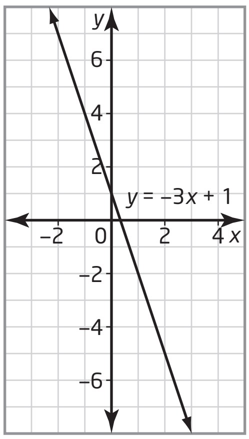
Answers
Chapter 1 Functions

1.1 Functions, Domain, and Range

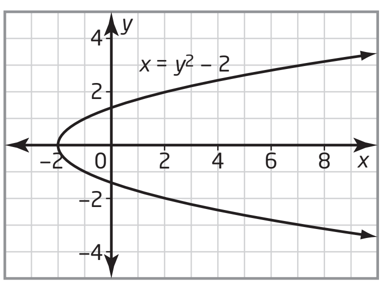
1. a) Yes, no vertical line will pass through more than one point.

b) No, any vertical line between $x = -6$ and $x = 6$ will pass through two points.

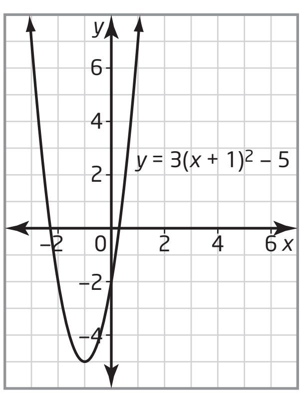
2. a) function



b) not a function



c) function



3. a) domain {2, 3, 4, 5, 6}, range {4, 6, 8, 10, 12}; function; for every element of the domain there is only one corresponding element of the range

b) domain {−6, −5, −4, −3}, range {2}, function; for every element of the domain there is only one corresponding element of the range

c) domain {5}, range {–4, −2, 0, 2, 4}; not a function; the single element of the domain corresponds with more than one element of the range

4. a) not a function; there are more range values than domain values

b) function; each domain value has only one range value

5. a) domain $\{x \in \mathbb{R}, -7 \le x \le 7\}$, range $\{y \in \mathbb{R}, -7 \le y \le 7\}$

b) domain $\{x \in \mathbb{R}, x \neq 5\}$, range $\{y \in \mathbb{R}, y \neq 0\}$

6. Answers may vary.

7. a) $A = -2x^2 + 60x$

b) domain $\{x \in \mathbb{R}, 0 < x < 30\}$, range $\{y \in \mathbb{R}, 0 < y < 450\}$

8. a) range {8}

b) range {−27, −18, −11, −6, −3}

c) range $\left\{\frac{8}{3}, 2, \frac{8}{5}, \frac{4}{3}, \frac{8}{7}\right\}$

9. a) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$

b) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \ge -1\}$

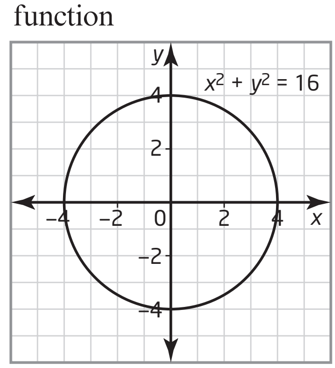
c) domain $\{x \in \mathbb{R}, x \le 3\}$, range $\{y \in \mathbb{R}\}$

d) domain $\{x \in \mathbb{R}, x \neq 4\}$, range $\{y \in \mathbb{R}, y \neq 0\}$

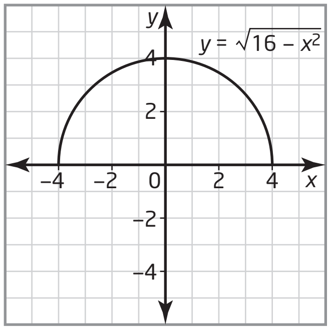
10. a) the graph is not a function; there are fewer elements in the domain than in the range

b) the graph is a function; for each value in the domain there is exactly one value in the range

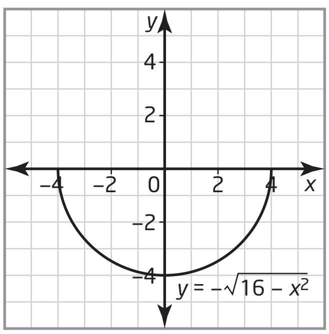
11. a) i) domain $\{x \in \mathbb{R}, -4 \le x \le 4\}$, range $\{y \in \mathbb{R}, -4 \le y \le 4\}$; not a function



ii) domain $\{x \in \mathbb{R}, -4 \le x \le 4\}$, range $\{y \in \mathbb{R}, 0 \le y \le 4\}$, function



iii) domain $\{x \in \mathbb{R}, -4 \le x \le 4\}$, range $\{y \in \mathbb{R}, -4 \le y \le 0\}$; function



b) Together, the semicircles in graphs ii) and iii) form the circle in graph i).

12. a) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, -3 \le x \le 7\}$; function

b) domain $\{y \in \mathbb{R}, x \le -3 \text{ or } x \le 3\}$, range $\{y \in \mathbb{R}\}$; not a function

1.2 Functions and Function Notation

1. a) $\frac{16}{5}, \frac{7}{5}, \frac{17}{10}; f : x \to -\frac{3}{5}x + 2$

b) $35, 5, \frac{5}{2}; f : x \to 6x^2 - 4x + 3$

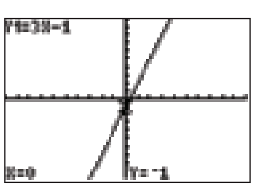
c) $-5, -14, -\frac{35}{4}; f : x \to -3(x + 1)^2 - 2$

d) $7, 7, 7; f : x \to 7$

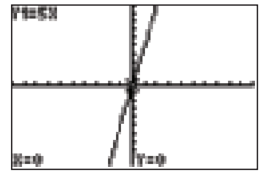
e) $-\frac{1}{7}, \frac{1}{5}, \frac{1}{3}; f : x \to \frac{1}{4x + 1}$

f) $\sqrt{7}, 1, \sqrt{2}; f : x \to \sqrt{3 - 2x}$

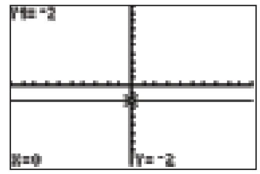
2. a) -1



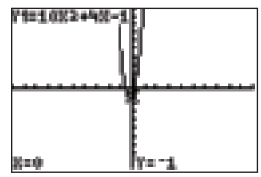
b) 0



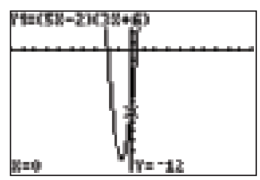
c) -2



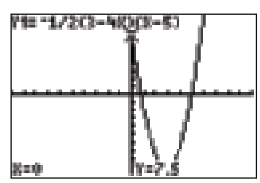
d) -1



e) -12



f) $\frac{15}{2}$



3. $x = 2$

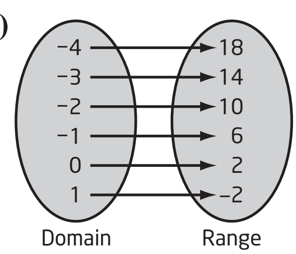
4. a) $a = -3; f(x) = -3x + 2$

b) $a = 6; f(x) = 6x + 2$

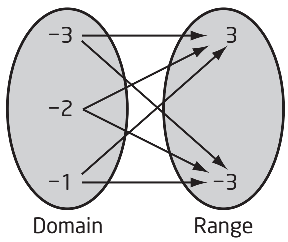
c) $a = 1.5; f(x) = 1.5x + 2$

d) $a = -6.2; f(x) = -6.2x + 2$

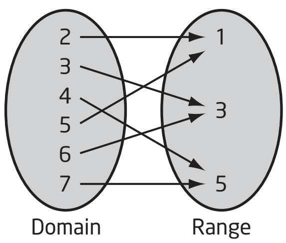
5. a)



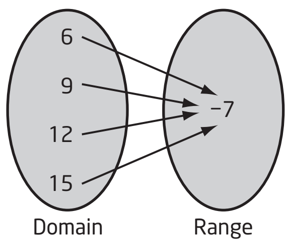
b)



c)



d)



6. **a)** function; each element of the domain corresponds with only one element of the range
b) not a function; some elements of the domain correspond with more than one element of the range
c) function; each element of the domain corresponds with only one element of the range
d) function; each element of the domain corresponds with only one element of the range

7. **a)** {(−8, 3), (−3, 1), (−1, 3), (2, 5), (2, −9)}
b) {(0, 3), (2, 8), (5, 7), (11, 3)}
c) {(−5, −2), (−2, 4), (0, 4), (7, −7), (7, −2)}
d) {(−5, 0), (−4, 2), (−3, 5), (−2, 6), (0, 7)}
e) {(−1, −3), (2, −3), (5, −3), (9, −3)}

8. **a)** not a function; some elements of the domain correspond with more than one element of the range
b) function; each element of the domain corresponds with only one element of the range
c) not a function; some elements of the domain correspond with more than one element of the range
d) function; each element of the domain corresponds with only one element of the range
e) function; each element of the domain corresponds with only one element of the range

9. **a)** $f : x \to -7x + 1$
b) $g : x \to x^2 + 7x - 5$
c) $h : b \to \sqrt{9b + 9}$
d) $r : k \to \dfrac{1}{5k - 3}$

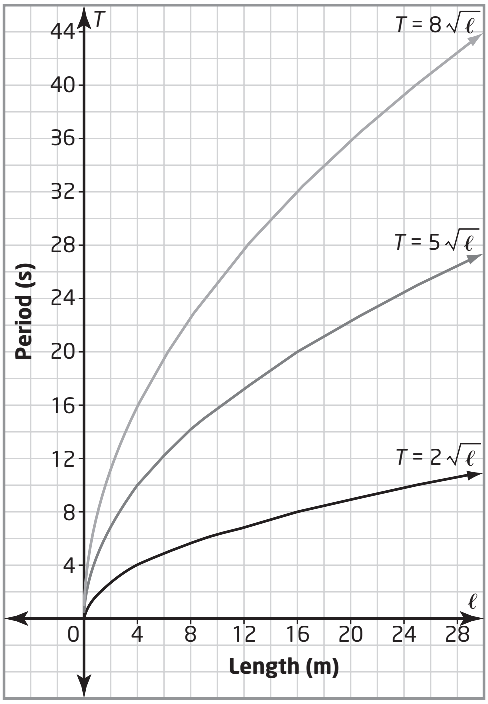
10. **a)** Substitute $\ell = 1.8$ in each equation.
Earth:
$T = 2\sqrt{1.8}$
$\doteq 2.7$
On Earth the period is approximately 2.7 s.
Moon:
$T = 5\sqrt{1.8}$
$\doteq 6.7$
On the moon the period is approximately 6.7 s.
Pluto:
$T = 8\sqrt{1.8}$
$\doteq 10.7$
On Pluto the period is approximately 10.7 s.

b) Substitute $T = 3$ in each equation and solve for ℓ.
Earth:
$5 = 2\sqrt{\ell}$
$2.5 = \sqrt{\ell}$ Square both sides.
$6.25 = \ell$
On Earth the length of the pendulum is 6.25 m.
Moon:
$5 = 5\sqrt{\ell}$
$1 = \sqrt{\ell}$
$1 = \ell$
On the moon the length of the pendulum is 1 m.
Pluto:
$5 = 8\sqrt{\ell}$
$\frac{5}{8} = \sqrt{\ell}$ Square both sides.
$\frac{25}{64} = \ell$ or $\ell \doteq 0.4$

On Pluto the length of the pendulum is approximately 0.4 m.

c) domain $\{\ell \in \mathbb{R}, \ell \geq 0\}$,
range $\{T \in \mathbb{R}, T \geq 0\}$

d)

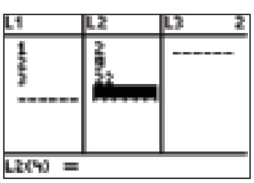


e) all are functions; each x-value corresponds with one y-value

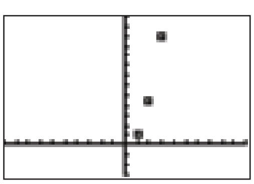
f) $T : \ell \rightarrow 2\sqrt{\ell}$
$T : \ell \rightarrow 5\sqrt{\ell}$
$T : \ell \rightarrow 8\sqrt{\ell}$

11. a) 60 000 **b) i)** 76 552 **ii)** 77 727
c) 2020 **d)** Yes

12. a)



b)



c) $y = 3x^2 - 2x + 1$

d) $f(-4) = 57, f(0) = 1, f(6) = 97$

13. a) domain $\{i \in \mathbb{R}, i \geq 0\}$,
range $\{A \in \mathbb{R}, 0 \leq A \leq 500\}$
b) Yes **c) i)** \$462.28 **ii)** \$428.67
d) i) 19.5% **ii)** 11.8%

14. a) Earth: $t : h \rightarrow \sqrt{\dfrac{80 - h}{4.9}}$;
Jupiter: $t : h \rightarrow \sqrt{\dfrac{80 - h}{12.8}}$

b) In each case, the expression under the radical sign cannot be less than zero. Since the denominators are both constants, $80 - h \geq 0$. Both h and t must be positive.
$80 - h \geq 0$ and $h \geq 0$
$-h \geq -80$ and $h \geq 0$
$0 \leq h \leq 80$
For both relations, the domain is $\{h \in \mathbb{R}, 0 \leq h \leq 80\}$ and the range is $\{t \in \mathbb{R}, t \geq 0\}$.

c) Yes; both relations are square root functions. For each relation, every value of the domain has exactly one value in the range.

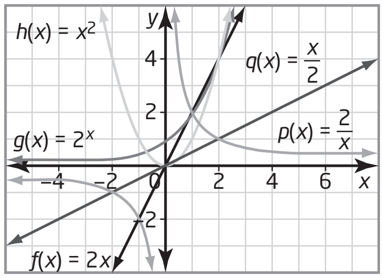
d) Substitute $h = 10$ in each equation and solve for t.
Earth:
$$t(10) = \sqrt{\frac{80 - 10}{4.9}}$$
$$\doteq 3.8$$
Jupiter:
$$t(10) = \sqrt{\frac{80 - 10}{12.8}}$$
$$\doteq 2.3$$
On Earth, the object reaches a height of 10 m after 3.8 s. On Jupiter, the object reaches a height of 10 m after 2.3 s.

15. a) Answers may vary. Sample answer:
$f(x) = 4x^2 - 3$
b) $f(x) = -0.5x^2 + 3$

16. a)

Function	Domain	Range
$f(x) = 2x$	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$
$g(x) = 2^x$	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}, y \geq 0\}$
$h(x) = x^2$	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}, y \geq 0\}$
$q(x) = \frac{x}{2}$	$\{x \in \mathbb{R}\}$	$\{y \in \mathbb{R}\}$
$p(x) = \frac{2}{x}$	$\{x \in \mathbb{R}, x \neq 0\}$	$\{y \in \mathbb{R}, y \neq 0\}$

b)



c) all are functions
d) $f(2) = g(2) = h(2) = 4$; $q(2) = p(2) = 1$
e) $f(0) = h(0) = q(0) = 0$;
$f(1) = g(1) = p(1) = 2$;
$g(-0.77) = h(-0.77) \doteq 0.59$;
$h(0.5) = q(0.5) = 0.25$;
$q(-2) = p(-2) = -1$;
$h(1.26) = p(1.26) \doteq 1.6$;
$f(-1) = p(-1) = -2$

17. a) $g(x) = \frac{-2}{x+3}$; $h(x) = 4x^2 - 5x$
b) i) −2 **ii)** −8 **iii)** 3 **iv)** 7

18. $f(x) = 2x^2 - 3x + 5$

1.3 Maximum or Minimum of a Quadratic Function

1. a) $y = (x - 4)^2 - 16$
b) $f(x) = (x + 8)^2 - 68$
c) $f(x) = \left(x + \frac{5}{2}\right)^2 + \frac{3}{4}$
d) $g(x) = \left(x - \frac{1}{2}\right)^2 + \frac{7}{4}$
e) $y = (x - 4)^2 - 22$
f) $y = \left(x - \frac{7}{2}\right)^2 - \frac{89}{4}$

2. a) (−2, −3); minimum
b) (3, 25); maximum
c) (−1, 8); maximum
d) $\left(\frac{5}{2}, -4\right)$; minimum
e) (2, 3); minimum
f) (−1, −1); maximum

3. a) (1, −3); minimum
b) (−3, 2); maximum
c) (−4, 5); maximum
d) (1, 3); minimum
e) (6, 0); minimum
f) (5, 11); maximum

4. Answers may vary.

5. $850 over cost

6. a) Let x represent the number of $1.25 price decreases and $D(x)$ represent the amount of the donation, in dollars.
$D(x)$ = (ticket price) × (number of tickets sold)
$D(x) = (31.25 - 1.25x)(104 + 8x)$
Since the equation is in factored form, solve $D(x) = 0$.
$0 = (31.25 - 1.25x)(104 + 8x)$
$31.25 - 1.25x = 0$ or $104 + 8x = 0$
$x = 25$ or $x = -13$
The maximum occurs at the average of the two zeros.
$x = \frac{25 - 13}{2}$
$= 6$
Tickets sold $= 104 + 8x$
$= 104 + 8(6)$
$= 152$
The maximum donation occurs when 152 tickets are sold.
b) ticket price $= 31.25 - 1.25x$ Substitute $x = 6$.
$= 31.25 - 1.25 \times 6$
$= 23.75$
The ticket price that maximizes the donation is $23.75.
c) $D(6) = 23.75 \times 152$
$= 3610$
The maximum donation is $3610.

7. a) 3.7 m **b)** after 0.8 s

8. 10 m by 20 m

9. a) Let x represent the larger number and let y represent the smaller number.
$x - y = 8$, so $y = x - 8$
Let $p(x)$ represent the product of the two numbers.
$p(x) = xy$
$= x(x - 8)$
$= x^2 - 8x$
Complete the square to determine the maximum product.
$p(x) = x^2 - 8x$
$= x^2 - 8x + 16 - 16$
$= (x - 4)^2 - 16$
The maximum product is −16.
b) The maximum product occurs when $x = 4$.
$y = 4 - 8$
$= -4$
The two numbers that produce the maximum product are 4 and −4.

10. a) 13 and 13 **b)** 338

11. 28.1 cm^2

12. a) 66.25 m **b)** 1.5 s **c)** 55 m

13. a) \$15.50 **b)** \$4805

14. a) 34.52 m **b)** 55 m

15. a) $y = -\frac{13}{16}x(x - 10)$
b) 20.3 m

16. a) 10.125 **b)** 2.25

17. $\frac{v^2}{2g} + h_0$

18. a) minimum = 1.875 V; maximum = 17 V
b) minimum: 2.25 min; maximum: 5 min

19. b must be an even integer

20. b must be divisible by 8

1.4 Skills You Need: Working With Radicals

1. a) $14\sqrt{3}$ **b)** $3\sqrt{30}$ **c)** $-4\sqrt{33}$
d) $-16\sqrt{21}$ **e)** $-15\sqrt{6}$ **f)** $6\sqrt{22}$
g) $-6\sqrt{35}$ **h)** $-49\sqrt{30}$

2. a) $3\sqrt{6}$ **b)** $7\sqrt{2}$ **c)** $12\sqrt{2}$
d) $5\sqrt{3}$ **e)** $6\sqrt{2}$ **f)** $5\sqrt{5}$
g) $4\sqrt{6}$ **h)** $3\sqrt{14}$ **i)** $4\sqrt{2}$ **j)** $6\sqrt{5}$

3. a) $13\sqrt{2}$ **b)** $14\sqrt{3}$
c) $-2\sqrt{11}$ **d)** $-4\sqrt{5} + 31\sqrt{6}$

4. a) $-33\sqrt{3}$ **b)** $-37\sqrt{2}$ **c)** $-23\sqrt{5}$
d) $31\sqrt{2}$ **e)** $-5\sqrt{3}$ **f)** $\sqrt{3} - 5\sqrt{5}$
g) $-6\sqrt{3} - 9\sqrt{2}$ **h)** $47\sqrt{2} - \sqrt{7}$

5. a) $5\sqrt{10}$ **b)** $-30\sqrt{2}$ **c)** $70\sqrt{2}$
d) $-240\sqrt{3}$ **e)** -126 **f)** -28
g) $-30\sqrt{10}$ **h)** $-162\sqrt{2}$

6. a) $2\sqrt{3} - \sqrt{6}$ **b)** $2\sqrt{15} - 4\sqrt{3}$
c) $6\sqrt{5} - 3\sqrt{10}$ **d)** $-2\sqrt{15} - 6\sqrt{21}$
e) $\sqrt{30} - 5\sqrt{2}$ **f)** $12\sqrt{22} + 20\sqrt{26}$
g) $6\sqrt{10} - 8\sqrt{15}$ **h)** $36\sqrt{3} - 72\sqrt{2}$

7. a) $1 - 5\sqrt{7}$ **b)** $2 + 6\sqrt{3}$
c) -43 **d)** $52 - 38\sqrt{2}$
e) $67 - 42\sqrt{2}$ **f)** 33

8. a) $-3\sqrt{3} - \frac{3}{2}\sqrt{2}$ **b)** $-4\sqrt{2}$

9. a)
$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(3\sqrt{5})(\sqrt{15}) \\ &= \frac{3}{2}(\sqrt{75}) \\ &= \frac{3}{2}\sqrt{25 \times 3} \\ &= \frac{3}{2}(5\sqrt{3}) \\ &= \frac{15\sqrt{3}}{2} \end{aligned}$$

b)
$$\begin{aligned} A &= \ell w \\ &= 7\sqrt{6}(3\sqrt{8}) \\ &= 21\sqrt{48} \\ &= 21\sqrt{16 \times 3} \\ &= 21(4\sqrt{3}) \\ &= 84\sqrt{3} \end{aligned}$$

c)
$$\begin{aligned} A &= \frac{h}{2}(b_1 + b_2) \\ &= \frac{\sqrt{20}}{2}(\sqrt{147} + \sqrt{48}) \\ &= \frac{\sqrt{4 \times 5}}{2}(\sqrt{49 \times 3} + \sqrt{16 \times 3}) \\ &= \frac{2\sqrt{5}}{2}(7\sqrt{3} + 4\sqrt{3}) \\ &= \sqrt{5}(11\sqrt{3}) \\ &= 11\sqrt{15} \end{aligned}$$

d)
$$\begin{aligned} A &= s^2 \\ &= (\sqrt{28} + \sqrt{54})^2 \\ &= (\sqrt{4 \times 7} + \sqrt{9 \times 6})^2 \\ &= (2\sqrt{7} + 3\sqrt{6})^2 \\ &= (2\sqrt{7})^2 + 2(2\sqrt{7})(3\sqrt{6}) + (3\sqrt{6})^2 \\ &= 4(7) + 12\sqrt{42} + 9(6) \\ &= 82 + 12\sqrt{42} \end{aligned}$$

10. $12\sqrt{35}$; explanations may vary

11. $6\sqrt{21}$ cm

12.
$$\begin{aligned} x^2 + x^2 &= (32\sqrt{2})^2 \\ 2x^2 &= 2048 \\ x^2 &= 1024 \\ x &= 32 \end{aligned}$$
The side length of the square game board is 32 cm.
Divide by 4 to determine the number of small squares along each side.
$32 \div 4 = 8$
Each side has 8 squares, so there are 64 squares in total.

13. $\sqrt{100-64}$ According to BEDMAS, subtract first, then take the square root.
$= \sqrt{36}$
$= 6$
$\sqrt{100} - \sqrt{64}$ According to BEDMAS, take the square root then subtract.
$= 10 - 8$
$= 2$
The order of the operations is reversed, so the answers are not the same.

14. **a)** $\frac{4+\sqrt{11}}{6}$ **b)** $\frac{7-2\sqrt{6}}{8}$ **c)** $\sqrt{13}$
d) $\frac{5+2\sqrt{7}}{9}$ **e)** $\frac{-2+\sqrt{5}}{4}$

15. **a)** **i)** $4\sqrt[3]{5}$ **ii)** $5\sqrt[3]{7}$ **iii)** 14
b) **i)** $2\sqrt[4]{15}$ **ii)** $3\sqrt[4]{10}$ **iii)** $6\sqrt[4]{7}$

16. **a)** $25\sqrt{m}$ **b)** $17\sqrt{c} - \sqrt{7ab}$
c) $8a\sqrt{5ab} + 10mn\sqrt{3n}$
d) $4b\sqrt{3ab} + 9c\sqrt{d}$

1.5 Solving Quadratic Equations

1. **a)** $-3, 2$ **b)** $-3, -4$ **c)** $-5, 5$
d) $-9, 3$ **e)** $-\frac{5}{3}, 3$ **f)** $\frac{2}{3}, -5$

2. **a)** $x = \frac{3}{2}$ or $-\frac{1}{3}$ **b)** $x = \frac{-3 \pm \sqrt{6}}{3}$
c) $x = \frac{-3 \pm \sqrt{3}}{2}$ **d)** $x = \frac{-7 \pm \sqrt{13}}{6}$
e) $x = -3 \pm \sqrt{5}$

3. **a)** no roots **b)** two roots
c) two roots **d)** one root
e) no roots **f)** no roots

4. **a)** $x = \frac{-5 \pm \sqrt{17}}{4}$ **b)** $x = 3 \pm \sqrt{2}$
c) $x = 3 \pm \sqrt{21}$ **d)** $x = \frac{10 \pm 2\sqrt{10}}{3}$
e) $x = \frac{24 \pm 4\sqrt{41}}{5}$

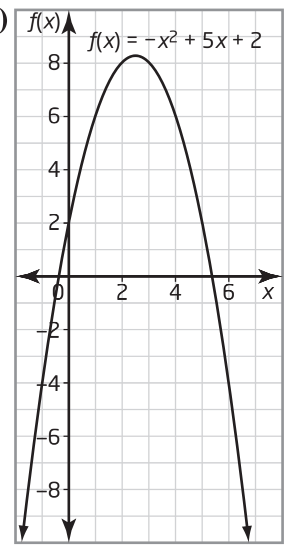
5. **a)** two distinct real roots
b) two equal real roots
c) no real roots
d) two distinct real roots
e) two equal real roots

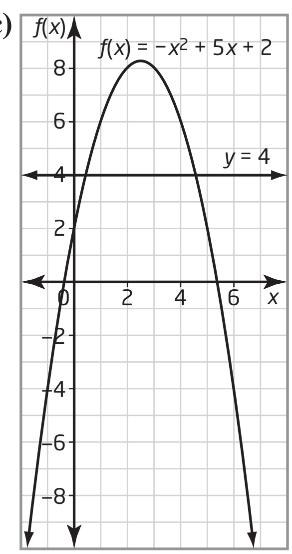
6. Methods may vary.
a) $-\frac{3}{2}, 4$ **b)** $\pm\frac{9}{2}$ **c)** $\frac{-5 \pm 2\sqrt{41}}{4}$
d) $0, \frac{3}{2}$ **e)** $3.78, -1.15$ **f)** $\frac{2}{3}, 1$
g) $\frac{5 \pm \sqrt{7}}{2}$ **h)** $-\frac{3}{4}, 1$

7. **a)** $k = \pm 4$
b) $k > 4$ or $k < -4$
c) $-4 < k < 4$

8. **a)**

x	$y = f(x)$
−2	−12
−1	−4
0	2
1	6
2	8
3	8
4	6
5	2
6	−4
7	−12

b)


c)


d) $x \doteq 0.5$, $x \doteq 4.5$ **e)** (0.4, 4), (4.6, 4)

9. **a)** $k = \pm 9, \pm 15$ **b)** $k = \pm 3, \pm 9$

c) Answers may vary. Possible values of k are $-4, -6, 14$.

10. Find the values of t when the height is 0.
Solve $h(t) = 0$.
$-4.9t^2 + 21.8t + 1.5 = 0$
Use the quadratic formula.

$$t = \frac{-21.8 \pm \sqrt{21.8^2 - 4(-4.9)(1.5)}}{2(-4.9)}$$
$$\doteq -0.068 \text{ or } 4.517$$

Time must be positive, so the ball will be in the air for approximately 4.5 s.

11. **a)** approximately 50 km/h
b) approximately 70 km/h
c) 100 km/h

12. Let x represent one of the numbers and let y represent the other number.
The sum of the numbers is 24, so $x + y = 24$.
Isolate y: $y = 24 - x$
The sum of the squares of the numbers is 306, so $x^2 + y^2 = 306$.
Substitute in .

$$x^2 + (24 - x)^2 = 306 \quad \text{Expand and simplify.}$$
$$x^2 + (576 - 48x + x^2) = 306$$
$$2x^2 - 48x + 270 = 0 \quad \text{Divide by 2.}$$
$$x^2 - 24x + 135 = 0$$
$$(x - 9)(x - 15) = 0$$
$$x = 9 \text{ or } x = 15$$

The two numbers are 9 and 15.

13. **a)** 498 m **b)** 10 s
c) 20 s **d)** 8 m

14. 9 cm, 12 cm

15. 5 cm

16. 11 and 12

17. 7.2 m by 7.2 m

18. 7, 8, 9 or −7, −8, −9

19. 18.6 A and 5.4 A

20. **a)** $x = \pm 3, \pm 1$ **b)** $x = -1 \pm \sqrt{3}$

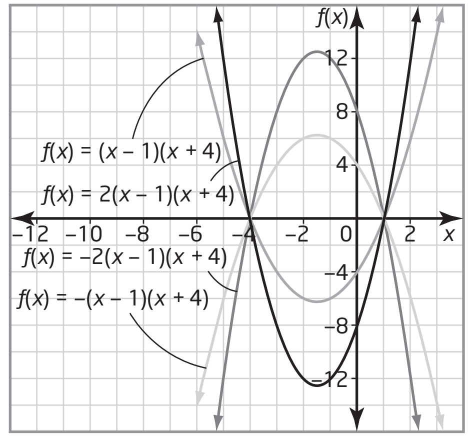
c) $x = \dfrac{-1 \pm \sqrt{37}}{6}$

21. **a)** $x = -1 \pm \sqrt{3}$ **b)** $x = \dfrac{2}{3}$

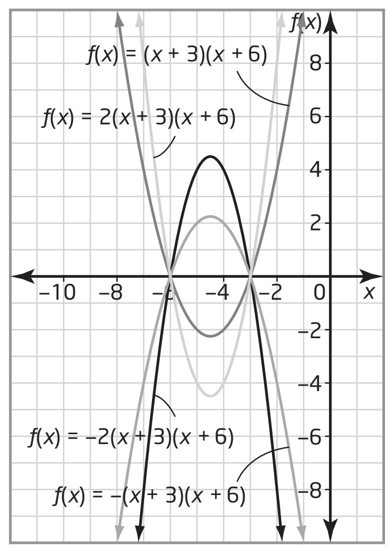
22. $a > 3 + 2\sqrt{2}$ and $a < 3 - 2\sqrt{2}$, $a \neq 0$

1.6 Determine a Quadratic Equation Given Its Roots

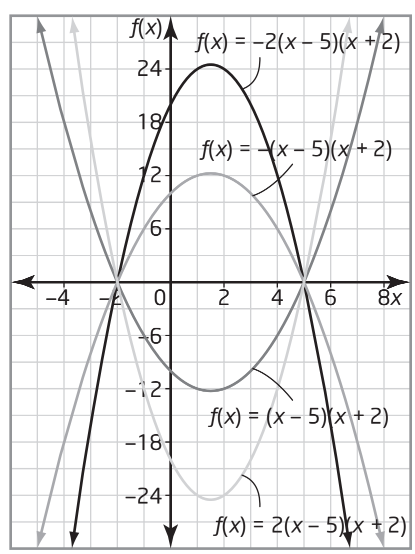
1. **a)** $f(x) = a(x - 1)(x + 4)$



b) $f(x) = a(x + 3)(x + 6)$



c) $f(x) = a(x - 5)(x + 2)$



2. a) $f(x) = a(x^2 + 3x - 4)$
b) $f(x) = a(x^2 + 9x + 18)$
c) $f(x) = a(x^2 - 3x - 10)$

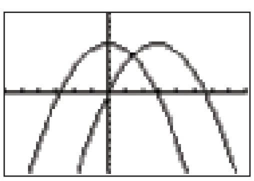
3. a) $y = 2(x - 4)(x + 3)$
b) $y = -0.5(x + 2)(x - 5)$
c) $y = 4x(3x - 2)$

4. a) $y = 2x^2 - 2x - 24$
b) $y = -0.5x^2 + 1.5x + 5$
c) $y = 12x^2 - 8x$

5. a) $y = -3x^2 + 12x + 3$
b) $y = -2x^2 - 12x - 6$
c) $y = 3x^2 - 24x + 52$
d) $y = -0.5x^2 - x + 6$

6. a) $y = 3(x - 2)^2 - 27$
b) $y = -2\left(x + \frac{1}{2}\right)^2 + \frac{49}{2}$
c) $y = 2(x + 1)^2 - 8$
d) $y = -3(x + 2)^2 + 48$

7. a) $y = -\frac{1}{32}x^2 + \frac{49}{8}$
b) $\frac{49}{8}$ m or 6.125 m
c) 14 m
d) $y = -\frac{1}{32}x^2 + \frac{7}{8}x$
e) The graph of $y = -\frac{1}{32}x^2 + \frac{7}{8}x$ is the graph of $y = -\frac{1}{32}x^2 + \frac{49}{8}$ translated 14 m to the right.
f)



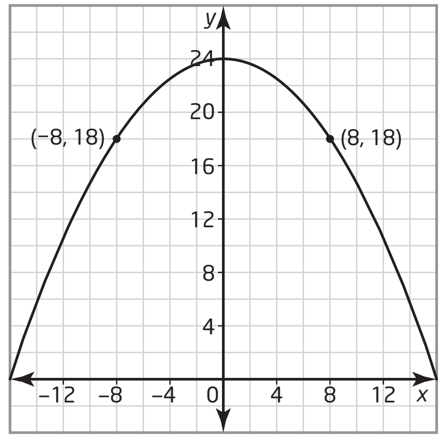
8. a) $y = -\frac{3}{16}x^2$
b) $y = \frac{1}{5}(x - 3)^2$
c) $y = -(x + 6)^2$
d) $y = -\frac{2}{3}(x - 4)^2$
e) $y = 5(x + 1)^2$

9. a) $y = -5x^2 - 5x + 10$
b) $y = 2x^2 - 2x - 12$
c) $y = \frac{3}{4}x^2 - 3x - 9$

10. If my graphs pass through the given points and have the same vertex and direction of opening as the given graphs, I know my equations are correct.

11. $ac > 9$

12. a) The width of the arch is 32 m, so half of the width is 16 m. When the vertex is on the y-axis, the x-intercepts are −16 and 16. A point that is 8 m from one end of the arch and 18 m high is (8, 18); another point on the other side is (−8, 18). The height of the arch is unknown.



b) Use the x-intercepts and the coordinates of the known point, (8, 18), to determine the factored form of the equation.

$y = a(x - 16)(x + 16)$ Substitute (8, 18) to find the value of a.

$18 = a(8^2 - 256)$
$18 = -192a$
$a = -\frac{3}{32}$

$f(x) = -\frac{3}{32}(x - 16)(x + 16)$ is the factored form of the equation.
The standard form, which in this case is also the vertex form, is
$f(x) = -\frac{3}{32}x^2 + 24$.

c) The vertex is (0, 24), so the maximum height of the arch is 24 m.

13. a) In this situation the x-intercepts are 0 and 32, and a point that is 8 m from one end of the arch is (8, 18). Use this information to write the factored form of the equation.

$y = ax(x - 32)$ Substitute $x = 8$ and $y = 18$.

$18 = a(8)(8 - 32)$
$18 = a(8)(-24)$
$18 = -192a$
$a = -\frac{18}{192}$

$a = -\frac{3}{32}$

The factored form of the equation is

$y = -\frac{3}{32}x(x - 32)$.

In standard form the equation is

$y = -\frac{3}{32}x^2 + 3x$.

b) The maximum height occurs at the vertex, which is halfway between the x-intercepts. The x-coordinate of the vertex is $x = 16$.

Substitute this value into the equation and solve for y.

$$\begin{aligned} y &= -\frac{3}{32}x^2 + 3x \\ &= -\frac{3}{32}(16)^2 + 3(16) \\ &= 24 \end{aligned}$$

The maximum height of the arch is 24 m. The height is the same as the one found in question 12, which makes sense since only the orientation has changed, not the size of the arch represented by the function.

14. Yes. Explanations may vary.

15. a) $f(x) = -2(x - 2)(x + 1)(x - 4)$;

$f(x) = -2x^3 + 10x^2 - 4x - 16$

b) $f(x) = a(2x - 1)(x + 2)(x - 3)$

c) $f(x) = -\frac{4}{3}x^3 + 2x^2 + \frac{22}{3}x - 4$

1.7 Solve Linear-Quadratic Systems

1. **a)** $(3, 24), (-2, -1)$ **b)** $\left(-\frac{4}{3}, -9\right), \left(\frac{5}{2}, \frac{5}{2}\right)$

c) $\left(\frac{3}{4}, \frac{5}{8}\right), (-1, 2)$ **d)** $(-7, -15), (2, -6)$

2. Answers may vary.

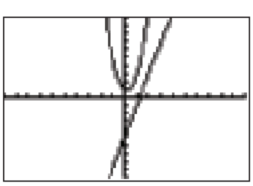
3. **a)** no intersection

b) two points of intersection

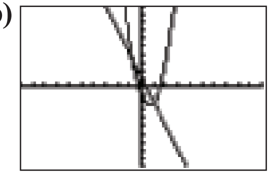
c) one point of intersection

d) two points of intersection

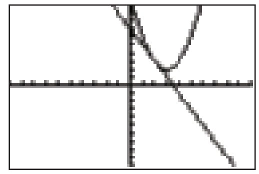
4. **a)**



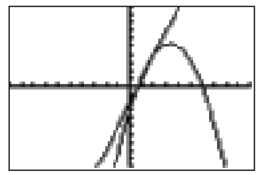
b)



c)



d)



5. **a)** $-\frac{41}{8}$ **b)** 3 **c)** 5 **d)** 8

6. Answers may vary.

7. $(4, 12), (-5, 57)$

8. To determine if the two paths intersect, set the equations equal.

$$\begin{aligned} -8x^2 + 720x + 56\,800 &= -960x + 145\,000 \\ -8x^2 + 1680x - 88\,200 &= 0 \quad \text{Divide by } -8. \\ x^2 - 210x + 11\,025 &= 0 \quad \text{Factor or use the quadratic formula.} \\ (x - 105)^2 &= 0 \\ x &= 105 \end{aligned}$$

Substitute $x = 105$ into $y = -960x + 145\,000$ and solve for y.

$$\begin{aligned} y &= -960(105) + 145\,000 \\ &= 44\,200 \end{aligned}$$

The paths will intersect at (105, 44 200).

9. Answers may vary.

10. **a)** $k > -\frac{4}{3}$ **b)** $k = -\frac{4}{3}$ **c)** $k < -\frac{4}{3}$

11. Equate the expressions and simplify.

$$\begin{aligned} kx^2 + 3x + 10 &= -5x + 3 \\ kx^2 + 8x + 7 &= 0 \quad \text{Use the discriminant; } a = k, b = 8, c = 7. \end{aligned}$$

$$\begin{aligned} &b^2 - 4ac \\ &= 8^2 - 4(k)(7) \\ &= 64 - 28k \end{aligned}$$

a) Two points of intersection occur when the discriminant is positive.

$$\begin{aligned} 64 - 28k &> 0 \\ 64 &> 28k \\ k &< \frac{16}{7} \end{aligned}$$

b) One point of intersection occurs when the discriminant is zero.

$$\begin{aligned} 64 - 28k &= 0 \\ 64 &= 28k \\ k &= \frac{16}{7} \end{aligned}$$

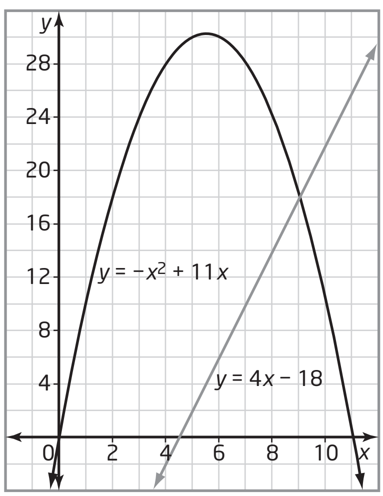
c) No points of intersection occur when the discriminant is negative.

$$\begin{aligned} 64 - 28k &< 0 \\ 64 &< 28k \\ k &> \frac{16}{7} \end{aligned}$$

12. (−16.8, −2.2), (24.5, 0.7)

13. Answers may vary. Sample answer: The line $x = 3$ is a vertical line that intersects or cuts through the parabola such that part of the line is above the parabola and part of it is below.

14. a)



b) (9, 18)

15. $10{:}30 < x < 14{:}30$, or between 10:30 a.m. and 2:30 p.m.

