9.3 Solving Problems Using Systems of Linear Equations

***Mathematics 10*, pages 492–501**

Suggested Timing

60–75 min

Materials

- graphing calculator or spreadsheet software

Blackline Masters

BLM 9–3 Chapter 9 Warm-Up
BLM 9–4 Chapter 9 Unit 4 Project
BLM 9–7 Section 9.3 Extra Practice

Mathematical Processes

- ✓ Communication (C)
- ✓ Connections (CN)
- Mental Math and Estimation (ME)
- ✓ Problem Solving (PS)
- ✓ Reasoning (R)
- ✓ Technology (T)
- Visualization (V)

Specific Outcome

RF9 Solve problems that involve systems of linear equations in two variables, graphically and algebraically.

Category	Question Numbers
Essential (minimum questions to cover the outcomes)	#1–3, 5, 8, 9, 14
Typical	#1–7, 9, 11, 14, 15
Extension/Enrichment	#2, 9–13, 15

Unit Project Note that #9 is a Unit 4 project question

Planning Notes

Have students complete the warm-up questions on **BLM 9–3 Chapter 9 Warm-Up** to reinforce prerequisite skills needed for this section.

Investigate Solving a Problem Involving a System of Linear Equations

Read through the opening paragraph with students. Ask students questions to activate their thinking:

- What is your ecological footprint?
- Do you know anyone who owns a hybrid vehicle? What were their reasons for purchasing a hybrid?

In the opening paragraph of the problem, there is a reference to the operating cost per kilometre. As a class, discuss the components of this figure: insurance costs, maintenance costs, fuel costs, and depreciation costs.

Have students work through the Investigate in pairs. Students may find it challenging to solve this question graphically due to the large values and similar slopes. It may benefit students to try solving the linear system algebraically first. However, seeing the visual relationship between the two graphs is important for students to understand the cost savings of the hybrid over the long term.

For #2, you may wish to ask these guiding questions:

- What values are you using for your window in the graphing calculator?
- What is the cost to operate each vehicle for 100 km, 1000 km, and 10 000 km?
- What is the difference in costs over 10 000 km?

At the end of the Investigate, discuss with the class which method was quicker and which method was more helpful for conceptually understanding the problem.

Meeting Student Needs

- Have students refer to the posters in the classroom illustrating the methods of solving. Ask students to predict when they would use each method.
- Have one or two students make a brief presentation about hybrid cars. Encourage them to include pros and cons as part of the presentation. You may wish to choose students who have strong presentation skills but are weaker in math.

- Depending on your class, you may wish to show examples of word problems before beginning this section. Students, particularly those with language difficulties, may find this section challenging. Consider having students work in pairs or small groups.
- Once students have completed the investigation, allow time for them to compare their summaries. Some students will benefit from verbally presenting their knowledge.

ELL

- You may wish to explain to students what hybrid cars are by comparing them to other cars.

Common Errors

- Some students may struggle with an appropriate window to see the two linear graphs on the calculator.

R_x Ensure that students have the correct equations entered. Then, help them to understand that appropriate sections of the graphs lie in quadrant I. Also, help them establish appropriate maximum values for the horizontal and vertical axes.

Answers

Investigate Solving a Problem Involving a System of Linear Equations

1. Let C represent the total cost, in dollars, of operating a car. Let k represent the number of kilometres driven.
Hybrid: $C = 28\,000 + 0.18k$.
Gas powered: $C = 21\,500 + 0.22k$

2. 162 500 km

3. a) 162 500 km

b) Example: I used the substitution method because the solution occurs when the costs are equal. So, one equation can be substituted easily into the other.

4. Example: To solve a system graphically, you must draw two graphs, either manually on a grid or on a graphing calculator, and determine the point of intersection. The coordinates of this point give the solution to the system. When solving by substitution, you must isolate one of the variables from one of the equations, substitute this expression into the other equation, and solve. This solution is then substituted into the other equation to solve for the second variable. When using elimination, the two equations are manipulated so that when they are added together one of the variables will be eliminated. Once you solve for the remaining variable, you then substitute that value into either equation to determine the value of the second variable.

Assessment	Supporting Learning
Assessment *as* Learning	
Reflect and Respond Listen as students discuss what they learned during the Investigate. Encourage them to generalize and reach a conclusion about their findings.	• Students may benefit from working with a partner. • Some students may need prompting in writing the system for #1. Ask students how they will need to rewrite their equations to graph them. • As a class, discuss student responses to #3. The discussion will assist students in answering #4. You may want to write the student responses to #3 on the board in point form for students to refer to. • For #4, you may want to have students refer back to #21 in section 9.2, as they have some similarities. Use students' response to #4 to determine whether they have clarified their thinking.

Link the Ideas

With the class, discuss what graphic organizer(s) students could use to display the advantages and disadvantages of the graphical method and the algebraic method. Have students work individually or in groups, but develop and complete individual organizers. You may wish to discuss with students how making their own organizers aids their concept development.

