Solving Systems of Linear Equations by Elimination

9.2

***Mathematics 10*, pages 480–491**

Suggested Timing

60–75 min

Blackline Masters

BLM 9–3 Chapter 9 Warm-Up
BLM 9–4 Chapter 9 Unit 4 Project
BLM 9–6 Section 9.2 Extra Practice

Mathematical Processes

- ✓ Communication (C)
- ✓ Connections (CN)
- Mental Math and Estimation (ME)
- ✓ Problem Solving (PS)
- ✓ Reasoning (R)
- Technology (T)
- ✓ Visualization (V)

Specific Outcome

RF9 Solve problems that involve systems of linear equations in two variables, graphically and algebraically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–4, 7–9, 13, 14, 20, 21
Typical	#1–3, 5, three of 7–12, 14–16, 20, 21
Extension/Enrichment	#2, 5, two of 10–13, 14–21

Unit Project Note that #14 is a Unit 4 project question.

Planning Notes

Have students complete the warm-up questions on **BLM 9–3 Chapter 9 Warm-Up** to reinforce prerequisite skills needed for this section.

If your school has an environmental club, you may wish to discuss ways in which the club raises money for their projects. Also, show students images or samples of organic coffee, reusable bags, and reusable food and drink containers.

Investigate Solving Systems of Equations by Elimination

As students work in pairs to answer the questions, circulate among them and ask questions.

To assist students with #1, ask how the shapes have changed on scale B from Diagram 1 to Diagram 2.

To assist students with #2, ask the following questions:

- In Diagram 2, how does dividing the 17-g mass into two 7-g masses and one 3-g mass make the two scale balances similar?
- What else is similar about the two scale balances in Diagram 2?
- What extra items are found on scale A compared to scale B in Diagram 2?

For students struggling with #3, check that they have correctly determined the mass of the block and then guide them back to Diagram 1.

Once students begin working on the algebraic section of the investigation, check that they translate the diagrams correctly and do not confuse the variables.

In the Reflect and Respond section, check that students construct the diagrams correctly to model the problem. With the class, go over #8 to 10. Ensure that students understand from the diagram why the substitution method would not work for this investigation (both variables appear on the same pan).

Meeting Student Needs

- Explain that a person's *ecological footprint* is a measure of that person's demand on Earth's ecosystems. It represents the amount of biologically productive land and sea area needed to regenerate the resources a person consumes.
- You may wish to further the discussion about ecological footprints. Ask students about the kinds of garbage they see when they are out in nature. Discuss whether the garbage has any effect on wildlife. Ask where the garbage might come from. Discuss whether switching to reusable shopping bags might be better for the environment and why.

- Some students may not be familiar with *hybrid vehicles*. Explain that a hybrid is a vehicle that runs off both an electric motor and a gas- or diesel-powered engine.
- When preparing students for this section, emphasize the notion of equivalent equations. Refer to doubling or tripling a recipe (all items in the recipe must be doubled or tripled, and so will the quantity), mixing gas and oil to a specific ratio (the amount of each will be determined by the total amount required), or other relevant examples. You can also demonstrate the concept of equivalent equations using algebra tiles and other manipulatives, including blocks.
- Work through the investigation by bringing reusable bags, packages of coffee, and play money to the classroom. This would be a helpful hands-on learning experience for some students.
- Depending on the needs of your class, you may wish to write a couple of equations on the board as examples for students prior to having them determine the equations for the Investigate.

ELL

- Show students samples of reusable bags, organic coffee packets, and reusable food and drink containers.
- Have students work with a partner on the Investigate.

Common Errors

- Some students may struggle with modelling #9 of the investigation.

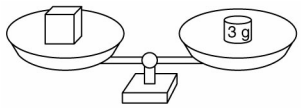
R$_x$ Guide students through translating the words into the diagram. Some students may benefit from translating into the algebraic equation first.

Answers

Investigate Solving Systems of Equations by Elimination

1. The mass of one cone and one block is 7 g so the mass of two cones and two blocks is twice as much.

2. From scale A in diagram 2, take two cones and two blocks from the left side and the two 7-g masses from the right side.



3. The mass of one cone added to the mass of one block is 7 g. Since the mass of one block is 3 g, the mass of one cone must be 4 g.

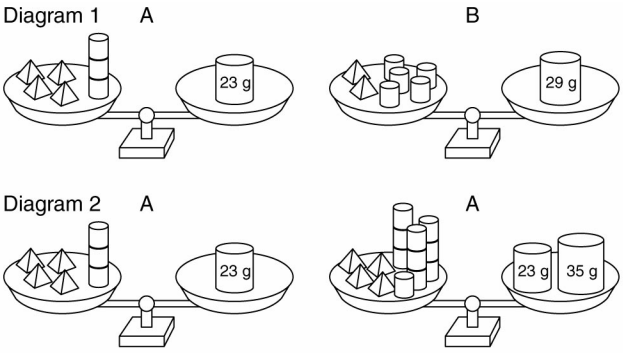
4. Let c represent the mass of one cone. Let b represent the mass of one block. The equations are $2c + 3b = 17$ and $c + b = 7$.