16. a) 7.1 s **b)** 112.8 m

17. 90 m by 160 m

18. $y = x - 10$

19. Two parabolas may have two, one, or no points of intersection. Equations and sketches will vary.

20. a) Estimates will vary. Sample answer: (−8, 6), (9.6, −2.8)

b) Estimates will vary. Sample answer: (3.5, 0.9), (1.5, −3)

21. There are no real roots.

Chapter 1 Review

1. a) Yes; vertical line test is satisfied

b) No; vertical line test is not satisfied

2. a) Yes; each domain value has only one range value

b) No; the domain value has four range values

3. a) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \leq 2\}$

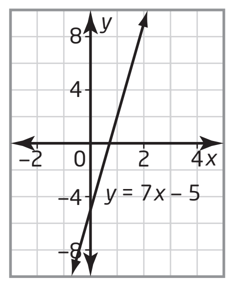
b) domain $\left\{x \in \mathbb{R}, x \leq \frac{4}{3}\right\}$, range $\{y \leq 0, y \in \mathbb{R}\}$

4. Answers may vary.

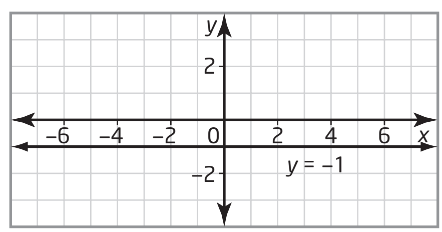
5. a) range {−5, −1, 3, 4, 11}

b) range {−7, −5, 1}

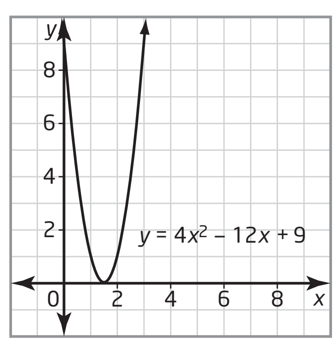
6. a) 16



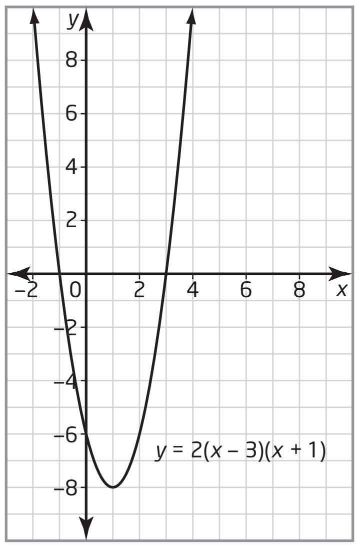
b) −1



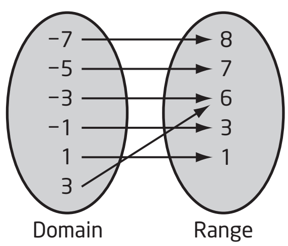
c) 9



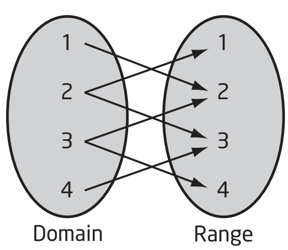
d) 0



7. **a)** function; each domain value has a single range value



b) not a function; the values 2 and 3 in the domain each have two values in the range



8. **a)** the width of the pool
b) domain $\{x \in \mathbb{R}, x > 0.2\}$, range $\{w \in \mathbb{R}, x \geq 0\}$, where w is the width of the pool
c) Yes; each element of the domain corresponds with one element of the range
d) $w = 18$ ft

9. **a)** (−3, 29); maximum
b) $\left(\frac{3}{2}, 16\right)$; maximum
c) (−6, 1); minimum

10. **a)** $\left(\frac{1}{2}, -3\right)$; minimum
b) $\left(-\frac{3}{4}, 2\right)$; minimum

11. **a)** \$25 000 **b)** \$3000

12. **a)** $7\sqrt{3}$ **b)** $2\sqrt{15}$

13. **a)** $2\sqrt{7} - 2\sqrt{3}$ **b)** $41\sqrt{6} + 36\sqrt{7}$
c) $-16\sqrt{2} - 2\sqrt{7}$ **d)** $36 - 40\sqrt{3}$
e) $48\sqrt{3} - 96\sqrt{2} - 96$ **f)** $\sqrt{10} - 2$
g) $3\sqrt{5}$

14. $A = 128$

15. $x = \frac{1}{3}$ or $-\frac{1}{6}$

16. $x = \dfrac{3 \pm \sqrt{7}}{2}$

17. two distinct real roots

18. 68.4 m by 131.6 m

19. **a)** $y = -24x^2 + 42x + 45$
b) $y = -x^2 + 10x - 22$

20. Answers may vary. Sample answer: $h = -4.8(x - 5)^2 + 120$

21. $y = 5(x + 1)^2$

22. $\left(-\frac{5}{2}, -17\right)$, (1, −3)

23. one

24. −3

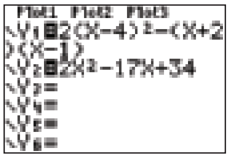
25. after 8.3 s

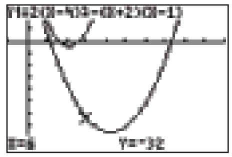
Chapter 1 Math Contest

1. B
2. D
3. A
4. B
5. C
6. A
7. C
8. B
9. D
10. C

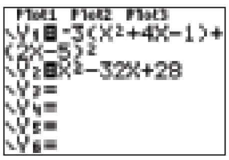
Chapter 2 Transformation of Functions

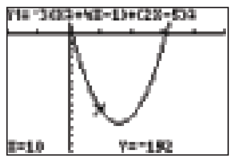
2.1 Functions and Equivalent Algebraic Expressions

1. a) 

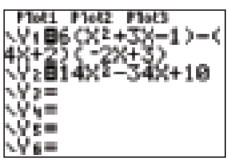


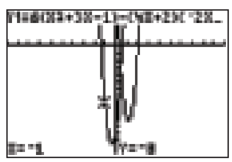
No. The functions do not appear to be equivalent.

b) 

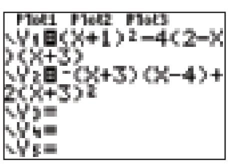


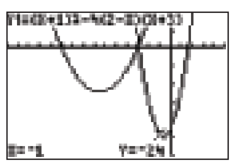
Yes. The functions appear to be equivalent.

c) 

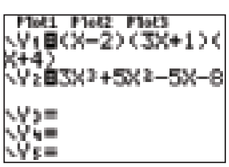


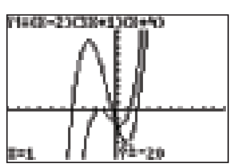
No. The functions do not appear to be equivalent.

d) 

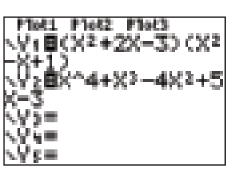


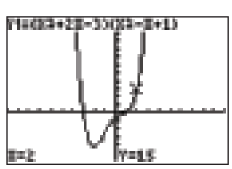
No. The functions do not appear to be equivalent.

e) 



No. The functions do not appear to be equivalent.

f) 



Yes. The functions appear to be equivalent.

2. a) $f(6) = -32$; $g(0) = 4$
b) Answers may vary.
c) $f(0) = -12$; $g(0) = 10$
d) $s(0) = -23$; $t(0) = 30$
e) $f(1) = -20$; $g(0) = -5$
f) Answers may vary.

3. a) $x \neq -3$
b) $x \neq -4$

4. a) $x \neq 3, x \geq 2$
b) $x \neq -2, x \neq 2$

5. a) Yes; $x \neq -5$
b) No; $g(x) = \frac{x+6}{x+3}$; $x \neq -6$; $x \neq -3$
c) No; $g(x) = x + 1$; $x \neq -5$
d) Yes; $x \neq -3$; $x \neq -\frac{1}{3}$

6. a) $\frac{1}{x+3}$; $x \neq -3, x \neq 7$
b) $-\frac{x+3}{2}$; $x \neq -3, x \neq 4$
c) $\frac{x+5}{x+3}$; $x \neq \frac{3}{2}, x \neq -3$
d) $\frac{x+4}{x+5}$; $x \neq -5, x \neq 9$
e) $\frac{x}{x+1}$; $x \neq -1, x \neq 2$
f) $\frac{2x-1}{2(x+1)}$; $x \neq -1, x \neq \frac{2}{3}$
g) $\frac{2x+1}{2x-1}$; $x \neq -\frac{1}{2}, x \neq \frac{1}{2}$
h) $\frac{x-17}{3-x}$; $x \neq -3, x \neq 0, x \neq 3$

7. a)

x	y
−4	7
−2	19
0	31
3	49
10	91

b)

x	y
−4	undefined
−2	undefined
0	0
3	$\frac{72}{5}$
10	$\frac{75}{2}$

$x \neq -4, x \neq -2$

c)

x	y
-4	0
-2	$-\frac{1}{6}$
0	undefined
3	$-\frac{7}{2}$
10	$\frac{7}{6}$

$x \neq 0$

d)

x	y
-4	39
-2	-11
0	9
3	24
10	-11

8. a) Area of large circle is $A = \pi r^2$
The diameter of the small circle is 8 cm, so the radius is 4 cm.
The area of the small circle is
$A = \pi(4)^2$
$= 16\pi$
Subtract the area of the small circle from the area of the large circle.
Shaded Area $= \pi r^2 - 16\pi$

b) Shaded Area $= \pi r^2 - 16\pi$
$= \pi(r^2 - 16)$
$= \pi(r - 4)(r + 4)$

c) Since this function represents area, it is restricted to positive r-values that result in positive area. The domain is $\{x \in \mathbb{R}, x > 4\}$.

9. a) $V = \pi(2x + 1)^2(x - 2)$;
$V = \pi(4x^3 - 4x^2 - 7x - 2)$

b) $SA = 2\pi(2x + 1)^2 + 2\pi(2x + 1)(x - 2)$;
$SA = 2\pi(6x^2 + x - 1)$

c) $56.5\ \text{m}^3$; $245.0\ \text{m}^2$

d) $\{x \in \mathbb{R}, x > 2\}$

10. a) $V(x) = (3x + 2)(2x - 0.5)(x + 1)$

b) $SA(x) = 2(3x + 2)(2x - 0.5) + 2(2x - 0.5)(x + 1) + 2(3x + 2)(x + 1)$

c) $25.875\ \text{m}^3$, $57.875\ \text{m}^2$; $84\ \text{m}^3$, $125\ \text{m}^2$; $149.625\ \text{m}^3$, $183.5\ \text{m}^2$

11. Expand and simplify each function.

$$\begin{aligned} f(x) &= x^2 + \left[\frac{1}{2}(x - 1)(x + 1)\right]^2 \\ &= x^2 + \left[\frac{1}{2}(x^2 - 1)\right]^2 \\ &= x^2 + \frac{1}{4}(x^2 - 1)^2 \\ &= x^2 + \frac{1}{4}(x^4 - 2x^2 + 1) \\ &= x^2 + \frac{1}{4}x^4 - \frac{1}{2}x^2 + \frac{1}{4} \\ &= \frac{1}{4}x^4 + \frac{1}{2}x^2 + \frac{1}{4} \end{aligned}$$

$$\begin{aligned} g(x) &= \left[\frac{1}{2}(x^2 + 1)\right]^2 \\ &= \frac{1}{4}(x^2 + 1)^2 \\ &= \frac{1}{4}(x^4 + 2x^2 + 1) \\ &= \frac{1}{4}x^4 + \frac{1}{2}x^2 + \frac{1}{4} \end{aligned}$$

$f(x)$ and $g(x)$ are equivalent expressions.

12. The graph of $f(x)$ is the line $y = 2x + 1$, with slope 2, y-intercept 1, and two holes, one at $\left(-\frac{2}{3}, -\frac{1}{3}\right)$ and the other at $(4, 9)$.

2.2 Skills You Need: Operations With Rational Expressions

1. a) $\frac{16y^2}{x^2}$; $x \neq 0$

b) $48x^4$; $x \neq 0$

c) $12b^3$; $b \neq 0$

d) $\frac{1}{3x}$; $x \neq 0$, $y \neq 0$

e) $16ab$; $a \neq 0$, $b \neq 0$

f) $18pr^2$; $p \neq 0$, $q \neq 0$, $r \neq 0$

2. a) 4; $x \neq 6$

b) 9; $x \neq -2$, $x \neq 0$

c) $\frac{x - 8}{x - 6}$; $x \neq -2$, $x \neq 6$

d) $\frac{7x}{x + 4}$; $x \neq -4$, $x \neq -\frac{1}{2}$, $x \neq 0$

e) $\frac{4x}{x + 6}$; $x \neq -6$, $x \neq 0$, $x \neq 5$

f) $\frac{x + 1}{x + 3}$; $x \neq -8$, $x \neq -4$, $x \neq -3$

g) $\frac{1}{x-4}$; $x \neq -2, x \neq -1, x \neq 1, x \neq 4$

h) $\frac{x-3}{x+6}$; $x \neq -6, x \neq -2, x \neq 4$

3. a) 2; $x \neq -1, x \neq 0$

b) x; $x \neq 3$

c) $\frac{x-5}{x+10}$; $x \neq -12, x \neq -10, x \neq 5$

d) $\frac{x+15}{4}$; $x \neq -6, x \neq 0$

e) $\frac{1}{6}$; $x \neq 0, x \neq 9$

f) $\frac{5x(x+13)}{x-5}$; $x \neq -2, x \neq 0, x \neq 5$

g) $\frac{3}{x+1}$; $x \neq -2, x \neq -1, x \neq 2$

h) $\frac{x+7}{x+5}$; $x \neq -5, x \neq -2, x \neq 1, x \neq 4$

4. a) $\frac{5x-2}{12}$

b) $\frac{-2x+75}{21}$

c) $-\frac{1}{28x}$; $x \neq 0$

d) $\frac{8+9a}{2ab}$; $a \neq 0, b \neq 0$

e) $\frac{44a-21b^2}{48a^2b^2}$; $a \neq 0, b \neq 0$

f) $\frac{3+7a}{10a}$; $a \neq 0$

g) $\frac{-3(x-7)}{(x-3)(x+3)}$; $x \neq -3, x \neq 3$

h) $\frac{9(2x+1)}{(x+4)(x-5)}$; $x \neq -4, x \neq 5$

i) $\frac{20x+37}{(x-2)(x+5)}$; $x \neq -5, x \neq 2$

5. a) $\frac{2x^2-3x+3}{(x-6)(x-3)}$; $x \neq 3, x \neq 6$

b) $\frac{-3x^2-8x-7}{(x-5)(x+3)}$; $x \neq -3, x \neq 5$

c) $\frac{2x^2+4x-3}{(x-2)(x+2)}$; $x \neq -2, x \neq 2$

d) $\frac{3x^2+4x}{(x-1)(x+2)}$; $x \neq -2, x \neq 1$

e) $\frac{-x-10}{(x+8)(x+6)}$; $x \neq -8, x \neq -6, x \neq -5,$ $x \neq -3$

f) $\frac{4x-7}{(x-4)(x+5)}$; $x \neq -7, x \neq -5, x \neq 4$

g) $\frac{1}{x+2}$; $x \neq -3, x \neq -2, x \neq 1$

h) $\frac{4x^2-37x+21}{(x+7)(x-10)}$; $x \neq -7, x \neq 2, x \neq 5,$ $x \neq 10$

6. a) Use the formula for time: $t = \frac{d}{v}$.
The total distance is 40 km, so half the total distance is 20 km.
Let t_1 represent David's time for the first half of the race and let t_2 represent David's time for the second half of the race. Let t represent the total time for the race.
The time for the first half of the race is represented by $t_1 = \frac{20}{x}$.
The time for the second half of the race is represented by $t_2 = \frac{20}{x-8}$.

$$\begin{aligned} t &= t_1 + t_2 \\ &= \frac{20}{x} + \frac{20}{x-8} \\ &= \frac{20(x-8)+20x}{x(x-8)} \\ &= \frac{20x-160+20x}{x(x-8)} \\ &= \frac{40x-160}{x(x-8)} \end{aligned}$$

The total time taken for the race is given by $t = \frac{40x-160}{x(x-8)}$.

b) Substitute $x = 35$ in t.

$$\begin{aligned} t &= \frac{40(35)-160}{35(35-8)} \\ &\doteq 1.3 \end{aligned}$$

David completed the race in approximately 1.3 h, which is 1 h and 18 min.

7. a) $\frac{2}{x-5}$; $x \neq 5$

b) $\frac{x+4}{x-4}$; $x \neq 4$

c) $\frac{2a-3}{5-2a}$; $a \neq \frac{5}{2}$

d) $\frac{b-4}{2b-3}$; $b \neq \frac{3}{2}$

e) $\frac{5x+2}{x-2}$; $x \neq 2$

f) $\frac{-x-1}{4x-3}$; $x \neq \frac{3}{4}$

g) $\frac{5b-8}{b+2}$; $x \neq -2$

h) $\frac{-3c+1}{5c-1}$; $x \neq \frac{1}{5}$

8. a) The dimensions of the box are
$\ell = 120 - 2x$, $w = 100 - 2x$, and $h = x$.
$V = \ell wh$
$V(x) = (120 - 2x)(100 - 2x)x$

b) $SA(x) = \ell w + 2\ell h + 2wh$
$SA(x) = (120 - 2x)(100 - 2x) + 2(120 - 2x)x + 2(100 - 2x)x$

c) The side length of the square must be positive, so $x > 0$. The shortest side is 100 cm, so $x < 50$ cm, since it would be impossible to cut two squares with sides larger than 50 cm from a side that is only 100 cm long. The domain is $\{x \in \mathbb{R}, 0 < x < 50\}$.

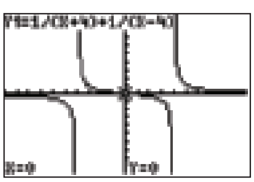
$$\frac{V}{SA} = \frac{x(120-2x)(100-2x)}{(120-2x)(100-2x) + 2x(120-2x) + 2x(100-2x)}$$
$$= \frac{4x(60-x)(50-x)}{4(60-x)(50-x) + 4x(60-x) + 4(50-x)}$$
$$= \frac{x(3000 - 60x - 50x + x^2)}{3000 - 6x - 50x + x^2 + 60x - x^2 + 50x - x^2}$$
$$= \frac{x^3 - 110x^2 + 3000x}{x^2 + 3000}$$

e) The denominator cannot be zero.
The restrictions are $x \neq \pm\sqrt{3000}$, or $x \neq \pm 54.77$.

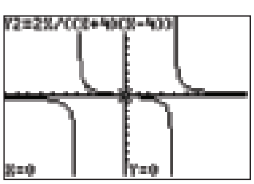
9. a) $\dfrac{6x^2 + x - 12}{10x - 2}$

b) $x \neq -\dfrac{3}{2}$, $x \neq \dfrac{1}{5}$

10. a)



b) $f(x) = \dfrac{2x}{(x-4)(x+4)}$



c) Answers may vary. Sample answer: The two graphs are the same. The restrictions for both graphs are $x \neq -4$ and $x \neq 4$. These are the vertical asymptotes of the graph.

11. $\dfrac{-19x}{3(x-1)}$; $x \neq -4$, $x \neq -3$, $x \neq -\dfrac{4}{3}$, $x \neq -1$, $x \neq 0$, $x \neq \dfrac{3}{2}$, $x \neq 1$, $x \neq 4$

12. a) $\dfrac{x+y}{y-x}$; $x \neq 0$, $y \neq 0$, $x \neq y$

b) $\dfrac{x^2 + y^2}{y^2 - x^2}$; $x \neq 0$, $y \neq 0$, $y \neq \pm x$

13. a) $\dfrac{-2a-3}{(a+2)^2}$; $a \neq -2$

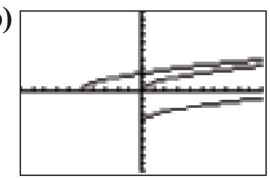
b) $\dfrac{2a+3}{3a+5}$; $a \neq -2$, $a \neq -\dfrac{5}{3}$

2.3 Horizontal and Vertical Translations of Functions

1. a)

x	$f(x) = \sqrt{x}$	$r(x) = f(x) - 4$	$s(x) = f(x+5)$
0	0	−4	$\sqrt{5}$
1	1	−3	$\sqrt{6}$
4	2	−2	3
9	3	−1	$\sqrt{14}$

b)



c) $r(x)$ translates the points 4 units down.
$s(x)$ translates the points 5 units to the left.

2. a) A′(−8, 3), B′(−5, 1), C′(−2, 1), D′(1, 10), E′(3, 10), F′(6, 8), G′(8, 8)

b) A′(−8, −7), B′(−5, −9), C′(−2, −9), D′(1, 0), E′(3, 0), F′(6, −2), G′(8, −2)

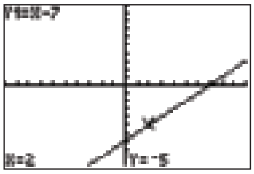
c) A′(−5, −3), B′(−2, −5), C′(1, −5), D′(4, 4), E′(6, 4), F′(9, 2), G′(11, 2)

d) A′(−9, −3), B′(−6, −5), C′(−3, −5), D′(0, 4), E′(2, 4), F′(5, 2), G′(7, 2)

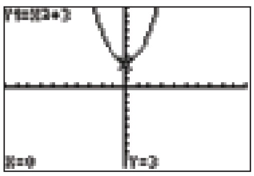
e) A′(−7, 6), B′(−4, 4), C′(−1, 4), D′(2, 13), E′(4, 13), F′(7, 11), G′(9, 11)

f) A′(−10, −7), B′(−7, −12), C′(−4, −12), D′(−1, −3), E′(1, −3), F′(4, −5), G′(6, −5)

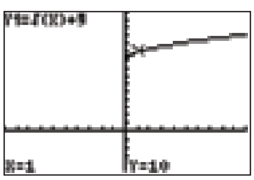
3. a) $f(x) = x$; $y = f(x) - 7$; translate the graph of $f(x)$ down 7 units;
$f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$;
$g(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$



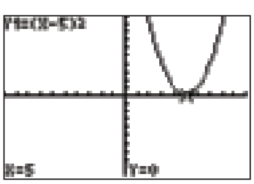
b) $f(x) = x^2$; $y = f(x) + 3$; translate the graph of $f(x)$ up 3 units;
$f(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \geq 0\}$;
$g(x)$:domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \geq 3\}$



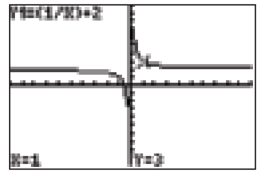
c) $f(x) = \sqrt{x}$; $y = f(x) + 9$; translate the graph of $f(x)$ up 9 units;
$f(x)$: domain $\{x \in \mathbb{R}, x \geq 0\}$,
range $\{y \in \mathbb{R}, y \geq 0\}$;
$g(x)$: domain $\{x \in \mathbb{R}, x \geq 0\}$,
range $\{y \in \mathbb{R}, y \geq 9\}$



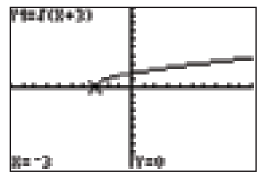
d) $f(x) = x^2$; $y = f(x - 5)^2$; translate the graph of $f(x)$ right 5 units;
$f(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \geq 0\}$;
$g(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \geq 0\}$



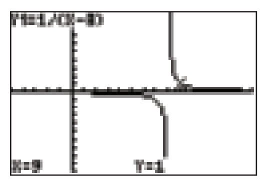
e) $f(x) = \frac{1}{x}$; $y = f(x) + 2$; translate the graph of $f(x)$ up 2 units;
$f(x)$: domain $\{x \in \mathbb{R}, x \neq 0\}$,
range $\{y \in \mathbb{R}, y \neq 0\}$;
$g(x)$: domain $\{x \in \mathbb{R}, x \neq 0\}$,
range $\{y \in \mathbb{R}, y \neq 0\}$



f) $f(x) = \sqrt{x}$; $y = f(x + 3)$; translate the graph of $f(x)$ left 3 units;
$f(x)$: domain $\{x \in \mathbb{R}, x \geq 0\}$,
range $\{y \in \mathbb{R}, y \geq 0\}$;
$g(x)$: domain $\{x \in \mathbb{R}, x \geq -3\}$,
range $\{y \in \mathbb{R}, y \geq 0\}$



g) $f(x) = \frac{1}{x}$; $g(x) = f(x - 8)$; translate the graph of $f(x)$ right 8 units;
$f(x)$: domain $\{x \in \mathbb{R}, x \neq 0\}$,
range $\{y \in \mathbb{R}, y \neq 0\}$;
$g(x)$: domain $\{x \in \mathbb{R}, x \neq 8\}$,
range $\{y \in \mathbb{R}, y \neq 0\}$



4. a) translate $f(x)$ left 3 units and up 1 unit;
$g(x) = f(x + 3) + 1$;
$f(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \geq 0\}$;
$g(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \geq 1\}$

b) translate $f(x)$ right 4 units and down 7 units; $g(x) = f(x - 4) - 7$;
$f(x)$: domain $\{x \in \mathbb{R}, x \geq 0\}$,
range $\{y \in \mathbb{R}, y \geq 0\}$;
$g(x)$: domain $\{x \in \mathbb{R}, y \geq 4\}$,
range $\{y \in \mathbb{R}, y \geq -7\}$

c) Answers may vary. Sample answer: translate $f(x)$ down 6 units;
$g(x) = f(x) - 6$; $f(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}\}$; $g(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}\}$

5. a) translate $f(x)$ right 4 units and up 3 units; $g(x) = f(x - 4) + 3$; $f(x)$: domain $\{x \in \mathbb{R}, x \neq 0\}$, range $\{y \in \mathbb{R}, y \neq 0\}$; $g(x)$:domain $\{x \in \mathbb{R}, x \neq 4\}$, range $\{y \in \mathbb{R}, y \neq 3\}$
   b) translate $f(x)$ left 2 units and down 2 units; $g(x) = f(x + 2) - 2$; $f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq -2\}$
   c) translate $f(x)$ left 3 units and up 5 units; $g(x) = f(x + 3) + 5$; $f(x)$: domain $\{x \in \mathbb{R}, x \geq 0\}$, range $\{y \in \mathbb{R}, y \geq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}, x \geq -3\}$, range $\{y \in \mathbb{R}, y \geq 5\}$

6. a) $b(x) = x + 2$
   b) $h(x) = x - 5$
   c) $m(x) = x + 9$
   d) $n(x) = x - 10$
   e) $r(x) = x + 10$
   f) $s(x) = x - 6$
   g) $t(x) = x - 4$

7. a) $b(x) = (x + 2)^2$
   b) $h(x) = x^2 - 5$
   c) $m(x) = x^2 + 9$
   d) $n(x) = (x - 3)^2 - 7$
   e) $r(x) = (x + 4)^2 + 6$
   f) $s(x) = (x + 2)^2 - 8$
   g) $t(x) = (x - 5)^2 + 1$

8. a) $b(x) = \sqrt{(x + 2)}$
   b) $h(x) = \sqrt{x} - 5$
   c) $m(x) = \sqrt{x} + 9$
   d) $n(x) = \sqrt{x - 3} - 7$
   e) $r(x) = \sqrt{x + 4} + 6$
   f) $s(x) = \sqrt{x + 2} - 8$
   g) $t(x) = \sqrt{x - 5} + 1$

9. a) $b(x) = \dfrac{1}{x + 2}$
   b) $h(x) = \dfrac{1}{x} - 5$
   c) $m(x) = \dfrac{1}{x} + 9$
   d) $n(x) = \dfrac{1}{x - 3} - 7$
   e) $r(x) = \dfrac{1}{x + 4} + 6$
   f) $s(x) = \dfrac{1}{x + 2} - 8$
   g) $t(x) = \dfrac{1}{x - 5} + 1$

10. a) False. The order does not matter. For example, if the base function $y = x^2$ is translated left 2 units and then up 1 unit, the image of the point (2, 4) is (0, 5). Similarly, if (2, 4) is translated up 1 unit and then left 2 units, the image point is also (0, 5). This will be true for all points on $y = x^2$.
   b) True. Consider the point (2, 2) on the graph of $y = x$. A translation of 2 units right results in the image point (4, 2), which is on the graph of $y = x - 2$. If the same point is translated 2 units down, the resulting point is (2, 0), which is also on the graph of $y = x - 2$. Thus, when translated horizontally the image points of the points on the line $y = x$ lie on the line $y = x - 2$. Similarly, when translated vertically the image points of the points on the line $y = x$ lie on the line $y = x - 2$.

11. When $f(x) = x$ is translated 5 units to the left the transformed function is
   $g(x) = f(x + 5)$
   $g(x) = x + 5$
   a) Equivalent; the transformed function is
   $g(x) = f(x) + 5$
   $= x + 5$
   b) Not equivalent; the transformed function is
   $g(x) = f(x) - 5$
   $= x - 5$
   c) Not equivalent; the transformed function is
   $g(x) = f(x - 2) + 3$
   $= (x - 2) + 3$
   $= x + 1$
   d) Equivalent; the transformed function is
   $g(x) = f(x + 3) + 2$
   $= (x + 3) + 2$
   $= x + 5$

e) Not equivalent; the transformed function is

$$\begin{aligned} g(x) &= f(x-4)+1 \\ &= (x-4)+1 \\ &= x-3 \end{aligned}$$

12. a) base function: $f(x) = x$; transformed function: $g(x) = x + 3$

b) base function: $f(x) = x^2$; transformed function: $g(x) = (x + 1)^2 + 2$

c) base function: $f(x) = \frac{1}{x}$; transformed function: $g(x) = \frac{1}{x-2} + 3$

d) base function: $f(x) = \sqrt{x}$; transformed function: $g(x) = \sqrt{x-1} + 4$

13. a) domain $\{x \in \mathbb{R}, x \geq 0\}$, the number of units of the product; range $\{y \in \mathbb{R}, y \geq 400\}$, the cost associated with producing x number of units

b) $g(x) = \sqrt{x-8} + 400$

c) translate right 8 units

d) domain $\{x \in \mathbb{R}, x \geq 8\}$, range $\{y \in \mathbb{R}, y \geq 400\}$

14. a) translate 6 units left and 5 units up

$f(x)$	$g(x) = f(x+6)+5$
$f(x) = x$	$g(x) = x + 11$
$f(x) = x^2$	$g(x) = (x+6)^2 + 5$
$f(x) = \sqrt{x}$	$g(x) = \sqrt{x+6} + 5$
$f(x) = \frac{1}{x}$	$g(x) = \frac{1}{x+6} + 5$

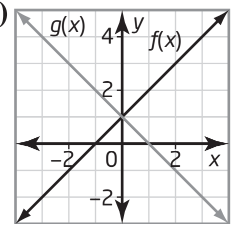
b) translate 4 units right and 3 units down

$g(x)$	$h(x) = g(x-4)-3$
$g(x) = x + 11$	$h(x) = x + 4$
$g(x) = (x+6)^2 + 5$	$h(x) = (x+2)^2 + 2$
$g(x) = \sqrt{x+6} + 5$	$h(x) = \sqrt{x+2} + 2$
$g(x) = \frac{1}{x+6} + 5$	$h(x) = \frac{1}{x+2} + 2$

c) $p(x) = f(x + 2) + 2$; Answers may vary.

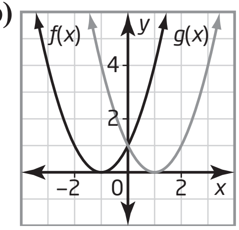
2.4 Reflections of Functions

1. a)



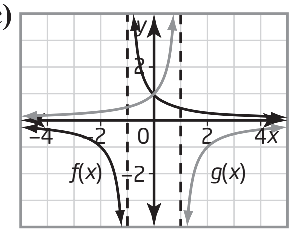
$f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$; $g(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$

b)



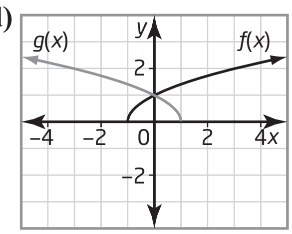
$f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 0\}$

c)



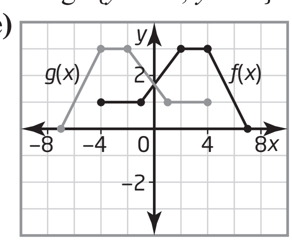
$f(x)$: domain $\{x \in \mathbb{R}, x \neq -1\}$, range $\{y \in \mathbb{R}, y \neq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}, x \neq 1\}$, range $\{y \in \mathbb{R}, y \neq 0\}$

d)



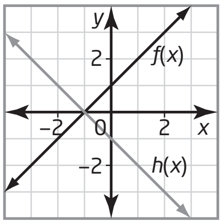
$f(x)$: domain $\{x \in \mathbb{R}, x \geq -1\}$, range $\{y \in \mathbb{R}, y \geq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}, x \leq 1\}$, range $\{y \in \mathbb{R}, y \geq 0\}$

e)



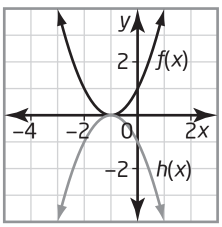
$f(x)$: domain $\{x \in \mathbb{R}, -4 \le x \le 7\}$,
range $\{y \in \mathbb{R}, 0 \le y \le 3\}$;
$g(x)$: domain $\{x \in \mathbb{R}, -7 \le x \le 4\}$,
range $\{y \in \mathbb{R}, 0 \le y \le 3\}$

2. a)



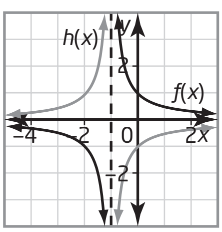
$f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$;
$h(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$

b)



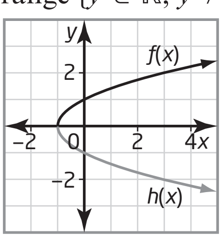
$f(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \ge 0\}$;
$h(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \le 0\}$

c)



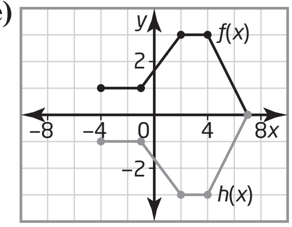
$f(x)$: domain $\{x \in \mathbb{R}, x \neq -1\}$,
range $\{y \in \mathbb{R}, y \neq 0\}$;
$h(x)$: domain $\{x \in \mathbb{R}, x \neq -1\}$,
range $\{y \in \mathbb{R}, y \neq 0\}$

d)



$f(x)$: domain $\{x \in \mathbb{R}, x \ge -1\}$,
range $\{y \in \mathbb{R}, y \ge 0\}$;
$h(x)$: domain $\{x \in \mathbb{R}, x \ge -1\}$,
range $\{y \in \mathbb{R}, y \le 0\}$

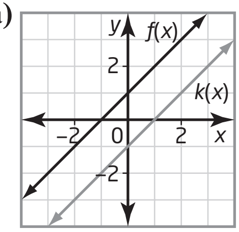
e)



$f(x)$: domain $\{x \in \mathbb{R}, -4 \le x \le 7\}$,
range $\{y \in \mathbb{R}, 0 \le y \le 3\}$;
$h(x)$: domain $\{x \in \mathbb{R}, -4 \le x \le 7\}$,
range $\{y \in \mathbb{R}, -3 \le y \le 0\}$

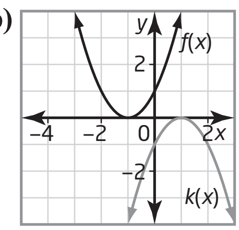
3. A reflection in the x-axis is represented by $k(x) = -f(-x)$.

a)



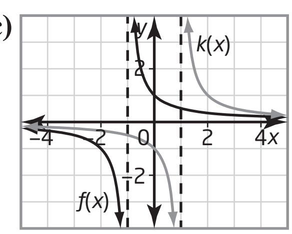
$f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$;
$k(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$

b)



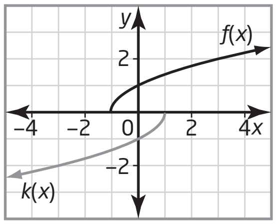
$f(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \ge 0\}$;
$k(x)$: domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y \le 0\}$

c)



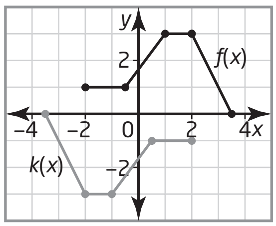
$f(x)$: domain $\{x \in \mathbb{R}, x \neq -1\}$,
range $\{y \in \mathbb{R}, y \neq 0\}$;
$k(x)$: domain $\{x \in \mathbb{R}, x \neq 1\}$,
range $\{y \in \mathbb{R}, y \neq 0\}$

d)



$f(x)$: domain $\{x \in \mathbb{R}, x \geq -1\}$, range $\{y \in \mathbb{R}, y \geq 0\}$;
$k(x)$: domain $\{x \in \mathbb{R}, x \leq 1\}$, range $\{y \in \mathbb{R}, y \leq 0\}$

e)



$f(x)$: domain $\{x \in \mathbb{R}, -4 \leq x \leq 7\}$, range $\{y \in \mathbb{R}, 0 \leq y \leq 3\}$;
$k(x)$: domain $\{x \in \mathbb{R}, -7 \leq x \leq 4\}$, range $\{y \in \mathbb{R}, -3 \leq y \leq 0\}$

4. **a)** $g(x) = -\sqrt{x-21} - 9$
b) $g(x) = (-x + 8)^2 - 17$
c) $g(x) = -(x + 1)^2 - 11$
d) $f(x) = \dfrac{1}{x-6} + 5$
Since $g(x) = -f(-x)$, replace x with $-x$ in $f(x)$ and multiply $f(x)$ by -1.

$$g(x) = -\left[\frac{1}{(-x)-6} + 5\right]$$
$$= -\left[\frac{1}{-x-6} + 5\right]$$
$$= -\left[\frac{1}{-(x+6)} + 5\right] \quad \text{Common factor } -1 \text{ in the denominator.}$$
$$= -\frac{1}{-(x+6)} - 5 \quad \text{Multiply each term in the brackets by } -1.$$
$$= \frac{1}{x+6} - 5 \quad \text{Simplify.}$$

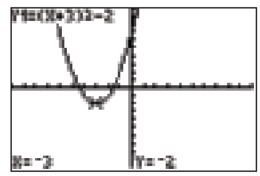
Therefore, $g(x) = \dfrac{1}{x+6} - 5$.
e) $g(x) = -\sqrt{-x+4} + 19$
f) $g(x) = \dfrac{1}{8-x} - 3$
g) $g(x) = x - 18$

5. **a)** $g(x) = f(-x)$; reflection in y-axis
b) $g(x) = -f(x)$; reflection in x-axis
c) $g(x) = -f(-x)$; reflection in x- and y-axes

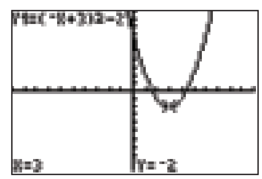
6. **a)** **i)** (0, 8)
ii) (−4, 0) and (−2, 0)
iii) There are no invariant points.
b) Answers may vary. Sample answer: The graph of $y = ax^2$, $\{a \in \mathbb{R}, a \neq 0\}$, will have (0, 0) as an invariant point under each type of reflection.

7. **a)** No
b) Yes; $-f(x)$
c) Yes; $f(-x)$
d) Yes; $-f(-x)$
e) No
f) Yes; $f(-x)$

8. **a)** Use a graphing calculator.



b) $g(x) = f(-x)$
$g(x) = (-x + 3)^2 - 2$



c) translate $f(x)$ 6 units to the right
d) $f(x - 6) = (x + 3 - 6)^2 - 2$
$= (x - 3)^2 - 2$

$$g(x) = (-x + 3)^2 - 2 \quad \text{Common factor } -1.$$
$$= [-1(x - 3)]^2 - 2$$
$$= (-1)^2(x - 3)^2 - 2 \quad \text{Apply the laws of exponents for powers.}$$
$$= (x - 3)^2 - 2$$
$$= f(x - 6)$$

A reflection of $f(x)$ in the y-axis is equivalent to a translation of 6 units right, $f(x - 6)$.
e) This would not be true for reflections in the x-axis because the direction of opening of the parabola would change, whereas translations do not change the direction of opening.
f) This would work for functions that have a vertical line as axis of symmetry. In this case reflection in the y-axis can

be obtained by applying a translation. For example, the quadratic function in part a) has a vertical line that passes through the vertex (−3, −2) as an axis of symmetry. A cubic function such as $f(x) = (x + 3)^3 - 2$ does not have a vertical line of symmetry, and so a reflection in the y-axis cannot be expressed as a translation.

