Chapter 1 Functions

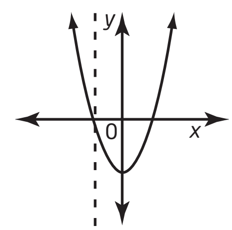
1.1 Functions, Domain, and Range

KEY CONCEPTS

- A relation is a function if for each value in the domain there is exactly one value in the range. This table of values models a function.

x	−2	−1	0	1	2
y	5	3	1	−1	−3

- The vertical line test can be used on the graph of a relation to determine if it is a function. If every vertical line passes through at most one point on the graph, then the relation is a function.



- The domain and the range of a function can be found by determining if there are restrictions based on the defining equation. Restrictions on the domain occur because division by zero is undefined and because expressions under a radical sign must be greater than or equal to zero. The range can have restrictions too. For example, a quadratic that opens upward will have a minimum value.
- Set notation is used to write the domain and range for a function. For example, for the function $y = x^2 + 2$:
  domain $\{x \in \mathbb{R}\}$ and range $\{y \in \mathbb{R}, y \geq 2\}$

Example

Write the domain and range of each relation, and then determine if the relation is a function.

a) {(8, 10), (1, −8), (5, 0), (−5, 6), (−6, −7), (−5, −2), (0, −9), (−9, 5)}

b)

x	−6	−4	−2	0	2	4	6	8
y	12	2	−4	−6	−4	2	12	26

c) $y = \dfrac{1}{x + 2}$

Solution

a) domain {−9, −6, −5, 0, 1, 5, 8}, range {−9, −8, −7, −2, 0, 5, 6, 10}
For some elements of the domain there is more than one corresponding element of the range ($x = -5$ goes with $y = -2$ and $y = 6$). This relation is not a function.

b) domain {−6, −4, −2, 0, 2, 4, 6, 8}, range {−6, −4, 2, 12, 26}
Each element of the domain corresponds with a unique element of the range. This relation is a function.

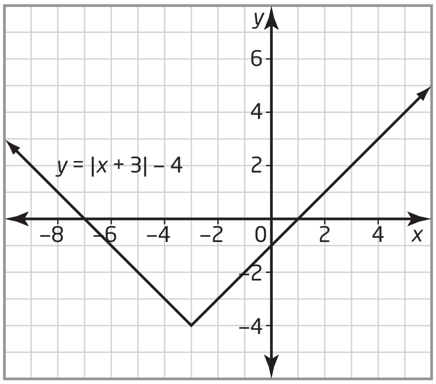
c) The denominator cannot be 0, so $x \neq -2$.
domain $\{x \in \mathbb{R}, x \neq -2\}$

Since the numerator is 1, the fraction can never equal 0.
range $\{y \in \mathbb{R}, y \neq 0\}$
For each x-value, there is only one possible y-value. This relation is a function.

A Practise

1. Does each graph represent a function? Explain.

a)



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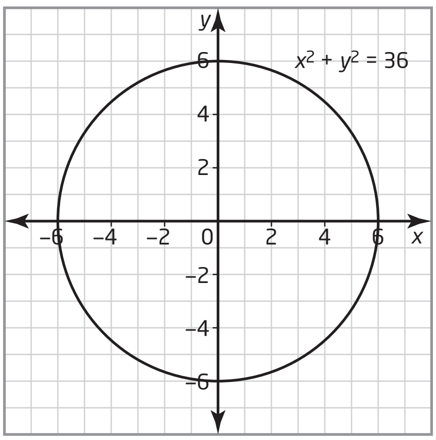


☆**2.** For each relation, determine if the relation is a function, and then sketch a graph of the relation.

a) $y = -3x + 1$

b) $x = y^2 - 2$

c) $y = 3(x + 1)^2 - 5$

3. State the domain and the range of each relation. Is each relation a function? Justify your answer.

a) $\{(2, 4), (3, 6), (4, 8), (5, 10), (6, 12)\}$

b) $\{(-6, 2), (-5, 2), (-4, 2), (-3, 2)\}$

c) $\{(5, -4), (5, -2), (5, 0), (5, 2), (5, 4)\}$

4. The domain and range of some relations are given. Each relation consists of only four points. Determine if each relation is a function. Explain.

a) domain $\{-4, -2\}$, range $\{1, 3, 5, 7\}$

b) domain $\{4, 5, 6, 7\}$, range $\{-1, 3, 7, 11\}$

B Connect and Apply

5. Determine the domain and the range of each relation.

a) $x^2 + y^2 = 49$

b) $y = \dfrac{2}{5 - x}$

6. For each given domain and range, draw two relations on the same axes: one that is a function and one that is not a function.

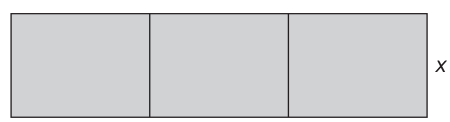
a) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}\}$

b) domain $\{x \in \mathbb{R}, x \leq 5\}$, range $\{y \in \mathbb{R}, y \geq -6\}$

c) domain $\{x \in \mathbb{R}\}$, range $\{y \in \mathbb{R}, y \leq 3\}$

d) domain $\{x \in \mathbb{R}, x \geq -4\}$, range $\{y \in \mathbb{R}, y \geq 1\}$

7. Hatia has 120 m of fencing. She plans to enclose a large rectangular garden and divide it into three equal parts.



a) Write an area function in terms of x to model the total area of the garden.

b) Determine the domain and the range for the area function.