Example 1

You may wish to assist students in determining the two initial equations in the linear system by prompting them with the following questions:

- Which unit of time will you use?
- How many trees will Jeremy plant in 1.5 h before Shilan starts?
- If time requires one variable, what will require the other variable for this system?

Note that once the equations are written, a graphical solution will likely provide students with a quicker insight into the problem. Encourage students to discuss with classmates their boundaries for the window on their graphing calculator.

Example 2

You may wish to discuss with your class what bannock is.

Solve the system algebraically using one of the two methods. Then, have students use the alternative algebraic method to solve the same problem. Discuss with the class which method was more efficient.

Key Ideas

The first bullet is the most important. Emphasize that students can use any of these methods to solve a linear system. However, there are problems that lend themselves to one particular method. Go over the other points in the Key Ideas, and then discuss examples of linear systems that lend themselves better to each method.

Meeting Student Needs

- For Example 2, have students research the Folklorama Festival in Winnipeg, Folkfest in Saskatoon, or other multicultural festivals in northern and western Canada. See the related Web Link that follows in this Teacher's Resource. You may wish to have students determine which cultural groups that are of interest to them participate in these festivals and what their pavilions offer to visitors.
- Organize students into groups. Have each group work through the examples provided in this section, discuss the various methods illustrated, and create a graphic organizer for use during the Check Your Understanding and Chapter 9 Review. Enlarge one copy of the graphic organizer and post it in the classroom for reference. Alternatively, students can use their chapter Foldable.

ELL

- For Example 1, you may need to clarify what a *fundraiser* and a *seedling* are. Also, students may not be familiar with the word *plant* as a verb. Use pictures and descriptions to explain these terms.
- For Example 2, show students pictures of bannock and buffalo stew so they understand that these are types of food.

WWW Web Link

For more information about the Folklorama Festival in Winnipeg and Folkfest in Saskatoon, go to www.mhrmath10.ca and follow the links.

Answers

Example 1: Your Turn

$x = 160$ and $y = -160$. Check that students' graphs show the correct equations and intersection point $(160, -160)$. Example: I prefer the algebraic method (substitution) because the second equation needs to be rearranged before it can be graphed.

Example 2: Your Turn

$x = 9$ and $y = 2.5$. Example: The substitution method is more complicated because there are several steps required to isolate either variable, and the result involves a complicated decimal expression. The elimination method is much simpler.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 1 Have students do the Your Turn related to Example 1.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may be unable to determine where the values of 90 and 1.5 came from. Prompt students with these questions: – How much time passes between when Jeremy starts and when Shilan starts? (90 min) – How many trees can Jeremy plant in 1 min? (1) – What does 90 represent for graphing an equation? (y-intercept) – How many trees can Shilan plant in 1 min? (3 trees in 2 min; therefore, 1.5 trees in 1 min) – What does 1.5 represent for graphing an equation? (slope) • Remind students of the importance of putting the equations in the same form before solving algebraically. You may wish to ask whether the form they use to graph could also be used to solve algebraically. Pointing out this possibility may save students some work when writing the terms in the same form.

Assessment	Supporting Learning
Assessment *for* Learning	
Example 2 Have students do the Your Turn related to Example 2.	• Encourage students to verbalize their thinking. • You may wish to have students work with a partner. • Some students may use addition and subtraction as two different methods of solving the system. You may wish to indicate that you want them to use substitution and elimination. • An alternative approach to the Your Turn is to ask students to solve the system using two different methods and leave it open. This allows the student who prefers a visual representation to use graphing as an approach. • In the example, the rationale given for solving for variable *B* and not *S* is that it leads to fractional numbers. However, ensure students understand that it is acceptable to have fractional numbers. Some students believe that if they end up with a fractional expression, they have made an error. It would be helpful to complete an example so students see that they can arrive at the answer through multiple approaches.

Check Your Understanding

Practise

For some parts of #1 and 2, one algebraic method will result in fewer steps than the alternative algebraic method.

Apply

In #3, using an algebraic method is the most straightforward approach.

In #4, students must convert percents, which results in coefficients that are decimal numbers.

In #9, students write one linear equation for the number of trout in the lake as a function of time and a second equation for the number of trout eaten as a function of time.

For #10, you may wish to note that the relationship between depth and time is only linear between 60 m and 90 m. Outside of this range, the relation is no longer linear.

Extend

In #12, students need to determine the intersection points of each pair of lines. These three intersection points will correspond to the vertices of the triangle bounded by the three lines.

In #13, the solutions are very different, even though the numbers are quite similar in the two linear systems. Students will get some insight into this phenomenon when they graph the systems in part c).

Create Connections

In #15, to assist students in creating a linear system with a solution that cannot be expressed exactly on the graphing calculator, encourage them to start with a solution that has a non-terminating decimal representation.

Unit Project

The Unit 4 project question, #9, presents students with a contextual question to be solved graphically. You might encourage students to also solve it algebraically using a method of their choice. Some students may have already demonstrated a preference for one method over another. Encourage them to use the method that would be the most appropriate or that they are most comfortable with.

If students find this question challenging, suggest that they draw the graph first. Students can plot the points in the table and proceed from there. They may wish to refer back to Chapter 8, their Foldable for Chapter 8, or section 9.1. Remind them that the slope-intercept form can also be used as a system of equations.