9. a) i) $f(x) = \frac{1}{x}$

ii) $f(x) = \sqrt{x}$

b) i) translate 9 units right and 4 units up

ii) translate 5 units left and 7 units down

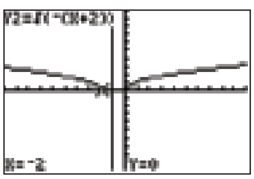
c) i) $k(x) = \frac{1}{9 - x} - 4$; $p(x) = \frac{-1}{x + 9} + 4$;

$q(x) = \frac{1}{x + 9} - 4$

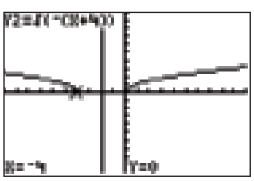
ii) $k(x) = -\sqrt{x + 5} + 7$;

$p(x) = \sqrt{-x + 5} - 7$; $q(x) = -\sqrt{-x + 5} + 7$

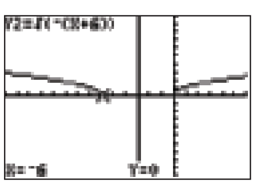
10. a) i) $g(x) = \sqrt{-(x + 2)}$



ii) $g(x) = \sqrt{-(x + 4)}$



iii) $g(x) = \sqrt{-(x + 6)}$



b) i) a reflection of $f(x) = \sqrt{x}$ in the y-axis and a translation of 2 units left

ii) a reflection of $f(x) = \sqrt{x}$ in the y-axis and a translation of 4 units left

iii) a reflection of $f(x) = \sqrt{x}$ in the y-axis and a translation of 6 units left

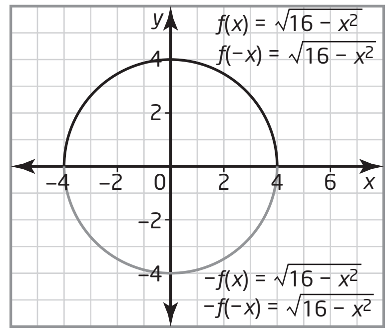
c) Yes; explanations may vary; $g(x) = \sqrt{-(x - 2a)}$

11. a) $f(-x) = \sqrt{16 - x^2}$; $f(-x) = -\sqrt{16 - x^2}$;

$-f(-x) = -\sqrt{16 - x^2}$;

$f(x) = f(-x)$ and $-f(x) = -f(-x)$ are equivalent

b) invariant points: (−4, 0) and (4, 0)



c) $f(x) = \sqrt{16 - x^2}$ and $f(-x) = -\sqrt{16 - x^2}$:

domain $\{x \in \mathbb{R}, -4 \le x \le 4\}$,

range $\{y \in \mathbb{R}, 0 \le y \le 4\}$

$-f(x) = -\sqrt{16 - x^2}$ and $-f(-x) = -\sqrt{16 - x^2}$:

domain $\{x \in \mathbb{R}, -4 \le x \le 4\}$,

range $\{y \in \mathbb{R}, -4 \le y \le 0\}$

12. a) All points are invariant under each reflection.

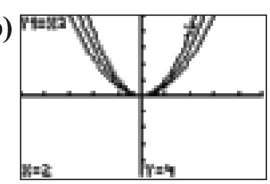
b) only true for circles with centre (0, 0)

2.5 Stretches of Functions

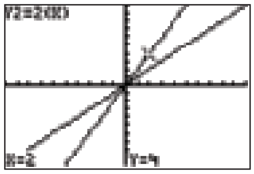
1. a)

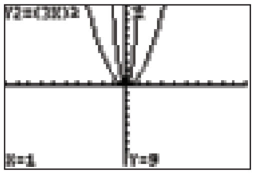
x	$f(x) = x^2$	$g(x) = \frac{3}{4}f(x)$	$h(x) = f\left(\frac{3}{4}x\right)$
−4	16	12	9
−2	4	3	$\frac{9}{4}$
0	0	0	0
2	4	3	$\frac{9}{4}$
4	16	12	9

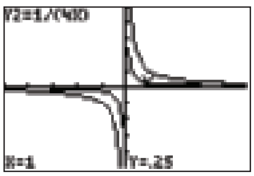
b)

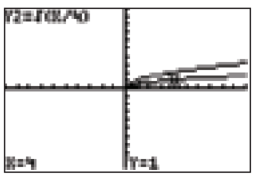


c) $g(x)$ represents a vertical compression of $f(x)$ by a factor of $\frac{3}{4}$; $h(x)$ represents a horizontal stretch of $f(x)$ by a factor of $\frac{4}{3}$

2. a) 

b) 

c) 

d) 

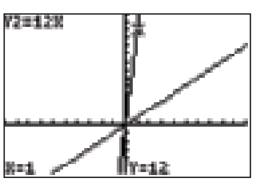
3. a) $a = 8$; $g(x)$ is a vertical stretch by a factor of 8 of $f(x)$

b) $k = 6$; $g(x)$ is a horizontal compression by a factor of $\frac{1}{6}$ of $f(x)$

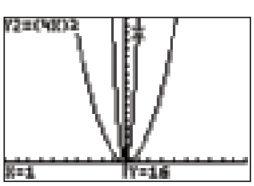
c) $a = \frac{2}{3}$; $g(x)$ is a vertical compression by a factor of $\frac{2}{3}$ of $f(x)$

d) $k = \frac{1}{9}$; $g(x)$ is a horizontal stretch by a factor of 9 of $f(x)$

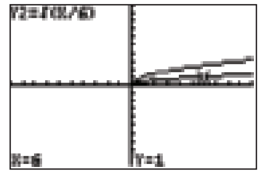
4. a) $g(x)$ is a vertical stretch by a factor of 12 of $f(x) = x$



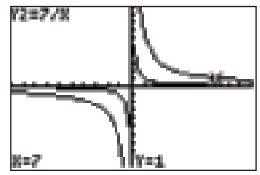
b) $g(x)$ is a horizontal compression by a factor of $\frac{1}{4}$ of $f(x) = x^2$



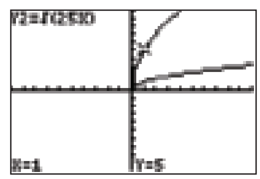
c) $g(x)$ is a horizontal stretch by a factor of 6 of $f(x) = \sqrt{x}$



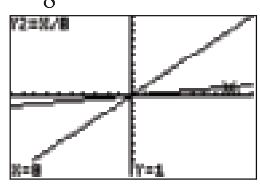
d) $g(x)$ is a vertical stretch by a factor of 7 of $f(x) = \frac{1}{x}$



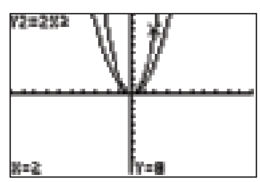
e) $g(x)$ is a horizontal compression by a factor of $\frac{1}{25}$ of $f(x) = \sqrt{x}$



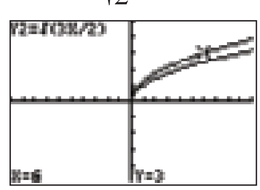
f) $g(x)$ is a vertical compression by a factor of $\frac{1}{8}$ of $f(x) = x$



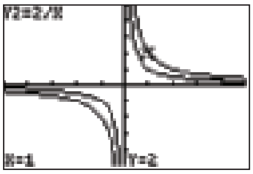
5. a) $g(x) = 2x^2$



b) $g(x) = \sqrt{\frac{3}{2}x}$



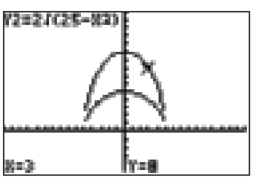
c) $g(x) = \frac{2}{x}$



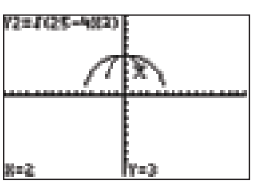
6. a) The base function is $f(t) = t^2$.
b) vertical stretch by a factor of $\frac{a}{2}$
c) Earth: $d(t) = 4.9t^2$;
Neptune: $d(t) = 5.7t^2$;
Mercury: $d(t) = 1.8t^2$
d) For all three planets, the domain is $\{t \in \mathbb{R}, t \geq 0\}$ and the range is $\{d \in \mathbb{R}, d \geq 0\}$.

7. a) compress $f(x)$ horizontally by a factor of $\frac{1}{8}$; $g(x) = \sqrt{8x}$
b) stretch $f(x)$ vertically by a factor of 7; $g(x) = \frac{7}{x}$

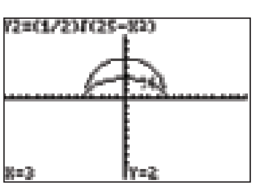
8. a) $g(x) = 2\sqrt{25 - x^2}$;
domain $\{x \in \mathbb{R}, -5 \leq x \leq 5\}$,
range $\{y \in \mathbb{R}, 0 \leq x \leq 10\}$



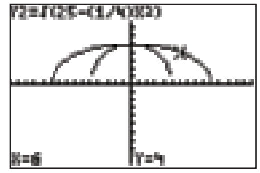
b) $g(x) = \sqrt{25 - 4x^2}$;
domain $\{x \in \mathbb{R}, -2.5 \leq x \leq 2.5\}$,
range $\{y \in \mathbb{R}, 0 \leq x \leq 5\}$



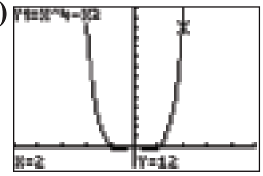
c) $g(x) = \frac{1}{2}\sqrt{25 - x^2}$;
domain $\{x \in \mathbb{R}, -5 \leq x \leq 5\}$,
range $\{y \in \mathbb{R}, 0 \leq x \leq 2.5\}$



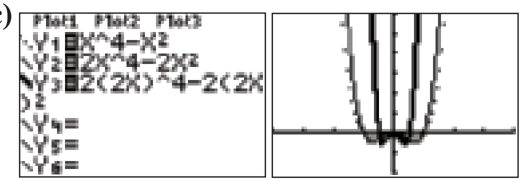
d) $g(x) = \sqrt{25 - \frac{1}{4}x^2}$;
domain $\{x \in \mathbb{R}, -10 \leq x \leq 10\}$,
range $\{y \in \mathbb{R}, 0 \leq y \leq 5\}$



9. a)



b) $g(x) = 2x^4 - 2x^2$; $h(x) = 2(2x)^4 - 2(2x)^2$
c)



d) No; simplified, $h(x) = 32x^4 - 8x^2$

10. The value of the parameter c is 18 in both $f(x)$ and $g(x)$.
In $g(x)$, the term $-10x^2$ may be written as follows:
$$\begin{aligned} -10x^2 &= -(0.4 \times 25x^2) \\ &= -0.4(25x^2) \\ &= -0.4(5x)^2 \end{aligned}$$
So $g(x) = f(5x)$,
since $f(5x) = -0.4(5x)^2 + 18$.
$$\begin{aligned} f(5x) &= -0.4(25x^2) + 18 \\ &= -10x^2 + 18 \end{aligned}$$
To obtain the graph of $g(x)$, apply a horizontal compression by a factor of $\frac{1}{5}$ to the graph of $f(x)$.

2.6 Combinations of Transformations

1. a) $a = 3$, $d = 5$; vertically stretch $f(x)$ by a factor of 3, then translate 5 units right
b) $a = \frac{1}{4}$, $c = 4$; vertically compress $f(x)$ by a factor of $\frac{1}{4}$, then translate 4 units up
c) $d = -6$, $c = 2$; translate $f(x)$ 6 units left and 2 units up

d) $k = \frac{1}{3}$, $c = 7$; horizontally stretch $f(x)$ by a factor of 3, then translate 7 units up

e) $k = 2$, $c = -8$; horizontally compress $f(x)$ by a factor of $\frac{1}{2}$, then translate down 8 units

f) $a = 5$, $c = -3$; vertically stretch $f(x)$ by a factor of 5, then translate down 3 units

2. a) $a = 4$, $k = 3$, $c = -2$; vertically stretch by a factor of 4, horizontally compress by a factor of $\frac{1}{3}$, and then translate 2 units down

b) $a = -5$, $c = 6$; vertically stretch by a factor of 5, reflect in the x-axis, and then translate 6 units up

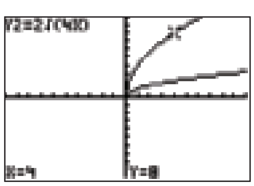
c) $a = \frac{1}{3}$, $d = 8$, $c = 1$; vertically compress by a factor of $\frac{1}{3}$, then translate 8 units right and 1 unit up

d) $k = -2$, $c = 6$; horizontally compress by a factor of $\frac{1}{2}$, reflect in the y-axis, and then translate 6 units up

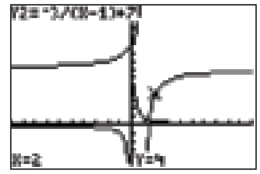
e) $a = -1$, $k = \frac{1}{4}$, $c = -1$; reflect in the x-axis, horizontally stretch by a factor of 4, and then translate 1 unit down

f) $a = \frac{2}{5}$, $k = 5$, $c = -7$; vertically compress by a factor of $\frac{2}{5}$, horizontally compress by a factor of $\frac{1}{5}$, and then translate 7 units down

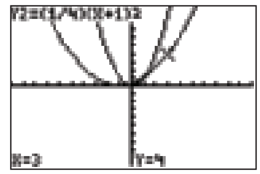
3. a) vertically stretch by a factor of 2 and horizontally compress by a factor of $\frac{1}{4}$; $g(x) = 2\sqrt{4x}$



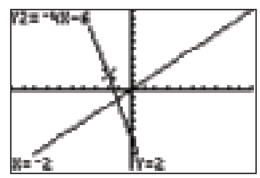
b) vertically stretch by a factor of 3, reflect in the x-axis, and then translate 1 unit right and 7 units up; $g(x) = \frac{-3}{x-1} + 7$



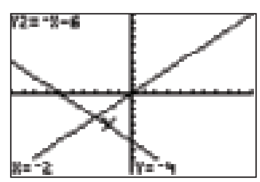
c) horizontally stretch by a factor of 2, then translate 1 unit left; $g(x) = \frac{1}{4}(x+1)^2$



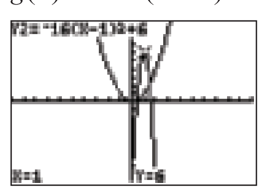
d) vertically stretch by a factor of 4, reflect in the x-axis, and then translate 6 units down; $g(x) = -4x - 6$



4. a) vertically compress by a factor of $\frac{1}{3}$, reflect in the x-axis, horizontally compress by a factor of $\frac{1}{3}$, and then translate 2 units left and 4 units down; $g(x) = -x - 6$

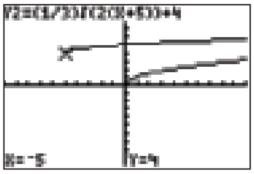


b) vertically stretch by a factor of 4, reflect in the x-axis, horizontally compress by a factor of $\frac{1}{2}$, and then translate 1 unit right and 6 units up; $g(x) = -16(x-1)^2 + 6$

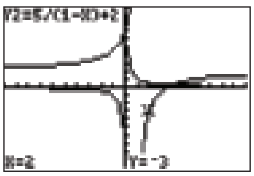


c) vertically compress by a factor of $\frac{1}{3}$, horizontally compress by a factor of

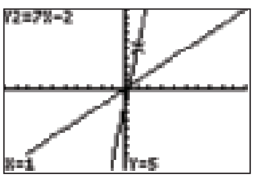
$\frac{1}{2}$, and then translate 5 units left and 4 units up; $g(x) = \frac{1}{3}\sqrt{2(x + 5)} + 4$



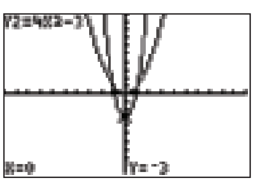
d) vertically stretch by a factor of 5, reflect in the y-axis, and then translate 1 unit right and 2 units up; $g(x) = \frac{5}{1 - x} + 2$



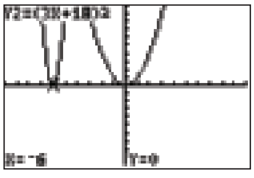
5. a) $f(x) = x$; $f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$; $g(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$



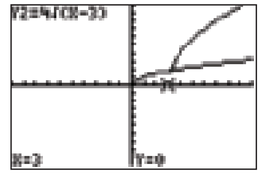
b) $f(x) = x^2$; $f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq -3\}$



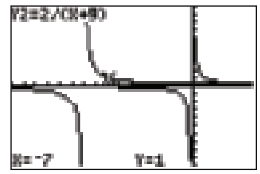
c) $f(x) = x^2$; $f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 0\}$



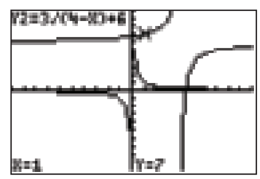
d) $f(x) = \sqrt{x}$; $f(x)$: domain $\{x \in \mathbb{R}, x \geq 0\}$, range $\{y \in \mathbb{R}, y \geq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}, x \geq 3\}$, range $\{y \in \mathbb{R}, y \geq 0\}$



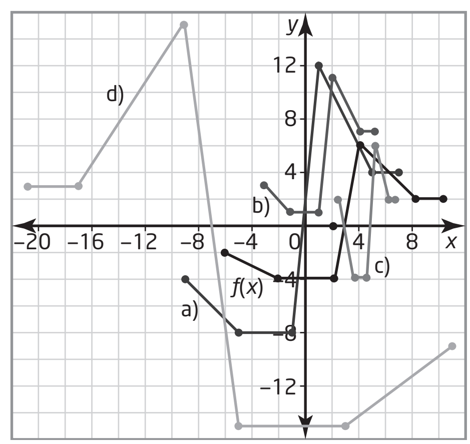
e) $f(x) = \frac{1}{x}$; $f(x)$: domain $\{x \in \mathbb{R}, x \neq 0\}$, range $\{y \in \mathbb{R}, y \neq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}, x \neq -9\}$, range $\{x \in \mathbb{R}, x \neq 0\}$



f) $f(x) = \frac{1}{x}$; $f(x)$: domain $\{x \in \mathbb{R}, x \neq 0\}$, range $\{y \in \mathbb{R}, y \neq 0\}$; $g(x)$: domain $\{x \in \mathbb{R}, x \neq 4\}$, range $\{x \in \mathbb{R}, x \neq 6\}$



6.



7. a) $y = \frac{2}{x + 4} - 3$; graph ii)

b) $y = \frac{1}{2}\sqrt{-x + 4} + 3$; graph iv)

The equation $y = \frac{1}{2}\sqrt{-x + 4} + 3$ may be expressed as $y = \frac{1}{2}\sqrt{-(x - 4)} + 3$. This equation matches graph iv) because the base function for graph iv) is $f(x) = \sqrt{x}$. The graph of $f(x) = \sqrt{x}$ is reflected in the y-axis, vertically compressed by a factor of $\frac{1}{2}$, and then translated right 4 units and up 3 units. The point (0, 4) satisfies the equation.

c) $y = 3\sqrt{x - 2} + 1$; graph i)

d) $y = \left[\frac{1}{4}(x - 5)\right]^2 + 2$; graph iii)

8. a) $y = -\frac{2}{x + 3} + 1$; graph ii)

b) $y = 2\sqrt{-x + 4} - 1$; graph i)

c) $y = \frac{4}{-x + 2} + 1$; graph iv)

d) $y = \frac{1}{2}\sqrt{x + 5} - \frac{5}{2}$; graph iii)

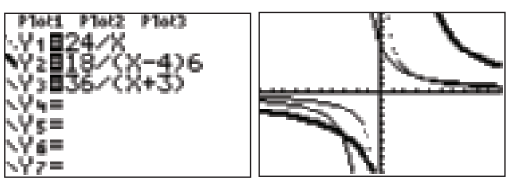
9. a) Since s is in the denominator, each time function is a reciprocal function, so the base function is $f(s) = \frac{1}{s}$.

b) In $t_1 = \frac{24}{s}$, $a = 24$, so t_1 is a vertical stretch by a factor of 24 of $f(s)$.

In $t_2 = \frac{18}{s - 4}$, $a = 18$ and $d = 4$, so t_2 is a vertical stretch of $f(s)$ by a factor of 18 and a translation of 4 units right.

In $t_3 = \frac{36}{s + 3}$, $a = 36$ and $d = -3$, so t_3 is a vertical stretch of $f(s)$ by a factor of 36 and a translation of 3 units left.

c)



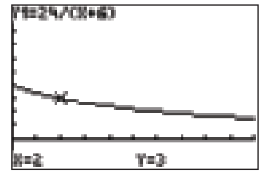
d) Substitute $s = 6$ in each time function.

$t_1 = \frac{24}{6}$
$= 4$ It will take Andrew and David 4 h to travel across the lake.

$t_2 = \frac{18}{6 - 4}$
$= 9$ It will take Andrew and David 9 h to travel up the river.

$t_3 = \frac{36}{6 + 3}$
$= 4$ It will take Andrew and David 4 h to travel down the river.

10. a) domain $\{t \in \mathbb{R}, t \geq 0\}$, range $\{V \in \mathbb{R}, 0 \leq V \leq 4000\}$



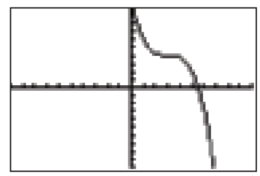
b) \$4000

c) i) \$3429

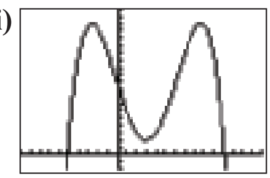
ii) \$2667

iii) \$1846

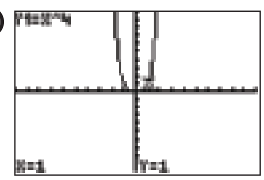
11. i)



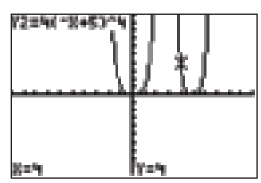
ii)



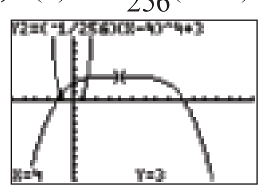
12. a)



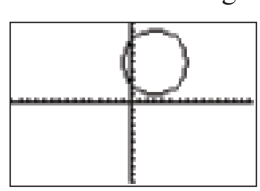
b) i) $g(x) = 4(-x + 5)^4$



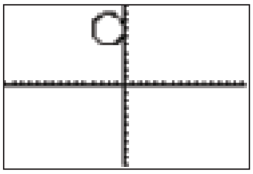
ii) $h(x) = -\frac{1}{256}(x - 4)^4 + 3$



13. a) translate 3 units right and 5 units up



b) translate 2 units left and 7 units up

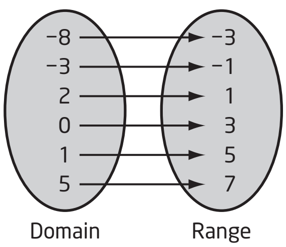


2.7 Inverse of a Function

1. a) {(6, 2), (1, 3), (−1, 4), (2, 5)};
function: domain {2, 3, 4, 5},
range {6, 1, −1, 2};
inverse: domain {6, 1, −1, 2},
range {2, 3, 4, 5}

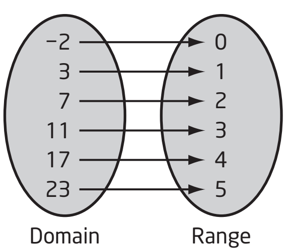
b) {(7, −3), (5, −2), (−2, −1), (−6, 0)};
function: domain {−3, −2, −1, 0},
range {7, 5, −2, −6};
inverse: domain {7, 5, −2, −6},
range {−3, −2, −1, 0}

c)



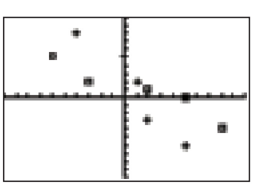
function: domain {−3, −1, 1, 3, 5, 7},
range {−8, −3, −2, 0, 1, 5};
inverse: domain {−8, −3, −2, 0, 1, 5},
range {−3, −1, 1, 3, 5, 7}

d)



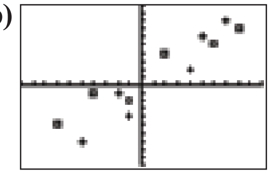
function: domain {0, 1, 2, 3, 4, 5},
range {−2, 3, 7, 11, 17, 23};
inverse: domain {−2, 3, 7, 11, 17, 23},
range {0, 1, 2, 3, 4, 5}

2. a)



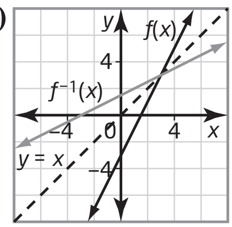
function: domain {−6, −3, 2, 5, 8},
range {−4, 0, 1, 2, 5};
inverse: domain {−4, 0, 1, 2, 5},
range {−6, −3, 2, 5, 8}

b)



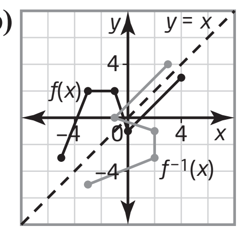
function: domain {−7, −4, −1, 2, 6, 8},
range {−5, −1, −2, 4, 5, 7};
inverse: domain {−5, −1, −2, 4, 5, 7},
range {−7, −4, −1, 2, 6, 8}

3. a)



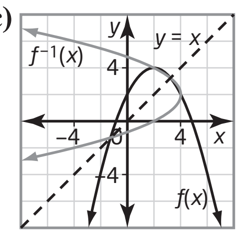
function

b)



not a function

c)



not a function

4. a) $f^{-1}(x) = \frac{x}{5}$

b) $f^{-1}(x) = \frac{x + 3}{4}$

c) $f^{-1}(x) = -x + 7$

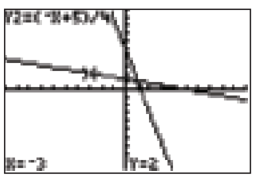
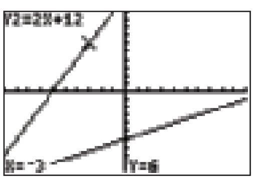
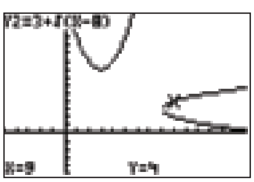
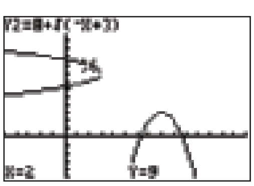
d) $f^{-1}(x) = \frac{-3x + 1}{2}$

5. a) $f^{-1}(x) = \pm\sqrt{x - 5}$

b) $f^{-1}(x) = \pm\sqrt{\frac{x}{7}}$

c) $f^{-1}(x) = \pm\sqrt{x} - 3$
d) $f^{-1}(x) = \pm\sqrt{3(x+4)}$

6. **a)** $f(x) = (x-2)^2 - 1$
$f^{-1}(x) = 2 \pm \sqrt{x+1}$
b) $f(x) = -(x-7)^2 + 10$
$f^{-1}(x) = 7 \pm \sqrt{-x+10}$
c) $f(x) = 2(x+4)^2 - 2$
$f^{-1}(x) = -4 \pm \sqrt{\dfrac{x+2}{2}}$
d) $f(x) = -3(x+4)^2 - 52$
$f^{-1}(x) = -4 \pm \sqrt{\dfrac{-x-52}{3}}$

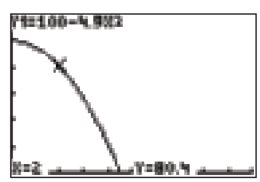
7. **a)** **i)** $f^{-1}(x) = \dfrac{-x+5}{4}$
ii) 
iii) function
b) **i)** $f^{-1}(x) = 2x + 12$
ii) 
iii) function
c) **i)** $f^{-1}(x) = 3 \pm \sqrt{x-8}$
ii) 
iii) not a function
d) **i)** $f^{-1}(x) = 8 \pm \sqrt{-x+3}$
ii) 
iii) not a function

8. **a)** domain $\{r \in \mathbb{R}, r \geq 0\}$, range $\{S \in \mathbb{R}, S \geq 0\}$
b) $r(S) = \sqrt{\dfrac{S}{4\pi}}$; domain $\{S \in \mathbb{R}, S \geq 0\}$, range $\{r \in \mathbb{R}, r \geq 0\}$

9. **a)** Let s represent Aubrey's weekly sales and let e represent her weekly earnings. The equation that represents her weekly earnings is $e = 450 + 0.08s$, where 450 corresponds to the \$450 she earns each week and $0.08s$ represents 8% of her sales.
b) To find the inverse, solve the equation for s.
$$e = 450 + 0.08s$$
$$e - 450 = 0.08s$$
$$\frac{e-450}{0.08} = s$$
$$s = 12.5e - 5625$$
c) The inverse represents Aubrey's weekly sales.
d) Substitute 1025 in s.
$$s = 12.5(1025) - 5625$$
$$= 7187.50$$
Aubrey's sales for the week were \$7187.50.

10. **a)** $u = 0.89c$
b) $c = 1.12u$; the value of the Canadian dollar in U.S. dollars
c) \$280.00

11. **a)** Since time must be positive, the domain is restricted to $t \geq 0$.
domain $\{t \in \mathbb{R}, t \geq 0\}$, range $\{h \in \mathbb{R}, 0 \leq h \leq 100\}$



b) Isolate t to determine the inverse.
$$h = 100 - 4.9t^2$$
$$h - 100 = -4.9t^2$$
$$t^2 = \frac{100-h}{4.9}$$
$$t = \sqrt{\frac{100-h}{4.9}}$$
Time is positive, so ignore the negative root.
domain $\{h \in \mathbb{R}, 0 \leq h \leq 100\}$, range $\{t \in \mathbb{R}, t \geq 0\}$

c) The inverse represents the time for an object to fall from a height of 100 m above the ground.

d) The object hits the ground when the height is 0 m. Substitute $h = 0$ in the inverse function and solve for t.

$$t = \sqrt{\frac{100 - 0}{4.9}}$$
$$= 4.5$$

The object hits the ground after 4.5 s.

12. a) Yes; $g(x)$ is a reflection of $f(x)$ in $y = x$

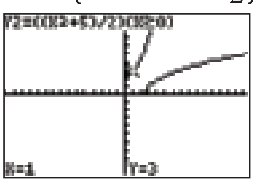
b) Yes; $g(x)$ is a reflection of $f(x)$ in $y = x$

c) No; $g(x)$ is a reflection of $f(x)$ in the x-axis

13. a) $f^{-1}(x) = \dfrac{x^2 + 5}{2}$

b) $f(x)$: domain $\left\{x \in \mathbb{R}, x \geq \dfrac{5}{2}\right\}$, range $\{y \in \mathbb{R}, y \geq 0\}$; $f^{-1}(x)$: domain $\{x \in \mathbb{R}, x \geq 0\}$, range $\left\{y \in \mathbb{R}, y \geq \dfrac{5}{2}\right\}$

c)



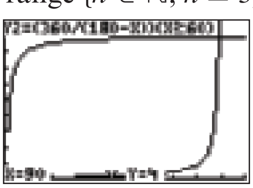
14. a) $f^{-1}(x) = \dfrac{2 + x}{3x}$

b) $f(x)$: domain $\left\{x \in \mathbb{R}, x \neq \dfrac{1}{3}\right\}$, range $\{y \in \mathbb{R}, y \neq 0\}$; $f^{-1}(x)$: domain $\{x \in \mathbb{R}, x \neq 0\}$, range $\left\{y \in \mathbb{R}, y \neq \dfrac{1}{3}\right\}$

15. a) $n(a) = \dfrac{360}{180 - a}$

b) function: domain $\{n \in \mathbb{N}, n \geq 3\}$, range $\{a \in \mathbb{R}, 60 \leq a < 180\}$; inverse: domain $\{a \in \mathbb{R}, 60 \leq a < 180\}$, range $\{n \in \mathbb{N}, n \geq 3\}$

c)



Chapter 2 Review

1. a) No

b) Yes

c) No

2. a) Yes; $x \neq \frac{1}{4}, x \neq 0$

b) No; $g(x) = \dfrac{2x + 1}{x}$; $x \neq 2, x \neq 0$

3. a) $\dfrac{1}{x + 2}$; $x \neq -8, x \neq -2$

b) $\dfrac{2x - 3}{x + 6}$; $x \neq -\frac{3}{4}, x \neq -6$

4. a) $24a^2b$; $a \neq 0, b \neq 0$

b) $\dfrac{5ab}{18c}$; $a \neq 0, b \neq 0, c \neq 0$

c) 2; $x \neq -9, x \neq \frac{4}{3}$

d) $\dfrac{x - 8}{x + 4}$; $x \neq -4, x \neq -3, x \neq 5$

e) -2; $x \neq 4, x \neq 3$

f) $\dfrac{x + 8}{x - 8}$; $x \neq -3, x \neq 1, x \neq 8$

5. a) $\dfrac{19}{15x}$; $x \neq 0$

b) $\dfrac{7a + 4a - ab}{12ab^2}$; $a \neq 0, b \neq 0$

c) $\dfrac{2x^2 + 8x + 23}{(x - 1)(x + 2)}$; $x \neq -2, x \neq 1$

d) $\dfrac{-2x^2 + 13x}{(x - 3)(x - 4)}$; $x \neq 3, x \neq 4$

e) $\dfrac{4x^2 - 10x - 9}{(x - 3)(x + 3)}$; $x \neq -3, x \neq 3$

f) $\dfrac{-5}{(x + 2)}$; $x \neq -5, x \neq -2, x \neq 2$

6. a) $s(x) = x - 6$

b) $t(x) = x - 4$

7. i) a) $s(x) = (x + 2)^2 - 8$

b) $t(x) = (x - 5)^2 + 1$

ii) a) $s(x) = \sqrt{x + 2} - 8$

b) $t(x) = \sqrt{x - 5} + 1$

iii) a) $s(x) = \dfrac{1}{x + 2} - 8$

b) $t(x) = \dfrac{1}{x - 5} + 1$

8. a) base function: $f(x) = \frac{1}{x}$; transformed function: $g(x) = \dfrac{1}{x + 6} - 3$

b) base function: $f(x) = \sqrt{x}$; transformed function: $g(x) = \sqrt{x + 4} + 1$

9. a) No

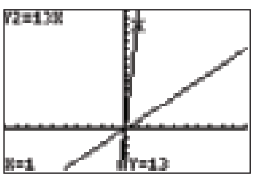
b) Yes; $g(x) = f(-x)$

c) No
d) Yes; $g(x) = -f(x)$

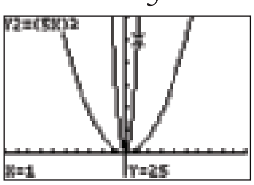
10. a) $g(x) = -\sqrt{x-1} - 8$
b) $g(x) = -(x + 3)^2 - 10$
c) $g(x) = \dfrac{1}{x-7} + 2$

11. a) $a = 9$; vertical stretch by a factor of 9
b) $k = 3$; horizontal compression by a factor of $\frac{1}{3}$
c) $a = \frac{2}{5}$; vertical compression by a factor of $\frac{2}{5}$
d) $k = \frac{1}{7}$; horizontal stretch by a factor of 7

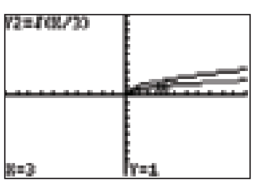
12. a) vertical stretch of $f(x) = x$ by a factor of 13



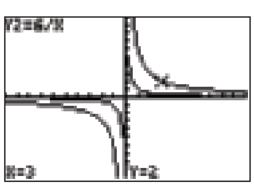
b) horizontal compression of $f(x) = x^2$ by a factor of $\frac{1}{5}$



c) horizontal stretch of $f(x) = \sqrt{x}$ by a factor of 3

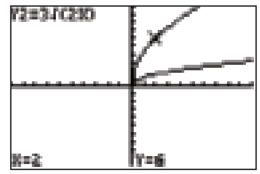


d) vertical stretch of $f(x) = \frac{1}{x}$ by a factor of 6

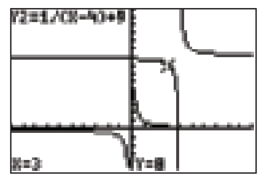


13. a) $a = 7$, $d = 1$; stretch vertically by a factor of 7, then translate 1 unit right
b) $a = \frac{1}{5}$, $c = -3$; compress vertically by a factor of $\frac{1}{5}$, then translate 3 units down
c) $d = -9$, $c = 8$; translate 9 units left and 8 units up
d) $k = \frac{1}{2}$, $c = 10$; stretch horizontally by a factor of 2, then translate 10 units up

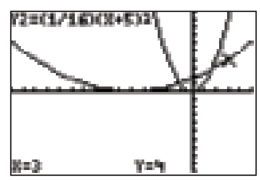
14. a) $g(x) = 3\sqrt{2x}$



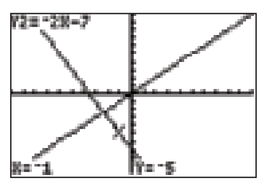
b) $g(x) = \dfrac{1}{x-4} + 9$



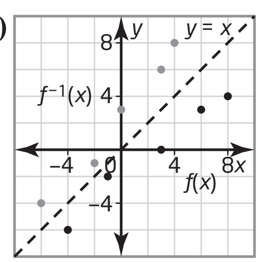
c) $g(x) = \frac{1}{16}(x + 5)^2$



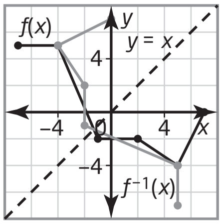
d) $g(x) = -2x - 7$



15. a)



function: domain {−4, −1, 3, 6, 8}, range {−6, −2, 0, 3, 4};
inverse: domain {−6, −2, 0, 3, 4}, range {−4, −1, 3, 6, 8}

b) 

function: domain $\{x \in \mathbb{R}, -7 \leq x \leq 7\}$, range $\{y \in \mathbb{R}, -4 \leq x \leq 5\}$;
inverse: domain $\{x \in \mathbb{R}, -4 \leq x \leq 5\}$, range $\{y \in \mathbb{R}, -7 \leq y \leq 7\}$

16. a) $f^{-1}(x) = \dfrac{-x + 7}{2}$

b) $f^{-1}(x) = \dfrac{4x + 3}{5}$

c) $f^{-1}(x) = 3 \pm \sqrt{x - 1}$

d) $f^{-1}(x) = \pm 2\sqrt{9 - x}$

Chapter 2 Math Contest

1. C
2. C
3. B
4. D
5. C
6. A
7. D
8. A
9. C
10. B
11. 8
12. $\dfrac{4b - 3a}{8a - 6b}$
13. 50; 50 = 1 + 49; 50 = 25 + 25
14. −25
15. 540

Chapter 3 Exponential Functions

3.1 The Nature of Exponential Growth

1. a)

Day	Population	First Differences	Second Differences
0	25		
1	75	50	
2	225	150	100
3	675	450	300
4	2025	1350	900
5	6075	4050	2700

b) Yes. For each additional day the ant population increases by a common factor of 3.

c) The ratio of the first differences is 3. The ratio of the second differences is also 3.

d) Answers may vary. Sample answer: Yes, the pattern will continue.

e)

Day	Population	First Differences	Second Differences	Third Differences	Fourth Differences
0	25				
1	75	50			
2	225	150	100		
3	675	450	300	200	
4	2025	1350	900	600	400
5	6075	4050	2700	1800	1200

2. a) The value 5 and the variable x have different positions in each function. In particular the value 5 is the coefficient in $y = 5x$, the exponent in $y = x^5$, and the base in $y = 5^x$. The variable x is multiplied with the 5 in $y = 5^x$, is the base in $y = x^5$ and is the exponent in $y = 5x$. The graph of $y = 5x$ is a straight line with slope 5. The graph of $y = x^5$ is a polynomial function of degree 5 and extends from quadrant 3 to quadrant 1. The graph of $y = 5^x$ is an exponential function that extends from quadrant 2 to quadrant 1.

b) $y = 5x$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$
$y = x^5$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$
$y = 5^x$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y > 0\}$

3. Justifications may vary.

4. a) $\dfrac{a^5}{a^5} = \dfrac{a \times a \times a \times a \times a}{a \times a \times a \times a \times a}$

b) 1

c) a^0 **d)** $a^0 = 1$

5. a) B

b) The constant 15 is the initial population. The power 4^n represents the daily quadrupling.

6. a)–e) 1 **f)** −1

7. **a)** exponential: $f(x) = 8^x$ is of the form $y = a^x$, where the base a is a constant, such that $a > 0$, and the exponent is a variable

   b) linear: $f(x) = 11 - 9x$ can be rewritten as $f(x) = -9x + 11$, which is of the form $y = mx + b$

   c) none of linear, quadratic, or exponential: $f(x) = \sqrt{x}$ is a square root function

   d) quadratic

8. Determine the first and second differences for each table of values.

   a)

x	y	First Differences	Second Differences	Ratio of First and Second Differences
−4	16			
−3	8	−8		
−2	4	−4	4	
−1	2	−2	2	0.5
0	1	−1	1	0.5
1	0.5	−0.5	0.5	0.5
2	0.25	−0.25	0.25	0.5

The table represents an exponential function. Neither the first differences nor the second differences are constant; however, the ratio of the first and second differences is constant.

   b)

x	y	First Differences
−4	13	
−3	10	−3
−2	7	−3
−1	4	−3
0	1	−3
1	−2	−3
2	−5	−3

The table represents a linear function because the first differences are constant.

   c)

x	y	First Differences	Second Differences
−4	−18		
−3	−11	7	
−2	−6	5	−2
−1	−3	3	−2
0	−2	1	−2
1	−3	−1	−2
2	−6	−3	−2

The table represents a quadratic function because the second differences are constant.

   d)

x	y	First Differences	Second Differences	Ratio of First and Second Differences
−4	625			
−3	125	−500		
−2	25	−100	400	
−1	5	−20	80	0.2
0	1	−4	16	0.2
1	0.2	−0.8	3.2	0.2
2	0.04	−0.16	0.64	0.2

The table represents an exponential function. Neither the first differences nor the second differences are constant; however, the ratio of the first and second differences is constant.