5. Multiplying both sides of the equation $c + b = 7$ by 2 gives $2c + 2b = 14$.

6.
$$\begin{array}{r} 2c + 3b = 17 \\ -(2c + 2b = 14) \\ \hline b = 3 \end{array}$$

7. Double the equation for scale B in diagram 1. Subtract this equation from the equation for scale A. The cost of a grocery bag is \$3, which equals the mass of a block, in grams. The cost of a packet of coffee is \$4, which equals the mass of a cone, in grams.

8.



9. The mass of one pyramid is 2 g, and the mass of one cone is 5 g.

10. Yes. Example: You could isolate p or c from either equation and substitute this expression into the other equation.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• Students may benefit from working with a partner. • Encourage students to label the pyramids p and the cylinders c to assist them in writing the system of equations. • Have students verbalize what *opposites* are. You may need to go over opposites of integer values and the sum of a number and its opposite value. • Remind students of the importance of multiplying every term on both sides of the equation. A common mistake is to multiply only one side of the equation.

Link the Ideas

Discuss with students that it may be easier to multiply one of the equations by a negative integer in order to eliminate the variable by addition.

This may be an appropriate point to introduce a system with fractional coefficients and explain how it may be necessary to multiply the equation in order to create integral coefficients.

Example 1

Discuss with students that an acceptable answer must be a whole number for this type of question, since the variables represent the number of downloads of each type. Focus students on using appropriate variables for a song (S) and a game (G).

After setting up the equations, ask students the following questions:

- Which variable should you eliminate? Why?
- Could the other variable be eliminated instead? Explain.

After students have determined the variable to be eliminated, ask them what constant multiplier(s) should be used, if any. Discuss whether the multiplier(s) should be negative or positive and what the implication is for each choice.

Reinforce the final step of checking the solution by substitution into the original two equations.

For the Your Turn question, the numbers are larger but should not cause a problem for students.

Example 2

Using the Did You Know?, discuss the concept of carbon offsets and their connection to global warming.

Help students understand why a farmer may choose to convert some cropland into woodland by discussing the work of organizations such as the Pacific Carbon Trust.

The Pacific Carbon Trust expects to purchase between 700 000 and 1 000 000 tonnes of carbon-dioxide equivalent offsets each year. Offsets or carbon savings are generated from changes made to avoid or absorb carbon dioxide. Typically, carbon offsets fall into three categories:

- renewable energy such as hydro power generation using natural river flow, wind farms, or solar installations
- energy efficiency such as increasing industrial energy efficiency by improving pollution control equipment, switching from oil to natural gas, or performing energy retrofits
- emissions storages or sinks, such as forestation of lands not previously forested

Land owners can develop projects that reduce carbon emissions and sell these carbon offsets to individuals and industry. As people become more concerned about climate change, more businesses and individuals are purchasing carbon savings to offset the environmental cost of such things as major sporting events (e.g., World Cup Soccer), travel, conferences (e.g., United Nations World Climate Research Programme), industrial operations (e.g., Google and Nike), and even weddings.

This provides opportunities for land owners, such as the farmer in this question, to invest in carbon reduction projects. They then sell the carbon savings they have generated to others to offset activities that increase carbon emissions. For more information on carbon offsets, use the Web Link at the end of this section.

In this problem, students use a table to organize the information. On the completed table, the two equations in this linear system appear vertically in the last two columns.

The numbers are large in these equations. Careful selection of the variable that will be eliminated can help minimize the size of the coefficients in the equivalent equations.

When taking up the problem in the Your Turn section, discuss whether students multiplied to eliminate the decimal coefficients. Discuss the advantages and disadvantages of this practice.

Example 3

For this question, emphasize the importance of drawing a diagram and labelling it with the assigned variables and given information. Students will need to rearrange one of the equations to ensure that both equations are in the same form, $ax + by = c$. You may wish to discuss with students that this form is not mandatory. What is mandatory is that both equations be in the same form. In other words, the form could be $ax = by + c$.

Key Ideas

As you go through the Key Ideas, ask students the following questions:

- In the second bullet, what does *elimination* mean?

- In the third bullet, what would be an alternative way to arrange the two equations in the same form? (See the Example 3 notes above.)
- In the fourth bullet, what multipliers would you use if you wished to eliminate the variable y instead?
- In the last bullet, how would you check your solution? Explain.