8. For each function, determine the range for the domain $\{-2, -1, 0, 1, 2\}$.

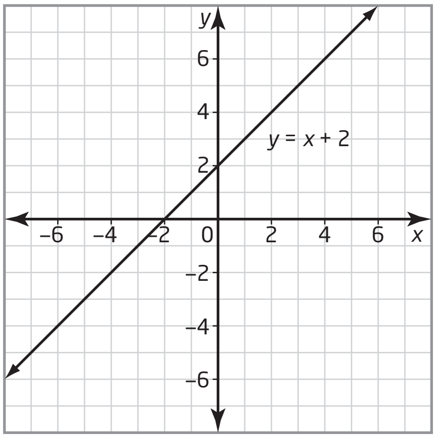
a) $y = 8$

b) $y = -(x + 3)^2 - 2$

c) $y = \dfrac{8}{x + 5}$

9. State the domain and the range of each relation.

a)



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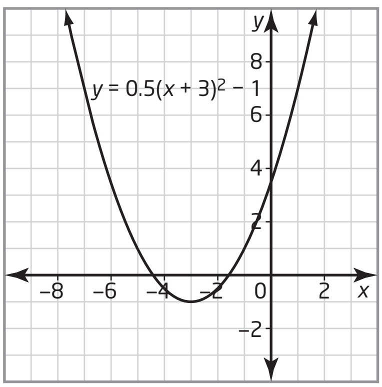

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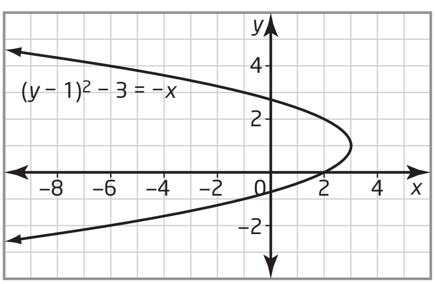

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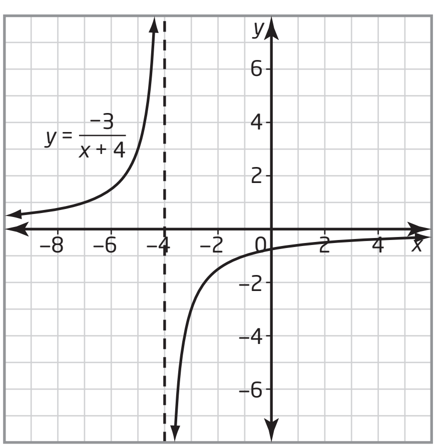


10. Describe the graph of a relation that satisfies each set of conditions.

a) There are two entries in the domain and five entries in the range.

b) There are three entries in the domain and three entries in the range.

C Extend

☆**11. a)** State the domain and range of each relation, and then identify if each relation is a function. Justify your answer graphically.

i) $x^2 + y^2 = 16$

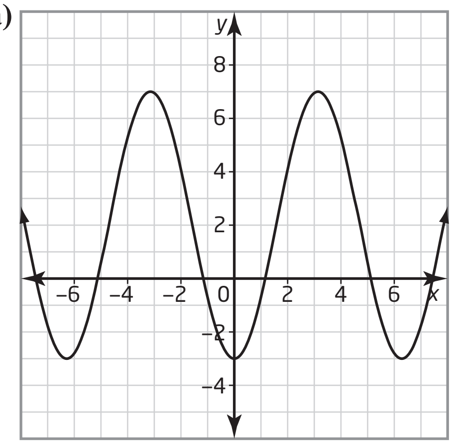
ii) $y = \sqrt{16 - x^2}$

iii) $y = -\sqrt{16 - x^2}$

b) Describe how the graphs of ii) and iii) are related to the graph of i).

12. State the domain and the range for each relation. Determine if each relation is a function.

a)



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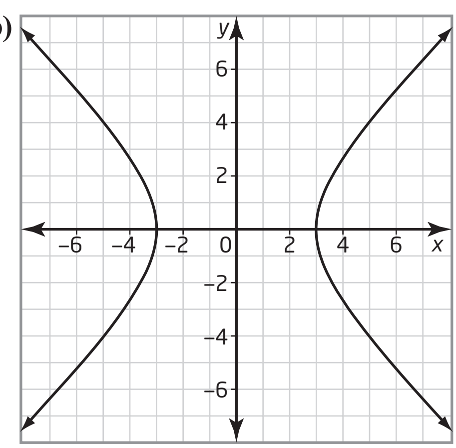
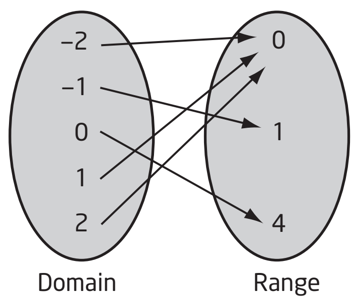


1.2 Functions and Function Notation

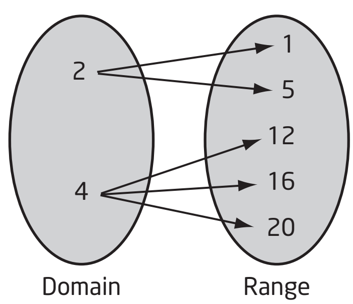
KEY CONCEPTS

- In function notation, the symbol $f(x)$ represents the dependent variable. It indicates that the function f is expressed in terms of the independent variable x. For example, $y = 3x^2 - 5$ is written as $f(x) = 3x^2 - 5$.