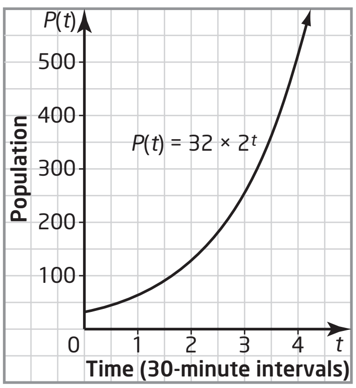
   e)

x	y	First Differences	Second Differences
−4	2.2		
−3	0.8	−1.4	
−2	−0.2	−1	0.4
−1	−0.8	−0.6	0.4
0	−1	−0.2	0.4
1	−0.8	0.2	0.4
2	−0.2	0.6	0.4

The table represents a quadratic function because the second differences are constant.

9. **a)** Answers may vary.
   b) Answers may vary. First 4 terms are 1, 4, 16, 64.
   c) i) 1024 **ii)** 4 194 304
   d) 5 terms **e)** 6 terms
   f) The answers differ by 1 term. This is an exponential function in which each term is multiplied by 4.

10. **a)** The initial population is 32; since it doubles every 30 minutes, multiply 32 by 2 for each 30-min interval. The equation is $p(t) = 32 \times 2^t$.
   b)



   c) Answers may vary. Sample answer: Since there are two 30-min intervals in each hour, 1 h on the graph corresponds to $t = 2$. By moving up to the curve and then across to the vertical axis, it can be estimated that at $t = 2$ the population is approximately 130 bacteria.
   This can be checked in the equation. To use the equation, substitute $t = 2$ to obtain $p(2) = 32 \times 2^2 = 128$, which is very close to 130.
   d) Since the graph does not extend to 3.5 h, it is easier to use the equation. In 3.5 h, there are seven 30-min intervals. Substitute $t = 7$ in $p(t) = 32 \times 2^t$:
   $$p(7) = 32 \times 2^7$$
   $$= 4096$$
   After 3.5 h there are 4096 bacteria.

11. **a)** $A(n) = 200(1.045)^n$
   b)

Number of Compounding Periods (years)	Amount ($)
0	200.00
1	209.00
2	218.41
3	228.23
4	238.50
5	249.24
6	260.45

   c)

Number of Compounding Periods (years)	Amount ($)	First Differences	Second Differences
0	200.00		
1	209.00	9.00	
2	218.41	9.41	0.41
3	228.23	9.82	0.41
4	238.50	10.27	0.45
5	249.24	10.74	0.47
6	260.45	11.21	0.47

   Neither. The first differences are not equal, nor are the second differences.
   d) The ratio of the first differences and second differences is approximately 1.04. The function is exponential.
   e) In the equation the constant ratio is represented by the value 1.045.
   f) No. The amount remains the same between compounding periods and increases only at the end of each compounding period.

12. **a)** $1663.08 **b)** $1975.21

13. **a) i)** 500 **ii)** 12 500
   b) $r(n) = 4 \times 5^n$, where r is the number of residents called in each interval, n
   c) approximately 5.4 intervals
   d) This is an example of exponential growth, because the number of residents notified increases by a factor of 5 with each interval.

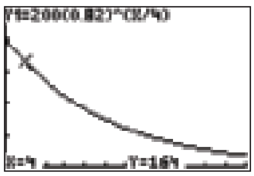
14. **a)** approximately 13 years; Answers may vary. Sample answer: Systematic trial.
   b) 5.18 or approximately 6 more years (since the interest is paid at the end of the year)

15. Prize A. The value of Prize A at week 26 alone is \$335 544.32. The value of Prize B for all 26 weeks is only \$260 000.

16.a) 6 days; population of A is 6400; population of B is 8192

b) 2 days sooner, on the 4th day

17. a)



b) approximately 14 h

c) For $h = 14$, $M \doteq 99.9$.

d) approximately 107 h

3.2 Exponential Decay: Connecting to Negative Exponents

1. a) $\frac{1}{7}$ **b)** $\frac{1}{10^2}$ **c)** $\frac{1}{a^4}$

d) $\frac{1}{mn}$ **e)** $-\frac{1}{4}$ **f)** $\frac{1}{2b}$

2. a) a^{-3} **b)** $2x^{-5}$

c) $-x^{-9}$ **d)** $\frac{2}{5}b^{-6}$

3. a) $\frac{1}{25}$ **b)** $\frac{1}{81}$ **c)** $\frac{1}{1000}$

d) $\frac{1}{16}$ **e)** $-\frac{1}{16}$ **f)** $-\frac{1}{64}$

g) $\frac{4}{81}$ **h)** $\frac{5}{48}$

4. a) 25 **b)** $\frac{1}{512}$ **c)** 729

d) $\frac{1}{36}$ **e)** 24 **f)** 64

5. a) a^4 **b)** $-\frac{12}{v^7}$

c) a^{10} **d)** $\frac{3}{m^2}$

6. a) 6 **b)** $\frac{49}{16}$

c) $\frac{64}{27}$ **d)** $\frac{81}{625}$

7. a) x^3y^3 **b)** $121b^2$

c) $\frac{b^8}{a^{12}}$ **d)** $\frac{8n^{12}}{125m^6}$

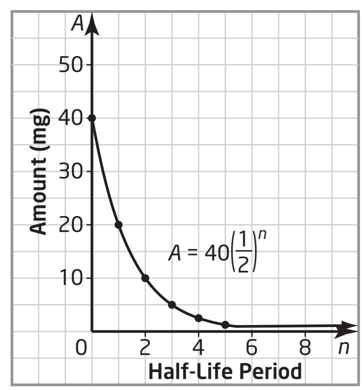
8. a)

Number of 20-day Intervals, n	Amount Remaining (mg)
0	40
1	20
2	10
3	5
4	2.5
5	1.25

Half the amount remains after each 20-day interval.

b) $A = 40\left(\frac{1}{2}\right)^n$, where n is the number of half-life periods, in 20-day intervals, and A is the amount of polonium-210 remaining, in milligrams

c) Plot the points from the table. Connect them with a smooth curve.



The graph starts at point (0, 40), decreases by a factor of $\frac{1}{2}$ with each 20-day interval, and has a horizontal asymptote, $y = 0$.

d) Since there are 7 days in a week, 10 weeks = 70 days. This represents 3.5 half-life periods. Substitute $n = 3.5$ into the equation to determine the amount remaining:

$$A = 40\left(\frac{1}{2}\right)^{3.5}$$
$$= 3.54$$

Approximately 3.54 mg of polonium-210 will remain after 10 weeks.

e) Answers may vary. Sample answer:
Determine 8% of 40 mg:
8% = 0.08, so 0.08(40) mg = 3.2 mg
Substitute $A = 6$ into the equation and solve for n:

$3.2 = 40\left(\frac{1}{2}\right)^n$ Divide each side by 40.

$0.08 = \left(\frac{1}{2}\right)^n$

Use systematic trial to solve for n.

Observe the chart in part a). Note that when $n = 3$ the amount remaining is 5 mg, and when $n = 4$ the amount remaining is 2.5 mg. Therefore, for 3.2 mg, the value of n is between 3 and 4.

Try $n = 3.5$ in $\left(\frac{1}{2}\right)^n$: $\left(\frac{1}{2}\right)^{3.5} \doteq 0.088$, which is a bit high.

Try $n = 3.64$ to get $\left(\frac{1}{2}\right)^{3.64} \doteq 0.0802$, which is very close, so $n = 3.64$.

Multiply to find the number of days.

$3.64(20) \doteq 73$ days

Therefore, it takes approximately 73 days for polonium-210 to decay to 8% of its initial mass.

f) Since $\frac{1}{2} = 2^{-1}$, an equivalent way to write the equation is $A = 40(2^{-1})^n$ or $A = 40(2^{-n})$.

Since $\frac{1}{2} = 0.5$, another equivalent way to write the equation is $A = 40(0.5)^n$.

9. **a)** In the formula 13 500 represents the initial value of the motorcycle. Since the value depreciates by 20% per year, 80% of the value remains. This amount is represented by the decimal 0.8.

b) i) \$10 800 **ii)** \$3538.94

c) Answers may vary. Sample answer: The equation is of the form $f(x) = ab^x$, where 0.8 is the constant ratio. This value is less than 1, so the initial amount is decreasing.

d) 3.1 years, or approximately 3 years 1 month

10. \$2720.63

11. **a)** The initial deposit was made 5 years ago, so determine the amount in the account when $n = 5$.
Substitute $A = 4200$, $i = 0.075$, and $n = 5$ in the formula $P = A(1 + i)^{-n}$.
$P = 4200(1 + 0.075)^{-5}$
$\doteq 2925.55$
Denise's initial deposit was \$2925.55.

b) Substitute $A = 4200$, $i = 0.075$, and $n = 2$ in the formula $P = A(1 + i)^{-n}$.
$P = 4200(1 + 0.075)^{-2}$
$\doteq 3634.40$
The amount in the account 2 years ago was \$3634.40.

c) Use the formula $A = P(1 + i)^n$ to determine the amount in the account 2 years from now. Substitute $P = 4200$, $i = 0.075$, and $n = 2$, and solve for A.
$A = 4200(1 + 0.075)^2$
$\doteq 4853.63$
Two years from now the amount in the account will be \$4853.63.
To determine the total interest earned, subtract the initial deposit from the amount at this point.
\$4853.63 − \$2925.55 = \$1928.08
The total interest earned up to this point is \$1928.08.

12. approximately 17.3%

13. **a)** $c = 100\left(2^{-\frac{t}{5}}\right)$

b) i) 16 h 36 min **ii)** 33 h 12 min

14. **a)** $T = 80\left(2^{-\frac{t}{5}}\right) + 20$ **b)** 15 min

3.3 Rational Exponents

1. **a)** 6 **b)** −11
c) $\frac{5}{7}$ **d)** $\frac{4}{9}$

2. **a)** 5 **b)** 2
c) 3 **d)** −4

3. **a)**
$$\begin{aligned} 32^{\frac{3}{5}} &= \left(32^{\frac{1}{5}}\right)^3 \\ &= \left(\sqrt[5]{32}\right)^3 \\ &= 2^3 \\ &= 8 \end{aligned}$$

b) $(-64)^{\frac{2}{3}} = \left(-64^{\frac{1}{3}}\right)^2$
$= \left(\sqrt[3]{-64}\right)^2$
$= (-4)^2$
$= 16$

c) $64^{\frac{5}{6}} = \left(64^{\frac{1}{6}}\right)^5$
$= \left(\sqrt[6]{64}\right)^5$
$= 2^5$
$= 32$

d) $6561^{\frac{5}{8}} = \left(6561^{\frac{1}{8}}\right)^5$
$= \left(\sqrt[8]{6561}\right)^5$
$= 3^5$
$= 243$

4. a) $1728^{-\frac{1}{3}} = \dfrac{1}{1728^{\frac{1}{3}}}$
$= \dfrac{1}{\left(\sqrt[3]{1728}\right)}$
$= \dfrac{1}{12}$

b) $36^{-\frac{3}{2}} = \dfrac{1}{36^{\frac{3}{2}}}$
$= \dfrac{1}{\left(36^{\frac{1}{2}}\right)^3}$
$= \dfrac{1}{\left(\sqrt{36}\right)^3}$
$= \dfrac{1}{6^3}$
$= \dfrac{1}{216}$

c) $\left(-\dfrac{8}{125}\right)^{\frac{5}{3}} = \dfrac{1}{\left(-\dfrac{8}{125}\right)^{\frac{5}{3}}}$
$= \dfrac{1}{\left(-\dfrac{8^{\frac{1}{3}}}{125^{\frac{1}{3}}}\right)^5}$
$= \dfrac{1}{\left(-\dfrac{\left(\sqrt[3]{8}\right)}{\left(\sqrt[3]{125}\right)}\right)^5}$
$= \dfrac{1}{\left(-\dfrac{2}{5}\right)^5}$
$= \dfrac{1}{-\dfrac{32}{3125}}$
$= -\dfrac{3125}{32}$

d) $\left(\dfrac{1024}{243}\right)^{-\frac{3}{5}} = \dfrac{1}{\left(\dfrac{1024}{243}\right)^{\frac{3}{5}}}$
$= \dfrac{1}{\left(\dfrac{\sqrt[5]{1024}}{\sqrt[5]{243}}\right)^3}$
$= \dfrac{1}{\left(\dfrac{4}{3}\right)^3}$
$= \dfrac{1}{\dfrac{64}{27}}$
$= \dfrac{27}{64}$

5. a) $8^{\frac{1}{3}} \times 8^{\frac{2}{3}}$
$= 8^{\frac{1}{3}+\frac{2}{3}}$ Use the product rule for exponents.
$= 8^{\frac{3}{3}}$
$= 8$

b) $16^{\frac{1}{4}} \div 16^{\frac{1}{2}} \times 16^{\frac{3}{4}}$
$= 16^{\frac{1}{4}-\frac{1}{2}+\frac{3}{4}}$ Find a common denominator for the fractional exponents.
$= 16^{\frac{1}{4}-\frac{1}{2}+\frac{3}{4}}$
$= 16^{\frac{2}{4}}$ Reduce the exponent.
$= 16^{\frac{1}{2}}$
$= \sqrt{16}$
$= 4$

c) $64^{\frac{1}{3}} \times 64^{\frac{1}{6}} \div 64^{\frac{2}{3}}$
$= 64^{\frac{1}{3}+\frac{1}{6}-\frac{2}{3}}$ Find a common denominator for the exponents.
$= 64^{\frac{2}{6}+\frac{1}{6}-\frac{4}{6}}$
$= 64^{\frac{1}{6}}$
$= \dfrac{1}{64^{\frac{1}{6}}}$ Take the reciprocal.
$= \dfrac{1}{\sqrt[6]{64}}$
$= \dfrac{1}{2}$

d) $3^{\frac{2}{3}} \times 27^{\frac{4}{9}}$
$= 3^{\frac{2}{3}} \times \left(3^3\right)^{\frac{4}{9}}$

$= 3^{\frac{2}{3}} \times 3^{\frac{4}{9}}$

$= 3^{\frac{2}{3}} \times 3^{\frac{4}{3}}$

$= 9$

6. a) $x^{\frac{2}{3}}$ **b)** $a^{\frac{7}{6}}$

c) $\frac{a^{\frac{8}{3}}}{b^{\frac{3}{2}}}$ **d)** $z^{\frac{8}{15}}$

7. a) $b^{\frac{2}{15}}$ **b)** $\frac{1}{a^{\frac{13}{12}}}$

c) $w^{\frac{4}{11}}$ **d)** $2a^{\frac{1}{2}}$

8. 1652.7 cm^3

9. a) $C = 2\pi\left(\frac{A}{\pi}\right)^{\frac{1}{2}}$ **b)** $C = 2(\pi A)^{\frac{1}{2}}$

c) i) 62.8 cm **ii)** 108.0 cm **iii)** 136.1 cm

d) Answers may vary.

10. a) 400 **b)** approximately 2 h 40 min

c) Yes. The exponents are equivalent because $\frac{t}{0.5} = 2t$.

d) Answers may vary.

11. a) $N = 1000(2^{\frac{t}{3}})$, where t is the time, in minutes

b) 32 000 **c)** approximately 18 min

12. a) i) 23 beats per min **ii)** 43 beats per min **iii)** 159 beats per min

b) i) 5 breaths per min **ii)** 10 breaths per min **iii)** 35 breaths per min

c) i) 5.08 kg **ii)** 0.987 kg or 987 g **iii)** 0.03 kg or 30.4 g

d) i) the larger the animal, the fewer beats per min

ii) the larger the animal, the fewer breaths per min

iii) the larger the animal, the greater the brain mass

e) Answers may vary.

13. a) $f^{-1}(x) = x^{\frac{2}{3}}$; 16

b) $f^{-1}(x) = (x + 2)^{\frac{5}{4}}$; 243

c) $f^{-1}(x) = \sqrt[4]{x^3 + 4}$; 268.014

14. a) $x^{\frac{29}{30}}$

b) $\frac{1}{x^{\frac{37}{12}}}$

c) $\frac{9}{7m}$

d) $\frac{1}{6b^{9x + 1}}$

e) $a^{\frac{5x}{8}}$

15. a) $P = P_0(10)^{\frac{t}{20}}$

b) approximately 3.2 times greater

16. a) No. Answers may vary.

b) No. Answers may vary.

3.4 Properties of Exponential Functions

1. Graph a) matches **B** $y = 3\left(\frac{1}{3}\right)^x$

Graph b) matches **D** $y = -3^x$

Graph c) matches **C** $y = \frac{1}{3}(3^x)$

Graph d) matches **A** $y = 3(3^x)$

2. Graph a) matches **C** $y = \frac{1}{2}(4)^x$

Graph b) matches **A** $y = -4(2)^x$

Graph c) matches **D** $y = 4\left(\frac{1}{2}\right)^x$

Graph d) matches **B** $y = -2(4)^x$

3. a) Answers may vary.

b) No. The conditions are satisfied by any curve whose equation is of the form $y = 2b^x$, where $b > 1$.

4. a) Answers may vary.

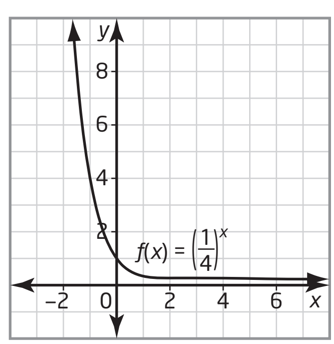
b) No. The conditions are satisfied by any curve whose equation is of the form $y = -3b^x$, where $b > 1$.

5. $y = 6 \times 2^x$

6. $y = 8 \times \left(\frac{1}{2}\right)^x$

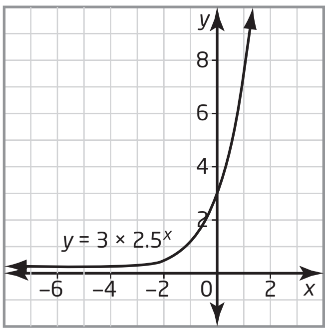
7. a) C **b)** 9.5 mg

8. a)



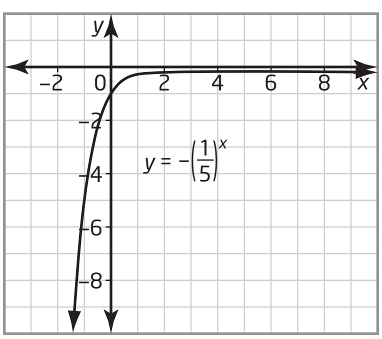
i) domain $\{x \in \mathbb{R}\}$
ii) range $\{y \in \mathbb{R}, y > 0\}$
iii) The y-intercept is 1. There is no x-intercept.
iv) The y-values are getting closer to zero as the x-values increase, so the graph is decreasing for $x \in \mathbb{R}$.
v) The equation of the horizontal asymptote is $y = 0$, which is the x-axis.

b)



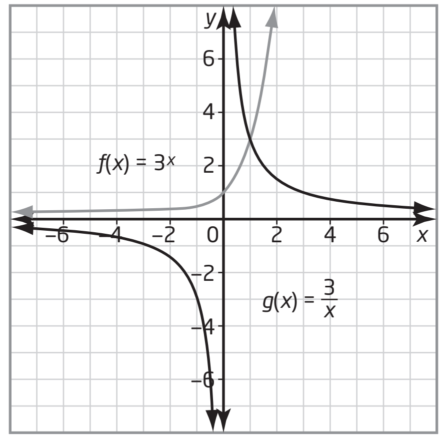
i) domain $\{x \in \mathbb{R}\}$
ii) range $\{y \in \mathbb{R}, y > 0\}$
iii) The y-intercept is 3. There is no x-intercept.
iv) The y-values are increasing as the x-values increase, so the graph is increasing for $x \in \mathbb{R}$.
v) The horizontal asymptote is $y = 0$, which is the x-axis.

c)



i) domain $\{x \in \mathbb{R}\}$
ii) range $\{y \in \mathbb{R}, y < 0\}$
iii) The y-intercept is −1. There is no x-intercept.
iv) The y-values are increasing as the x-values increase, so the graph is increasing for $x \in \mathbb{R}$.
v) The horizontal asymptote is $y = 0$, which is the x-axis.

9. a)



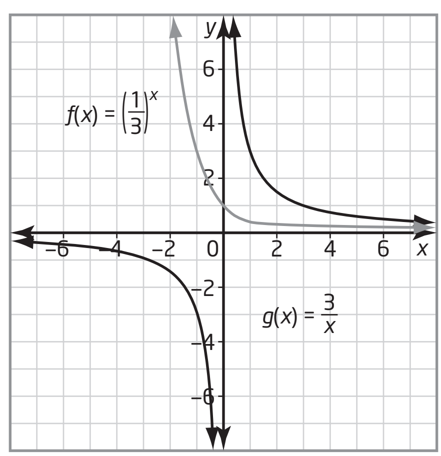
b) i) They have the same horizontal asymptote, $y = 0$. Both graphs pass through the point (1, 3). The curves have a similar shape, one end near the x-axis and the other end moving away from it, either upward or downward.

ii) Differences:

- the graph of $f(x) = 3x$ increases as x increases, it has only one branch, and has no vertical asymptotes
- the graph of $g(x) = \frac{3}{x}$ decreases as x increases, it has two branches, and has a vertical asymptote, $x = 0$

c) Both graphs have $y = 0$ (the x-axis) as a horizontal asymptote. The graph of $g(x) = \frac{3}{x}$ also has a vertical asymptote at $x = 0$ (the y-axis).

10.a)



b) i) Both graphs decrease as x increases. They have the same horizontal asymptote, $y = 0$. The curves have a similar shape, one end near the x-axis and the other end moving away from it, either upward or downward.

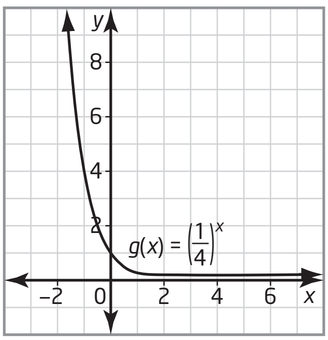
ii) Differences:

- the graph of $f(x) = \left(\frac{1}{3}\right)^x$ has only one branch and no vertical asymptotes
- the graph of $g(x) = \frac{3}{x}$ has two branches and a vertical asymptote, $x = 0$

c) Both graphs have $y = 0$ (the x-axis) as a horizontal asymptote. The graph of $g(x) = \frac{3}{x}$ also has a vertical asymptote at $x = 0$ (the y-axis).

11. a) The graphs are the same.

b)



c) $f(x) = 4^{-x}$

$$= \frac{1}{4x}$$

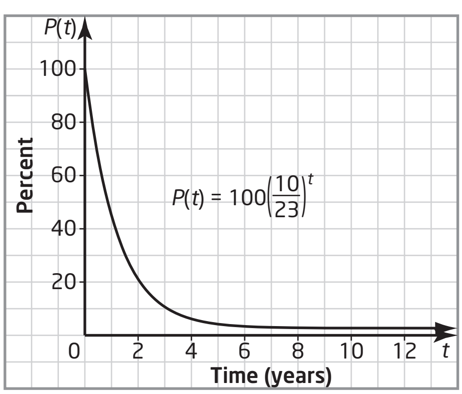
$$= \left(\frac{1}{4}\right)^x$$

$$= g(x)$$

12. a) $P(t) = 100\left(\frac{10}{23}\right)^t$

b) The common ratio $\frac{10}{23}$ is a proper fraction (value is between 0 and 1), so when multiplied with 100 will make the value smaller.

c)



The graph falls from left to right, which means that as time passes the percent of radioactive substance remaining is decreasing.

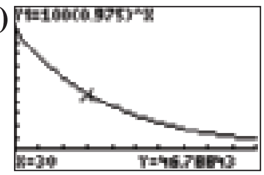
d) domain $\{t \in \mathbb{R}, t \geq 0\}$, range $\{P \in \mathbb{R}, 0 < P \leq 100\}$

e) As time passes the percent remaining of the substance decreases, becoming closer to 0 but never actually reaching 0.

f) i) Answers may vary. Approximately 3% of the initial amount of the substance remains.

ii) $P(4) = 3.6\%$

13. a)



b) 27 m

14. a) 372 m^3

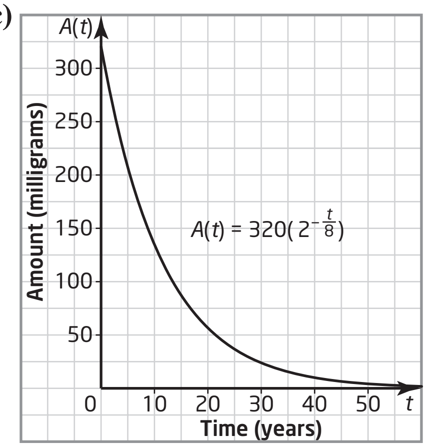
b) $S(V) = (3V)^{\frac{2}{3}}(4\pi)^{\frac{1}{3}}$

c) 137 m^2

15. a) 8 days

b) $A(t) = 320\left(2^{-\frac{t}{8}}\right)$; $A(t) = 320\left(\frac{1}{2}\right)^{\frac{t}{8}}$

c)



domain $\{t \in \mathbb{R}, t \geq 0\}$, range $\{A \in \mathbb{R}, 0 < A \leq 320\}$

d) 20 days. Answers may vary (estimating from the graph or using systematic trial in the equation).

16. a) $r = \dfrac{(3V)^{\frac{1}{2}}}{5\pi^{\frac{1}{2}}}$

b) Answers may vary. Sample answer: domain $\{V \in \mathbb{R}, V > 0\}$

c) The radius of the base will increase by a factor of $\sqrt{2}$.

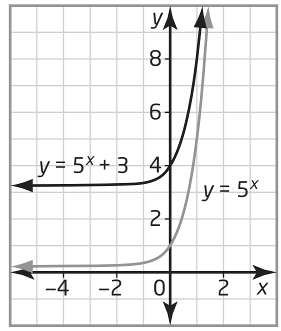
3.5 Transformations of Exponential Functions

1.

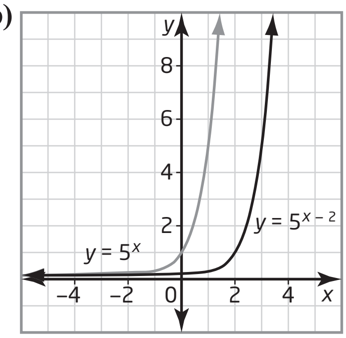
The Roles of the Parameters *a*, *k*, *d*, and *c* in Exponential Functions of the Form $y = ab^{k(x-d)} + c$ ($b > 0$, $b \neq 1$)	
Role of *c*	**Transformation on Graph of $y = b^x$**
$c > 0$	Translation c units up
$c < 0$	Translation c units down
Role of *d*	
$d > 0$	Translation d units right
$d < 0$	Translation d units left
Role of *a*	
$a > 1$	Vertical stretch by a factor of a
$0 < a < 1$	Vertical compression by a factor of a
$-1 < a < 0$	Vertical compression by a factor of $\|a\|$ and a reflection in the x-axis
$a < -1$	Vertical stretch by a factor of $\|a\|$ and reflection in the x-axis
Role of *k*	
$k > 1$	Horizontal compression by a factor of $\frac{1}{k}$
$0 < k < 1$	Horizontal stretch by a factor of $\frac{1}{k}$
$-1 < k < 0$	Horizontal stretch by a factor of $\frac{1}{\|k\|}$ and a reflection in the y-axis
$k < -1$	Horizontal compression by a factor of $\frac{1}{\|k\|}$ and a reflection in the y-axis
Domain and Range of $y = ab^{k(x-d)} + c$	
The domain is always $\{x \in \mathbb{R}\}$.	i) When the graph is below its horizontal asymptote the range is $\{y \in \mathbb{R}, y < c\}$. ii) When the graph is above its horizontal asymptote the range is $\{y \in \mathbb{R}, y > c\}$.

2. **a)** translate $y = 5x$ up 3 units
b) translate $y = 5x$ right 2 units
c) translate $y = 5x$ left 1 unit
d) translate $y = 5x$ right 4 units and down 6 units

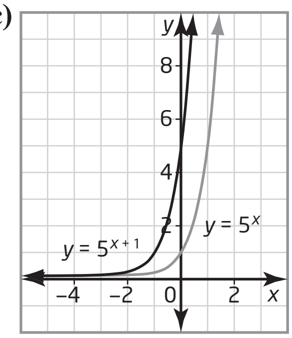
3. **a)**



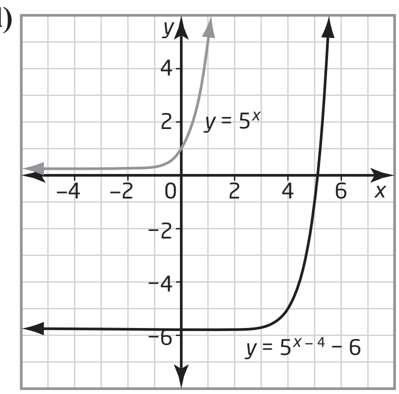
b)



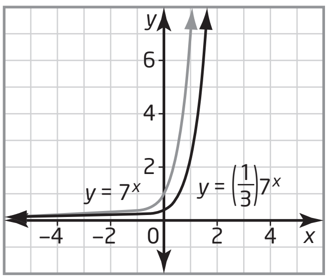
c)

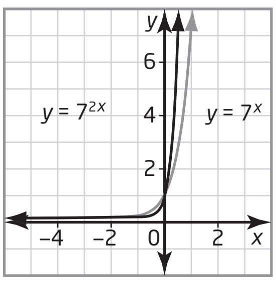


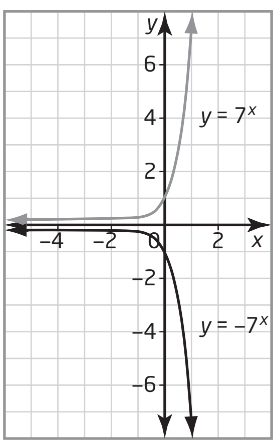
d)

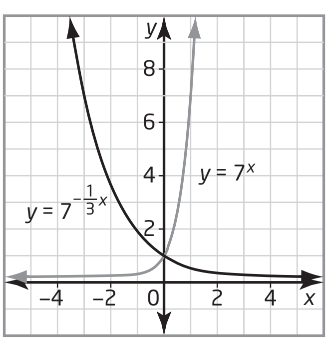


4. **a)** vertical compression by a factor of $\frac{1}{3}$
b) horizontal compression by a factor of $\frac{1}{2}$
c) reflection in the x-axis
d) horizontal stretch by a factor of 3 and a reflection in the y-axis

5. **a)**


b)


c)


d)


6. **a)** $y = 11^{-x}$ **b)** $y = 4(11^x)$
c) $y = 11^{\frac{x}{3}}$ **d)** $y = -115^x$

7. **a)** Compare $y = -3[4^{2(x+1)}] + 5$ to $y = ab^{k(x-d)} + c$. The parameters are $a = -3$, $k = 2$, $d = -1$, and $c = 5$.

i) $k = 2$ corresponds to a horizontal compression by a factor of $\frac{1}{2}$. Divide the x-coordinates of the points in column 1 by 2.

$y = 4^x$	$y = 4^{2x}$
(−1, 0.25)	$\left(-\frac{1}{2}, 0.25\right)$
(0, 1)	(0, 1)
(1, 4)	$\left(\frac{1}{2}, 4\right)$
(2, 16)	(1, 16)
(3, 64)	$\left(\frac{3}{2}, 64\right)$

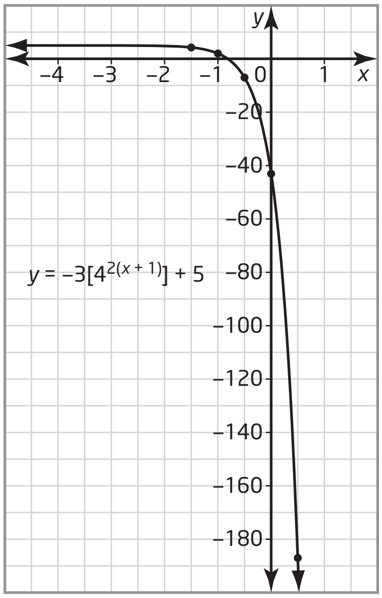
ii) $a = -3$ corresponds to a vertical stretch by a factor of 3 and a reflection in the x-axis. Multiply the y-coordinates of the points in column 2 by −3.

$y = 4^x$	$y = 4^{2x}$	$y = -3[4^{2x}]$
(−1, 0.25)	$\left(-\frac{1}{2}, 0.25\right)$	$\left(-\frac{1}{2}, 0.75\right)$
(0, 1)	(0, 1)	(0, −3)
(1, 4)	$\left(\frac{1}{2}, 4\right)$	$\left(\frac{1}{2}, -12\right)$
(2, 16)	(1, 16)	(1, −48)
(3, 64)	$\left(\frac{3}{2}, 64\right)$	$\left(\frac{3}{2}, -192\right)$

iii) $d = -1$ corresponds to a translation of 1 unit to the left, so add −1 to each x-value in column 3. $c = 5$ corresponds to a translation of 5 units up, so add 5 to each y-value in column 3.

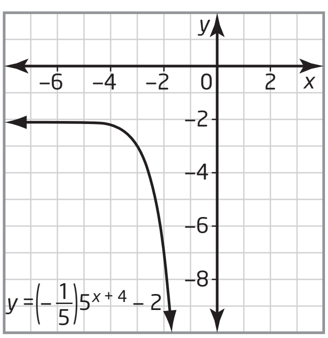
$y = 4^x$	$y = 4^{2x}$	$y = -3[4^{2x}]$	$y = -3[4^{2(x+1)}] + 5$
(−1, 0.25)	$\left(-\frac{1}{2}, 0.25\right)$	$\left(-\frac{1}{2}, 0.75\right)$	$\left(-\frac{3}{2}, 4.25\right)$
(0, 1)	(0, 1)	(0, −3)	(−1, 2)
(1, 4)	$\left(\frac{1}{2}, 4\right)$	$\left(\frac{1}{2}, -12\right)$	$\left(-\frac{1}{2}, -7\right)$
(2, 16)	(1, 16)	(1, −48)	(0, −43)
(3, 64)	$\left(\frac{3}{2}, 64\right)$	$\left(\frac{3}{2}, -192\right)$	$\left(\frac{1}{2}, -187\right)$

To sketch the graph, plot the points in column 4 and draw a smooth curve through them.



b) The domain is $\{x \in \mathbb{R}\}$. Since the graph is below its horizontal asymptote, which is $y = 5$, the range is $\{y \in \mathbb{R}, y < 5\}$.

8.



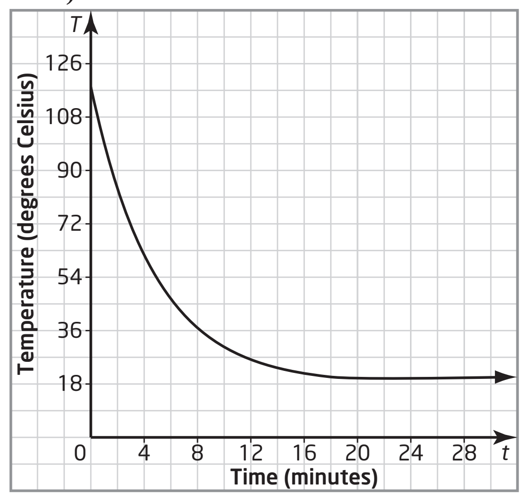
9. a) Compare the transformed equation with $y = ab^{k(x-d)} + c$ to determine the values of a, k, d, and c.
For $y = -f(4x) - 7$ the parameters are $a = -1$, $k = 4$, $d = 0$, and $c = -7$.
The function $f(x) = 3^x$ is reflected in the x-axis, compressed horizontally by a factor of $\frac{1}{4}$, and shifted down 7 units. The equation of the corresponding transformed function is $f(x) = -(3^{4x}) - 7$.

b) The domain is $\{x \in \mathbb{R}\}$. Since a is negative, the graph will be below its horizontal asymptote, which is $y = -7$, so the range is $\{y \in \mathbb{R}, y < -7\}$.

10. a) $f(x) = -\left(2^{-\frac{1}{5}(x-1)}\right) - 3$

b) domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y < -3\}$; The equation of the horizontal asymptote is $y = -3$.

11. a)



b) $T = 18$, the initial temperature that the bar will cool down to

c) approximately 33 min

12. a) Answers may vary. Sample answer:
$y = 2^{-4x}$; $y = 4^{-2x}$; $y = 16^{-x}$

b) For $y = 2^{-4x}$, the base function $y = 2^x$ is compressed horizontally by a factor of $\frac{1}{4}$ and reflected in the y-axis.
For $y = 4^{-2x}$, the base function $y = 4^x$ is compressed horizontally by a factor of $\frac{1}{2}$ and reflected in the y-axis.
For $y = 16^{-x}$, the base function $y = 16^x$ is reflected in the y-axis.

13. a) Answers may vary. Sample answer:
Equations of the form $y = 8(2^x) - 5$ satisfy the given conditions.

b) No. Equations of the form $y = 8(a^x) - 5$, where $a > 0$, will satisfy the given conditions. The y-intercept indicates that the base function is stretched vertically by a factor of 8, and the asymptote indicates that the base function is translated down 5 units.

14. a) $A = 250\left(\frac{1}{2}\right)^{\frac{t}{138}}$; $A = 175\left(\frac{1}{2}\right)^{\frac{t}{16}}$

b) $A = \left(\frac{1}{2}\right)^t$

c) For $A = 250\left(\frac{1}{2}\right)^{\frac{t}{138}}$, the base function $A = \left(\frac{1}{2}\right)^t$ is stretched vertically by a factor of 250 and stretched horizontally by a factor of 138.

For $A = 175\left(\frac{1}{2}\right)^{\frac{t}{16}}$, the base function $A = \left(\frac{1}{2}\right)^t$ is stretched vertically by a factor of 175 and stretched horizontally by a factor of 16.

d) $A = 250(2)^{\frac{t}{138}}$; $A = 175(2)^{\frac{t}{16}}$

i) $A = 2^t$

ii) The vertical stretch and horizontal stretch remain the same.

iii) The new transformation is a reflection in the y-axis.

e) 123.8 g; 0.4 g

f) 458.4 days; 53.2 days

15. a) $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$

b) The base function $A = \left(\frac{1}{2}\right)^t$ is stretched vertically by a factor of A_0 and stretched horizontally by a factor of h.

c) $A = A_0(2)^{-\frac{t}{h}}$

3.6 Making Connections: Tools and Strategies for Applying Exponential Models

1. Graph A: $y = 2 \times 2.2^x$; Graph B: $y = 6 \times 1.6^x$; Graph C: $y = 9 \times 0.8^x$

2. Answers may vary.

3. a) Yes. The points increase at a constant ratio.

b) Answers may vary.

c) $V(n) = 150 \times 1.12^n$

d) $584.40

e) approximately 17 years

4. a) $C(t) = 100(0.98)^t$

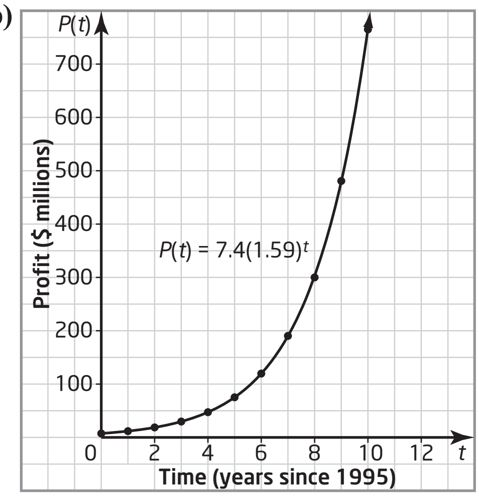
b) i) 60% **ii)** 36% **iii)** 22% **iv)** 13%

c) The half-life of the battery is 34 days.

5. a)

t	$P(t)$
0	7.4
1	11.8
2	18.7
3	29.8
4	47.3
5	75.2
6	119.6
7	190.2
8	302.3
9	480.6
10	764.2

b)



c) The profit in 1995 corresponds to $t = 0$. Substitute $t = 0$ in the equation. Solve for P.

$P(0) = 7.4(1.59)^0$
$= 7.4$

The profit was $7.4 million.

d) 2015 is 20 years from 1995. Substitute $t = 20$ in the equation. Solve for P.

$P(20) = 7.4(1.59)^{20}$
$= 78\,917.66$

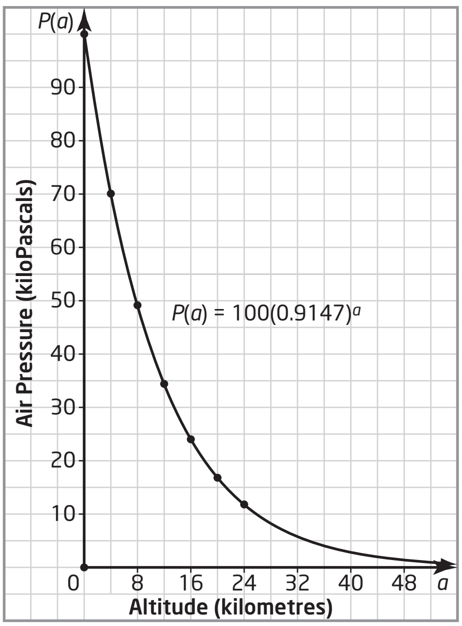
In 2015 the predicted profit is $78 917.66 million (or approximately $79 billion).

e) Substitute $P = 500$ in the equation. Solve for t.