For the algebraic approach, remind them to identify the meaning of their variables and what they are trying to determine, and to write the equations in the same form.

Meeting Student Needs

- Provide **BLM 9–7 Section 9.3 Extra Practice** to students who would benefit from more practice.
- For #1, ask each student to choose one of either part a), b), or c) and write a problem containing the information presented in the two equations given. Have them exchange the problem with another student to check that the material is accurate. Save these problems to use as examples or assignments in future classes.
- For #3 and 4, you may wish to have students research mean temperatures and transportation habits in their own community. They might then create questions based on the information they find.
- As an alternative to #5, if a class in the school is running a fundraiser based on sales, gather the data from that class on the cost and selling price of each item being sold. Then, present a problem to the class using the information gathered.
- For #7, have students research the history of Cirque du Soleil. They may be interested to know that the organization has its roots in Québec. See the related Web Link that follows in this Teacher's Resource.
- Allow students to choose three questions from #3 to 8. Tell students to solve each question using a different method—one graphically, one algebraically using substitution, and one algebraically using elimination. Ensure that students can justify the method of choice for each question.
- Some students will benefit from completing the unit project question, #9.

ELL

- Since #6 and 9 contain some difficult language, you may wish to have students work with a partner on these questions.
- In #8, some students may not be familiar with car rental companies. Use descriptions and pictures to assist their understanding.
- For #11, you may wish to have a student volunteer explain cross-country skiing and squash to students who are not familiar with these sports. Make available pictures and Internet video footage to help them visualize.

Enrichment

- Have students solve the following problem, then create several of their own, including complete solutions: A school store ordered 13 packages of paper and 4 packages of pencils, which came to $48.70. A second order of 6 packages of paper and 2 packages of pencils arrived, which cost $23.20. Determine the cost of one package of paper and one package of pencils. ($2.50 for paper and $4.05 for pencils)

Gifted

- Challenge students to create and solve a problem that meets the following criteria:
  - It involves two sizes of drink bottles.
  - The two sentences that follow must be filled in and included in the problem.
    _____ small bottle(s) and one large bottle can hold 4 L of water.
    One large bottle subtract _____ small bottle(s) results in 1 L of water.
  - The question must be "How many litres of water does each bottle hold?"

  (Example: Two small bottles and one large bottle can hold 4 L of water. One large bottle subtract one small bottle results in 1 L of water. How many litres of water does each bottle hold? Answer: The small bottle holds 1 L of water and the large bottle holds 2 L of water.)

Web Link

For more information about Cirque du Soleil, go to www.mhrmath10.ca and follow the links.

For graphing calculator activities related to systems of equations, go to www.mhrmath10.ca and follow the links.

Assessment	Supporting Learning
Assessment *for* Learning	
Practise and Apply Have students do #1–3, 5, and 8. Students who have no problems with these questions can go on to the remaining questions.	• For #1, remind students of the importance of writing the equations in the same form. Emphasize that they are checking their equations graphically. You may wish to go through all three and have students suggest which algebraic method might be the most appropriate to use for each question and explain why. • For #2, you may wish to review how to multiply out the decimal and fractional values before solving. Again, have students verbalize which method they feel would work best for solving each system. • For #3, ask students what they are looking for. Have them identify what the variables will represent. Ask which variable will have a larger number. Coach them through the first equation. These prompts should also assist them in explaining the variables in #5. • For #8, some students may find the question easier to approach if they think of it graphically. Ask them to identify the slope of each line. Then, ask them to identify the fixed number, or *y*-intercept, for each line. Have them verbalize what their window setting will be and compare with a partner before solving.
Unit 4 Project If students complete #9, which is related to the Unit 4 project, take the opportunity to assess how their understanding of the chapter outcomes is progressing.	• You may wish to provide students with **BLM 9–4 Chapter 9 Unit 4 Project** and have them finalize their answers. • Some students will find it easier to solve the project question by graphing first. Prompt students to consider what they have learned about graphing that would help them. You may wish to use the following prompts: – Is the trout population in the lake increasing or decreasing? – Plot the points for year and number of trout in the lake. Is the slope of this line positive or negative? – How can you determine the slope? – What is the slope? – What is the *y*-intercept, or starting value, of trout in the lake? – How can this information be used in an equation of the form $y = mx + b$? Have students follow the same process for the second equation. Then, on the same graph, have them graph the points and line for the number of trout eaten by osprey. • Students should now be able to answer parts a) and b). They can use their graph to answer what the point of intersection means.
Assessment *as* Learning	
Create Connections Have all students complete #14 and most students complete #15.	• For #14, guide students not to write a system of equations that has coefficients that are all 1. Suggest that only one coefficient can have that value. This will allow you and students themselves to better assess their understanding of the methods for solving systems and of the steps involved. • You may wish to brainstorm with the class rational numbers that cannot be expressed exactly on a graphing calculator (e.g., $\frac{1}{3}$, $\frac{1}{7}$, $\frac{2}{9}$).