- Relations and functions given as ordered pairs can be represented using mapping diagrams. This involves using directed arrows from each value in an oval representing the domain to the corresponding value or values in an oval representing the range.



- In a mapping diagram, a relation is not a function when an element from the domain has two or more arrows leading to different elements of the range.



- Mapping notation can replace function notation. For example, $f(x) = 3x^2 - 5$ can be written as $f : x \rightarrow 3x^2 - 5$.

Example

A quadratic function machine uses a function of the form $f(x) = ax^2 + b$. The points (1, 3) and (−2, −3) lie on the function. Determine the values of a and b, and then write the defining equation of the function.

Solution

For (1, 3), $f(1) = 3$.
$a(1)^2 + b = 3$
$a + b = 3$ ①

For (−2, −3), $f(-2) = -3$.
$a(-2)^2 + b = -3$
$4a + b = -3$ ②

Subtract equation ① from equation ②.

$4a + b = -3$ ②
$a + b = 3$ ①
$3a = -6$ ③
$a = -2$

Substitute $a = -2$ in ① to solve for b.

$-2 + b = 3$
$b = 5$

The defining equation is $f(x) = -2x^2 + 5$.

A Practise

1. For each function below,
   i) determine $f(-2)$, $f(1)$, and $f\left(\frac{1}{2}\right)$
   ii) write the function in mapping notation
   a) $f(x) = -\frac{3}{5}x + 2$
   b) $f(x) = 6x^2 - 4x + 3$
   c) $f(x) = -3(x + 1)^2 - 2$
   d) $f(x) = 7$
   e) $f(x) = \frac{1}{4x + 1}$
   f) $f(x) = \sqrt{3 - 2x}$

2. Find the value of each function at $x = 0$. Sketch the graph of each function.
   a) $f(x) = 3x - 1$
   b) $k(x) = 5x$
   c) $p(x) = -2$
   d) $g(x) = 10x^2 + 4x - 1$
   e) $h(x) = (5x - 2)(3x + 6)$
   f) $q(x) = -\frac{1}{2}(3 - 4x)(x - 5)$

3. Given $f(x) = 4x + 7$, determine the value of x if $f(x) = 15$.

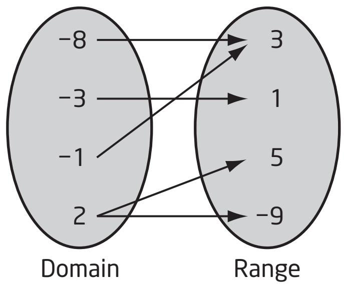
4. A linear function machine uses a function of the form $f(x) = ax + 2$. In each case, suppose the given point is on the function. Find the value of a, and then write the defining equation.
   a) (4, −10)
   b) (1, 8)
   c) (3, 6.5)
   d) (−5, 33)

5. Represent each set of data in a mapping diagram.
   a) {(−4, 18), (−3, 14), (−2, 10), (−1, 6), (0, 2), (1, −2)}
   b) {(−3, 3), (−2, −3), (−1, 3), (−1, −3), (−2, 3), (−3, −3)}
   c) {(2, 1), (3, 3), (4, 5), (5, 1), (6, 3), (7, 5)}
   d) {(6, −7), (9, −7), (12, −7), (15, −7)}

6. Refer to question 5. Determine if each relation is a function. Justify your answer.

7. Write the ordered pairs associated with each mapping diagram.

a)



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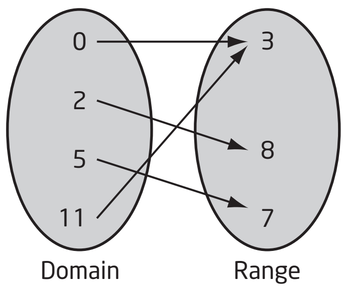

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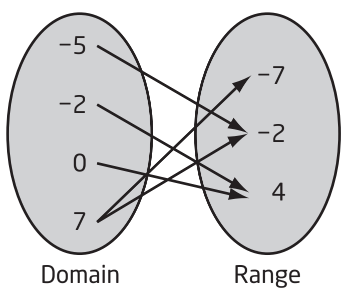

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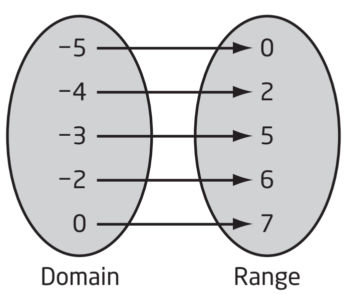
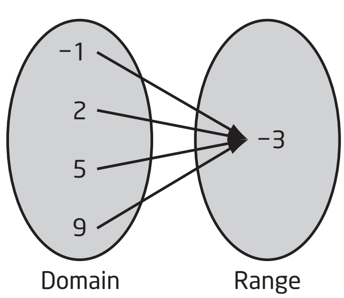


e)