$500 = 7.4(1.59)^t$ Divide each side by 7.4.
$67.6 = (1.59)^t$

Use systematic trial to find a value of t so that $(1.59)^t = 67.6$. From the table of values in part a), $h = 500$ lies between $t = 9$ and $t = 10$, so the nearest whole number is $t = 9$.
Add 9 years to 1995 to find the answer.
$1995 + 9 = 2004$
The profit reached \$500 million in 2004.

6. a)



b) Use an exponential regression. The equation $p(a) = 100(0.9147)^a$ represents the air pressure, measured in kiloPascals (kPa), at an altitude of a km above sea level.

c) Use the equation to determine air pressure for each location. Convert each altitude to kilometres. Since $1000 \text{ m} = 1 \text{ km}$, then $1 \text{ m} = \frac{1}{1000}$ or 0.001 km. Multiply each altitude by 0.001.

i) Mount Logan, 6050 m = 6.050 km

$$p(6.050) = 100(0.9147)^{6.050}$$
$$= 100(0.583\,090\,6)$$
$$= 58.309\,06$$

Therefore, the air pressure on Mount Logan is approximately 58.3 kPa.

ii) Mount Everest, 8848 m = 8.848 km

$$p(8.848) = 100(0.9147)^{8.848}$$
$$= 100(0.454\,353\,4)$$
$$= 45.435\,34$$

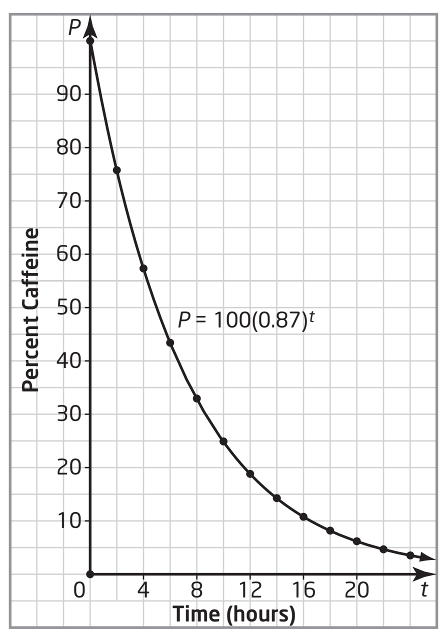
Therefore, the air pressure on Mount Everest is approximately 45.4 kPa.

d) To determine the altitude when the air pressure is 20 kPa, substitute $p = 20$ into the equation and solve for a. From the table we know that $p = 24$ when $a = 16$ and also $p = 16.8$ when $a = 20$. Since $p = 20$ is between 16.8 and 24, try values of a between 16 and 20. Try $a = 18$. $100(0.9147)^{18} = 20.09$. This value is very close to 20. Therefore, the air pressure is 20 kPa at an altitude of approximately 18 km above sea level.

7. a)

t (hours)	P (% caffeine remaining)
0	100
2	75.7
4	57.3
6	43.4
8	32.8
10	24.8
12	18.8
14	14.2
16	10.8
18	8.2
20	6.2
22	4.7
24	3.5

b) exponential decay



c) **i)** 87% **ii)** 28.6% **iii)** 12.4%

d) approximately 5 h

8. **a)** $l = p(1.20)^n$

b) $t = q(0.83)^n$

c) $l = p(1.20)^n$ represents exponential growth; the ratio 1.20 is greater than 1. $t = q(0.83)^n$ represents exponential decay; the ratio 0.83 is between 0 and 1.

d) $l = 2.00(1.20)^n$; $t = 0.50(0.83)^n$

e) $l = 12.38$ m; $t = 0.08$ m

f) 13 passes; approximately 0.04 m

g) 33 passes; approximately 820 m

Chapter 3 Review

1. **a)** C

b) 85 is the initial population; 3^n represents the constant ratio for tripling

2. **a)–c)** 1 **d)** −1

3. **a)** **i)** exponential **ii)** linear **iii)** quadratic

b) **i)** For exponential functions the constant ratio of the first and second differences is equal.

ii) For linear functions the first differences are equal.

iii) For quadratic functions the second differences are equal.

4. **a)** $\frac{1}{x^3}$ **b)** $\frac{3}{b^2}$

5. **a)** w^{-4} **b)** $-3b^{-8}$

6. **a)** $\frac{1}{216}$ **b)** $\frac{6}{125}$

c) $\frac{1}{12}$ **d)** 8

e) $\frac{125}{216}$ **f)** $\frac{49}{64}$

7. **a)** $\frac{b^{10}}{9}$ **b)** $-\frac{b^6}{8a^6}$

c) $\frac{x^{18}}{729}$ **d)** $\frac{243d^{10}}{32c^{20}}$

8. **a)** $\frac{16}{9}$ **b)** $\frac{2187}{128}$

c) −5 **d)** 9

9. **a)** $(-3125)^{\frac{4}{5}}$ **b)** $32^{\frac{3}{5}}$

10. **a)** $\sqrt[5]{(-32)^4}$; 16 **b)** $\sqrt[3]{\left(\frac{1}{343}\right)^2}$; $\frac{1}{49}$

c) $\sqrt[3]{(-125)^2}$; 25

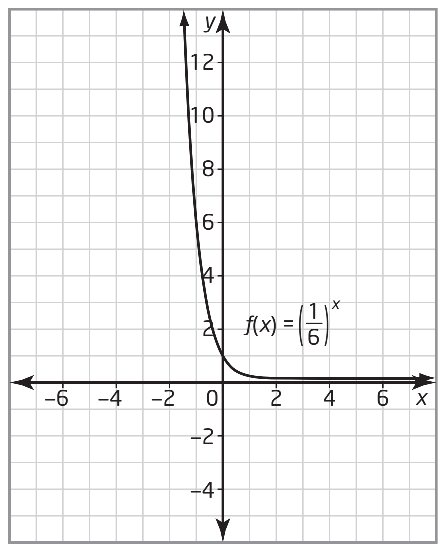
11. **a)** $16^{-\frac{1}{2}}$; $\frac{1}{4}$ **b)** $81^{\frac{3}{2}}$; 729 **c)** $256^{\frac{1}{8}}$; 2

12. **a)** $s^{\frac{5}{12}}$ **b)** $m^{\frac{2}{7}}n^{\frac{8}{15}}$ **c)** $\frac{1}{k^{\frac{3}{14}}}$

d) $\frac{2}{32}v^{\frac{9}{10}}$

13. **a)** $s = \frac{2\sqrt{A}}{3^{\frac{1}{4}}}$ **b)** 5.39 m

14.a)



i) domain $\{x \in \mathbb{R}\}$

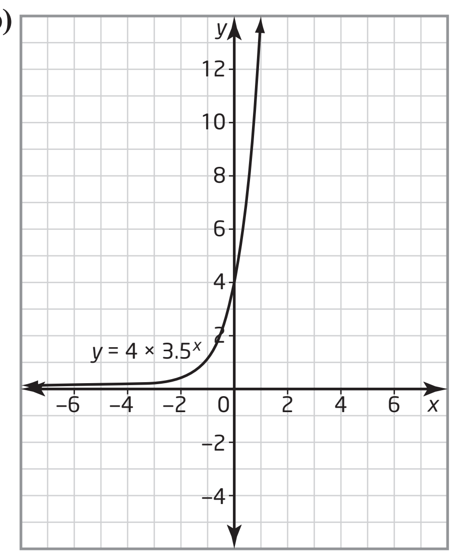
ii) range $\{y \in \mathbb{R}, y > 0\}$

iii) y-intercept is 1; no x-intercept

iv) decreasing for $x \in \mathbb{R}$

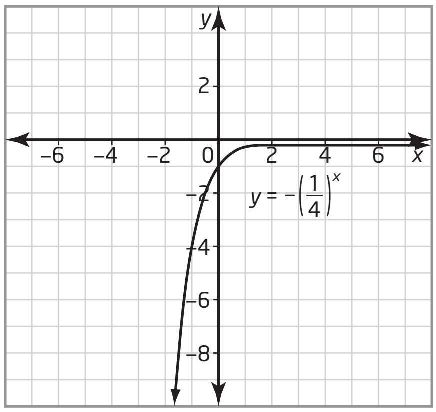
v) horizontal asymptote $y = 0$

b)



i) domain $\{x \in \mathbb{R}\}$
ii) range $\{y \in \mathbb{R}, y > 0\}$
iii) y-intercept is 4; no x-intercept
iv) increasing for $x \in \mathbb{R}$
v) horizontal asymptote $y = 0$

c)



i) domain $\{x \in \mathbb{R}\}$
ii) range $\{y \in \mathbb{R}, y < 0\}$
iii) y-intercept is −1; no x-intercept
iv) increasing for $x \in \mathbb{R}$
v) horizontal asymptote $y = 0$

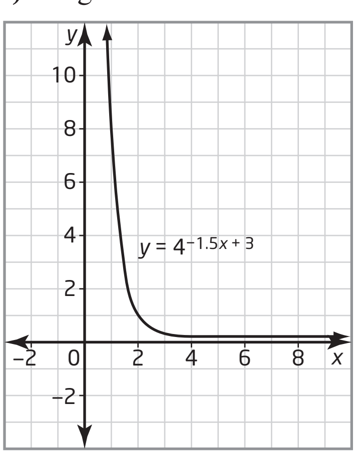
15.a) $A = 28\left(\frac{1}{2}\right)^{\frac{t}{5}}$

b) Answers may vary. Sample answer: In 5 days half the amount, or 14 grams, of the sample remain. The amount is reduced as time passes.

c) The graph falls to the right, since the amount of radioactive sample decreases. The first point on the graph is (0, 28) and the x-axis, or line $y = 0$, is the horizontal asymptote.
domain $\{t \in \mathbb{R}, t > 0\}$,
range $\{A \in \mathbb{R}, A > 0\}$

d) 4 mg

16.



17. a) The function $f(x) = 0.5^x$ is stretched vertically by a factor of 2, stretched horizontally by a factor of 3, and shifted right 5 units. The corresponding transformed equation is $f(x) = 2\left(0.5^{\frac{1}{3}(x-5)}\right)$.

b) domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y > 0\}$. The equation of the horizontal asymptote is $y = 0$.

18. a) The equation of the transformed function is

$$f(x) = -\frac{1}{3}\left[\left(\frac{1}{4}\right)^{2(x+4)}\right] + 6$$

b) domain $\{x \in \mathbb{R}\}$,
range $\{y \in \mathbb{R}, y < 6\}$. The equation of the horizontal asymptote is $y = 6$.

19. a)

t (years since 1981)	P (population in millions)
0	24.0
2	24.7
4	25.4
6	26.1
8	26.8
10	27.6
12	28.4
14	29.2
16	30.0
18	30.8
20	31.7

b) $P(t) = 24.0(1.014)^t$
c) 31.3 million
d) 36.9 million
e) approximately 2018

Chapter 3 Math Contest

1. D
2. B
3. C
4. D
5. A
6. 6
7. $x = 0, 1$
8. D
9. C
10. −20
11. A
12. D
13. $\{y \in \mathbb{R}, y < 2\}$

Chapter 4 Trigonometry

4.1 Special Angles

1.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	undefined
180°	0	–1	0
270°	–1	0	undefined
360°	0	1	0

2.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0	1	0
30°	0.5000	0.8660	0.5774
45°	0.7071	0.7071	1
60°	0.8660	0.5000	1.7321
90°	1	0	undefined
180°	0	–1	0
270°	–1	0	undefined
360°	0	1	0

3. **a)** 45° **b)** 135°, 315°
c) $\sin 225° = -\frac{1}{\sqrt{2}}$, $\cos 225° = -\frac{1}{\sqrt{2}}$, $\tan 225° = 1$
d) $\sin 135° = \frac{1}{\sqrt{2}}$, $\cos 135° = -\frac{1}{\sqrt{2}}$, $\tan 135° = -1$, $\sin 315° = -\frac{1}{\sqrt{2}}$, $\cos 315° = \frac{1}{\sqrt{2}}$, $\tan 315° = -1$

4. **a)** 30° **b)** 210°, 330°
c) $\sin 150° = \frac{1}{2}$, $\cos 150° = -\frac{\sqrt{3}}{2}$, $\tan 150° = -\frac{1}{\sqrt{3}}$
d) $\sin 210° = -\frac{1}{2}$, $\cos 210° = -\frac{\sqrt{3}}{2}$, $\tan 210° = \frac{1}{\sqrt{3}}$, $\sin 330° = -\frac{1}{2}$, $\cos 330° = \frac{\sqrt{3}}{2}$, $\tan 330° = -\frac{1}{\sqrt{3}}$

5. **a)** 60° **b)** 120°, 240°
c) $\sin 300° = -\frac{\sqrt{3}}{2}$, $\cos 300° = \frac{1}{2}$, $\tan 300° = -\sqrt{3}$
d) $\sin 120° = \frac{\sqrt{3}}{2}$, $\cos 120° = -\frac{1}{2}$, $\tan 120° = -\sqrt{3}$, $\sin 240° = -\frac{\sqrt{3}}{2}$, $\cos 240° = -\frac{1}{2}$, $\tan 240° = \sqrt{3}$

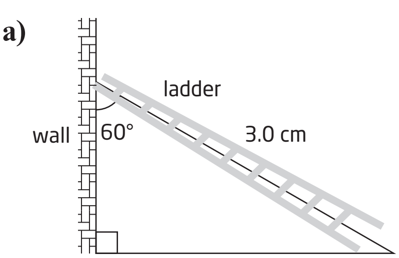
6. Answers may vary. Sample answer: $\sin 70° = 0.94$, $\cos 70° = 0.34$, $\tan 70° = 2.75$

7. Answers may vary. Sample answer: $\sin 220° = -0.64$, $\cos 220° = -0.77$, $\tan 220° = 0.84$

8. **a)** The CAST rule identifies the quadrant in which each trigonometric ratio is positive. In the first quadrant, all the trigonometric ratios are positive. The sine ratio is positive in the second quadrant. The tangent is positive in the third quadrant, and the cosine is positive in the fourth quadrant.
b)

Ratio	Positive in Quadrants:	Negative in Quadrants:
sin	first and second	third and fourth
cos	first and fourth	second and third
tan	first and third	second and fourth

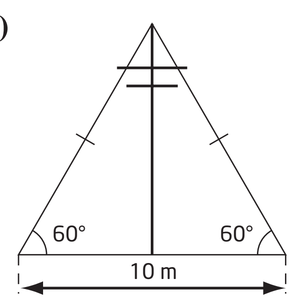
9. **a)**



b) $\frac{3\sqrt{3}}{2}$ m **c)** 1.5 m

10. **a)** $14\sqrt{2}$ km **b)** Pythagorean theorem

11. **a)**



b) Let h represent the height of the hydro pole. The wires are of equal length, so the large triangle formed by the two wires is isosceles. The pole, which represents the altitude of the triangle, is halfway between the two points where the wires are secured to the ground. Therefore, the distance from the bottom of the pole to one of the secured points is 5 m.

Solve $\tan 60° = \frac{h}{5}$.

Substitute $\tan 60° = \sqrt{3}$.

$\sqrt{3} = \frac{h}{5}$

$h = 5(\sqrt{3})$

The pole is $5(\sqrt{3})$ m tall.

c) Let w represent the length of each wire. To determine the length of each wire, find the length of the hypotenuse of one of the right triangles. From part a) we now know the height of the pole.

Solve $\sin 60° = \frac{h}{w}$.

Substitute $\sin 60° = \frac{\sqrt{3}}{2}$ and $h = 5(\sqrt{3})$.

$\frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{w}$ Multiply each side by $2w$.

$\sqrt{3}w = 10(\sqrt{3})$ Simplify.

Each wire is 10 m long.

12. $96\sqrt{3}$ cm^2

13. a) $\frac{-(1+\sqrt{3})}{2}$ **b)** $\frac{1}{2\sqrt{3}}$ **c)** $3\sqrt{3} + \frac{1}{\sqrt{2}}$

14. Answers may vary.

15. a) 60°, 120° **b)** 45°, 135°, 225°, 315° **c)** 150°, 330°

16. $15 + 5\sqrt{3}$ m

17. b) i) $A = s^2$ **ii)** $A = \frac{3\sqrt{3}}{2}s^2$ **iii)** $A = \frac{\sqrt{3}}{4}s^2$

4.2 Co-terminal and Related Angles

1. a) $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$

b) $\sin\theta = -\frac{2}{\sqrt{53}}$, $\cos\theta = -\frac{7}{\sqrt{53}}$, $\tan\theta = \frac{2}{7}$

c) $\sin\theta = \frac{3}{\sqrt{45}}$, $\cos\theta = -\frac{6}{\sqrt{45}}$, $\tan\theta = -\frac{1}{2}$

d) $\sin\theta = -\frac{5}{\sqrt{29}}$, $\cos\theta = \frac{2}{\sqrt{29}}$, $\tan\theta = -\frac{5}{2}$

2. a) $\sin\theta = \frac{5}{\sqrt{34}}$, $\cos\theta = -\frac{3}{\sqrt{34}}$, $\tan\theta = -\frac{5}{3}$

b) $\sin\theta = \frac{8}{17}$, $\cos\theta = -\frac{15}{17}$, $\tan\theta = -\frac{8}{15}$

c) $\sin\theta = -\frac{4}{5}$, $\cos\theta = -\frac{3}{5}$, $\tan\theta = \frac{4}{3}$

d) $\sin\theta = \frac{12}{13}$, $\cos\theta = -\frac{5}{13}$, $\tan\theta = -\frac{12}{5}$

e) $\sin\theta = \frac{3}{\sqrt{58}}$, $\cos\theta = \frac{7}{\sqrt{58}}$, $\tan\theta = \frac{3}{7}$

f) $\sin\theta = -\frac{9}{\sqrt{82}}$, $\cos\theta = \frac{1}{\sqrt{82}}$, $\tan\theta = -9$

3. a) $\sin A = \frac{15}{17}$, $\tan A = -\frac{15}{18}$

b) $\cos B = -\frac{3}{5}$, $\tan B = \frac{4}{3}$

c) $\sin C = -\frac{12}{13}$, $\cos C = \frac{5}{13}$

d) $\cos D = \frac{6}{\sqrt{85}}$, $\tan D = \frac{7}{6}$

e) $\sin E = \frac{4\sqrt{10}}{13}$, $\tan E = -\frac{4\sqrt{10}}{3}$

f) $\sin F = -\frac{8}{\sqrt{233}}$, $\cos F = \frac{13}{\sqrt{233}}$

4. Answers may vary. Sample answers:

a) sin 120°, sin (−240°)

b) cos 150°, cos (−210°)

c) tan 135°, tan (−45°)

d) sin 40°, sin (−220°)

e) cos 75°, cos (−75°)

f) tan 10°, tan (−170°)

5. a) To determine three positive angles that are co-terminal with 205°, add multiples of 360° to 205°. The expression 205° + n360°, where n is a positive integer, will result in positive co-terminal angles.

Let $n = 1$ to obtain one angle:
205° + (1)360° = 565°.

Let $n = 2$ to obtain a second angle:
205° + (2)360° = 925°.

Let $n = 3$ to obtain a third angle:
205° + (3)360° = 1285°.

b) To determine three negative angles that are co-terminal with 310°, subtract multiples of 360° from 310°. The expression 310° − n360°, where n is a positive integer, will result in negative co-terminal angles.

Let $n = 1$ to obtain one angle:
310° − (1)360° = −50°.

Let $n = 2$ to obtain a second angle:
310° − (2)360° = −410°.

Let $n = 3$ to obtain a third angle:
310° − (3)360° = −770°.

6. **a)** not co-terminal; Their difference is not a multiple of 360°.
   b) co-terminal; Their difference is 360°.
   c) co-terminal; Their difference is a multiple of 360°.
   d) not co-terminal; Their difference is not a multiple of 360°.
   e) co-terminal; Their difference is 360°.
   f) co-terminal; Their difference is a multiple of 360°.
   g) not co-terminal; Their difference is not a multiple of 360°.
   h) co-terminal; Their difference is 360°.

7. **a)** $\sin A = -\frac{1}{2}$, $\cos A = \frac{\sqrt{3}}{2}$, $\tan A = -\frac{1}{\sqrt{3}}$
   b) $\sin B = \frac{\sqrt{3}}{2}$, $\cos B = -\frac{1}{2}$, $\tan B = -\sqrt{3}$
   c) $\sin C = 0$, $\cos C = -1$, $\tan C = 0$
   d) $\sin D = \frac{1}{\sqrt{2}}$, $\cos D = \frac{1}{\sqrt{2}}$, $\tan D = 1$
   e) $\sin E = \frac{1}{2}$, $\cos E = \frac{\sqrt{3}}{2}$, $\tan E = \frac{1}{\sqrt{3}}$
   f) $\sin F = -\frac{1}{2}$, $\cos F = -\frac{\sqrt{3}}{2}$, $\tan F = \frac{1}{\sqrt{3}}$

8. From the CAST rule we know that sine is negative in the third and fourth quadrants. From our special triangles we know that $\sin 45° = \frac{1}{\sqrt{2}}$.
   In the third quadrant, the angle A that results in $\sin A = -\frac{1}{\sqrt{2}}$ is $A = 225°$.
   In the fourth quadrant, the angle B that results in $\sin B = -\frac{1}{\sqrt{2}}$ is $B = 315°$.

9. 45°, 225°

10. 240°, 300°

11. 0°, 180°, 360°; sine

12. **a)** $\sin A = -\frac{7}{\sqrt{74}}$, $\cos A = \frac{5}{\sqrt{74}}$, $\tan A = -\frac{7}{5}$;
    $\sin B = -\frac{7}{\sqrt{74}}$, $\cos B = -\frac{5}{\sqrt{74}}$, $\tan B = \frac{7}{5}$
    b) $\angle A = 306°$, $\angle B = 234°$

13. **a)** $\sin C = -\frac{1}{\sqrt{37}}$, $\cos C = -\frac{6}{\sqrt{37}}$, $\tan C = \frac{1}{6}$;
    $\sin D = \frac{1}{\sqrt{37}}$, $\cos D = -\frac{6}{\sqrt{37}}$, $\tan D = -\frac{1}{6}$
    b) $\angle C = 189°$, $\angle D = 171°$

14. **a)** $\sin E = \frac{3}{\sqrt{13}}$, $\cos E = -\frac{2}{\sqrt{13}}$, $\tan E = -\frac{3}{2}$;
    $\sin F = -\frac{3}{\sqrt{13}}$, $\cos F = \frac{2}{\sqrt{13}}$, $\tan F = -\frac{3}{2}$
    b) $\angle E = 124°$, $\angle F = 304°$

15. **a)** $\theta = 229°$ or $\theta = 311°$
    b) $\theta = 66°$ or $\theta = 294°$
    c) $\theta = 144°$ or $\theta = 324°$

16. **a)** 27.7 square units
    b) 304.0 square units
    c) 1383.9 square units

17. 146.3°

18. **a)** $P\left(-\frac{3}{\sqrt{58}}, -\frac{7}{\sqrt{58}}\right)$
    b) $\sin\theta = \frac{7}{\sqrt{58}}$, $\cos\theta = -\frac{3}{\sqrt{58}}$
    c) The corresponding trigonometric ratios are equal.

4.3 Reciprocal Trigonometric Ratios

1. $\sin C = \frac{7}{25}$, $\csc C = \frac{25}{7}$, $\cos C = \frac{24}{25}$,
   $\sec C = \frac{25}{24}$, $\tan C = \frac{7}{24}$, $\cot C = \frac{24}{7}$

2. $\sin A = \frac{24}{25}$, $\cos A = \frac{7}{25}$, $\tan A = \frac{24}{7}$,
   $\csc A = \frac{25}{24}$, $\sec A = \frac{25}{7}$, $\cot A = \frac{7}{24}$

3. **a)** $\csc\theta = \frac{5}{3}$ **b)** $\sec\theta = \sqrt{2}$
   c) $\cot\theta = \frac{3}{7}$ **d)** $\sec\theta = -\frac{\sqrt{61}}{6}$
   e) $\cot\theta = -\frac{1}{5}$ **f)** $\csc\theta = -\frac{13}{12}$
   g) $\sec\theta =$ undefined
   h) $\csc\theta = 1$

4. **a)** $\cos\theta = \frac{3}{8}$ **b)** $\sin\theta = \frac{4}{5}$
   c) $\tan\theta = -\sqrt{3}$ **d)** $\cos\theta = -\frac{15}{17}$
   e) $\csc\theta = \frac{1}{\sqrt{2}}$ **f)** $\tan\theta = -\frac{4}{9}$
   g) $\cos\theta = -1$ **h)** $\sin\theta = 0$

5. **a)** $\sin 40° = 0.643$, $\cos 40° = 0.766$, $\tan 40° = 0.839$, $\csc 40° = 1.556$, $\sec 40° = 1.305$, $\cot 40° = 1.192$
   b) $\sin 36° = 0.588$, $\cos 36° = 0.809$, $\tan 36° = 0.727$, $\csc 36° = 1.701$, $\sec 36° = 1.236$, $\cot 36° = 1.376$

c) sin 88° = 0.999, cos 88° = 0.035, tan 88° = 28.636, csc 88° = 1.001, sec 88° = 28.654, cot 88° = 0.035

d) sin 110° = 0.940, cos 110° = −0.342, tan 110° = −2.747, csc 110° = 1.064, sec 110° = −2.924, cot 110° = −0.364

e) sin 237° = −0.839, cos 237° = −0.545, tan 237° = 1.540, csc 237° = −1.192, sec 237° = −1.836, cot 237° = 0.649

f) sin 319° = −0.656, cos 319° = 0.755, tan 319° = −0.869, csc 319° = −1.524, sec 319° = 1.325, cot 319° = −1.150

g) sin 95° = 0.996, cos 95° = −0.087, tan 95° = −11.430, csc 95° = 1.004, sec 95° = −11.474, cot 95° = −0.087

h) sin 67° = 0.921, cos 67° = 0.391, tan 67° = 2.356, csc 67° = 1.086, sec 67° = 2.559, cot 67° = 0.424

i) sin 124° = 0.829, cos 124° = −0.559, tan 124° = −1.483, csc 124° = 1.206, sec 124° = −1.788, cot 124° = −0.675

6. **a)** 38° **b)** 56° **c)** 19° **d)** 61° **e)** 41° **f)** 50° **g)** 15° **h)** 74°

7. $\sin 210° = -\frac{1}{2}$, $\cos 210° = -\frac{\sqrt{3}}{2}$, $\tan 210° = \frac{1}{\sqrt{3}}$, $\csc 210° = -2$, $\sec 210° = -\frac{2}{\sqrt{3}}$, $\cot 210° = \sqrt{3}$

8. $\sin 225° = -\frac{1}{\sqrt{2}}$, $\cos 225° = -\frac{1}{\sqrt{2}}$, $\tan 225° = 1$, $\csc 225° = -\sqrt{2}$, $\sec 225° = -\sqrt{2}$, $\cot 225° = 1$

9. sin 90° = 1, cos 90° = 0, tan 90° = undefined, csc 90° = 1, sec 90° = undefined, cot 90° = 0

10. 240°, 300°

11. **a)** 33° **b)** 37° **c)** 69° **d)** 44° **e)** 51° **f)** 33° **g)** 71° **h)** 35°

12. 45°, 225°

13. **a)** $\sin\theta = \frac{4}{5}$, $\csc\theta = \frac{5}{4}$, $\cos\theta = -\frac{3}{5}$, $\sec\theta = -\frac{5}{3}$, $\tan\theta = -\frac{4}{3}$, $\cot\theta = -\frac{3}{4}$

b) $\sin\theta = -\frac{5}{13}$, $\csc\theta = -\frac{13}{5}$, $\cos\theta = -\frac{12}{13}$, $\sec\theta = -\frac{13}{12}$, $\tan\theta = \frac{5}{12}$, $\cot\theta = \frac{12}{5}$

c) $\sin\theta = \frac{15}{17}$, $\csc\theta = \frac{17}{15}$, $\cos\theta = -\frac{8}{15}$, $\sec\theta = -\frac{15}{8}$, $\tan\theta = -\frac{15}{8}$, $\cot\theta = -\frac{8}{15}$

d) $\sin\theta = \frac{1}{\sqrt{10}}$, $\csc\theta = \sqrt{10}$, $\cos\theta = \frac{3}{\sqrt{10}}$, $\sec\theta = \frac{\sqrt{10}}{3}$, $\tan\theta = \frac{1}{3}$, $\cot\theta = 3$

e) $\sin\theta = -\frac{3}{\sqrt{13}}$, $\cos\theta = \frac{2}{\sqrt{13}}$, $\tan\theta = -\frac{3}{\sqrt{13}}$, $\csc\theta = -\frac{3}{\sqrt{13}}$, $\sec\theta = \frac{\sqrt{13}}{2}$, $\cot\theta = -\frac{2}{3}$

f) $\sin\theta = -\frac{12}{\sqrt{193}}$, $\cos\theta = -\frac{7}{\sqrt{193}}$, $\tan\theta = \frac{12}{7}$, $\csc\theta = -\frac{\sqrt{193}}{12}$, $\sec\theta = -\frac{\sqrt{193}}{7}$, $\cot\theta = \frac{7}{12}$

g) $\sin\theta = \frac{5}{\sqrt{26}}$, $\cos\theta = \frac{1}{\sqrt{26}}$, $\tan\theta = 5$, $\csc\theta = \frac{\sqrt{26}}{5}$, $\sec\theta = \sqrt{26}$, $\cot\theta = \frac{1}{5}$

h) $\sin\theta = \frac{11}{\sqrt{157}}$, $\cos\theta = \frac{6}{\sqrt{157}}$, $\tan\theta = \frac{11}{6}$, $\csc\theta = \frac{\sqrt{157}}{11}$, $\sec\theta = \frac{\sqrt{157}}{6}$, $\cot\theta = \frac{6}{11}$

i) $\sin\theta = \frac{1}{\sqrt{2}}$, $\cos\theta = -\frac{1}{\sqrt{2}}$, $\tan\theta = -1$, $\csc\theta = \sqrt{2}$, $\sec\theta = -\sqrt{2}$, $\cot\theta = -1$

j) $\sin\theta = -\frac{3}{\sqrt{73}}$, $\cos\theta = \frac{8}{\sqrt{73}}$, $\tan\theta = -\frac{3}{8}$, $\csc\theta = -\frac{\sqrt{73}}{3}$, $\sec\theta = \frac{\sqrt{73}}{8}$, $\cot\theta = -\frac{8}{3}$

14. **a)** $\csc\theta = \sqrt{37}$, $\sec\theta = \frac{\sqrt{37}}{6}$, $\cot\theta = 6$; $\theta = 9°$

b) $\csc\theta = \frac{\sqrt{29}}{5}$, $\sec\theta = \frac{\sqrt{29}}{2}$, $\cot\theta = \frac{2}{5}$; $\theta = 68°$

c) $\csc\theta = \frac{\sqrt{113}}{7}$, $\sec\theta = \frac{\sqrt{113}}{8}$, $\cot\theta = \frac{8}{7}$; $\theta = 41°$

d) $\csc\theta = \frac{\sqrt{73}}{3}$, $\sec\theta = \frac{\sqrt{73}}{8}$, $\cot\theta = \frac{8}{3}$; $\theta = 21°$

e) $\csc\theta = \frac{\sqrt{202}}{11}$, $\sec\theta = \frac{\sqrt{202}}{9}$, $\cot\theta = \frac{9}{11}$; $\theta = 51°$

f) $\csc\theta = \frac{\sqrt{65}}{4}$, $\sec\theta = \frac{\sqrt{65}}{7}$, $\cot\theta = \frac{7}{4}$; $\theta = 30°$

15. First determine the value of the smallest positive angle θ such that $\csc \theta = 3.5$.
Solve $\sin \theta = \frac{1}{3.5}$ to get $\theta = 17°$. Using the CAST rule we know that cosecant is negative where sine is negative, that is, in the third and fourth quadrants.
In the third quadrant, the angle is $180° + 17° = 197°$.
In the fourth quadrant, the angle is $360° - 17° = 343°$.
The two angles are 197° and 343°.

16. 99°, 261°

17. 14°, 194°

18. $\sin \theta = -\frac{24}{25}$, $\cos \theta = \frac{7}{25}$, $\csc \theta = -\frac{25}{24}$, $\sec \theta = \frac{25}{7}$, $\cot \theta = -\frac{7}{24}$

19. $\sin \theta = \frac{4}{5}$, $\cos \theta = -\frac{3}{5}$, $\tan \theta = -\frac{4}{3}$, $\sec \theta = -\frac{5}{3}$, $\cot \theta = -\frac{3}{4}$

20. $\sin \theta = -\frac{\sqrt{8}}{3}$, $\tan \theta = \sqrt{8}$, $\csc \theta = -\frac{3}{\sqrt{8}}$, $\sec \theta = -3$, $\cot \theta = \frac{1}{\sqrt{8}}$

21. $\sin \theta = \frac{\sqrt{33}}{7}$, $\cos \theta = \frac{4}{7}$, $\tan \theta = \frac{\sqrt{33}}{4}$, $\csc \theta = \frac{7}{\sqrt{33}}$, $\cot \theta = \frac{4}{\sqrt{33}}$

22. **a)** $\sec 48° = \frac{w}{16.7}$, where w represents the length of the wire
b) 25.0 m

23. **a)** $\csc 14.5° = \frac{r}{1.3}$, where r represents the length of the ramp
b) 5.2 m, or 5 m and 20 cm

24. **a)** $\frac{171}{196}$
b) $\frac{16}{153}$
c) $\frac{25}{169}$
d) $\frac{153}{16}$

25. Substitute $\sin \theta = \frac{y}{r}$ and $\cos \theta = \frac{x}{r}$ into the left side of the given equation.

$$\begin{aligned}
\text{L.S.} &= \sin^2 \theta + \cos^2 \theta \\
&= \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2 \\
&= \frac{y^2}{r^2} + \frac{x^2}{r^2} \\
&= \frac{x^2 + y^2}{r^2} && \text{Substitute } x^2 + y^2 = r^2 \text{ in the numerator.} \\
&= \frac{r^2}{r^2} \\
&= 1 \\
&= \text{R.S.}
\end{aligned}$$

The name is appropriate because the identity represents the Pythagorean theorem in terms of the trigonometric ratios.

b) Substitute $\sin \theta = \frac{1}{\cos \theta}$ and $\cos \theta = \frac{1}{\sec \theta}$ in $\sin^2 \theta + \cos^2 \theta = 1$.
An equivalent equation is
$\frac{1}{\cos^2 \theta} + \frac{1}{\sec^2 \theta} = 1$.

26. $\sin \theta = -\frac{\sqrt{a^2 - b^2}}{a}$, $\cos \theta = -\frac{b}{a}$, $\tan \theta = \frac{\sqrt{a^2 - b^2}}{b}$, $\csc \theta = -\frac{a}{\sqrt{a^2 - b^2}}$, $\cot \theta = \frac{b}{\sqrt{a^2 - b^2}}$

27. $\csc \theta = \frac{\sqrt{5x^2 + 2x + 1}}{x + 1}$, $\sec \theta = \frac{\sqrt{5x^2 + 2x + 1}}{2x}$

4.4 Problems in Two Dimensions

1. **a)** Because ΔABC is a right triangle, use the cosine ratio.
b) Because ΔPQR is a right triangle, use the tangent ratio.
c) ΔABC is an oblique triangle where three sides are known, so use the cosine law.
d) ΔDEF is an oblique triangle where two sides and the contained angle are known, so use the cosine law.
e) ΔXYZ is a right triangle, so use the tangent ratio.

2. **a)** $a = 10.6$ cm **b)** $r = 7.3$ m
c) $\angle B = 44.0°$ **d)** $d = 16.3$ cm
e) $\angle Y = 51.7°$

3. 61.1°

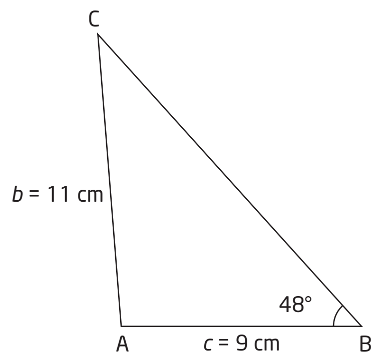
4. **a)** $a = 8.6$ cm, $\angle B = 85°$, $\angle C = 60°$
b) $\angle A = 117.3°$, $\angle B = 26.4°$, $\angle C = 36.3°$
c) $b = 19.8$ m, $c = 17.7$ m, $\angle C = 63°$

5. 4.7 m

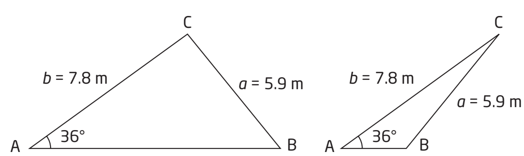
6. 6.0 km

7. **a)** 4.5 km **b)** 92.4°

8. **a)** This triangle has only one solution, $a = 14.7$ cm.



b) $c = 10.0$ m or $c = 2.6$ m



9. **a)** $\angle C = 39°$, $b = 4.7$ cm, $\angle C = 141°$, $b = 1.0$ cm
   b) $\angle F = 56°$, $e = 6$ cm, $\angle F = 124°$, $e = 2.7$ cm

10. **a)** $b \sin A = 2.7 \sin 32°$
$= 1.4$
   b) Since $a = 1.2$ and $1.2 < 1.4$, then $a < b \sin A$, so there is no solution.

11. **a)** 2.4 **b)** one solution

12. **a)** 4.3 **b)** two solutions

13. 8.1 km

14. 147.5 km or 64.4 km

15. 122 m

16. Aqua: 31.6 km; Belli: 22.8 km

17. 5.8 km or 50.1 km

18. 9.2 units, 12.4 units

19. $\angle A = 49°$, $\angle C = 101°$, AB = 7.8 cm OR $\angle A = 131°$, $\angle C = 19°$, AB = 2.6 cm

20. Answer may vary.

21. **a)** Two solutions will occur when $a < b$ and $a > b \sin A$. Substitute $\angle A = 40°$ and $b = 50.0$ to get $a > 50 \sin 40°$, where $50 \sin 40° = 32.1$. Note, however, that since $b = 50.0$ and $a < b$, then $a < 50$. The range of values for a is $32.1 < a < 50.0$.
   b) $a < 104.2$ **c)** $a = 61.8$ or $a > 73.7$

4.5 Problems in Three Dimensions

1. 43°

2. **a)** 18.8 m **b)** 74.4°

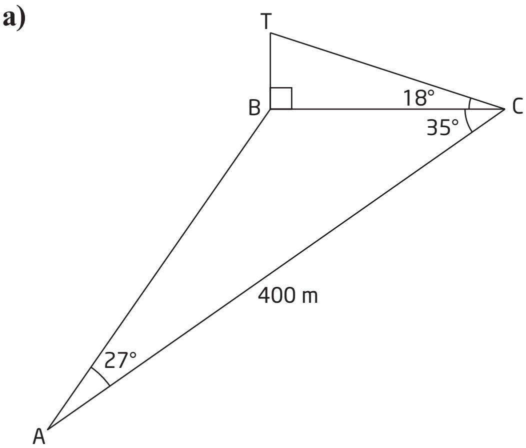
3. **a)** 3.4 m **b)** 6.8 m **c)** 9.6 m

4. 14.8 cm

5. 4.2 km

6. **a)** 13.7 cm **b)** 21°

7. **a)**



b) There is insufficient information in $\triangle TBC$ to find the height TB directly; however, since BC is in common to $\triangle ABC$ and $\triangle TBC$, use $\triangle ABC$ to find the length of BC. Then, find the height TB in $\triangle TBC$.

In $\triangle ABC$, $\angle B = 180° - 27° - 35°$
$= 118°$

Use the sine law to find the length of BC.

$$\frac{BC}{\sin 27°} = \frac{400}{\sin 118°}$$
$$BC = \frac{400 \sin 27°}{\sin 118°}$$
$$BC \doteq 205.7$$

$\triangle TBC$ is a right triangle. Use the tangent ratio to determine the height TB.

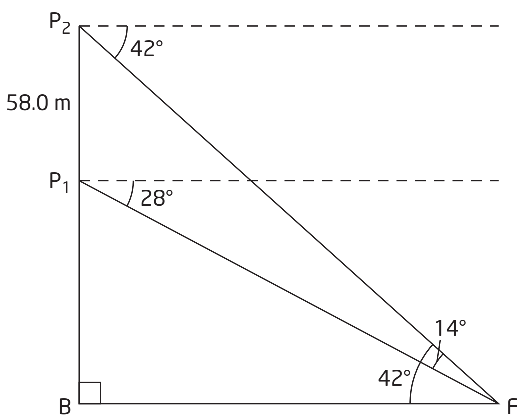
$$\tan 18° = \frac{TB}{205.7}$$
$$TB = 205.7 \tan 18°$$
$$\doteq 66.8$$

The height of the cliff is approximately 67 m.