Meeting Student Needs

- You may wish to encourage students to always add the two equations. Discuss the idea of opposites having a sum of zero. In Example 1, draw attention to equation ② being multiplied by –2 resulting in $-4s$, which is the opposite of $+4s$ in equation ①. This allows students to add the two equations. If you have students who repeatedly subtract incorrectly, the option of multiplying to produce opposites may be helpful.
- Ensure that students understand what *common multiples* are.
  Multiples of 2 are 2, 4, **6**, 8, 10, **12**, 14, 16, **18**, …
  Multiples of 3 are 3, **6**, 9, **12**, 15, **18**, …
  The multiples that are in boldface are the common multiples. When using the elimination method, a common multiple is necessary.
- For Example 2, assist students in understanding what *revenue* is: the entire amount of income before deductions. *Gross* is another word often used to describe revenue.
- Examples 2 and 3 demonstrate two methods of creating the equations necessary to solve the problem. Encourage students to try both methods to allow for individual learning styles.
- In Example 3, work through the translation of the words into algebraic equations. For example, "2.46 m longer than five times the width" needs to translate to $5w + 2.46$.
- For Example 3, give students a chance to practise rearranging equations. To do this, students perform the same operation on both sides of the equation. For example, the following equation needs to be rewritten in the form $ax + by = c$:

$$\begin{aligned} 3x + 5 &= 2y \\ 3x + 5 - 2y &= 2y - 2y \\ 3x + 5 - 2y &= 0 \\ 3x + 5 - 2y - 5 &= 0 - 5 \\ 3x - 2y &= -5 \end{aligned}$$

- Have students create a poster stating the steps for solving linear systems using elimination. They may wish to refer to the Key Ideas.

ELL

- Ensure that students add the term *elimination* to their vocabulary dictionary.
- For the Example 1 Your Turn, have a volunteer explain basketball to students who may not be familiar with the sport. Ensure that pictures or Internet video footage is available to assist their understanding.
- In Example 2, students may not be familiar with a number of terms: *crop*, *converting*, *cropland*, *woodland*, *carbon sink*, *revenue*, etc. Use verbal descriptions, examples, and visuals to assist student understanding.
- For Example 3 and the Your Turn problem, encourage students to draw their own diagram to assist them as they work through the questions.

WWW Web Link

For more information on carbon offsets, go to www.mhrmath10.ca and follow the links.

Answers

Example 1: Your Turn

The price for an adult was $14. The price for a student was $9.

Example 2: Your Turn

Let m represent the number of muffins sold. Let y represent the number of yogurts sold. Solve the linear system $m + y = 160$ and $1.5m + 2y = 273.5$. 93 muffins and 67 yogurts

Example 3: Your Turn

The length is 3.4 m. The width is 2.7 m.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Have students verbalize what variables they will use. • Some students may find it easier to add integers than to subtract integers, especially where negative values are concerned. In this case, explain that the elimination method can also be referred to as the addition method since you add opposite terms to eliminate them. Review the notion of opposite integers and provide several examples. • The side notes beside the worked example ask important questions that should be addressed and discussed as a class before students go on to the Your Turn. Have students model their solution after the example. • Remind students to multiply both sides of an equation when multiplying by a constant value.
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Encourage students to use a table or other graphic organizer to help organize their thinking. Draw attention to the table in Example 2, which enables you to generate the system from the last two columns. Some students may find it easier to fill in the total row first and then work backwards. Encourage them to use whichever approach works best for them. • Assist students in developing a table for the Your Turn question. • Ask them to verbalize the selection of variables and what the system would be, using inspection.
Example 3 Have students do the Your Turn related to Example 3.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Students may benefit from a discussion of the meaning and implications of the words *double*, *triple*, *quadruple*, *half*, *one third*, etc. • Encourage the use of diagrams to solve the problem. • Ensure that students understand the importance of writing the equations in the same form. It does not need to necessarily be $ax + by = c$; it could be $by = ax + c$ or $by + ax = c$ or $ax = by + c$. Emphasize that what is important is that the forms match. • Remind students of the importance of verifying their solutions.

Check Your Understanding

Practise

In #1, none of the equations require rearranging to solve with elimination. Then, in #2, students need to rearrange in each problem.

In #3, the data appear in a table in order to remind students how data can be organized for a word problem.

In #4, both equations in each problem must be multiplied by a constant before elimination can occur.

For #5c), refer students to the work they did for #24 of section 9.1. Have them compare the two solutions and then explain their solution for #5c).

In #6, students will find that there is no solution. Students may wish to graph the two equations with their graphing calculator in order to better visualize the problem.

Apply

For #7, you may wish to assist students in setting up the second equation. Students should realize that $b + t = 30$, since each vehicle will have only one seat. For setting up the second equation, ask what $2b$ and $3t$ would represent.

For #8, refer students to the Did You Know? in the margin for some context to the Communities in Bloom project. You may wish them to learn more about this project. See the related Web Link that follows in the Teacher's Rescource.