8. Refer to question 7. Determine if each relation is a function. Justify your answer.

9. Write each function in mapping notation.

a) $f(x) = -7x + 1$

b) $g(x) = x^2 + 7x - 5$

c) $h(b) = \sqrt{9b + 9}$

d) $r(k) = \dfrac{1}{5k - 3}$

B Connect and Apply

☆**10.** The period of a pendulum varies on different planets due to the different gravitational forces. On Earth, the period, T, in seconds, of a pendulum is given by the relation $T = 2\sqrt{\ell}$, where ℓ is the length of the pendulum, in metres. On the moon, the relation is $T = 5\sqrt{\ell}$, and on Pluto the relation is $T = 8\sqrt{\ell}$.

a) For each location, determine the period when the length of the pendulum is 1.8 m.

b) For each location, determine the length of the pendulum that results in a period of 3 s.

c) Determine the domain and the range of each relation.

d) Graph each relation on the same set of axes.

e) Is each relation a function? Explain.

f) Write each relation in mapping notation.

11. The population of a town, p, in thousands, t years since 1995, is modelled by the relation $p(t) = \dfrac{240t + 300}{3t + 5}$.

a) What was the population of the town in 1995?

b) Determine the population of the town in each year.

i) 2003

ii) 2008

c) In what year will the population reach 78 750?

d) Is $p(t)$ a function? Justify your answer.

12. Use Technology A quadratic function machine uses a function of the form $f(x) = ax^2 + bx + c$. Given the data $\{(1, 2), (2, 9), (3, 22)\}$, you can use a graphing calculator to determine the equation.

a) Enter the values of the domain in **L1** and the values of the range in **L2**.

b) Plot the data.

c) Run quadratic regression and record the resulting equation.

d) Use this function to determine the range values for the domain values $x = -4$, $x = 0$, and $x = 6$.

13. Michael needs \$500 two years from now. The amount to be invested, A, at an interest rate i is given by the relation $A(i) = \frac{500}{(1 + i)^2}$. Note that i must be expressed as a decimal.

a) Determine the domain and range for this relation.

b) Is this relation a function? Explain.

c) How much money needs to be invested at each interest rate?

i) 4%

ii) 8%

d) At what rate of interest would each amount need to be invested?

i) \$350

ii) \$400

☆**14.** An object is dropped from a height of 80 m above the surface of a planet. On Earth, the relation $t(h) = \sqrt{\frac{80 - h}{4.9}}$ represents the time, t, in seconds, when the object is at a height of h metres above the ground. On Jupiter, the relation is $t(h) = \sqrt{\frac{80 - h}{12.8}}$.

a) Express each relation using mapping notation.

b) Determine the domain and range of each relation.

c) Is each relation a function? Explain.

d) Determine the times when the object is 10 m above the ground on Earth and on Jupiter.

15. A quadratic function machine uses a function of the form $f(x) = ax^2 + b$. Determine the values of a and b for each pair of points, and then write the defining equation of the function.

a) $(1, 1)$ and $(-1, 1)$

b) $(2, 1)$ and $(-4, -5)$

C Extend

16. Consider the relations $f(x) = 2x$, $g(x) = 2^x$, $h(x) = x^2$, $q(x) = \frac{x}{2}$, and $p(x) = \frac{2}{x}$.

a) State the domain and range of each relation.

b) Graph each relation on the same set of axes.

c) Is each relation a function? Justify your answer.

d) Which relations have the same value when $x = 2$? Justify your answer.

e) Which relations are equal for other values of x? Justify your answer.

17. a) Express $g : x \rightarrow \frac{-2}{x + 3}$ and $h : x \rightarrow 4x^2 - 5x$ in function notation.

b) Determine each value.

i) $g(-1) + h(1)$

ii) $4g(1) - h(2)$

iii) $\sqrt{h(-1)}$

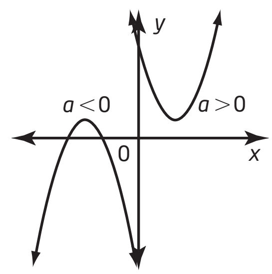
iv) $7[g(-5)]^2$

18. A quadratic function of the form $f(x) = ax^2 + bx + c$ satisfies the conditions $f(1) = 4$, $f(-1) = 10$, and $f(2) = 7$. Determine the values of a, b, and c, and state the defining equation.

1.3 Maximum or Minimum of a Quadratic Function

KEY CONCEPTS

- The minimum or maximum value of a quadratic function occurs at the vertex of the parabola.
- The vertex of a quadratic function can be found by
  - graphing
  - completing the square: for $f(x) = a(x - h)^2 + k$, the vertex is (h, k)
  - partial factoring: for $f(x) = ax\left(x + \frac{b}{a}\right) + k$, the x-coordinate of the vertex is $-\frac{b}{2a}$
- The sign of the coefficient a in the quadratic function $f(x) = ax^2 + bx + c$ or $f(x) = a(x - h)^2 + k$ determines whether the vertex is a minimum or a maximum.
  If $a > 0$, then the parabola opens upward and has a minimum.
  If $a < 0$, then the parabola opens downward and has a maximum.