8. 120 m

9. First draw a diagram to represent this situation.

Let P_1 represent the initial position of the balloon before it rises.
Let P_2 represent the balloon's position after it rises vertically.
Let B represent the barn and let F represent the farmhouse.
Mark the angles of depression.
Use parallel lines to determine that $\angle BFP_2 = 42°$ and $\angle BFP_1 = 28°$.
Subtract $42° - 28°$ to determine that $\angle P_1FP_2 = 14°$.



a) There is insufficient information to determine the initial height of the balloon, P_1B.
In ΔP_1P_2F,

$$\angle P_2P_1F = 90° + 28° \doteq 118°$$

Determine the distance P_2F by using the sine law in ΔP_1P_2F.

$$\frac{58}{\sin 14°} = \frac{P_2F}{\sin 118°}$$

$$P_2F = \frac{58 \sin 118°}{\sin 14°} = 211.7$$

ΔP_2BF is a right triangle. Use the sine ratio to determine the distance P_1B.

$$\sin 42° = \frac{P_1B + 58}{211.7}$$

$$P_1B = 211.7 \sin 42° - 58 \doteq 83.7$$

The balloon is at a height of approximately 83.7 m before it rises.

b) ΔP_1BF is a right triangle. Use the cotangent ratio to determine the distance BF.

$$\cot 28° = \frac{BF}{83.7}$$

$$BF \doteq 157.4$$

The barn is approximately 157.4 m from the farmhouse.

10. 37.1 m

11. 37.4 m

12. 43.7 m

13. 1.36 km

4.6 Trigonometric Identities

1. Answers may vary. Sample answers:
a) $1 - \cos^2 \theta$ **b)** $\sec^2 \theta - 1$ **c)** $1 + \tan^2 \theta$
d) $\cos^2 \theta$ **e)** $-\cot^2 \theta$ **f)** -1
g) $\dfrac{-1}{1 + \cot^2 \theta}$ **h)** $\cot^2 \theta$

2. **a)** 1 **b)** $\sin \theta$ **c)** $\dfrac{1}{\sin \theta \cos \theta}$
d) $|\sin \theta|$ **e)** -1 **f)** $\dfrac{2}{\cos \theta}$

3. **a)** $\sin^2 \theta$
b) $\tan^2 \theta + \sin^2 \theta$
c) $4(1 + 2 \cos \theta \sin \theta)$
d) $\sin^2 \theta - \cos^2 \theta$ or $1 - 2 \cos^2 \theta$ or $2 \sin^2 \theta - 1$

4.–5. Answer may vary.

6. Technology may be used to graph each side of the identity. If the graphs are the same, then the identity is verified.

7. Answer may vary.

8.
$$\text{L.S.} = \frac{1 + \cot \theta}{\csc \theta} = \frac{1 + \frac{\cos \theta}{\sin \theta}}{\frac{1}{\sin \theta}} = \frac{\sin \theta}{1} \times \left(\frac{\sin \theta + \cos \theta}{\sin \theta}\right) = \sin \theta + \cos \theta$$

Therefore, L.S. = R.S., and the statement is true for all θ.

9. Answers may vary.

10. Prove $\sin^4\theta + 2\cos^2\theta - \cos^4\theta = 1$ for all θ.

$\text{L.S.} = \sin^4\theta + 2\cos^2\theta - \cos^4\theta$	
$= (\sin^2\theta)^2 + 2\cos^2\theta - \cos^4\theta$	
$= (1 - \cos^2\theta)^2 + 2\cos^2\theta - \cos^4\theta$	Substitute $\sin^2\theta = 1 - \cos^2\theta$.
$= 1 - 2\cos^2\theta + \cos^4\theta + 2\cos^2\theta - \cos^4\theta$	Expand $(1 - \cos^2\theta)^2$.
$= 1$	Simplify.

Therefore, L.S. = R.S., and the statement is true for all θ.

11. Answers may vary.

12. $1 + \cot^2\theta$

13.–14. Answers may vary.

15. When $\theta = 90°$, $\sin 90° = 1$, so $\frac{\cos\theta}{1-\sin\theta}$ has a value of 0 in the denominator. Therefore, this part of the expression is undefined. Similarly, when $\theta = 270°$, $\sin 270° = -1$, so $\frac{\cos\theta}{1+\sin\theta}$ has a value of 0 in the denominator. Therefore, this part of the expression is undefined.

16.–25. Answers may vary.

26. a) No **b)** Yes
c) Yes **d)** No

27. b) Answers may vary. Sample answer: Substitute an angle value and show that the left side is not equal to the right side.

28.–34. Answers may vary.

35. $\cot^2\theta(1 + \tan^2\theta) = \csc^2\theta$

36.–39. Answers may vary.

40. 1

Chapter 4 Review

1. Answers may vary.

2. a) $-\frac{1}{2\sqrt{6}}$ **b)** $1 - \sqrt{3}$ **c)** $\sqrt{3} + 1$

3. a) i) $6\sqrt{2}$ m **ii)** 6 m
b) i) $\frac{12}{\sqrt{3}}$ m **ii)** $\frac{6}{\sqrt{3}}$ m
c) i) 12 m **ii)** $6\sqrt{3}$ m

4. $\sqrt{\frac{20}{\sqrt{3}}}$ cm

5. a) $\sin\theta = \frac{12}{13}$, $\cos\theta = -\frac{5}{13}$, $\tan = -\frac{12}{5}$
b) $\sin = -\frac{3}{5}$, $\cos = \frac{4}{5}$, $\tan = -\frac{3}{4}$

6. a) $\cos G = -\frac{\sqrt{96}}{11}$, $\tan G = \frac{5}{\sqrt{96}}$
b) $\sin E = \frac{\sqrt{33}}{7}$, $\tan E = \frac{\sqrt{33}}{4}$

7. a) $\theta = 39°$ or $\theta = 141°$
b) $\theta = 110°$ or $\theta = 250°$
c) $\theta = 24°$ or $\theta = 204°$

8. $\sin 120° = \frac{\sqrt{3}}{2}$, $\cos 120° = -\frac{1}{2}$, $\tan 120° = -\sqrt{3}$, $\csc 120° = \frac{2}{\sqrt{3}}$, $\sec 120° = -2$, $\cot 120° = -\frac{1}{\sqrt{3}}$

9. a) $\sin\theta = -\frac{7}{25}$, $\csc\theta = -\frac{25}{7}$, $\cos\theta = \frac{24}{25}$, $\sec\theta = \frac{25}{24}$, $\tan\theta = -\frac{7}{24}$, $\cot\theta = -\frac{24}{7}$
b) $\sin\theta = -\frac{3}{\sqrt{34}}$, $\csc\theta = -\frac{\sqrt{34}}{3}$, $\cos\theta = -\frac{5}{\sqrt{34}}$, $\sec\theta = -\frac{\sqrt{34}}{5}$, $\tan\theta = \frac{3}{5}$, $\cot\theta = \frac{5}{3}$
c) $\sin\theta = -\frac{9}{\sqrt{97}}$, $\csc\theta = -\frac{\sqrt{97}}{9}$, $\cos\theta = \frac{4}{\sqrt{97}}$, $\sec\theta = \frac{\sqrt{97}}{4}$, $\tan\theta = -\frac{9}{4}$, $\cot\theta = -\frac{4}{9}$
d) $\sin\theta = \frac{3}{\sqrt{10}}$, $\csc\theta = \frac{\sqrt{10}}{3}$, $\cos\theta = \frac{1}{\sqrt{10}}$, $\sec\theta = \sqrt{10}$, $\tan\theta = 3$, $\cot\theta = \frac{1}{3}$

10. a) one solution; ∠B = 62°, ∠C = 47°, c = 9.4 m
b) no solution
c) two solutions; ∠B = 67°, ∠C = 69°, c = 12.5 mm *or* ∠B = 113°, ∠C = 23°, c = 5.2 mm
d) one solution; ∠E = 103°, ∠F = 35°, e = 12.4 km
e) two solutions; ∠D = 50°, ∠F = 92°, f = 21.8 mm *or* ∠D = 130°, ∠F = 12°, f = 4.5 mm
f) no solution

11. 10.3 km or 1.0 km

12. a) 9.2 m **b)** 49°

13. 97.6 m

14. 111.0 km

15. 84 km

16. a) $1 - \sin^2 \theta$ **b)** $1 + \cot^2 \theta$

c) $\csc^2 \theta - 1$ **d)** $\tan^2 \theta$

e) 1 **f)** $\dfrac{1}{1 + \tan^2\theta}$

17. Answers may vary.

Math Contest

1. 2:1

2. $3\sqrt{3}:4\pi$

3. $\dfrac{10\sqrt{3}}{3}$ cm

4. D

5. B

6. C

7. B

8. A

9. B

10. D

11. C

Chapter 5 Trigonometric Functions

5.1 Modelling Periodic Behaviour

1. a) periodic; pattern of y-values repeats at regular intervals

b) non-periodic; pattern of y-values does not repeat at regular intervals

c) non-periodic; pattern of y-values does not repeat at regular intervals

d) periodic; pattern of y-values repeats at regular intervals

e) periodic; pattern of y-values repeats at regular intervals

f) non-periodic; pattern of y-values does not repeat at regular intervals

g) periodic; pattern of y-values repeats at regular intervals

h) non-periodic; pattern of y-values does not repeat at regular intervals

2. a) period is 6; amplitude is 1.5

c) period is 360; amplitude is approximately 5

e) period is 6; amplitude is 3.5

g) period is 3; amplitude is 3.5

3.–5. Answers may vary.

6. a) 6

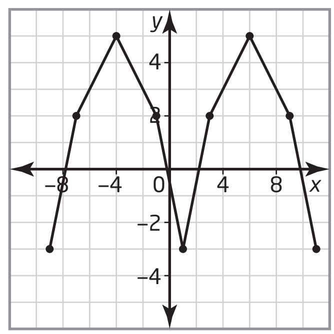
b) −4

c) 3

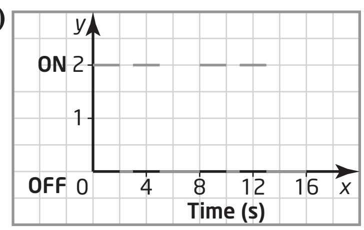
d) 2

7. Answers may vary.

8. No, the period is not necessarily 6. For instance, the following graph shows a periodic function with $f(3) = f(9) = 2$, but the period is 10.



9. a)



b) It is periodic because the pattern for the length of the flashes repeats at regular intervals.

c) 8 s

d) 1

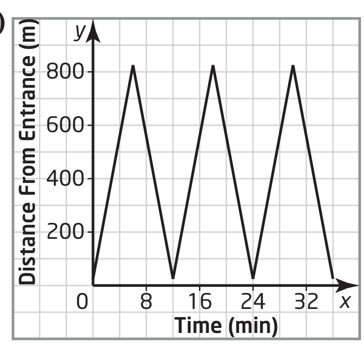
10. a) No

b) Yes

c) Yes

11. Answers may vary.

12. a)



b) period is 12 min; amplitude is 400 m

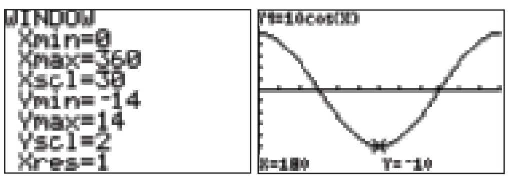
c) Let t represent the time, in minutes, and let d represent the distance from the entrance, in metres.
The domain is $\{t \in \mathbb{R}, 0 \leq t \leq 36\}$.
The range is $\{d \in \mathbb{R}, 25 \leq d \leq 825\}$.

13. The length of the interval that gives the domain is a whole number multiple of the period. Examples may vary.

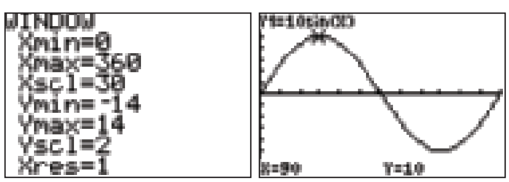
14. The amplitude is half the length of the interval that gives the range. Examples may vary.

15. Yes; period is 10

5.2 The Sine Function and the Cosine Function

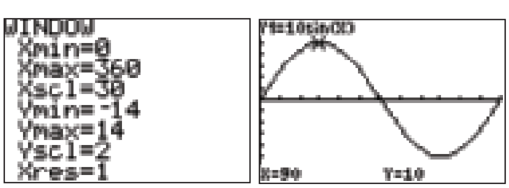
1. a) 

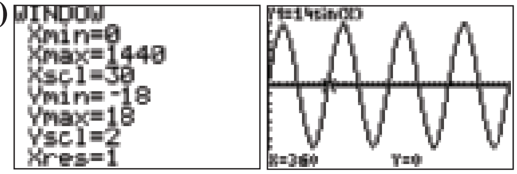
A cosine function models the horizontal displacement, because the horizontal displacement starts at 10 m and decreases to 0 m at 90°, a characteristic of the cosine function.

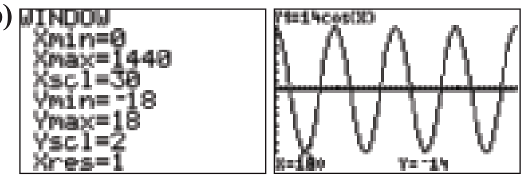
b) 

A sine function models the vertical displacement, because the vertical displacement starts at 0 m and moves through to a maximum at 90°, a characteristic of the sine function.

2. The graph has zeros at 0°, 180°, and 360°, and reaches the highest point, 10, at 90°, and the lowest point at 270°, so a sine function models the vertical distance.

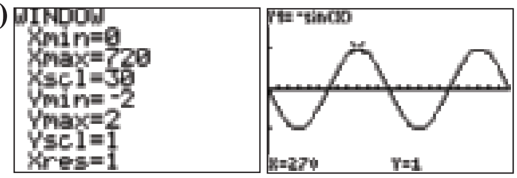


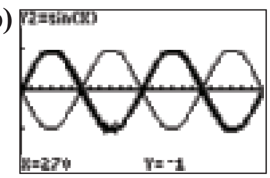
3. a) 

b) 

c) 4 cycles

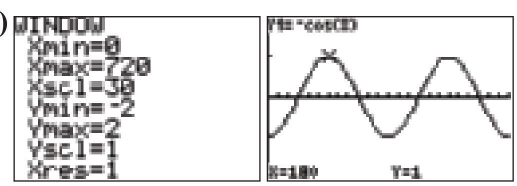
d) 48 cycles

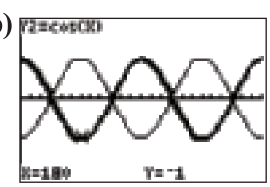
4. a) 

b) 

c) Similarities: The amplitude for each graph is 1. The period for each graph is 360°. The range for each graph is $\{y \in \mathbb{R}, -1 \leq y \leq 1\}$. The zeros, or x-intercepts, of each graph are the same.

Differences: The graph of $y = -\sin x$ is a reflection in the x-axis of the graph of $y = \sin x$. The minimum points on $y = -\sin x$ are the maximum points on $y = \sin x$, and the maximum points on $y = -\sin x$ are the minimum points on $y = \sin x$.

5. a) 

b) 

c) Similarities: The amplitude for each graph is 1. The period for each graph is 360°. The range for each graph is $\{y \in \mathbb{R}, -1 \le y \le 1\}$. The zeros, or x-intercepts, of each graph are the same.
Differences: The graph of $y = -\cos x$ is a reflection in the x-axis of the graph of $y = \cos x$. The minimum points on $y = -\cos x$ are the maximum points on $y = \cos x$, and the maximum points on $y = -\cos x$ are the minimum points on $y = \cos x$.

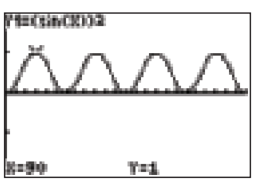
6. **a)** The y-intercept of the function $y = (\sin x)^2$ is the same as that of $y = \sin x$, which is 0.

b) The x-intercepts occur when $y = 0$. Solve $(\sin x)^2 = 0$. This is equivalent to solving $\sin x = 0$. The x-intercepts are $x = 0°, 180°, 360°, 540°$, and $720°$.

c) Since $(\sin x)^2$ is always positive, the minimum value of $y = (\sin x)^2$ is 0 and the maximum value is 1.

d) The range is $\{y \in \mathbb{R}, 0 \le y \le 1\}$. The amplitude is half of $\frac{1-0}{2}$, or 0.5.

e)



f) Similarities: The graphs of the two functions both begin at 0, they have a maximum of 1, and they have the same x-intercepts.
Differences: The amplitude, range, and period of the two function are different.

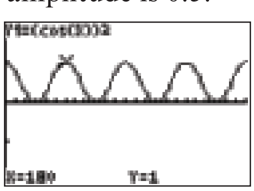
7. **a)** 1

b) 90°, 270°, 450°, and 630°

c) The minimum value is 0 and the maximum value is 1.

d) The range is $\{y \in \mathbb{R}, 0 \le y \le 1\}$. The amplitude is 0.5.

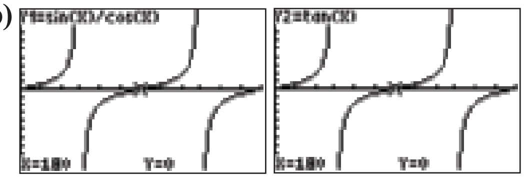
e)



f) Similarities: The graphs of the two functions both begin at 1. They have a maximum of 1 and the same x-intercepts.
Differences: The amplitude, range, and period of the two functions are different.

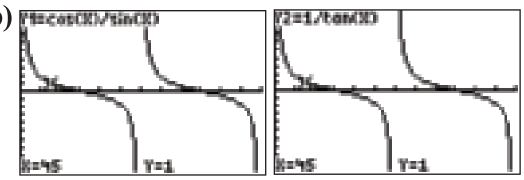
8. **a)** Answers may vary. Sample answer: The graphs are the same because $\frac{\sin x}{\cos x} = \tan x$.

b)



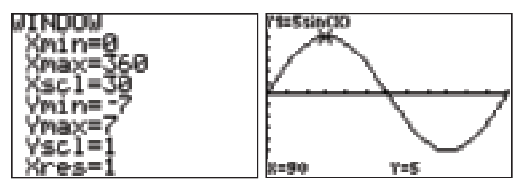
9. **a)** Answers may vary. Sample answer: The graphs are the same because $\frac{\cos x}{\sin x} = \cot x$.

b)

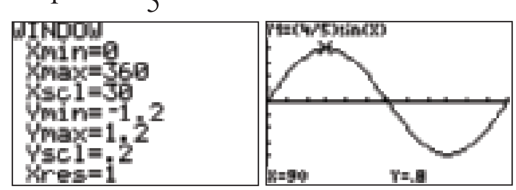


5.3 Investigate Transformations of Sine and Cosine Functions

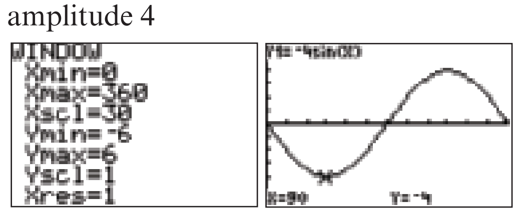
1. **a)** vertical stretch by a factor of 5, amplitude 5



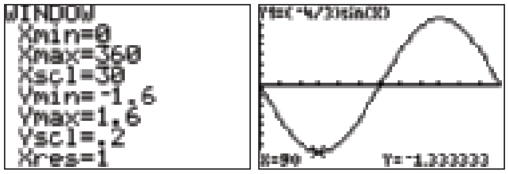
b) vertical compression by a factor of $\frac{4}{5}$, amplitude $\frac{4}{5}$



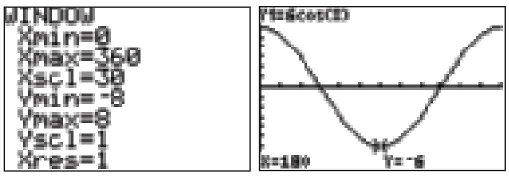
c) vertical stretch by a factor of 4, amplitude 4



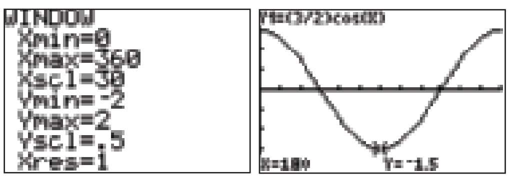
d) vertical stretch by a factor of $\frac{4}{3}$, amplitude $\frac{4}{3}$



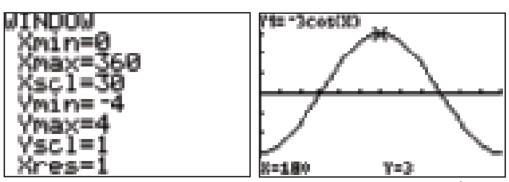
2. a) vertical stretch by a factor of 6, amplitude 6



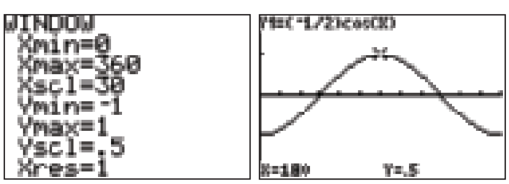
b) vertical stretch by a factor of $\frac{3}{2}$, amplitude $\frac{3}{2}$



c) vertical stretch by a factor of 3, amplitude 3



d) vertical compression by a factor of $\frac{1}{2}$, amplitude $\frac{1}{2}$



3. a) horizontal compression by a factor of $\frac{1}{2}$, period 180°

b) horizontal stretch by a factor of 2, period 720°

c) horizontal stretch by a factor of $\frac{3}{2}$, period 540°

d) horizontal stretch by a factor of 4, period 1440°

e) horizontal compression by a factor of $\frac{1}{3}$, period 120°

f) horizontal compression by a factor of $\frac{1}{5}$, period 72°

g) horizontal stretch by a factor of $\frac{4}{3}$, period 480°

h) horizontal compression by a factor of $\frac{2}{3}$, period 240°

4. a) The amplitude of the graph is 1, since this is half the difference between the maximum value of −2 and the minimum value of −4. The graph of $y = \sin x$ has a maximum value of 1 and a minimum value of −1, so this graph represents a vertical shift down of 3 units of the graph of $y = \sin x$. The equation that matches this graph is B, $y = \sin x - 3$.

b) The amplitude of the graph is 1, since this is half the difference between the maximum value of 3.5 and the minimum value of 1.5. The graph of $y = \sin x$ has a maximum value of 1 and a minimum value of −1, so this graph represents a vertical shift up of 2.5 units of the graph of $y = \sin x$. The equation that matches this graph is A, $y = \sin x + 2.5$.

5. a) Since the point (0°, 1) on the graph of $y = \cos x$ is now located at (−60°, 1), the equation that matches this graph is B, $y = \cos(x + 60°)$.

b) Since the point (0°, 1) on the graph of $y = \cos x$ is now located at (30°, 1), the equation that matches this graph is A, $y = \cos(x - 30°)$.

6. a) phase shift right 40°; vertical shift up 2 units

b) phase shift left 60°; vertical shift down 3 units

c) phase shift right 38°; vertical shift up 5 units

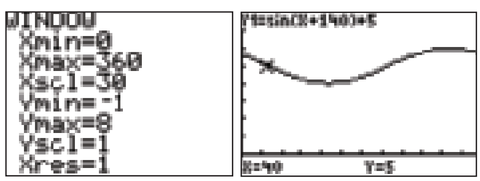
d) phase shift left 30°; vertical shift down 6 units

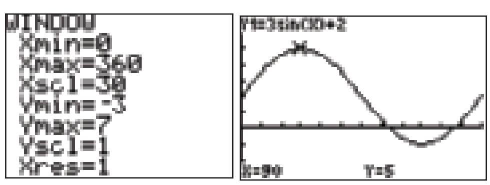
7. **a)** phase shift left 70°; no vertical shift
   b) phase shift right 82°; vertical shift up 8 units
   c) phase shift left 100°; vertical shift down 1 unit
   d) phase shift right 120°; vertical shift up 9 units

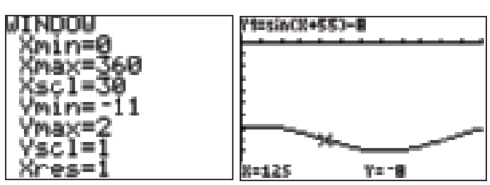
8. Answers may vary. Sample answers:
   a) $y = 4 \sin 2x$; $y = 4 \cos [2(x - 45°)]$
   b) $y = -2 \sin 3x$; $y = -2\cos [3(x - 30°)]$
   c) $y = 8 \sin 6x$; $y = 8 \cos [6(x - 15°)]$

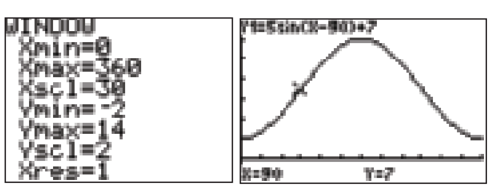
9. Answers may vary. Sample answers:
   a) $y = 5 \cos 3x$; $y = 5 \sin [3(x - 90°)]$
   b) $y = -4 \cos 2x$; $y = 4 \sin [2(x - 45°)]$
   c) $y = 10 \cos 8x$; $y = 10 \sin [8(x + 11.25°)]$

10. **a) i)** phase shift left 140°, vertical shift up 5 units
   ii) no phase shift, vertical shift up 2 units
   iii) phase shift left 55°, vertical shift down 8 units
   iv) phase shift right 90°, vertical shift up 7 units

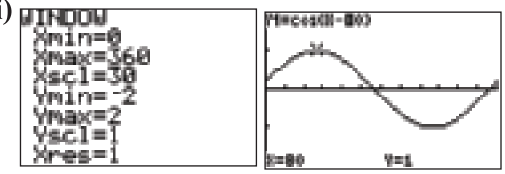
   b) i) 

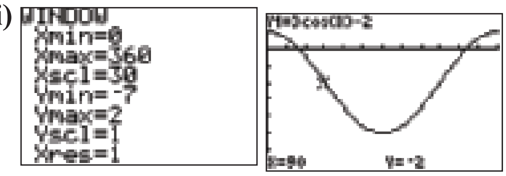
   ii) 

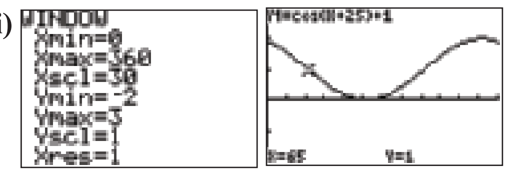
   iii) 

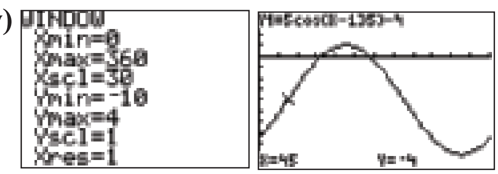
   iv) 

11. **a) i)** no vertical shift, amplitude 1
   ii) vertical shift down 2 units, amplitude 3
   iii) vertical shift up 1 unit, amplitude 1
   iv) vertical shift down 4 units, amplitude 5

   b) i) 

   ii) 

   iii) 

   iv) 

12. **a)** Since the amplitude is 30, the graph would fluctuate between 30 and −30. However, there is a vertical shift up of 45, making the lowest value −30 + 45, or 15.
   So, 15 cm is the lowest vertical position that the point reaches.
   b) Since the amplitude is 30, the graph would fluctuate between 30 and −30. However, there is a vertical shift up of 45, making the highest value 30 + 45, or 75.
   So, 75 cm is the highest vertical position that the point reaches.
   c) The period is $\frac{360}{540}$, or $\frac{2}{3}$ s.
   d) The value of k will change from 540 to 270, so the equation becomes $y = 30 \sin (270t) + 45$.

13. **a)** Since $d = -90°$ and $c = -2$, the graph of $y = \sin x$ must be shifted left 90° and down 2 units.
   b) Since $a = -1$, $d = 60°$, and $c = 4$, the graph of $y = \sin x$ must be reflected in the x-axis, and shifted right 60° and up 4 units.

14. a) The horizontal shift is 25° to the left, so $d = -25°$. The vertical shift is 5 units up, so $c = 5$. The equation of the transformed graph is $y = \sin(x + 25°) + 5$.

b) The graph is reflected in the x-axis, so $a = -1$. The horizontal shift is 42° to the right, so $d = 42°$. The vertical shift is 2 units down, so $c = -2$. The equation of the transformed graph is $y = -\sin(x - 42°) - 2$.

15. a) amplitude 3, period 180°, phase shift right 30°, vertical shift up 1 unit

b) amplitude $\frac{1}{2}$, period 120°, phase shift left 120°, vertical shift down 6 units

c) amplitude 4, period 1440°, phase shift left 45°, vertical shift down 3 units

d) amplitude 0.6, period 300°, phase shift right 90°, vertical shift up 2 units

16. $f = 490\cos(90°t) + 610$

17. a) 2.7 m

b) 0.3 m

c) 1.2 s

d) 2.1 m

18. a) 25 cm

b) 5 cm

c) 1 s

d) at approximately 0.5 s, 1.0 s, 1.5 s, 2.0 s, etc.

5.4 Graphing and Modelling with $y = a\sin[k(x - d)] + c$ and $y = a\cos[k(x - d)] + c$

1. a) amplitude 2, period 120°, phase shift 15° right, vertical shift 4 units up

b) amplitude 5, period 30°, phase shift 60° left, vertical shift 2 units down

c) amplitude 6, period 20°, phase shift 45° left, vertical shift 1 unit up

d) amplitude $\frac{2}{3}$, period 600°, phase shift 30° right, vertical shift $\frac{4}{5}$ units up

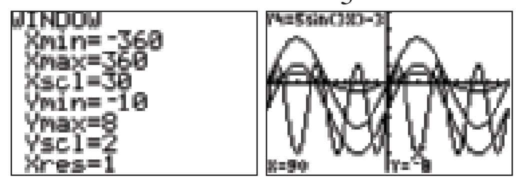
2. a) amplitude $\frac{4}{7}$, period 36°, phase shift 16° left, vertical shift 9 units up

b) amplitude 11, period 10°, phase shift 75° left, vertical shift 3 units down

c) amplitude 8, period 6°, phase shift 12° right, vertical shift 7 units up

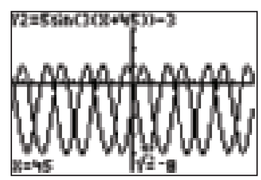
d) amplitude $\frac{5}{9}$, period 900°, phase shift 85° right, vertical shift $\frac{3}{8}$ units up

3. a) Answers may vary. Sample answer: Apply the amplitude of 5, apply the vertical shift of 3 units down, and apply the horizontal compression by a factor of $\frac{1}{3}$.

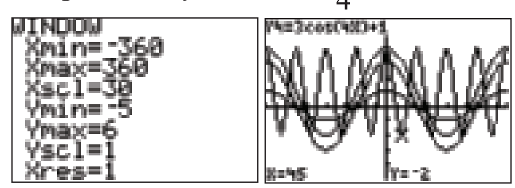


b) $f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, -1 \le y \le 1\}$; $g(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, -8 \le y \le 2\}$

c) $h(x) = 5\sin[3(x + 45°)] - 3$

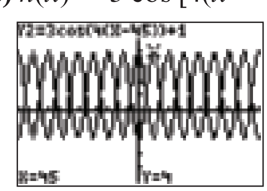


4. a) Answers may vary. Sample answer: Apply the amplitude of 3, apply the vertical shift of 1 unit up, and apply the horizontal compression by a factor of $\frac{1}{4}$.



b) $f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, -1 \le y \le 1\}$; $g(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, -2 \le y \le 4\}$

c) $h(x) = 3\cos[4(x - 45°)] + 1$



5. Answers may vary. Sample answers:

a) $y = 6\sin[2(x + 45°)] - 4$

b) $y = 6\cos 2x - 4$

6. Answers may vary. Sample answers:

   a) $y = \frac{3}{4}\sin\left[\frac{1}{3}(x + 270°)\right] + 2$

   b) $y = \frac{3}{4}\cos\frac{1}{3}x + 2$

7. a) amplitude 8, period 120°, phase shift 20° left, vertical shift down 8 units

   b) maximum 0, minimum −16

   c) 10°, 130°, 250°

   d) −1.07

8. a) amplitude 6, period 72°, phase shift 60° right, no vertical shift

   b) maximum 6, minimum −6

   c) 6°, 42°, 78°

   d) 3

9. Answers may vary.

10. The graph is reflected in the x-axis, and vertically stretched by a factor of 3, so $a = -3$.
    The graph is horizontally compressed by a factor of $\frac{1}{4}$, so $k = 4$.
    The graph is shifted 35° to the left, so $d = -35°$.
    The vertical shift is 8 units down, so $c = -8$.
    The equation of the transformed graph is $y = -3\sin[4(x + 35°)] - 8$.

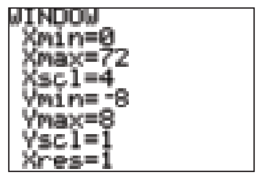
11. a) Apply the amplitude of 3, $y = 3\sin x$; apply the reflection in the x-axis, $y = -3\sin x$; apply the vertical shift of 7 units up, $y = -3\sin x + 7$; apply the horizontal stretch of factor 4, $y = -3\sin\frac{1}{4}x + 7$; translate the function 50° right, $g(x) = -3\sin\left[\frac{1}{4}(x - 50°)\right] + 7$.

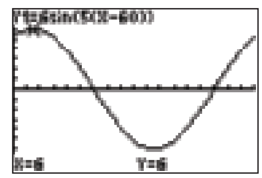
    b) $f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, -1 \le y \le 1\}$; $g(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, 4 \le y \le 10\}$

12. a) Apply the amplitude of $\frac{3}{4}$, $y = \frac{3}{4}\cos x$; apply the vertical shift of 2 units down, $y = \frac{3}{4}\cos x - 2$; apply the horizontal compression of factor $\frac{1}{6}$, $y = \frac{3}{4}\cos 6x - 2$; translate the function 45° left, $g(x) = \frac{3}{4}\cos[6(x + 45°)] - 2$

    b) $f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, -1 \le y \le 1\}$; $g(x)$: domain $\{x \in \mathbb{R}\}$, range $\left\{y \in \mathbb{R}, \frac{5}{4} \le y \le \frac{11}{4}\right\}$

13. The equation of the cosine function is of the form $g(x) = a\cos[k(x - d)]$, since the sine function does not have a vertical shift. The cosine function will have the same amplitude and period as the sine function, so $a = 6$ and $k = 5$. Thus far the equation is $y = 6\cos[5(x - d)]$.
    Determine the phase shift. Sketch a graph of the sine function to find d.





Since the first maximum occurs at 6°, the phase shift is 6° to the right, so $d = 6°$.
A cosine equation that models this function is $y = 6\cos[5(x - 6°)]$.

14. a) $y = 4\sin[3(x - 30°)] + 1$

    b) $y = 4\cos[3(x - 60°)] + 1$

15. Answers may vary. Sample answers:

    a) $y = 5\cos 30x + 11$

    b) $y = 5\sin[30(x + 3)] + 11$

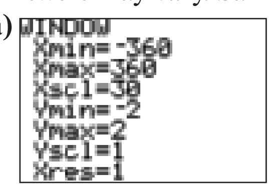
    c) $y = 5\sin[30(x - 3)] + 11$

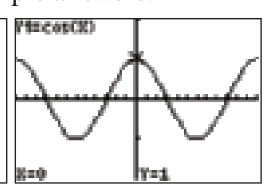
    d) $y = 5\cos[30(x - 6)] + 11$

    e) Answers may vary.

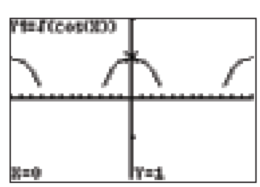
16. Answers may vary. Sample answers:

    a)



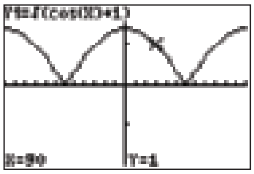


    b), c) The square root function will produce only positive y-values, so the graph of $y = \sqrt{\cos x}$ will have missing sections where the graph of $y = \cos x$ has negative y-values.



d) The graph of $y = \sqrt{\cos x + 1}$ will not have any missing parts, since no part of the graph will be below the x-axis.

e)



17. a) maximum: $y = a + c$ for $x = d + \frac{90°(1 + 4n)}{k}$, where n is an integer

b) minimum: $y = c - a$ for $x = d + \frac{90°(3 + 4n)}{k}$, where n is an integer

18. a) maximum: $y = a + c$ for $x = d + \frac{n360°}{k}$, where n is an integer

b) minimum: $y = c - a$ for $x = d + \frac{180°(1 + 2n)}{k}$, where n is an integer

5.5 Data Collecting and Modelling

1. a) maximum 10.5 m, minimum 1.5 m

b) high tide at 7 a.m. and 7 p.m.; low tide at 1 a.m. and 1 p.m.

c) 3.75 m

d) 4 a.m., 10 a.m., 4 p.m., 10 p.m.

2. a) maximum 9100 tourists; minimum 2100 tourists

b) maximum in November, at $t = 11$; minimum in May, at $t = 5$

c) 3850

d) March 18 (3 months, 18 days), June 12 (6 months, 12 days)

3. Answers may vary. Sample answers:

a) maximum 3.5 m, minimum 0.5 m, $a = 1.5$

b) $c = 2$

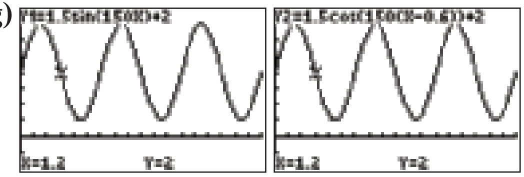
c) $d = 0$

d) period 2.4 s, $k = 150$

e) $y = 1.5 \sin 150x + 2$

f) $y = 1.5 \cos [150(x - 0.6)] + 2$

g)



4. Answers may vary. Sample answers:

a) The maximum value on the graph is 8 m and the minimum value is 2 m. Subtract and divide by 2 to find the amplitude, $\frac{8 - 2}{2}$, or 3.

b) Since the amplitude is 3, the horizontal reference line is $y = 5$, which is 3 below the maximum and 3 above the minimum. The value of the horizontal reference line indicates that the graph has been shifted up 5 units, so $c = 5$.

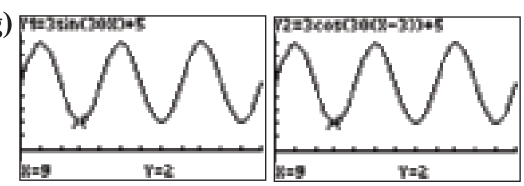
c) Since the y-intercept is at (0, 5) and the points (6, 5) and (12, 5) are all on the horizontal reference line, there is no phase shift, so $d = 0$.

d) Since the period is 12 s, $\frac{360}{k} = 12$ and $k = 30$.

e) Substitute the values $a = 3$, $k = 30$, $d = 0$, and $c = 5$ in $y = a \sin [k(x - d)] + c$. A sine equation that models the motion is $y = 3 \sin 30x + 5$.

f) The part of the graph from 3 s to 12 s represents a cosine function. The values of a, k, and c remain the same but $d = 3$, since the first maximum occurs at (3, 8). A cosine equation that models the motion is $y = 3 \cos [30(x - 3)] + 5$.

g)



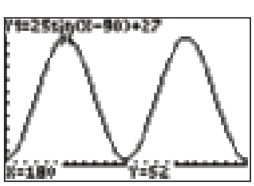
5. **a)** Substitute each value of x from the first column to determine the corresponding *height* value in the second column.

x	$h(x) = 25 \sin (x - 90°) + 27$
0°	$25 \sin (0° - 90°) + 27 = 2$
30°	$25 \sin (30° - 90°) + 27 = 5.35$
60°	$25 \sin (60° - 90°) + 27 = 14.50$
90°	$25 \sin (90° - 90°) + 27 = 27$
120°	$25 \sin (120° - 90°) + 27 = 39.50$
150°	$25 \sin (150° - 90°) + 27 = 48.65$
180°	$25 \sin (180° - 90°) + 27 = 52$
210°	$25 \sin (210° - 90°) + 27 = 48.65$
240°	$25 \sin (240° - 90°) + 27 = 39.50$
270°	$25 \sin (270° - 90°) + 27 = 27$
300°	$25 \sin (300° - 90°) + 27 = 14.50$
330°	$25 \sin (330° - 90°) + 27 = 5.35$
360°	$25 \sin (360° - 90°) + 27 = 2$

b) The cycle will repeat itself after 1 revolution, so the values will follow the pattern in the above table.

x	$h(x) = 25 \sin (x - 90°) + 27$
390°	5.35
420°	14.50
450°	27
480°	39.50
510°	48.65
540°	52
570°	48.65
600°	39.50
630°	27
660°	14.50
690°	5.35
720°	2

c)



d) From the graph and the table, the maximum height of the rider is 52 m and the minimum height of the rider is 2 m.

e) From the graph, the height of the rider is 40 m in 4 places: approximately 120°, 240°, 480°, and 600°.

f) The start of the first cosine wave is at 180°, so the phase shift is 180° to the right. The period, amplitude, and vertical shift remain the same as in the given sine function. A cosine equation that models the height is $h(x) = 25 \cos (x - 180°) + 27$.

6. **a)** $\sqrt{3}$

b) multiply by a factor of 4

7. **a)** The period of the graph is 360 days, so the equation will be of the form $y = a \sin (x - d) + c$. The maximum value is 20 and the minimum value is 16. So, $a = 2$. The horizontal reference line is $y = 18$, which means there is a vertical shift of 18 units up, so $c = 18$. There is no phase shift, since the first point is at (0, 18), on the horizontal reference line. A sine equation that models the time of sunset in Saskatoon is $y = 2 \sin x + 18$.

b) The first point on the cosine wave is at 90 days, so a cosine equation that models the time of sunset in Saskatoon is $y = 2 \cos (x - 90) + 18$.

c) The range of the function is $\{y \in \mathbb{R}, 16 \leq y \leq 20\}$.

