with these questions can go on to the remaining questions.	• For the initial Practise questions, you may wish to have students work with a partner of similar ability. • For students who find #1 challenging, prompt and coach them through part a) of this question. Then, have them solve parts b) and c) on their own. • For #3 and 8, students may find it easier to eliminate the decimals. Coach students through #3 and have them complete #8 to check for understanding. This question provides prompts so students should be able to complete it alone. • Some students may benefit from drawing a model diagram, for example, for #6 and 12. Prompt them to select variables that are easy to remember for the problem. Have them translate the diagram into a system of equations. Ask students to verbalize what they are looking for and what the variables represent. • For #7, ask students to explain how they know the graph is a solution to the system. Challenge them to support their explanation with two different approaches. • For #9, students who have difficulty writing the linear system may benefit from first drawing a diagram and labelling the parts with variables. • Some students are confident using the graphing method but have difficulty writing a linear system. For a question such as #13, ask them which part of the question would represent the slope of the line and which part the *y*-intercept. Ask them to write the equations in slope-intercept form and then determine the system and use substitution to solve. Remind them that they can use graphing to verify the solution.
Unit 4 Project If students complete #15, which is related to the Unit 4 project, take the opportunity to assess how their understanding of chapter outcomes is progressing.	• You may wish to provide students with **BLM 9–4 Chapter 9 Unit 4 Project** and have them finalize their answers. • Prompt students to select variables appropriate for the question. They may wish to draw a diagram first and then translate it into a system. • Students may find that the approaches they used in #6 and 12 may help them complete this question. • Remind students to store all project-related materials in their project portfolio.
Assessment *as* Learning	
Create Connections Have all students complete #25 and 26.	• Encourage students to verbalize their thinking. • Allow students to work with a partner to discuss the questions, and then have them provide individual responses orally or in written form. • For #25, you may wish to have students revisit #8 before responding, since this question also involves a graphical representation and solving using substitution. Ensure that students answer with both similarities and differences. Have students use this comparison as a springboard to explaining the advantage of an algebraic approach. • For #26, some students may be uncertain which question is appropriate to choose. Write a list of possible questions on the board or direct them to any part of #1. You might have students record their response to this question in their Foldable and use it as a study tool.