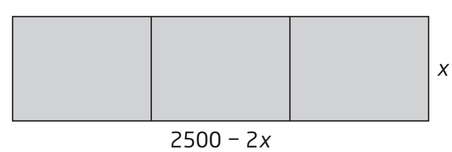


Example

A farmer has 5000 m of fencing to enclose a rectangular field and subdivide it into three equal plots. The enclosed area is to be a maximum. Determine the dimensions of one plot of land, to the nearest metre.

Solution

Let x represent the width of the rectangular field.
Write an expression for the length of the field.



$\frac{5000 - 4x}{2} = 2500 - 2x$

$A(x) = x(2500 - 2x)$

$0 = x(2500 - 2x)$ Use the x-intercepts to find the x-coordinate of the vertex.

$x = 0$ or $2500 - 2x = 0$

$x = 0$ or $x = 1250$

The x-coordinate of the vertex is $x = 625$. This is the width of the field with maximum area.

Length of one plot $= \frac{2500 - 2(625)}{3}$ Length of one plot $= \frac{\text{length of field}}{3}$

$\doteq 417$

The dimensions of each plot of land are approximately 417 m by 625 m.

A Practise

1. Complete the square for each function.

   a) $y = x^2 - 8x$

   b) $f(x) = x^2 + 16x - 4$

   c) $f(x) = x^2 + 5x + 7$

   d) $g(x) = x^2 - x + 2$

   e) $y = x^2 - 8x - 6$

   f) $y = x^2 - 7x - 10$

2. Complete the square to determine the coordinates of the vertex. State if the vertex is a minimum or a maximum.

   a) $f(x) = x^2 + 4x + 1$

   b) $f(x) = -2x^2 + 12x + 7$

   c) $f(x) = -5x^2 - 10x + 3$

   d) $f(x) = 3x^2 - 15x + \frac{59}{4}$

   e) $f(x) = \frac{3}{4}x^2 - 3x + 6$

   f) $f(x) = -\frac{2}{5}x^2 - \frac{4}{5}x - \frac{7}{5}$

3. Use partial factoring to determine the vertex of each function. State if the vertex is a minimum or a maximum.

   a) $f(x) = 4x^2 - 8x + 1$

   b) $f(x) = -3x^2 - 18x - 25$

   c) $f(x) = -\frac{1}{2}x^2 - 4x - 3$

   d) $f(x) = \frac{4}{7}x^2 - \frac{8}{7}x + \frac{25}{7}$

   e) $f(x) = 0.3x^2 - 3.6x + 10.8$

   f) $f(x) = -0.4x^2 + 4x + 1$

4. **Use Technology** Use a graphing calculator to verify your answers to questions 2 and 3.

B Connect and Apply

5. An electronics store sells an average of 52 laptops per month at an average selling price that is $660 more than the cost price. For every $40 increase in the selling price, the store sells two fewer laptops. What amount over the cost price will maximize revenue?

☆6. The student council is organizing a trip to a rock concert. All proceeds from ticket sales will be donated to charity. Tickets to the concert cost $31.25 per person if a minimum of 104 people attend. For every 8 extra people that attend, the price will decrease by $1.25 per person.

   a) How many tickets need to be sold to maximize the donation to charity?

   b) What is the price of each ticket that maximizes the donation?

   c) What is the maximum donation?

7. A ball is kicked into the air. It follows a path given by $h(t) = -4.9t^2 + 8t + 0.4$, where t is the time, in seconds, and $h(t)$ is the height, in metres.

   a) Determine the maximum height of the ball to the nearest tenth of a metre.

   b) When does the ball reach its maximum height?

8. A lifeguard has 40 m of rope to enclose a rectangular swimming area at a small lake. One side of the rectangle is a straight sandy beach. What are the dimensions of the largest swimming area that she can enclose?

☆9. Two numbers have a difference of 8.

   a) What is the maximum product of these numbers?

   b) What are the numbers that produce the maximum product?

10. The sum of two numbers is 26, and the sum of their squares is a minimum.

    a) Determine the numbers.

    b) What is the minimum sum of their squares?

11. Determine the maximum area of a triangle, in square centimetres, if the sum of its base and its height is 15 cm.

12. A ball is thrown upward from the balcony of an apartment building and falls to the ground. The height of the ball, h metres, above the ground after t seconds is modelled by the function $h(t) = -5t^2 + 15t + 55$.

a) Determine the maximum height of the ball.

b) How long does it take the ball to reach its maximum height?

c) How high is the balcony?

13. The parent council is planning the annual spaghetti supper to raise money for new school bleachers. Last year, the tickets sold for \$11 each, and 400 people attended. This year the parent council has decided to raise the ticket price. They know that for every \$1 increase in price, 20 fewer people will attend the supper.

a) What ticket price would maximize the revenue?

b) What is the maximum revenue?