8. **a)** $y = 9 \sin (x + 230°) + 12$

b) $y = 9 \cos (x + 140°) + 12$

c) The phase shift of the curves will be altered by an additional 10°; $y = 9 \sin (x + 220°) + 12$; $y = 9 \cos (x + 130°) + 12$

9. **a)** $y = 9 \sin (x + 230°) + 14.5$

b) $y = 9 \cos (x + 140°) + 14.5$

10. **a)** $\frac{1}{25}$ s

b) maximum 63 cm, minimum −27 cm, amplitude 45

c) maximum at $\frac{1}{100}$ s, minimum at $\frac{3}{100}$ s

d) 63 cm

11. 59.8 cm

12. $h(t) = 8 \cos (19.9t) + 9.5$

13. a) $y = 1100 \sin (93\,600t)$

b) $y = 550 \sin (70\,200t)$

5.6 Use Sinusoidal Functions to Model Periodic Phenomena Not Involving Angles

1. a) maximum height 7 m, minimum height −7 m

b) high tide at 5:30 a.m. and 5:30 p.m.; low tide at 11:30 a.m. and 11:30 p.m.

c) $h(t) = 7 \cos [30(t + 6.5)]$

2. a) $h(t) = 4.5 \sin [30(t + 4)]$

b) low tide at 5 p.m.; high tide at 11 a.m. and 11 p.m.

c) $h(t) = 4.5 \sin [30(t + 3.5)]$

d) $h(t) = 4.5 \cos [30(t + 0.5)]$

3. a) maximum 1050, minimum 250, amplitude 400

b) $c = 650$

c) $d = 0$

d) 5 years, $k = 72$

e) $y = 400 \sin 72x + 650$

f) Answers may vary, but all graphs should match the shape of the graph given.

4. For 8 years, $k = 45$. The new equation is $y = 400 \sin 45x + 650$.

5. a) The graph begins at March 21. There are 31 days in March, so April 1 occurs 11 days after March 21. Substitute $x = 11$ into $y = 4.5 \sin \frac{72}{73}x + 12$ to find y.

$$\begin{aligned} y &= 4.5 \sin \frac{72}{73}(11) + 12 \\ &= 4.5 \sin \frac{792}{73} + 12 \\ &\doteq 12.8 \end{aligned}$$

Convert 12.8 h to hours and minutes.
$0.8 \times 60 = 48$
There are about 12.8 h, or 12 h 48 min of daylight on April 1.

b) First, determine how many days September 1 falls after March 21.

March 21–31	= 10 days
April	= 30 days
May	= 31 days
June	= 30 days
July	= 31 days
August	= 31 days
September	= 1 day
Total	= 164 days

Substitute $x = 164$ in $y = 4.5 \sin \frac{72}{73}x + 12$ to find y.

$$\begin{aligned} y &= 4.5 \sin \frac{72}{73}(164) + 12 \\ &= 4.5 \sin \frac{11\,808}{73} + 12 \\ &\doteq 13.4 \end{aligned}$$

Determine the number of minutes.
$0.4 \times 60 = 24$
There are approximately 13.4 h, or 13 h 24 min of daylight on September 1.

6. a) The amplitude is $\frac{12 - 2}{2}$, or 5. So, $a = 5$.
The period is 12 h.

$$\begin{aligned} \frac{360}{k} &= 12 \\ k &= 30 \end{aligned}$$

The vertical shift is $2 + 5$, or 7.
So, $c = 7$.
Since x represents the hours after midnight, the first maximum point is (6, 12) and the point at the beginning of the first sine wave is (3, 7), so the phase shift is $d = 3$.
Substitute the values $a = -5$, $k = 30$, $d = 3$, and $c = 7$ into the equation $y = a \sin [k(x - d)] + c$.
A sinusoidal equation that represents the depth of the water is
$y = 5 \sin [30(x - 3)] + 7$.

b) 4:15 a.m. occurs 4.25 h after 12 midnight. Substitute $x = 4.25$ into the equation and solve for y.

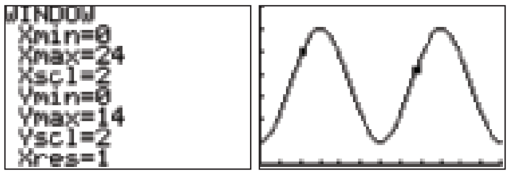
$$\begin{aligned} y &= 5 \sin [30(4.25 - 3)] + 7 \\ &\doteq 10.0 \end{aligned}$$

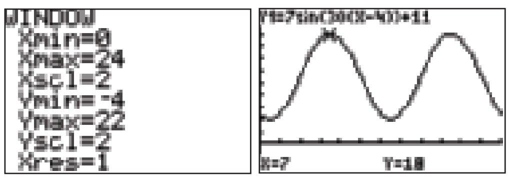
At 4:15 a.m., the depth of the water is approximately 10.0 m.

3:30 p.m. occurs 15.5 h after 12 midnight. Substitute $x = 15.5$ into the equation and solve for y.

$y = 5 \sin [30(15.5 - 3)] + 7$
$\doteq 8.3$

At 3:30 p.m. the depth of the water is approximately 8.3 m.

c) 

7. **a)** maximum 18 m, minimum 4 m
b) 12 h
c) 

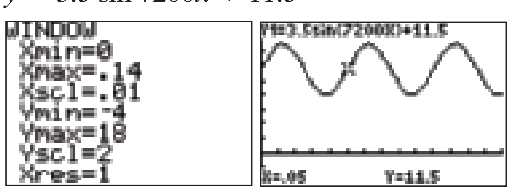
d) midnight, 2 a.m., 12 noon, and 2 p.m.

8. **a)** $\frac{1}{5}$ s
b) $k = 1800$
c) 8 V
d) $V = 8 \sin 1800t$

9. **a)** Answers may vary. Sample answer:
$y = 3 \cos \left[\frac{72}{73}(x - 172)\right] + 12.3$
b) 12.8 h
c) Answers may vary. Sample answer: April 26

10. **a)** period 0.017 s, amplitude 160 V
b) 160 V
c) $V = 160 \sin \frac{360}{0.017}t$

11. **a)** $y = 6.9 \sin [15(x + 4)] + 22$
b) Plot the points in the table and sketch a graph of the equation on the same set of axes. Then, check to see if the points lie on the graph of the equation.
c) **i)** 15.3 °C
ii) 28.7 °C
iii) 22.9 °C

12. **a)** $y = 3.5 \sin 7200x + 11.5$
b) 

c) 0.029 min, 0.046 min

13. **a)** Graph ①: $y = 10 \sin 2400x + 12$
Graph ②:
$y = 10 \sin [2400(x - 0.05)] + 12$
Graph ③:
$y = 10 \sin [2400(x - 0.1)] + 12$
b) 0.15 cycles per second

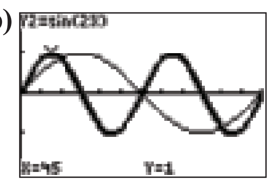
Chapter 5 Review

1. **a)** Yes; pattern of y-values repeats at regular intervals
b) No; pattern of y-values does not repeat at regular intervals
c) No; pattern of y-values does not repeat at regular intervals

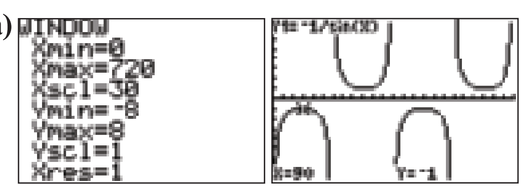
2. Answers may vary.

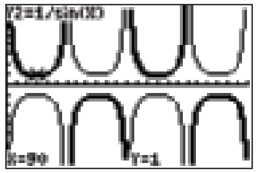
3. **a)**

x	$2x$	$y = \sin 2x$
0°	0°	0
45°	90°	1
90°	180°	0
135°	270°	–1
180°	360°	0
225°	450°	1
270°	540°	0
315°	630°	–1
360°	720°	0

b) 

c) Similarities: The amplitude for each graph is 1. The range for each graph is $\{y \in \mathbb{R}, -1 \le y \le 1\}$.
Differences: The period for $y = \sin x$ is 360°, but the period for $y = \sin 2x$ is 180°. The zeros, or x-intercepts, of $y = \sin x$ are 0°, 180°, and 360°. The x-intercepts of $y = \sin 2x$ are 0°, 90°, 180°, 270°, and 360°.

4. **a)** 

b) 

c) Similarities: The period of each graph is 360°. The vertical asymptotes for each graph are $x = n180°$, where n is an integer. The range for each graph is $\{y \in \mathbb{R}, y \leq -1 \text{ or } y \geq 1\}$.
Differences: The graph of $y = -\csc x$ is a reflection of the graph of $y = \csc x$ in the x-axis, so each branch of the graph opens in the opposite direction.

5. Answers may vary. Sample answers:
$y = \frac{3}{4}\sin 4x$; $y = \frac{3}{4}\cos [4(x - 22.5°)]$

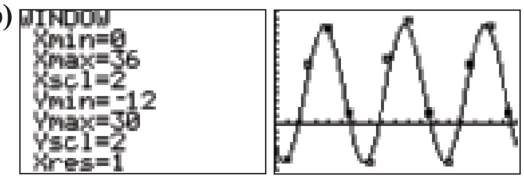
6. Answers may vary. Sample answers:
$y = 6 \cos 4x$; $y = 6 \sin [4(x - 67.5°)]$

7. a) amplitude 5, period 120°, phase shift right 40°, vertical shift up 6 units
b) amplitude $\frac{1}{4}$, period 90°, phase shift left 100°, vertical shift down 2 units
c) amplitude 7, period 540°, phase shift left 75°, vertical shift down 1 unit
d) amplitude 0.4, period $\frac{720°}{7}$, phase shift right 60°, vertical shift up 5.6 units

8. a) Apply the amplitude of 7, $y = 7 \sin x$; apply the reflection in the x-axis, $y = -7 \sin x$; apply the vertical shift of 1 unit up, $y = -7 \sin x + 1$; apply the horizontal stretch by a factor of 2, $y = -7 \sin \frac{1}{2}x + 1$; translate the function 30° right,
$g(x) = -7 \sin \left[\frac{1}{2}(x - 30°)\right] + 1$.
b) $f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, -1 \leq y \leq 1\}$;
$g(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, -6 \leq y \leq 8\}$

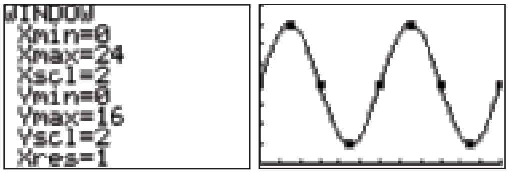
9. a) Apply the amplitude of $\frac{2}{3}$, $y = \frac{2}{3}\cos x$; apply the vertical shift of 4 units down, $y = \frac{2}{3}\cos x - 4$; apply the horizontal compression by a factor of $\frac{1}{5}$, $y = \frac{2}{3}\cos 5x - 4$; translate the function 28° left, $g(x) = \frac{2}{3}\cos [5(x + 28°)] - 4$.
b) $f(x)$: domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, -1 \leq y \leq 1\}$;
$g(x)$: domain $\{x \in \mathbb{R}\}$,
range $\left\{y \in \mathbb{R}, -\frac{14}{3} \leq y \leq -\frac{10}{3}\right\}$

10. Answers may vary. Sample answers:
a) $y = -7 \sin [2(x - 30°)]$
b) $y = -7 \cos [2(x - 75°)]$

11. Answers may vary. Sample answers:
a) $y = 18 \sin [30(x - 4.4)] + 8$
b) 

c) There is a good fit because the graph passes through most of the points.

12. a) Answers may vary. Sample answer:
The depth of the water at low tide is a minimum of 2 m, which is 6 m below average sea level. So, average sea level is 2 + 6, or 8 m. This represents the horizontal reference line of the graph.
At high tide the maximum depth of the water is 14 m.
Since the maximum and minimum values are 6 m from the horizontal reference, the amplitude is $a = 6$.
The cycle takes 12 h to complete, so the period is 12 h and $k = 30$. At times 0 h, 6 h, and 12 h, the water depth is 8 m. Since the tide is coming in, it is high tide 3 h past midnight, so the water depth is 14 m. Low tide occurs 3 h before the end of the 12-h cycle, so at 9 h the water depth is 2 m. The graph represents a 24-h period, so there will be 2 cycles on the graph.
Points on the graph will be (0, 8), (3, 14), (6, 8), (9, 2), (12, 8), (15, 14), (18, 8), (21, 2), (24, 8). Plot these points and draw a sine curve to obtain the following graph.



b) i) $y = 6 \sin 30x + 8$
ii) $y = 6 \cos [30(x - 3)] + 8$

c) Answers may vary. Sample answer: In this situation the depth of the water at 3 a.m. is 8 m, the average sea level, and the tide is coming in, so the maximum will occur 3 h later at 6:00 a.m. This indicates that the graph is now translated 3 units to the right, so $d = 3$, but all the other values remain the same. A sine equation that represents this situation is $y = 6 \sin [30(x - 3)] + 8$, and a cosine equation is $y = 6 \cos [30(x - 6)] + 8$.

Math Contest

1. B
2. D
3. $x = 3, y = 2, z = 5$
4. A
5. 8, 9, 10
6. C
7. D
8. C
9. A
10. B
11. A
12. D
13. $3\sqrt{2}$
14. B

Chapter 6 Discrete Functions

6.1 Sequences as Discrete Functions

1. a) 3, 5, 7
b) 1, −2, −5
c) −1, 1, 3
d) 9, 11, 13
e) 1, 2, 4
f) −18, −54, −162

2. a) −27
b) 50
c) 6
d) $\frac{1}{64}$

3. a) The first term is 5. Multiply each term by 5 to get the next term. Next three terms: 3125, 15 625, 78 125
b) The first term is 9. Subtract 2 from each term to get the next term. Next three terms: 1, −1, −3
c) The first term is −4. Multiply each term by 2 to get the next term. Next three terms: −64, −128, −256
d) The first term is 300. Divide each term by 10 to get the next term. Next three terms: 0.03, 0.003, 0.0003
e) The first term is 3. Multiply each term by 3 to get the next term. Next three terms: 729, 2187, 6561
f) The first term is ar^3. Divide each term by r to get the next term. Next three terms: $\frac{a}{r}, \frac{a}{r^2}, \frac{a}{r^3}$
g) The first term is 0.11. Multiply each term by −3 to get the next term. Next three terms: 8.91, −26.73, 80.19

4. a) $f(n) = 6n$; domain $\{n \in \mathbb{N}\}$

Term Number, n	Term, t_n	First Differences
1	6	
		6
2	12	
		6
3	18	
		6
4	24	

b) $f(n) = 10 - 3n$; domain $\{n \in \mathbb{N}\}$

Term Number, n	Term, t_n	First Differences
1	7	
		−3
2	4	
		−3
3	1	
		−3
4	−2	

c) $f(n) = n^2 + 1$; domain $\{n \in \mathbb{N}\}$

Term Number, n	Term, t_n	First Differences	Second Differences
1	2		
		3	
2	5		2
		5	
3	10		2
		7	
4	17		

d) $f(n) = 2n^2 + 3n - 1$; domain $\{n \in \mathbb{N}\}$

Term Number, n	Term, t_n	First Differences	Second Differences
1	4		
		9	
2	13		4
		13	
3	26		4
		17	
4	43		

5. **a)** $f(n) = 15n$; domain $\{n = 1, 2, 3, 4\}$
b) $f(n) = n^2 - 3$; domain $\{n = 1, 2, 3, 4\}$
c) $f(n) = 1 - \frac{1}{2}n$; domain $\{n = 1, 2, 3, 4\}$

6. **a)** $f(n) = -\frac{1}{4}n^2$; domain $\{n = 1, 2, 3, 4\}$
b) $f(n) = n^2 + 3n$; domain $\{n = 1, 2, 3, 4\}$
c) $f(n) = -3^n$; domain $\{n = 1, 2, 3, 4\}$

7. **a)** discrete; set of disconnected points
b) continuous; line is made up of connected points
c) discrete; set of disconnected points
d) continuous; curve is made up of connected points

8. **a)** The first term is 2 and further odd terms are multiples of 2. The second term is −5 and further even terms are multiples of −5. The next four terms are −25, 12, −30, 14.
b) The first term is −7 and further odd terms increase by 6. The second term is 9 and further even terms decrease by 7. The next four terms are −12, 17, −19, 23.
c) The first term is 4 and further odd terms increase by 5. The second term is $\frac{1}{27}$ and further even terms are multiplied by 3. The next four terms are 1, 24, 3, 29.
d) The first term is 8 and further odd terms are multiplied by 0.1. All even terms are −1. The next four terms are 0.008, −1, 0.0008, −1.

9. **a)** Yes; a multiple of 9
b) No; not a multiple of 9
c) No; not a multiple of 9
d) Yes; a multiple of 9

10. **a)** $t_n = -16(-2)^{-n+1}$ or $t_n = \frac{-16}{(-2)^{n-1}}$; $t_{15} = \frac{-16}{(-2)^{15}}$
b) $t_n = \frac{n}{2n-1}$; $t_{15} = \frac{15}{29}$
c) $t_n = \frac{2}{\sqrt{n}}$; $t_{15} = \frac{2}{\sqrt{15}}$
d) $t_n = 3^{n-1}$; $t_{15} = 3^{15}$
e) $t_n = \frac{n+1}{n}$; $t_{15} = \frac{16}{15}$

11. Answers may vary. Sample answers:
a) 9, 5, 1, −3; $f(n) = 13 - 4n$
b) −3, 9, −27, 81; $f(n) = -3(-3)^{n-1}$

12. **a)** The sales on the first day are \$20, so the first term is 20.
Since the sales triple each day, multiply the first term and subsequent terms by 3 to obtain the sequence 20, 60, 180, 540, 1620.
b) Since 20 is repeatedly multiplied by 3, the explicit formula is $f(n) = 20(3^{n-1})$.
c) Substitute $n = 14$ to obtain the 14th term.
$$f(14) = 20(3^{14-1}) = 31\,886\,460$$
Since this is a new small business, it is unreasonable to expect sales of \$31 886 460 on the 14th day. It is unlikely that sales will triple every day for 2 weeks. Most likely sales will level off at a certain amount.

13. **a)** $f(n) = 3400 - 260(n - 1)$
b) $f(6) = 2100$
c) 9 years

14. **a)** $\sqrt{\sqrt{\sqrt{5}}}$, $\sqrt{\sqrt{\sqrt{\sqrt{5}}}}$
b) $5^{\frac{1}{2}}$, $5^{\frac{1}{4}}$, $5^{\frac{1}{8}}$, $5^{\frac{1}{16}}$, $5^{\frac{1}{32}}$
c) $f(n) = 5^{\frac{1}{2^n}}$
d) $f(n) = 5^{2^{-n}}$
e) $f(50) = 5^{2^{-50}}$

15. a) $t_n = (2n - 1) \times (3n - 2)$

b) $t_n = \dfrac{(2n - 1) \times (2n + 1)}{(2n) \times (2n + 2)}$

6.2 Recursive Procedures

1. a) 2, 8, 14, 20 **b)** 4, 10, 28, 82
c) −1, 0, 0.5, 0.75 **d)** 500, 100, 20, 4
e) 3, 0, 9, −6 **f)** 90, 120, 160, $\frac{640}{3}$

2. a) −3, 1, 5, 9 **b)** 0.25, −0.50, 1.0, −2.0
c) 4, $\frac{4}{3}$, $\frac{1}{3}$, $\frac{1}{15}$ **d)** −7, 10, −7, 10
e) −1.5, 0.5, 3.5, 7.5

3. a) $f(1) = 2, f(n) = 2f(n - 1) + 1$
b) $f(1) = 3, f(n) = f(n - 1) - 2n$
c) $f(1) = -2, f(n) = f(n - 1) - n + 1$

4. a) 32, 16, 8, 4; $f(1) = 32$; $f(n) = 0.5f(n - 1)$
b) 2, −3, 7, −13; $f(1) = 2$; $f(n) = 1 - 2f(n - 1)$

5. a) $f(n) = 3.14$
b) 3.14, 3.14, 3.14, 3.14, 3.14

6. a) 0.035
b)

Year	House Value ($)
0	275 000
1	275 000 + 0.035 × 275 000 = 284 625
2	284 625 + 0.035 × 284 625 = 294 586.88
3	294 586.88 + 0.035 × 294 586.88 = 304 897.42
4	304 897.42 + 0.035 × 304 897.42 = 315 568.83
5	315 568.83 + 0.035 × 315 568.83 = 326 613.74
6	326 613.74 + 0.035 × 326 613.74 = 338 045.22
7	338 045.22 + 0.035 × 338 045.22 = 349 876.80
8	349 876.80 + 0.035 × 349 876.80 = 362 122.49

c) 275 000, 284 625, 294 586.88, 304 897.42, 315 568.83, 326 613.74, 338 045.22, 349 876.80, 362 122.49
d) $t_1 = 275\,000$, $t_n = 1.035t_{n-1}$
e) $547 191.94

7. a) 42, 51, 65, 84
b) Answers may vary. Sample answer: There are 42 seats in the row 1. The number of seats in row 2 is equivalent to the number of seats in row 1 reduced by 1 and then increased by 5 times the row number. The number of seats in each subsequent row is found by taking the number of seats of the previous row, reduced by 1, and then increased by 5 times the row number.
c) $t_1 = 42$, $t_n = t_{n-1} + 5n - 1$

8. a) The number of bacteria doubles every hour. Multiply each term by 2 to obtain the next term, starting with 12. The first seven terms of the sequence are 12, 24, 48, 96, 192, 384, and 768.
b) Since each term is multiplied by 2 to get the next term, the terms of the sequence may be written as follows:
$t_1 = 12(2^0)$, $t_2 = 12(2^1)$, $t_3 = 12(2^2)$,
$t_4 = 12(2^3)$, ….
By following the above pattern, the explicit formula for the nth term is
$t_n = 12(2^{n-1})$.
c) Each subsequent term is found by multiplying each preceding term by 2, so the recursion formula is $t_1 = 12$, $t_n = 2t_{n-1}$.
d) Answers may vary. Sample answer: It is easier to find the recursion formula because the pattern depends on the preceding term.
e) Answers may vary. Sample answer: Substitute $n = 12$ in the explicit formula.
$$\begin{aligned} t_{12} &= 12(2^{12-1}) \\ &= 12(12^{11}) \\ &= 24\,576 \end{aligned}$$
After 12 h, there are 24 576 bacteria.
I used the explicit formula because the number of bacteria can be found by substituting $n = 12$. To use the recursion formula it is necessary to find the previous 11 terms.
f) Since the number of bacteria is known, find n.
Substitute $t_n = 1\,572\,864$ into $t_n = 12(2^{n-1})$ and solve for n.

$1\ 572\ 864 = 12(2^{n-1})$
$131\ 072 = 2^{n-1}$
Use systematic trial to find n.
When $n = 18$, $2^{18-1} = 2^{17} = 131\ 072$.
There are 1 572 864 bacteria after approximately 18 h.

9. **a)** 1, 7, 13, 19, 25; $t_1 = 1$, $t_n = t_{n-1} + 6$
b) 2, 5, 8, 11, 14; $t_1 = 2$, $t_n = t_{n-1} + 3$
c) $-\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \frac{1}{16}, -\frac{1}{32}$; $t_1 = -\frac{1}{2}$, $t_n = -\frac{1}{2}t_n - 1$
d) −1, 0, 3, 8, 15; $t_1 = -1$, $t_n = t_{n-1} + 2n - 3$

10. **a)** 1, 1, 2, 3, 5
b) It is given that $f(1) = 4$ and $f(2) = -1$, so the first two terms are 4, −1.
To find the third term, substitute $n = 3$ into $f(n) = f(n-1) - 2f(n-2)$.
$$\begin{aligned} f(3) &= f(3-1) - 2f(3-2) \\ &= f(2) - 2f(1) && \text{Substitute } f(2) = -1 \text{ and } f(1) = 4. \\ &= -1 - 2(4) \\ &= -9 \end{aligned}$$
To find the fourth term, substitute $n = 4$ into $f(n) = f(n-1) - 2f(n-2)$.
$$\begin{aligned} f(4) &= f(4-1) - 2f(4-2) \\ &= f(3) - 2f(2) && \text{Substitute } f(3) = -9 \text{ and } f(2) = -1. \\ &= -9 - 2(-1) \\ &= -9 + 2 \\ &= -7 \end{aligned}$$
To find the fifth term, substitute $n = 5$ into $f(n) = f(n-1) - 2f(n-2)$.
$$\begin{aligned} f(5) &= f(5-1) - 2f(5-2) \\ &= f(4) - 2f(3) && \text{Substitute } f(4) = -7 \text{ and } f(3) = -9. \\ &= -7 - 2(-9) \\ &= -7 + 18 \\ &= 11 \end{aligned}$$
The first five terms of the sequence are 4, −1, −9, −7, and 11.
c) 2, 3, 3, 6, 3 **d)** −1, 4, 6, −2, −14
e) 3, −2, −3, 10, −25 **f)** 5, 1, −1, 2, 5

11. **a)** 1, 0, 1, 0 **b)** 64, −16, 4, −1
c) −3, 15, −75, 375
d) −2, 5, 26, 89
e) $\frac{1}{8}, -\frac{1}{2}, -3, -13$
f) $a - 2b$, $a + b$, $a + 4b$, $a + 7b$
g) $2c + 3d$, $c + 3d$, $3d$, $3d - c$
h) $m - 5n$, $3m - 4n$, $5m - 3n$, $7m - 2n$

12. **a)** 4, −3, −10, −17; $t_n = 11 - 7n$
b) 81, −27, 9, −3; $t_n = 81(-3)^{1-n}$
c) 0, 3, 8, 15; $t_n = n^2 - 1$
d) −5, −2, 1, 4; $t_n = 3n - 8$

13. **a)** 2, −5, 16, −47, 142
b) 17, 13, 7, −1, −11

14. Yes. Examples may vary. Sample answer:
Example: 2, 4, 8, 16, 32, …
$t_1 = 2$, $t_n = 2t_{n-1}$ or $t_1 = 2$, $t_n = t_{n-1} + 2^{n-1}$

6.3 Pascal's Triangle and Expanding Binomials

1. **a)** Due to the symmetry of Pascal's triangle, there is an equivalent hockey stick pattern that begins with the first 1 in row 3 and ends with the first 35 in row 7, as shown.

```
            1
           1 1
          1 2 1
         1 3 3 1
        1 4 6 4 1
      1 5 10 10 5 1
     1 6 15 20 15 6 1
    1 7 21 35 35 21 7 1
  1 8 28 56 70 56 28 8 1
```

b)

```
                     1
                   1   1
                 1   2   1
               1   3   3   1
             1   4   6   4   1
           1   5   10  10  5   1
         1   6   15  20  15  6   1
       1   7   21  35  35  21  7   1
     1   8   28  56  70  56  28  8   1
   1   9  36  84  126 126 84  36  9   1
 1  10  45  120  210  252  210  120  45  10  1
```

i) 84 = 56 + 21 + 6 + 1
ii) 120 = 84 + 28 + 7 + 1
iii) 56 = 35 + 15 + 5 + 1
iv) 252 = 126 + 70 + 35 + 15 + 5 + 1

2. **a)** 2048 **b)** 524 288
c) 4 194 304 **d)** 1 073 741 824
e) 2^{n+1}

3. **a)** $t_{6,8}$ **b)** $t_{10,3}$
c) $t_{4,5}$ **d)** $t_{11,8}$
e) $t_{16,10}$ **f)** $t_{a+2,b+1}$

4. **a)** $t_{18,10} + t_{18,11}$ **b)** $t_{24,15} + t_{24,16}$
c) $t_{13,6} + t_{13,7}$ **d)** $t_{a-1,1} + t_{a-1,2}$
e) $t_{x+1,x-4} + t_{x+1,x-3}$

5. **a)** $(x+1)^8 = x^8 + 8x^7 + 28x^6 + 56x^5 + 70x^4 + 56x^3 + 28x^2 + 8x + 1$
b) $(y-2)^7 = y^7 - 14y^6 + 84y^5 - 280y^4 + 560y^3 - 672y^2 + 448y - 128$
c) $(2+t)^6 = 64 + 192t + 240t^2 + 160t^3 + 60t^4 + 12t^5 + t^6$
d) $(1-m^2)^4 = 1 - 4m^2 + 6m^4 - 4m^6 + m^8$
e) $(a+2b)^3 = a^3 + 6a^2b + 12ab^2 + 8b^3$

6. **a)** 8 **b)** 21 **c)** 2 **d)** 39
e) 55 **f)** $n+2$ **g)** 1

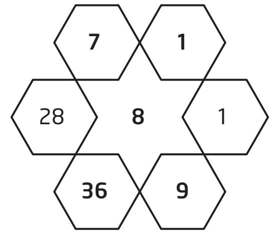
7. **a)** 1 **b)** 5 **c)** 3
d) 210 **e)** 45 **f)** 4

8. **a)** 15 **b)** 7 **c)** 17 **d)** 10
e) 20 **f)** 13 **g)** 9 **h)** 12

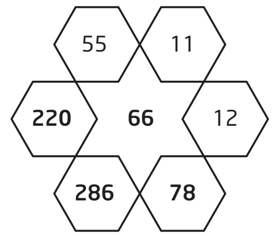
9. **a)** $t_{8,3} - t_{7,2}$ **b)** $t_{10,5} - t_{9,4}$
c) $t_{14,2} - t_{13,1}$ **d)** $t_{27,17} - t_{26,16}$
e) $t_{25,3} - t_{24,2}$ **f)** $t_{11,0} - t_{10,9}$
g) $t_{n+1,r} - t_{n,r-1}$

10. **a)** Refer to Pascal's triangle. Look for the row where the number 28 appears. This is in row 8. The number 28 is the 3rd entry and the 7th entry. To decide which is accurate for the given diagram, note that in the diagram the last number is 1. So, use the 28 in row 8 that is the 7th entry. It is followed by an 8, so in the middle position, write the number 8. Find the numbers in row 7 of Pascal's triangle to insert in the top two hexagons. These are 7 and 1. Use row 9 of Pascal's triangle to find the values that belong in the bottom two hexagons. These are 36 and 9.

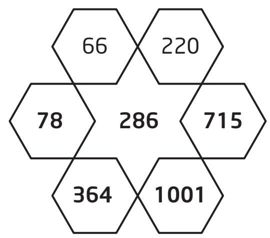
a)



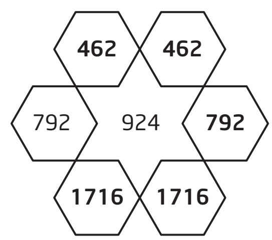
b)



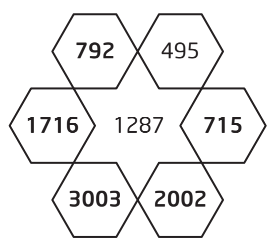
c)



d)



e)



11. **a)** $2x^4 + 12x^2y^2 + 2y^4$
b) $8x^3y + 8xy^3$

12. **a)** $64x^6 + 576x^4 + 2160x^2 + 4320 + \frac{4860}{x^2} + \frac{2916}{x^4} + \frac{729}{x^6}$
b) $x^3 - 12x^2 + 60x - 160 + \frac{240}{x} - \frac{192}{x^2} + \frac{64}{x^3}$

13. Since Pascal's triangle is for binomials, group together two of the three terms to express the trinomial as a binomial, and then expand.

Use $(a + b + c)^3 = [(a + b) + c]^3$ and treat $(a + b)$ as a single term, such as x. To expand $[(a + b) + c]^3$ think of this as $[x + c]^3$. Once this is expanded using Pascal's triangle, replace x with $(a + b)$ and further expand and simplify.

b) $(a + b + c)^3$
$= [(a + b) + c]^3$
$= (a + b)^3 + 3(a + b)^2c + 3(a + b)c^2 + c^3$
$= a^3 + 3a^2b + 3ab^2 + b^3 + 3(a^2 + 2ab + c^2)c + 3ac^2 + 3bc^2 + c^3$
$= a^3 + 3a^2b + 3ab^2 + b^3 + 3a^2c + 6abc + 3ac^2 + 3bc^2 + c^3$

14. $a = b = 2$

15. –467

16. a) The coefficients of the expansion of $(a + b)^5$ are 1, 5, 10, 10, 5, 1.

$\binom{5}{0} = \frac{5!}{0!(5-0)!} = 1, \binom{5}{1} = 5, \binom{5}{2} = 10,$
$\binom{5}{3} = 10, \binom{5}{4} = 5, \binom{5}{5} = 1$

b) $\binom{10}{0} = 1, \binom{10}{1} = 10, \binom{10}{2} = 45,$
$\binom{10}{3} = 120, \binom{10}{4} = 210, \binom{10}{5} = 252,$
$\binom{10}{6} = 210, \binom{10}{7} = 120, \binom{10}{8} = 45,$
$\binom{10}{9} = 10, \binom{10}{10} = 1$

6.4 Arithmetic Sequences

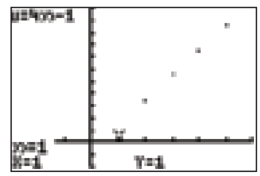
1. a) $a = 5, d = 3$; 14, 17, 20, 24
b) $a = -3, d = 5$; 12, 17, 22, 27
c) $a = 1.5, d = -0.8$; –0.9, –1.7, –2.5, –3.3
d) $a = 33, d = -1.8$; 27.6, 25.8, 24, 22.2
e) $a = \frac{1}{4}, d = \frac{1}{4}$; $1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}$
f) $a = 0.25, d = 0.01$; 0.28, 0.29, 0.30, 0.31

2. a) arithmetic; $a = 1, d = 3$
b) not arithmetic; no common difference between terms
c) arithmetic; $a = -12, d = 7$
d) arithmetic; $a = 0.41, d = -0.09$
e) not arithmetic; no common difference between terms
f) arithmetic; $a = \frac{19}{12}, d = -\frac{1}{3}$

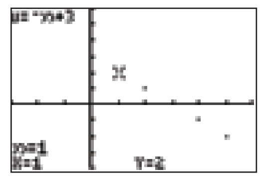
3. a) 12, 7, 2; $t_n = -5n + 17$
b) $-9, -7, -5$; $t_n = 2n - 11$
c) $11, \frac{81}{8}, \frac{37}{4}$; $t_n = -\frac{7}{8}n + \frac{95}{8}$
d) $-\frac{2}{3}, -\frac{1}{6}, \frac{1}{3}$; $t_n = -\frac{1}{2}n - \frac{7}{6}$
e) $x^2, 2.3x^2, 3.6x^2$; $t_n = 1.3x^2n - 0.3x^2$

4. a) 39 **b)** –77 **c)** –9
d) –28.8 **e)** 75 **f)** –173

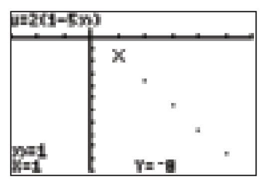
5. a) 3, 7, 11, 15, 19



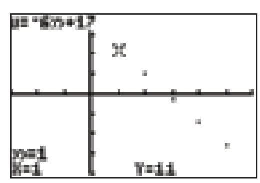
b) 2, 1, 0, –1, –2



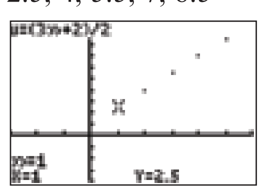
c) –8, –18, –28, –38, –48



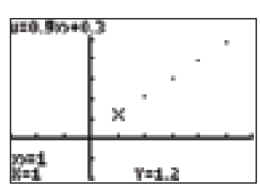
d) 11, 5, –1, –7, –13



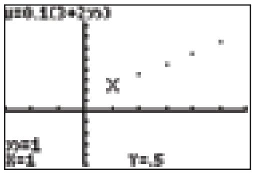
e) 2.5, 4, 5.5, 7, 8.5



f) 1.2, 2.1, 3, 3.9, 4.8



g) 0.5, 0.7, 0.9, 1.1, 1.3



6. t^{21}

7. a) 30 b) 53 c) 86
d) 33 e) 65 f) 38

8. a) Find the terms of the sequence using the recursion formula $t_1 = -5$, $t_n = t_{n-1} + 3$.
$t_1 = -5$
$t_2 = -5 + 3 = -2$
$t_3 = -2 + 3 = 1$
$t_4 = 1 + 3 = 4$
The sequence is −5, −2, 1, 4, ….
The first term is −5 and the common difference is 3, so this is an arithmetic sequence.
b) Substitute $a = -5$ and $d = 3$ into $t_n = a + (n - 1)d$.
$t_n = -5 + (n - 1)(3)$
$= -5 + 3n - 3$
$= 3n - 8$

9. a) $a = \frac{1}{5}, d = \frac{4}{15}; 1, \frac{19}{15}, \frac{23}{15}$
b) $a = -1, d = -\frac{2}{3}; -3, -\frac{11}{3}, -\frac{13}{3}$
c) $a = 2, d = -\frac{3}{4}; -\frac{1}{4}, -1, -\frac{7}{4}$
d) $a = -\frac{3}{8}, d = \frac{7}{8}; \frac{9}{4}, \frac{25}{8}, 4$
e) $a = -4x + y, d = x + 4y; -x + 13y, 17y, x + 21y$
f) $a = 3m - \frac{5}{6}n, d = -6m + \frac{1}{6}n;$
$-15m - \frac{1}{3}n, -21m - \frac{1}{6}n, -27m$

10. a) $a = -3, d = 7; t_n = 7n - 10$
b) $a = -3, d = 5; t_n = 5n - 8$
c) $a = 42, d = 2; t_n = 2n + 40$
d) $a = -3, d = 3; t_n = 3n - 6$
e) $a = 4.5, d = 1.5; t_n = 1.5n + 3$
f) $a = 16.2, d = -6; t_n = -6n + 22.2$
g) $a = x + 29, d = 5x; t_n = 5nx - 4x + 29$
h) $a = 3x^3 - 2, d = -x^3 - 1;$
$t_n = 4x^3 - nx^3 - n - 1$

11. a) $t_1 = -3, t_n = t_{n-1} + 7$
b) $t_1 = -3, t_n = t_{n-1} + 5$
c) $t_1 = 42, t_n = t_{n-1} + 2$
d) $t_1 = -3, t_n = t_{n-1} + 3$
e) $t_1 = 4.5, t_n = t_{n-1} + 1.5$
f) $t_1 = 16.2, t_n = t_{n-1} - 6$
g) $t_1 = x + 29, t_n = t_{n-1} + 5x$
h) $t_1 = 3x^3 - 2, t_n = t_{n-1} - x^3 - 1$

12. a) 13 m
b) 0, 13, 26, 39, 52, 65, 78, 91, 104, 117
c) $t_n = 13n - 13$
d) 13 is the common difference, d

13. a) The salary the first 6 months is \$73 000. Add \$2275 for each subsequent 6-month period.
The sequence is 73 000, 75 275, 77 550, 79 825, ….
The sequence is arithmetic because there is a common difference of 2275 between each term, beginning with the first term, 73 000.
b) Substitute $a = 73\,000$ and $d = 2275$ into $t_n = a + (n - 1)d$.
$t_n = 73\,000 + (n - 1)(2275)$
$= 2275n + 70\,725$
The general term is $t_n = 2275n + 70\,725$.
c) Since each term is 2275 greater than the previous term, the recursion formula is $t_1 = 73\,000$, $t_n = t_{n-1} + 2275$.
d) 8 years is equivalent to 16 6-month periods. Substitute $n = 16$ into $t_n = 2275n + 70\,725$.
$t_{16} = 2275(16) + 70\,725$
$= 107\,125$
After 8 years, the architect's salary will be \$107 125.
e) Substitute $t_n = 127\,600$ into $t_n = 2275n + 70\,725$ and solve for n.
$127\,600 = 2275n + 70\,725$
$56\,875 = 2275n$
$n = 25$
25 6-month periods is equivalent to 12.5 years.
The architect's salary will be \$127 600 after 12.5 years.

14. a) 15
b) 9, 13, 17, 21, 25

15. $t_n = 5n - 2$

16. a) \$786.60
b) 21 years

17. 8, 3, −2 and −2, 3, 8

18. 11, 5, −1 or −1, 5, 11

6.5 Geometric Sequences

1. a) arithmetic; The first term is $a = 7$ and the common difference is $d = -2$.
b) geometric; The first term is $a = 4$ and the common ratio is $r = -4$.
c) geometric; The first term is $a = 3$ and the common ratio is $r = 0.1$.
d) neither; The first term is $a = 8$, but there is no common difference or common ratio between the consecutive terms.
e) geometric; The first term is $a = 1$ and the common ratio is $r = 3$.
f) geometric; The first term is $a = ab$ and the common ratio is $r = b$.