For #12, students may benefit from creating a table to arrange their information. The table might include speed, time, and distance as headings.

Extend

In #15, students have an opportunity to practise previously learned algebraic skills of simplifying expressions within equations.

For #17, students complete a typical mixture-type problem. Students need to convert the percents to decimals before setting up the equations in the linear system.

Create Connections

At this point in the section, it might be beneficial to have students take a question that they have solved using graphing and substitution and now complete it using elimination. They could include this work in their Foldable.

Unit Project

The Unit 4 project question, #14, provides an opportunity for students to determine water usage and the number of loads of laundry of two women in a week based on their habits and the type of washing machine they use. Some students will find this easier to complete using a table. Have them review Example 2 and use it as a model. Other students may find it easier to interpret graphically, especially part b). Although the section focuses on the elimination method, encourage students to use whatever approach works best for them. Emphasize that using more than one method allows them to verify their solution.

Meeting Student Needs

- Provide **BLM 9–6 Section 9.2 Extra Practice** to students who would benefit from more practice.
- Have students work in pairs to demonstrate their understanding prior to doing individual work on the Check Your Understanding questions.
- Students may wish to check their work by entering the equations into a graphing calculator.
- For one part of #1 or another question of your choice, do the following activity: If you have a tiled floor in your school, use masking tape to outline the axes of a graph. Then, after writing the equations in $y = mx + b$ form, have one student start at the y-intercept and move the number of squares indicated by the slope. Have a different student create a straight line of tape that forms the line of equation 1. Follow the same procedure with two other students and equation 2. Students can then determine the point of intersection. The graph on the floor would serve as a reminder of the definition of the solution of a system of equations.
- For #4 and 5, students multiply both equations by a constant. You may wish to refer students back to Example 3 prior to assigning these questions.
- For #6, students need to understand that if there is no solution, the two lines are parallel. A system of parallel lines will never intersect. You may want students to solve this problem graphically once they have tried solving it algebraically. They will then be able to see why there is no solution.
- Before students begin the Apply questions, show a couple of examples of solving word problems. Students, particularly those with language difficulties, may not understand what is expected of them. You may wish to have students create a word problem to help them in their understanding.
- Some students will benefit from completing the unit project question, #14.

ELL

- For #7, show students pictures of a bicycle and a tricycle to ensure they understand the number of wheels that each one has.
- For #8, refer students back to the Investigate, which is a similar question.
- For #11, explain to students that an example of a passenger vehicle is a car and that a ferry is a type of boat that carries cars and trucks.
- Since #12–14 and 16 include challenging language, you may wish to have students work with a partner to complete them.

Enrichment

- Ask students to examine the following system and predict whether it has one solution, no solution, or an infinite number of solutions. Have them check their prediction using the elimination method.

$$0.1x + \frac{2}{5}y = 1000$$

$$\frac{1}{10}x - 0.4y = 1000$$

(An infinite number of solutions. Since they are equivalent equations, elimination eliminates everything, showing that any value for x or y solves both equations.)

WWW Web Link

For more information about Communities in Bloom, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1–4, 7–10, 13, 20, and 21. Students who have no problems with these questions can go on to the remaining questions.	• For #1, students complete an entry level set of questions in which the variables do not require multiplication by a constant. Have students verbalize which variable would be the easier one to eliminate and whether they would add or subtract. • For #2, ensure that students understand the importance of writing the equations in the same form when they begin a question. Clearing up any misunderstandings at this point will assist them in writing their systems for the Apply questions. • In #3, students have the opportunity to use a table to set up and then solve a system. Many students find a table helpful in organizing their information. For students who are overwhelmed by the language in a problem, suggest that they use a table.
Unit 4 Project If students complete #14, which is related to the Unit 4 project, take the opportunity to assess how their understanding of chapter outcomes is progressing.	• You may wish to provide students with **BLM 9–4 Chapter 9 Unit 4 Project** and have them finalize their answers. • Students may wish to refer to Example 1 to assist them in setting up the equations. Alternatively, since it may help students to use a table, they might review Example 2. • Have students identify variables and what the variables will represent in the answer. • Remind students that they can use a variety of approaches. Some students may find answering parts b) and c) easier from a graphical representation. Ask students how they would have to change their equations to represent them as graphs.
Assessment *as* Learning	
Create Connections Have all students complete #20 and 21.	• For #20, students use the substitution method on questions they solved using the elimination method. Good choices to direct them to are #1 or 2. In part c), if students have difficulty identifying what they are looking for, suggest that they refer to #4 and 5 before answering. • As an assessment question, #21 is useful for identifying whether students have a good grasp of the methods in Chapter 9. You may wish to collect their responses to get an idea of their understanding at this point and to identify methods that students are not comfortable with. You might have students include their responses in their Foldable for future reference. Students need a clear understanding of each method before moving on to section 9.3.