14. The arch of a bridge is modelled by the function $h(d) = 2 - 0.043d^2 + 2.365d$, where h is the height, in metres, and d is the horizontal distance, in metres, from the origin of the arch.

a) Determine the maximum height of the arch, to the nearest hundredth of a metre.

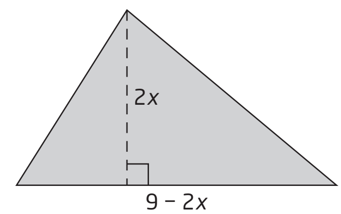
b) What is the width of the arch at its base?

15. A parabolic arch supporting a bridge over a canal is 10 m wide. The height of the arch 2 m from the edge of the canal is 13 m.

a) Determine an equation to represent the arch, assuming that the edge of the canal is at the origin.

b) Determine the maximum height of the arch, to the nearest tenth of a metre.

16. a) Determine the maximum area of this triangle.



b) What value of x produces the maximum area?

C Extend

17. A ball is thrown vertically upward from an initial height of h_0 metres, with an initial velocity of v m/s, and is affected by the acceleration due to gravity, g. The height function of the ball is given by $h(t) = -\frac{1}{2}gt^2 + vt + h_0$, where $h(t)$ is the height, in metres, and t is the time, in seconds. Write an expression for the maximum height of the ball.

18. In an electrical circuit, the voltage, V volts, as a function of time, t minutes, is modelled by the quadratic function $V(t) = 2t^2 - 9t + 12$.

a) Determine the minimum and maximum voltages during the first 5 min.

b) At what times do the values found in part a) occur?

19. Determine the condition on the value of b so that the minimum value of the quadratic function $f(x) = x^2 + bx + 5$ is an integer.

20. Determine the condition on the value of b so that the maximum value of the quadratic function $f(x) = -4x^2 + bx - 3$ is an integer.

1.4 Skills You Need: Working With Radicals

KEY CONCEPTS

- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ for $a \geq 0$ and $b \geq 0$.

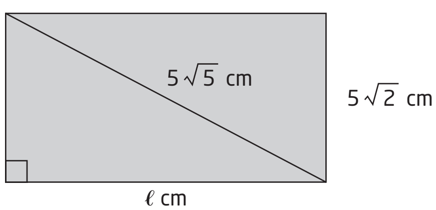
- An entire radical can be simplified to a mixed radical in simplest form by removing the largest perfect square from under the radical to form a mixed radical.
  For example, $\sqrt{50} = \sqrt{25 \times 2}$
  $= 5\sqrt{2}$

- Like radicals can be combined through addition and subtraction. For example, $3\sqrt{7} + 2\sqrt{7} = 5\sqrt{7}$.

- Radicals can be multiplied using the distributive property.
  For example, $4\sqrt{2}\left(5\sqrt{3} - 3\right) = 20\sqrt{6} - 12\sqrt{2}$ and
  $$\begin{aligned}\left(\sqrt{2} - 3\right)\left(\sqrt{2} + 1\right) &= \sqrt{4} + \sqrt{2} - 3\sqrt{2} - 3\\ &= 2 - 2\sqrt{2} - 3\\ &= -2\sqrt{2} - 1\end{aligned}$$

Example

Find the length, area, and perimeter of this rectangle. Express your answers in simplified radical form.



Solution

$$\begin{aligned}\ell^2 + (5\sqrt{2})^2 &= (5\sqrt{5})^2\\ \ell^2 &= 125 - 50\\ \ell^2 &= 75\\ \ell &= 5\sqrt{3}\end{aligned}$$

$$\begin{aligned}A &= \ell w\\ &= \left(5\sqrt{3}\right)\left(5\sqrt{2}\right)\\ &= 25\sqrt{6}\end{aligned}$$

$$\begin{aligned}P &= 2\ell + 2w\\ &= 2\left(5\sqrt{3}\right) + 2\left(5\sqrt{2}\right)\\ &= 10\sqrt{3} + 10\sqrt{2}\end{aligned}$$

The length of the rectangle is $5\sqrt{3}$ cm, the area is $25\sqrt{6}$ cm^2, and the perimeter is $10\sqrt{3} + 10\sqrt{2}$ cm.

A Practise

1. Simplify.
   a) $2(7\sqrt{3})$
   b) $\sqrt{5}(3\sqrt{6})$
   c) $-\sqrt{11}(4\sqrt{3})$
   d) $8\sqrt{3}(-2\sqrt{7})$
   e) $-3\sqrt{3}(5\sqrt{2})$
   f) $-6\sqrt{2}(-\sqrt{11})$
   g) $2\sqrt{5}(-3\sqrt{7})$
   h) $-7\sqrt{6}(7\sqrt{5})$

2. Express each as a mixed radical in simplest form.
   a) $\sqrt{54}$
   b) $\sqrt{98}$
   c) $\sqrt{288}$
   d) $\sqrt{75}$
   e) $\sqrt{72}$
   f) $\sqrt{125}$
   g) $\sqrt{96}$
   h) $\sqrt{126}$
   i) $\sqrt{32}$
   j) $\sqrt{180}$