2. a) 2; 48, 96, 192
b) −4; −1280, 5120, −20 480
c) −2; 8, −16, 32 **d)** 0.1; 0.8, 0.08, 0.008
e) 3; $-\frac{27}{2}, -\frac{81}{2}, -\frac{243}{2}$
f) 0.2; 0.004, 0.0008, 0.000 16
g) $\frac{(x+3)^3}{4}; \frac{(x+3)^{14}}{768}, \frac{(x+3)^{17}}{3072}, \frac{(x+3)^{20}}{12\,288}$

3. a) $t_n = 4096\left(\frac{1}{4}\right)^{n-1}; \frac{1}{4}$
b) $t_n = 12\left(\frac{1}{2}\right)^{n-1}; \frac{3}{512}$
c) $t_n = 6\left(-\frac{1}{3}\right)^{n-1}; -\frac{2}{729}$
d) $t_n = 13.45(0.2)^{n-1}$; 0.000 006 886 4
e) $t_n = \frac{1}{32}(4)^{n-1}$; 524 288
f) $t_n = \frac{a^2}{b}\left(\frac{a}{2}\right)^{n-1}; \frac{a^{17}}{32\,768b}$

4. a) 3, −3, 3, −3 **b)** 22, −44, 88, −176
c) $\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{8}{3}$, **d)** $4, 4\sqrt{5}, 20, 20\sqrt{5}$
e) $-2, -\frac{4}{3}, -\frac{8}{9}, -\frac{16}{27}$
f) −1111, 333.3, −99.99, 29.997

5. a) 8 **b)** 15 **c)** 8
d) 7 **e)** 13

6. a) arithmetic; $a = 3m, d = 4m$
b) geometric; $a = -1, r = \frac{2}{x}$
c) arithmetic; $a = 3x - 4y, d = 2x - 2y$
d) geometric; $a = 5.440, r = 10$
e) neither
f) arithmetic; $a = 7, d = -3 + s$

7. a) $m = 21, n = 189$ or $m = -21, n = -189$
b) $m = -50, n = -250$
c) $m = \frac{1}{3}, n = \frac{3}{4}$
d) $m = 1, n = 36$
e) $m = 32, n = 8$
f) $m = 20, n = 100$

8. First determine a and r to find the general term.
$a = \frac{2}{81}$
Divide the second term by the first term to find r.

$$r = \frac{\frac{4}{27}}{\frac{2}{81}} = \frac{4}{27} \times \frac{81}{2} = 6$$

$$t_n = \frac{2}{81}(6)^{n-1}$$

To determine the n-value that results in 6912, solve $6912 = \frac{2}{81}(6)^{n-1}$.

$$6912 = \frac{2}{81}(6)^{n-1}$$
$$\frac{6912(81)}{2} = 6^{n-1}$$
$$279\,936 = 6^{n-1}$$

Use systematic trial to find n. Since $6^7 = 279\,936$, $n - 1 = 7$ and $n = 8$. Therefore, 6912 is eighth term in the sequence.

9. 9

10. In each recursion formula the first term, t_1, corresponds with the value of a in the formula for the general term of a geometric sequence, and the coefficient of t_{n-1} corresponds with the value of r.
a) Since $t_1 = 4$, then $a = 4$. The coefficient of t_{n-1} is $-3x$, so $r = -3x$. Substitute a and r into $t_n = ar^{n-1}$. The general term is $t_n = 4(-3x)^{n-1}$.

b) Since $t_1 = -28m^3$, then $a = -28m^3$. The coefficient of t_{n-1} is $\frac{1}{2}$, so $r = \frac{1}{2}$. Substitute a and r in $t_n = ar^{n-1}$. The general term is $t_n = -28m^3\left(\frac{1}{2}\right)^{n-1}$.

c) Since $t_1 = \frac{5}{3}$, then $a = \frac{5}{3}$. The coefficient of t_{n-1} is $\frac{3}{4} + c$, so $r = \frac{3}{4} + c$. Substitute a and r in $t_n = ar^{n-1}$. The general term is $t_n = \frac{5}{3}\left(\frac{3}{4} + c\right)^{n-1}$.

11. a) $t_n = 2(2)^{n-1}$
b) i) 64 **ii)** 1024 **iii)** 16 384
c) 13 generations

12. a) 2048 bacteria **b)** $t_{72} = 8(2)^{71}$

13. a) 20 **b)** 32

14. a) 24 m **b)** 13th bounce

15. a) 0 or $-\frac{19}{3}$ **b)** −9

16. $p = \frac{1}{3}$, $q = 2$, $r = 12$, $s = 72$

17. a) $t_n = 9\left(\frac{1}{3}\right)^{n-1}$ **b)** t_9

18. 2, 6, 18

19. a) $\frac{9}{5}, \frac{6}{5}, \frac{4}{5}, \frac{8}{15}, \frac{16}{45}$ or $9, -6, 4, -\frac{8}{3}, \frac{16}{9}$
b) 1, −2, 4, −8, 16 or $\frac{3}{7}, \frac{6}{7}, \frac{12}{7}, \frac{24}{7}, \frac{48}{7}$

6.6 Arithmetic Series

1. a) 72 **b)** −136 **c)** −240
d) 105 **e)** $\frac{53}{3}$ **f)** $331.5x$

2. a) $a = 3$, $d = 4$; $S_{10} = 210$
b) $a = 5$, $d = 7$; $S_{10} = 365$
c) $a = 2$, $d = 6$; $S_{10} = 290$
d) $a = 6$, $d = 12$; $S_{10} = 600$
e) $a = \frac{3}{2}$, $d = -1$; $S_{10} = -30$
f) $a = 5.6$, $d = 0.3$; $S_{10} = 69.5$

3. a) $\frac{8752}{3}$ **b)** 510 **c)** −1368
d) −151.8 **e)** $168\sqrt{5}$ **f)** $\frac{3280}{81}$

4. a) 1 001 000 **b)** 20 200 **c)** −1430
d) 399 **e)** 416 **f)** −383.5

5. a) 7155 **b)** −110 **c)** −137.6
d) $\frac{55}{6}$ **e)** $210\sqrt{3}$ **f)** 780

6. a) 528 **b)** −850 **c)** 462
d) 235 **e)** −40 **f)** −285
g) 63 **h)** $231a + 210b$ **i)** 495

7. a) 12 **b)** 30 **c)** 20
d) 25 **e)** 6 or 10 **f)** 15
g) 7 or 22 **h)** 12 **i)** 21

8. a) 10 000 **b)** 4 002 000 **c)** 9560

9. a) 25 **b)** 1111

10. −1575

11. 1035

12. a) $286\sqrt{6}$ **b)** $-19x$
c) $1479a + 261b$
d) $\frac{1520}{x^2}$

13. a) not arithmetic; The sequence of terms does not have a common difference.
b) arithmetic; The sequence of terms has a common difference of $-4x^2$.
c) arithmetic; The sequence of terms has a common difference of $2m - b$.
d) not arithmetic; The sequence of terms does not have a common difference.

14. 1540

15. a) The profit in the first week is \$350. This is the first term of the series.
The profit in the second week is \$350 + \$75, or \$425. This is the second term of the series.
The profit in the third week is \$425 + \$75, or \$500. This is the third term of the series.
Each term is \$75 more than the previous term. The constant difference between terms is 75. Therefore, the terms form an arithmetic sequence and the sum of the terms form an arithmetic series. The last term represents the profit in the 16th week. The last term is \$350 + 15(\$75), or \$1475.
The arithmetic series that represents the total profit is
$350 + 425 + 500 + \cdots + 1475$.

b) Substitute $a = 350$, $t_{16} = 1475$, and $n = 16$ into the formula $S_n = \frac{n}{2}(a + t_n)$.

$$\begin{aligned} S_{16} &= \frac{16}{2}(350 + 1475) \\ &= 8(1825) \\ &= 14\,600 \end{aligned}$$

The total profit for the season is \$14 600.

16. Since there are 8 two-week periods in 16 weeks, in Job A Bashira would earn \$450 × 8, or \$3600. In Job B, Bashira would earn \$100 the first week, \$125 the second week, \$150 the third week, and so on until week 16 when he earns \$100 + 15(\$25), or \$475.
The total amount of money is represented by the arithmetic series $100 + 125 + 150 + \cdots + 475$.
The sum of the series is $\frac{16}{2}(100 + 475)$, or 4600.
Bashira should accept Job B since he would earn \$4600, which is \$1000 more than the amount he would earn in Job A.

17. a) $n^2 + 4n$ **b)** $2n^2 + 3n$ **c)** $\frac{5n^2 + n}{2}$ **d)** $\frac{7n^2 + 15n}{2}$ **e)** $\frac{-3n^2 + 21n}{2}$

18. a) $n^2 + n$ **b)** n^2

19. $28 + 25 + 22 + 19 + \cdots$

20. $t_n = 4n + 3$

21. 5, 7, 9, 11

22. $2 + 5 + 8 + 11 + 14 + \cdots$

23. $t_n = 3n - 2$

6.7 Geometric Series

1. a) geometric; The first term is $a = 1$ and the common ratio is $r = 10$.
b) arithmetic; The first term is $a = 6$ and the common difference is $d = 6$.
c) geometric; The first term is $a = -4$ and the common ratio is $r = -2$.
d) neither; The first term is $a = 7$ and there is no common ratio or common difference.
e) neither; The first term is $a = 6571$ and there is no common ratio or common difference.
f) neither; The first term is $a = \frac{1}{8}$ and there is no common ratio or common difference.

2. a) $a = 3, r = 2; S_9 = 1533$
b) $a = 5, r = 2; S_{12} = -6825$
c) $a = \frac{1}{8}, r = 2; S_8 = \frac{255}{8}$
d) $a = -0.2, r = -3; S_{13} = -79\,716.2$
e) $a = 9, r = -1; S_{50} = 0$
f) $a = 2, r = 0.1; S_{17} = \frac{20}{9}$

3. a) $\frac{1093}{81}$ **b)** $\frac{2049}{512}$ **c)** -1364 **d)** $-524\,286$ **e)** 1 562 496 **f)** $-16\,382$ **g)** $\frac{133}{16}$ **h)** 0 **i)** 31 100

4. a) 3906 **b)** 88 572 **c)** $\frac{29\,524}{3}$ **d)** 27 305 **e)** 6560 **f)** 2999.9997 **g)** $\frac{9841}{27}$

5. a) $\frac{511}{128}$ **b)** 9840 **c)** $\frac{8525}{64}$ **d)** $\frac{5115}{512}$ **e)** $\frac{1705}{512}$

6. a) $\frac{-624\sqrt{5}}{\sqrt{5} + 1}$
b) The first term is $a = x$. The common ratio is $r = \sqrt{7}$. To find S_{13}, $n = 13$. Substitute these values in the formula

$$S_n = \frac{a(r^n - 1)}{r - 1}.$$

$$S_{13} = \frac{x[(\sqrt{7})^{13} - 1]}{\sqrt{7} - 1}$$

First, simplify $(\sqrt{7})^{13}$.

$$\begin{aligned} (\sqrt{7})^{13} &= (\sqrt{7})^{12 + 1} \\ &= (\sqrt{7})^{12}\sqrt{7} \\ &= 117\,649\sqrt{7} \end{aligned}$$

Then,

$$S_{13} = \frac{x(117\,649\sqrt{7} - 1)}{\sqrt{7} - 1}$$

c) $\frac{4(x^{14} - 1)}{x - 1}$
d) $\frac{2(x^{22} - 1)}{x^2 - 1}$

7. a) $\frac{25\,999}{64}$

b) $\frac{3\sqrt{6}(279\,936\sqrt{6} + 1)}{\sqrt{6} + 1}$

c) 19 790.251 12 or $2500(1.2^{12} - 1)$

d) $\frac{8(8192x^{39} - 1)}{2x^3 - 1}$

8. a) From the general term $t_n = -2(3)^{n-1}$, $a = -2$ and $r = 3$. Substitute these values in $S_n = \frac{a(r^n - 1)}{r - 1}$ and simplify, if possible.

$$S_n = \frac{-2(3^n - 1)}{3 - 1} = \frac{-2(3^n - 1)}{2} = 1 - 3^n$$

Therefore, an expression for the sum of the series is $S_n = 1 - 3^n$. Substitute $n = 9$ to obtain $S_9 = 1 - 3^9$, or −19 682.

b) $S_n = 54\left(1 - \left(\frac{2}{3}\right)^n\right)$, $S_9 = \frac{38\,342}{729}$

c) $S_n = \frac{x^2(x^{2n} - 1)}{x^2 - 1}$, $S_9 = \frac{x^2(x^{18} - 1)}{x^2 - 1}$

9. $\frac{1023}{32}$

10. $a = 13$

11. 27

12. 717 km

13. 10 prizes

14. $10 737 418.23; Answers may vary. Sample answer: Her dad would probably not be able to afford this amount.

15. $t_n = 3(5)^{n-1}$ or $t_n = 75\left(\frac{1}{5}\right)^{n-1}$

16. Answers may vary.

17. 9841

18. a) $S_n = 2(2^n - 1) - \frac{1}{2}(3^n - 1)$

b) −238

Chapter 6 Review

1. a) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}$ b) $\frac{5}{2}, \frac{7}{3}, \frac{9}{4}$

2. a) 250 b) $\frac{7}{8}$

3. a) The first term is 7. Add 7 to the absolute value of each term and then multiply the result by $(-1)^{n+1}$ to get the next term. Next three terms: 35, −42, 49

b) The first term is $-\frac{1}{2}$. Divide each term by −2 (or multiply by $-\frac{1}{2}$) to get the next term. Next three terms: $-\frac{1}{32}, \frac{1}{64}, -\frac{1}{128}$

c) The first term is $3x$. Multiply each term by $2x$ to get the next term. Next three terms: $48x^5$, $96x^6$, $192x^7$

4. a) $f(n) = -5n$, domain $\{n \in \mathbb{N}\}$

Term Number, n	Term, t_n	First Differences
1	−5	
		−5
2	−10	
		−5
3	−15	
		−5
4	−20	

b) $f(n) = -3n^2 + 10n + 15$, domain $\{n \in \mathbb{N}\}$

Term Number, n	Term, t_n	First Differences	Second Differences
1	22		
		1	
2	23		−6
		−5	
3	18		−6
		−11	
4	7		

5. a) 2, 8, 7, 13 b) 5, 2, 0.8, 0.32

c) 10, −10, 5, $-\frac{5}{3}$

6. a) $f(1) = 1, f(n) = 3f(n - 1) + 1$

b) $f(1) = 3, f(n) = f(n - 1) + 2$

c) $f(1) = -2, f(n) = -4 - 3f(n - 1)$

7. a) $\frac{1}{8}, -\frac{1}{2}, -3, -13$

b) $a - 2b$, $a + b$, $a + 4b$, $a + 7b$

8. a) 5, 5.60, 6.27, 7.02

b) $t_1 = 5$, $t_n = 1.12t_{n-1}$

c) $t_n = 5(1.12)^{n-1}$

d) $13.87

e) 25 weeks

9. a) 210 = 126 + 56 + 21 + 6 + 1

b) 70 = 35 + 20 + 10 + 4 + 1

c) 126 = 70 + 35 + 15 + 5 + 1

10. a) 2 097 152 b) 262 144

11. a) $t_{9,\,7}$ b) $t_{13,\,3}$ c) $t_{24,\,22}$

12. a) $a^5 + 5a^4 + 10a^3 + 10a^2 + 5a + 1$
b) $256x^8 - 786x^6y^3 + 864x^4y^6 - 432x^2y^9 + 81y^{12}$
c) $1 - \frac{5}{x} + \frac{10}{x^2} - \frac{10}{x^3} + \frac{5}{x^4} - \frac{1}{x^5}$

13. a) $a = -19$, $d = -6$, $t_n = -13 - 6n$; $-37, -43, -49, -55$
b) $a = \frac{8}{3}$, $d = -\frac{2}{5}$, $t_n = \frac{46}{15} - \frac{2}{5}n$; $\frac{22}{15}, \frac{16}{15}, \frac{2}{3}, \frac{4}{15}$

14. a) 9 b) 52

15. a) $t_n = 437 - 14n$ b) 269 m
c) 20th minute

16. a) neither; The first term is $a = \frac{3}{5}$, but there is no common difference or common ratio between successive terms.
b) geometric; The first term is $a = 2$ and the common ratio is $r = \sqrt{3}$.
c) arithmetic; The first term is $a = x + 7y$ and the common difference is $d = x + 3y$.

17. a) $t_n = -3(-5)^{n-1}$; $-1\,171\,875$
b) $t_n = -\frac{2}{625}(-5)^{n-1}$; $-31\,250$

18. 1 171 875 e-mail messages

19. a) $a = 21$, $d = -6$; -60
b) $a = -4$, $d = -5$; -265

20. a) 42 b) 3604

21. 420 m

22. $2 + 6 + 10 + \cdots$

23. $a = 100$, $r = -2$; $S_8 = -8500$

24. a) 2186 b) $\frac{14\,762}{19\,683}$

25. $2 + 10 + 50 + \cdots$ or $72 - 60 + 50 - \cdots$

Math Contest

1. B
2. D
3. $8x^2 - 10x + 5$
4. $x = 4$, $y = 5$
5. C
6. A
7. D
8. B
9. A
10. $t_n = 1$ and $t_n = 4(-2)^{1-n}$
11. $x = -16$, $y = -\frac{128}{3}$
12. A
13. $1, -2, 4, -8, \ldots$ and $\frac{3}{7}, \frac{6}{7}, \frac{12}{7}, \frac{24}{7}, \ldots$
14. A
15. C
16. B

Chapter 7 Financial Applications

7.1 Simple Interest

1. a) \$108 b) \$33.25
c) \$18.17 d) \$24.09

2. a) \$535, \$570, \$605, \$640, \$675
b) $a = 535$, $d = 35$
c) $t_n = 500 + 35n$; The nth term is the linear model that represents the amount of the \$500 investment at the end of the nth year.

3. a) All first differences are \$40, representing the amount of simple interest added to the GIC each year.
b) \$1000; $t = 0$ is the start of the investment
c) 4%

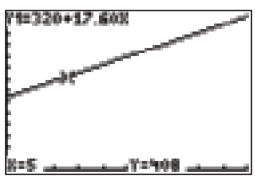
4. a) $A = 1000 + 40t$
b) Answers may vary. Sample answer: This is a partial variation since the initial amount is not \$0. The linear model contains a fixed part and a variable part; fixed part = 1000, variable part = $40t$.
c) 25 years

5. The principal is $P = 2100$.
The interest rate is 18%, so $r = 0.18$.
The time is 23 days. Since there are 365 days in a year, then in days, $t = \frac{23}{365}$.
Substitute these values into the formula $I = Prt$ and solve for I.
$I = 2100 \times 0.18 \times \frac{23}{365}$
$\doteq 23.82$
The company would charge \$23.82 in interest.

6. **a)** \$800 **b)** 5%
c) $A = 800 + 40t$ **d)** 20 years

7. **a)** $I = 40t$
b) 20 years, the same as in 6d)

8. Determine the principal, P.
The interest rate is $2\frac{3}{4}$%. So, $r = 0.0275$.
The time is $t = 4$.
The interest is $I = 165$.
Substitute these values into the formula $I = Prt$ and solve for P.
$165 = P \times 0.0275 \times 4$
$165 = 0.11P$
$P = \frac{165}{0.11}$
$P = 1500$
Katio borrowed \$1500.

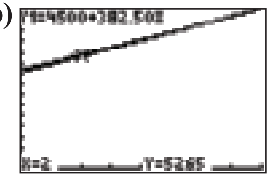
9. **a)** $A = 320 + 17.60t$
b)


c) 7 years 5 months **d)** 7.54%

10. Determine the interest rate, r.
The principal is $P = 1350$.
The time is 8 months. Since there are 12 months in a year, then in years,
$t = \frac{8}{12}$, or $\frac{2}{3}$.
The interest is $I = 38.50$.
Substitute these values into the formula $I = Prt$ and solve for r.
$38.25 = 1350 \times r \times \frac{2}{3}$
$38.25 = 900r$
$r = \frac{38.25}{900}$
$r = 0.0425$
The annual rate of interest of Lorilo's loan was 4.25%.

11. **a)** \$4213.25 **b)** \$413.25
c) Rosalie should repay the loan 6 months sooner, that is, in approximately 1 year.

12. 8 years

13. **a)** Option 1: $A = 4500 + 382.50t$;
Option 2: $A = 4650 + 360t$
b)


c) The loan payment for the two options is equal at 6 years 8 months. Option 1 is less costly, so it is the better option if Arash pays the loan in less than 6 years 8 months. For a longer repayment period, Option 2 is the better option.

14. **a)** \$63.75 **b)** \$850 **c)** 7.5%

15. \$611.76

16. \$2400

17. **a)** \$1419.01 **b)** \$56.61
c) Answers may vary. Sample answer: The interest is "compounded" on the previous interest as well as the initial principal.

7.2 Compound Interest

1. **a)** \$914.62 **b)** \$264.62

2. **a)** \$1466.07 **b)** \$491.07

3. **a)** 0.005 83 **b)** 0.0225
c) 0.04 **d)** 0.0042

4. **a)** 16 **b)** 10 **c)** 8
d) 21 **e)** 7

5. **a)** $n = 4$, $i = 0.085$ **b)** $n = 20$, $i = 0.0175$
c) $n = 36$, $i = 0.003$ **d)** $n = 13$, $i = 0.0275$
e) $n = 730$, $i = 0.000\,169\,86$

6. **a)** \$1601.59 **b)** \$301.59
c) simple interest is \$273; compound interest earns \$28.59 more

7. \$8.81

8. **a)** The principal is \$6800, so $P = 6800$.
When the interest is compounded semi-annually, it is added twice a year.
The semi-annual rate is $\frac{5.2\%}{2}$, or 2.6%.
So, $i = 0.026$.
i) In 4 years, there are 4×2, or 8 compounding periods. So, $n = 8$.
Substitute the known values into the compound interest formula.

$A = P(1 + i)^n$
$= 6800(1 + 0.026)^8$
$= 6800(1.026)^8$
$\doteq 8350.03$
The amount after 4 years is \$8350.03.

ii) In 7 years, there are 7×2, or 14 compounding periods.
Substitute the known values into the compound interest formula.
$A = P(1 + i)^n$
$= 6800(1 + 0.026)^{14}$
$= 6800(1.026)^{14}$
$\doteq 9740.29$
The amount after 7 years is \$9740.29.

b) Determine the difference between the amounts for the 4th year and 7th year.
\$9740.29 − \$8350.03 = \$1390.26
Therefore, \$1390.26 was earned in interest between the 4th year and the 7th year.

9. a) i) \$555
ii) \$700.31
iii) \$720.39
iv) \$730.97
v) \$738.24
vi) \$741.82

b) The best scenario for Kara is simple interest, since this is the least interest payment. The worst is the daily compounded interest, since this is the greatest interest payment.

c) The shorter the compounding period, the greater the interest.

10. The principal is $P = 5000$.
Since Isabella wants her investment to double, then $A = 10\ 000$.
The daily interest rate is $\frac{6\%}{365}$, so $i \doteq 0.000\ 164$.
Let x represent the number of years it takes for the investment to double in value.
In x years, there are $365x$ compounding periods. So, $n = 365x$.
Use the compound interest formula.
$A = P(1 + i)^n$
$10\ 000 = 5000(1 + 0.000\ 164)^{365x}$
$2 = (1.000\ 164)^{365x}$
Use systematic trial to find the value of $365x$.
Since $(1.000\ 164)^{4230} \doteq 2.001$, then $x \doteq 11.6$.
Convert to years and months.
$0.6 \times 12 = 7.2$
Therefore, it takes approximately 11 years and 7 months for Isabella's investment to double.

11. a) The best choice is the one that earns the most interest. Determine the amount at the end of 7 years for each option. The principal is $P = 8000$.

Option 1:
When the interest is compounded semi-annually, it is added twice a year.
The semi-annual rate is $\frac{6\%}{2}$, or 3%. So, $i = 0.03$.
In 7 years, there are 7×2, or 14 compounding periods. So, $n = 14$.
Substitute the known values into the compound interest formula.
$A = P(1 + i)^n$
$= 8000(1 + 0.03)^{14}$
$= 8000(1.03)^{14}$
$\doteq 12\ 100.72$
If Meg invests her money in Option 1, she will have \$12 100.72 after 7 years.

Option 2:
Calculate the interest first. Substitute $P = 8000$, $r = 0.075$, and $t = 7$ into $I = Prt$.
$I = 8000(0.075)(7)$
$= 4200$
Now, find the amount.
$A = P + I$
$= 8000 + 4200$
$= 12\ 200$
If Meg invests her money in Option 2, she will have \$12 200 after 7 years.
Therefore, Meg should choose Option 2 because she will earn more interest.

b) In Option 1, the investment earns compound interest. This represents an exponential function because interest is earned on interest. The amount grows exponentially because it is multiplied by

1.03 for each compounding period.
In Option 2, the investment earns simple interest. This is a constant amount paid out at the end of each year, so it represents a linear function.

12. 7.25%

13. 5.8%

14. a) i) 18 years
ii) 24 years
iii) 12 years
b) Answers may vary. Sample answer: The Rule of 72 is close but not exact. The results using the compound interest formula are
i) 17 years 8 months
ii) 23 years 5 months
iii) 11 years 11 months

15. 7 years

16. Maxime should choose Option 1 because the amount at the end of 4 years is \$6194.12, whereas the amount at the end of 4 years with Option 2 is \$6189.87.

17. Answers may vary. Sample answer: James takes out a \$1200 loan at 6.5% per year, compounded annually.

18. 17 years 6 months

19. \$5420.75

20. a) i) 6.09% **ii)** 8.24%
b) Answers may vary. Sample answer: No, the principal amount does not influence the effective rate. However, the effective rate of interest is higher the more often the nominal rate is compounded.

7.3 Present Value

1. a) \$633.67 **b)** \$950

2. \$47.21

3. a) \$1100 **b)** \$514.63

4. a) \$2000 **b)** \$1151.68

5. The future value of the investment is \$10 000. So, $FV = 10\ 000$.
The monthly rate is $\frac{6.3\%}{12}$, or $i = 0.005\ 25$.
In 5 years there are 5×12, or 60 compounding periods. So, $n = 60$.
Substitute the known values into the formula $PV = \frac{FV}{(1+i)^n}$.

$$\begin{aligned} PV &= \frac{10\ 000}{(1 + 0.005\ 25)^{60}} \\ &= \frac{10\ 000}{(1.005\ 25)^{60}} \\ &\doteq 7303.90 \end{aligned}$$

Therefore, Tara should invest \$7303.90 at 6.3% today to have \$10 000 in 5 years.

6. quarterly

7. 7 years

8. a) Investment A:
Substitute $FV = 8000$, $n = 24$, and $i = 0.013\ 75$ into $PV = \frac{FV}{(1+i)^n}$.

$$\begin{aligned} PV &= \frac{8000}{(1 + 0.013\ 75)^{24}} \\ &= \frac{8000}{(1.013\ 75)^{24}} \\ &\doteq 5764.33 \end{aligned}$$

With Investment A, Paula would have to invest \$5764.33 to have \$8000 in 6 years.

Investment B:
Substitute $FV = 8000$, $n = 72$, and $i \doteq 0.004\ 417$ into $PV = \frac{FV}{(1+i)^n}$.

$$\begin{aligned} PV &= \frac{8000}{(1 + 0.004\ 417)^{72}} \\ &= \frac{8000}{(1.004\ 417)^{72}} \\ &\doteq 5824.76 \end{aligned}$$

With Investment B, Paula would have to invest \$5824.76 to have \$8000 in 6 years.
b) Investment A is the better choice for Paula since she would have to invest less money to reach her goal of \$8000 in 6 years.

9. a) \$4083.50 **b)** \$4053.39 **c)** \$4037.83

10. \$757.88

11. 6%

12. a) \$3759.68 **b)** 9.7%

13. a) \$2.63 **b)** \$0.45

14. \$49 295.23

15. \$7651.41

16. \$16 137.80

17. \$473.06

7.4 Annuities

1. a) semi-annually; The annual interest is 10%, but in the time line, $(1.05)^n$ indicates that for each payment period, 5% interest is paid. This means that interest is paid twice a year.

b) 6 years; The time line shows 12 payments, with payments made twice a year.

c) \$4775.14

2. a) quarterly; The annual interest is 8%, but in the time line, $(1.02)^n$ indicates that for each payment period, 2% interest is paid. This means that interest is paid four times a year.

b) 5 years; The time line shows 20 payments, with payments made four times a year.

c) \$9718.95

3. a) a time line for the future value of an annuity with $R = 1200$, $n = 10$, and $i = 0.0375$

b), c) \$14 241.41 **d)** \$2241.41

4. a) a time line for the future value of an annuity with $R = 40$, $n = 24$, and $i = 0.005$

b) \$1017.28 **c)** \$57.28

5. \$899.65

6. \$560.19

7. a) \$174.48 **b)** 10%

8. a) 3 years **b)** \$86.95

9. 170 weeks, or 3 years 4 months

10. Substitute $A = 500\,000$, $i \doteq 0.004\,667$, and $n = 420$ into $A = \dfrac{R[(1 + i)^n - 1]}{i}$, and then solve for R.

$$500\,000 = \frac{R[(1 + 0.004\,667)^{420} - 1]}{0.004\,667}$$

$$2333.5 = R[(1.004\,667)^{420} - 1]$$

$$R = \frac{2333.5}{(1.004\,667)^{420} - 1}$$

$$R \doteq 384.56$$

Daniel should make monthly deposits of \$384.56 so that he will have \$500 000 at retirement.

11. a) \$24 000 **b)** Answers may vary.

c) Anna: \$27 908.01; Donella: \$32 775.87; Tina: \$46 204.09

12. a) Option A: \$480 000; Option B: \$960 000

b) Mick should choose Option A. When he is 65, the value of Option A will be \$3 491 007.83, which is \$1 143 926.17 more than Option B, which will be worth \$2 356 081.66.

13. i) Substitute $R = 1000$, $n = 45$, and $i = 0.06$ into $A = \dfrac{R[(1 + i)^n - 1]}{i}$.

$$A = \frac{1000[(1 + 0.06)^{45} - 1]}{0.06}$$

$$= \frac{1000[(1.06)^{45} - 1]}{0.06}$$

$$\doteq 212\,743.51$$

In this situation, the amount at age 65 is \$212 743.51.

A total of \$1000 × 45, or \$45 000, was deposited into the account.

ii) Substitute $R = 3000$, $n = 15$, and $i = 0.06$ into $A = \dfrac{R[(1 + i)^n - 1]}{i}$.

$$A = \frac{3000[(1 + 0.06)^{15} - 1]}{0.06}$$

$$= \frac{3000[(1.06)^{15} - 1]}{0.06}$$

$$\doteq 69\,827.91$$

In this situation, the amount at age 65 is \$69 827.91.

A total of \$3000 × 15, or \$45 000, was deposited into the account.

In each situation, the deposit amount is equal but the final amount is much higher for the \$1000 deposited since age 20 than for the \$3000 deposited since age 50.

14. a) \$13 971.65 **b)** \$13 180.79

c) $A = \dfrac{R(1 + i)[(1 + i)^n - 1]}{i}$

15. \$3799.47

16. \$4199.32

17. \$1230.17

7.5 Present Value of an Annuity

1. **a)** 14%; The time line shows an interest rate of 7%, which is paid every 6 months.
b) 4; There are 4 compound interest periods.
c) \$10 161.63

2. **a)** \$200 159.61

3. **a)** a time line for present value of an annuity with $R = 500$, $n = 5$, and $i = 0.09$
b) \$1944.83

4. **a)** a time line for the present value of an annuity with $R = 300$, $n = 36$, and $i = 0.005\,75$
b) \$9730.34 **c)** \$1069.66

5. \$361.52

6. **a)** \$2766.21 **b)** \$50 648.40

7. **a)** Substitute $PV = 12\,000$, $i \doteq 0.006\,667$, and $n = 60$ into $PV = \dfrac{R[1-(1+i)^{-n}]}{i}$.

$$12\,000 = \frac{R[1-(1+0.006\,667)^{-60}]}{0.006\,667}$$
$$80.004 = R[1-(1.006\,667)^{-60}]$$
$$R = \frac{80.004}{1-(1.006\,667)^{-60}}$$
$$R \doteq 243.32$$

Therefore, Jessica's monthly payments are \$243.31.

b) To calculate the interest paid, multiply the amount of the payments by the number of payments and subtract the amount borrowed.

$$\begin{aligned} I &= n \times R - PV \\ &= 60 \times 243.32 - 12\,000 \\ &= 14\,599.20 - 12\,000 \\ &= 2599.20 \end{aligned}$$

Jessica is paying \$2599.20 in interest on the loan.

8. **a)** \$33 982.11
b) \$26 017.89

9. **a)** Substitute $R = 100$, $i = 0.005$, and $n = 120$ into $PV = \dfrac{R[1-(1+i)^{-n}]}{i}$.

$$\begin{aligned} PV &= \frac{100[1-(1+0.005)^{-120}]}{0.005} \\ &= \frac{100[1-(1.005)^{-120}]}{0.005} \\ &\doteq 9007.35 \end{aligned}$$

The charitable organization needs \$9007.35 to fund this prize.

b)
$$\begin{aligned} A &= n \times R \\ &= 120 \times 100 \\ &= 12\,000 \end{aligned}$$

The winner receives \$12 000.

c)
$$\begin{aligned} I &= A - PV \\ &= 12\,000 - 9007.35 \\ &= 2992.65 \end{aligned}$$

Therefore, \$2992.65 of the winnings was earned as interest.

10. Substitute $PV = 7500$, $i \doteq 0.002\,417$, and $n = 36$ into $PV = \dfrac{R[1-(1+i)^{-n}]}{i}$.

$$7500 = \frac{R[1-(1+0.002\,417)^{-36}]}{0.002\,417}$$
$$18.1275 = R[1-(1.002\,417)^{-36}]$$
$$R = \frac{18.1275}{1-(1.002\,417)^{-36}}$$
$$R \doteq 217.78$$

The customer's monthly payment will be \$217.78.

11. 9.8% compounded monthly

12. \$498.43

13. 7.93%

14. **a)** \$854.45 at 8%; \$846.26 at 7.5%
b) \$41 013.60 at 8%; \$40 620.48 at 7.5%
c) \$393.12

15. \$3260.67

16. 10 years

17. \$4866.72

Chapter 7 Review

1.

Principal, P	Interest Rate, r	Time, t	Simple Interest, I
\$627.00	6.5%	2 months	\$6.79
\$389.15	9.25%	58 days	\$5.72
\$270.00	8%	3 years	\$64.80
\$425.00	$7\frac{1}{2}$%	145 days	\$12.66
\$380.21	$4\frac{3}{4}$%	6 months	\$9.03
\$178.50	8.6%	245 days	\$10.30
\$3200.00	11.5%	4.5 months	\$138.00

2. \$29.07
3. **a)** \$960 **b)** \$4960
4. Answers may vary. Sample answer: Simple interest is paid annually on the principal and is not reinvested. Compound interest is earned annually but is reinvested with the principal, so interest is earned on interest. Simple interest accumulates at the same rate and represents linear growth. Compound interest accumulates at a rate that has a constant ratio and represents exponential growth.
5. **a)** \$750.00 **b)** \$936.28 **c)** \$186.28 **d) i)** \$2250 **ii)** \$2436.28
6. **a)** \$1773.95 **b)** \$173.95
7. \$1432.49
8. 7.72%
9. 15 years 1 month
10. 10.78%
11. **a)** \$12 918.79 **b)** \$1918.79
12. \$933.09
13. 8.14%
14. **a)** \$2896.99 **b)** \$3000 **c)** \$103.01
15. \$26 050.33
16. 3.5%

Math Contest

1. B
2. C
3. B
4. $-\frac{1}{8}$
5. A
6. D
7. 12
8. $\frac{9}{49}$
9. 0, 3, −7
10. $t_n = 1$ and $t_n = 4(-2)^{1-n}$
11. 30°
12. B

Practice Exam

1. $\frac{81}{625}$
2. $(-7)^{65}$
3. $\frac{x^{28}}{y^{42}}$
4. $\frac{1}{(5)^9}$
5. $48^{\frac{1}{5}}$
6. −8
7. exponential
8. 1
9. $y = 0$
10. $\{y \in \mathbb{R}, y > 0\}$
11. $y = 2^{x-3}$
12. $y = 6^{2x}$
13. increasing
14. $y = 2^{-3x}$
15. $y = -1$
16. **a)** $y = 7 \sin [2(x - 30°)]$
 b) $y = -4 \sin 3x + 2$
17. $\frac{5x^2 + 44x}{(x-2)(x-3)(x+4)}$; the restrictions are $x \neq -4, x \neq 2, x \neq 3$
18. When simplified, $f(x) = 11(x + 4)$ and $g(x) = 11(x - 4), x \neq -1$. Therefore, $f(x)$ is not equivalent to $g(x)$.

19. a) $a = -1$; this is a reflection in the x-axis
$k = 4$; this is a horizontal compression by a factor of $\frac{1}{4}$
$d = -3$; this is a horizontal translation of 3 units to the left
$c = -1$; this is a vertical translation of 1 unit down
The equation is $y = -\sqrt{4(x + 3)} - 1$.

b) $a = \frac{1}{2}$; this is a vertical compression of factor $\frac{1}{2}$
$k = -1$; this is a reflection in the y-axis
$d = -5$; this is a horizontal translation of 5 units to the left
$c = 2$; this is a vertical translation of 2 units up
The equation is $y = \frac{-1}{2(x + 5)} + 2$.

20. a) base function $f(x) = \frac{1}{x}$; transformed function $g(x) = \sqrt{x - 2} - 5$

b) base function $f(x) = \frac{1}{x}$; transformed function $g(x) = \frac{1}{x - 1} + 4$

21. i) a) domain $\{x \in \mathbb{R}, x \neq 2\}$, range $\{y \in \mathbb{R}, y \neq 0\}$

b) $f(-5) = \frac{1}{7}$

c) $f^{-1}(x) = \frac{2x - 1}{x}$

ii) a) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \geq 1\}$

b) $f(-5) = 76$

c) $f^{-1}(x) = \pm\sqrt{\frac{x - 1}{3}}$

22. a) $f(x) = -3(x - 1)^2 + 7$; V(1, 7) is a maximum

b) $f(x) = 2(x - 2)^2 - 1$; V(2, −1) is a minimum

23. $f(x) = 2x^2 - 12x + 8$

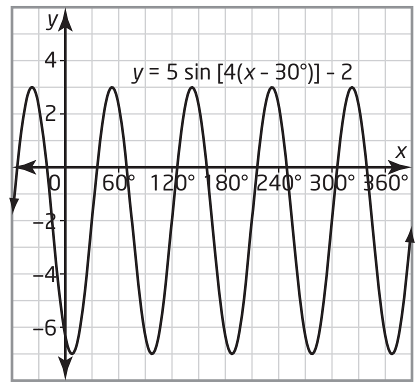
24. a) The value of the discriminant is 17, which is greater than zero, so the line and the quadratic function intersect at two points.

b) The value of the discriminant is −27, which is less than zero, so the line and the quadratic function do not intersect.

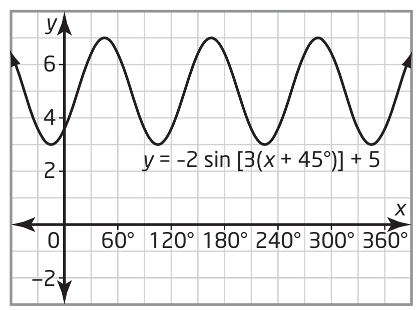
25. Ambiguous case; there are two answers: 21.9 km or 12.7 km

26. 4.2 m

27. a) amplitude is 5, period is 90°, horizontal translation is 30° to the right, range $\{y \in \mathbb{R}, -7 \leq y \leq 3\}$



b) amplitude is 2, period is 120°, horizontal translation is 45° to the left, range $\{y \in \mathbb{R}, 3 \leq y \leq 7\}$



28. $10\sqrt{3} + 40$ m

29. Answers may vary.

30. a) maximum: 29 °C, in July

b) minimum: −7 °C, in January

c) 36 °C

d) twice the value of 18, the coefficient of the sine term

e) 22

f) twice the amount of 11, the constant term

g) October is month 10; substitute $t = 10$ in equation to get 11 °C

h) month 5, May, and month 9, September

31. $t_{19,\,8} - t_{18,\,7}$

32. $(4x - y)^5 = 1024x^5 - 1280x^4y + 640x^3y^2 - 160x^2y^3 + 20xy^4 - y^5$

33. a) 82 **b)** 19

34. $a = -5$, $d = 12$; $t_n = 12n - 17$

35. 10

36. $3 + 10 + 17 + 24 + 31 + 38 + \cdots$

37. 180 m

38. a) approximately 13.4 m
b) approximately 201.2 m

39. $S_{64} = 2^{64} - 1$

40. a) arithmetic; $a = 100$, $d = -10$; sum $= -1980$
b) geometric; $a = 1$, $r = 3$; sum $= 3280$

41. $t_4 = 250$; $S_6 = 7812$

42. a) 24 800 is the purchase value of the car; 0.78 is the constant factor or ratio as the value depreciates 22% each year
b) i) \$15 088.32 **ii)** \$7160.19
c) The initial value of the car depreciates by a constant factor each year.
d) 3.7 years or approximately 3 years 8 months

43. \$131.25

44. a) \$2030 **b)** \$7030

45. \$8121.66

46. 9.51% per annum, compounded quarterly

47. \$801.40

48. approximately 12.3 years

49. \$1799.05