3. Simplify.
   a) $7\sqrt{2} - 9\sqrt{2} + 15\sqrt{2}$
   b) $-5\sqrt{3} + 11\sqrt{3} - 9\sqrt{3} + 17\sqrt{3}$
   c) $16\sqrt{11} + 3\sqrt{11} - 25\sqrt{11} + 6\sqrt{11} - 2\sqrt{11}$
   d) $9\sqrt{5} + 8\sqrt{6} - 13\sqrt{5} + 19\sqrt{6} + 4\sqrt{6}$

4. Simplify each radical first, and then add or subtract.
   a) $5\sqrt{12} - 2\sqrt{48} - 7\sqrt{75}$
   b) $2\sqrt{8} - 3\sqrt{98} - 2\sqrt{200}$
   c) $\sqrt{20} - 3\sqrt{245} - 2\sqrt{20}$
   d) $-3\sqrt{50} - \sqrt{32} - 5\sqrt{200}$
   e) $-3\sqrt{12} + 5\sqrt{27} - 6\sqrt{48} + 2\sqrt{75}$
   f) $\sqrt{48} - \sqrt{20} - \sqrt{27} - \sqrt{45}$
   g) $2\sqrt{12} + 3\sqrt{50} - 2\sqrt{75} - 6\sqrt{32}$
   h) $4\sqrt{18} - 2\sqrt{63} + \sqrt{175} + 5\sqrt{98}$

5. Simplify.
   a) $\sqrt{5}(\sqrt{50})$
   b) $5\sqrt{3}(-2\sqrt{6})$
   c) $5\sqrt{7}(2\sqrt{14})$
   d) $-6\sqrt{5}(8\sqrt{15})$
   e) $3\sqrt{3}(-7\sqrt{12})$
   f) $4\sqrt{7}(-\sqrt{7})$
   g) $-2\sqrt{15}(5\sqrt{6})$
   h) $9\sqrt{24}(-3\sqrt{3})$

6. Expand. Simplify where possible.
   a) $\sqrt{2}(\sqrt{6} - \sqrt{3})$
   b) $2\sqrt{3}(\sqrt{5} - 2)$
   c) $3\sqrt{5}(2 - \sqrt{2})$
   d) $-2\sqrt{3}(\sqrt{5} + 3\sqrt{7})$
   e) $\sqrt{5}(\sqrt{6} - \sqrt{10})$
   f) $4\sqrt{2}(3\sqrt{11} + 5\sqrt{13})$
   g) $2\sqrt{5}(3\sqrt{2} - 4\sqrt{3})$
   h) $6\sqrt{6}(3\sqrt{2} - 4\sqrt{3})$

7. Expand. Simplify where possible.
   a) $(\sqrt{7} - 6)(\sqrt{7} + 1)$
   b) $(2 + \sqrt{12})(4 - \sqrt{3})$
   c) $(4\sqrt{2} - 5\sqrt{3})(4\sqrt{2} + 5\sqrt{3})$
   d) $(6 - 4\sqrt{2})(2 - 5\sqrt{2})$
   e) $(7 - 3\sqrt{2})^2$
   f) $(3\sqrt{5} - 2\sqrt{3})(3\sqrt{5} + 2\sqrt{3})$

8. Simplify.

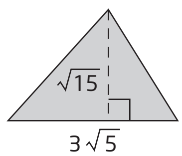
a) $\frac{1}{3}\sqrt{27} - \frac{2}{3}\sqrt{108} - \frac{1}{4}\sqrt{72}$

b) $-\frac{3}{4}\sqrt{32} + \frac{3}{7}\sqrt{98} - \frac{4}{5}\sqrt{50}$

B Connect and Apply

☆**9.** Find a simplified expression for the area of each shape.

a)



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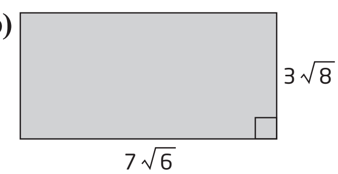

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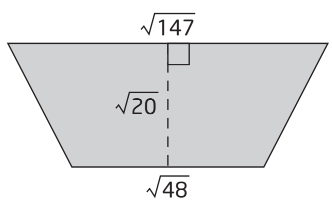

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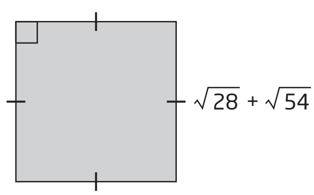


10. Fully simplify $\sqrt{5040}$. Explain your steps.

11. A square has area 756 cm^2. Write an expression in simplified radical form for the length of one side.

☆**12.** A square game board is made up of small squares, each with side length 4 cm. The diagonal of the game board measures $32\sqrt{2}$ cm. How many small squares are on the game board?

☆**13.** Explain why $\sqrt{100 - 64}$ is not equal to $\sqrt{100} - \sqrt{64}$.

C Extend

14. Simplify.

a) $\dfrac{16 + \sqrt{176}}{24}$

b) $\dfrac{35 - 2\sqrt{150}}{40}$

c) $\dfrac{\sqrt{39}}{\sqrt{3}}$

d) $\dfrac{15 + \sqrt{252}}{27}$

e) $\dfrac{-18 + \sqrt{405}}{36}$

15. a) Simplifying a cube root requires the factor to appear three times under the cube root sign. Any factor that does not appear three times is left under the cube root. Simplify each cube root.

i) $\sqrt[3]{320}$

ii) $\sqrt[3]{875}$

iii) $\sqrt[3]{2744}$

b) Explain how you can extend the method of part a) to simplify each radical. Use your method to simplify each fourth root.

i) $\sqrt[4]{240}$

ii) $\sqrt[4]{810}$

iii) $\sqrt[4]{9072}$

16. Simplify.

a) $\sqrt{16m} - \sqrt{9m} + 2\sqrt{25m} + 7\sqrt{4m}$

b) $\sqrt{28ab} + 5\sqrt{c} - 3\sqrt{7ab} + 2\sqrt{36c}$

c) $\sqrt{45a^3b} - \sqrt{12m^2n^3} + 3n\sqrt{48m^2n} + a\sqrt{125ab}$

d) $\sqrt{75ab^3} - \sqrt{c^2d} - b\sqrt{3ab} + 5c\sqrt{4d}$

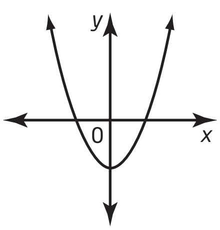
1.5 Solving Quadratic Equations

KEY CONCEPTS

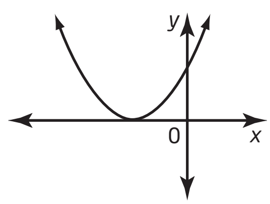
- A quadratic equation can be solved by
  - completing the square
  - factoring
  - using the quadratic formula
  - graphing

- The number of solutions to a quadratic equation and the number of zeros of the related quadratic function can be determined using the discriminant.

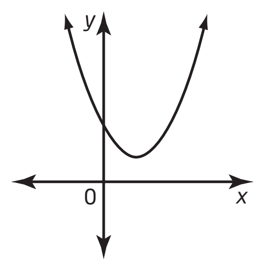
If $b^2 - 4ac > 0$, there are two solutions (two distinct real roots).



If $b^2 - 4ac = 0$, there is one solution (two equal real roots).



If $b^2 - 4ac < 0$, there are no real solutions.



Example

a) Make a table of values for the function $f(x) = -2x^2 + 5x$ for the domain $\{-2, -1, 0, 1, 2, 3, 4\}$.

b) Graph this quadratic function.

c) On the same set of axes, graph the line $y = -7$.

d) Use your graph to estimate the x-coordinates of the points of intersection of $y = -7$ and $f(x) = -2x^2 + 5x$.

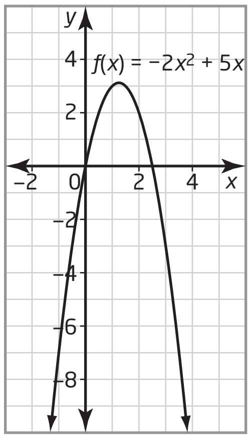
e) Use algebra to determine the coordinates of the points of intersection of $y = -7$ and $f(x) = -2x^2 + 5x$.

Solution

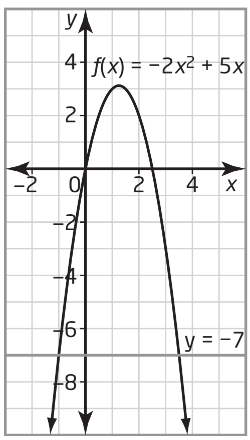
a)

x	$y = f(x)$
−2	−18
−1	−7
0	0
1	3
2	2
3	−3
4	−12

b)



c)



d) The line appears to intersect the parabola at $x = -1$ and $x = 3.5$.

e) To determine the x-coordinates of the points of intersection, substitute $y = -7$ into $f(x) = -2x^2 + 5x$ and solve.

$$-7 = -2x^2 + 5x$$
$$2x^2 - 5x - 7 = 0$$
$$(2x - 7)(x + 1) = 0$$
$$x = \frac{7}{2} \text{ or } x = -1$$

Every point on the line has y-coordinate -7, so the y-coordinates of the points of intersection are -7. The points of intersection are $\left(\frac{7}{2}, -7\right)$ and $(-1, -7)$.

A Practise

1. Solve each quadratic equation by factoring. Check your answers.

   a) $x^2 + x - 6 = 0$

   b) $x^2 + 7x + 12 = 0$

   c) $3x^2 - 75 = 0$

   d) $2x^2 + 12x - 54 = 0$

   e) $3x^2 - 4x - 15 = 0$

   f) $3x^2 + 13x - 10 = 0$

2. Solve each quadratic equation using the quadratic formula. Give exact answers.

   a) $6x^2 - 7x - 3 = 0$

   b) $3x^2 + 6x + 1 = 0$

   c) $2x^2 + 6x + 3 = 0$

   d) $3x^2 + 7x + 3 = 0$

   e) $x^2 + 6x + 4 = 0$

3. **Use Technology** Using a graphing calculator, graph a related function to determine the number of roots for each quadratic equation.

   a) $7x^2 - 2x + 3 = 0$

   b) $-3x^2 + 10x + 8.5 = 0$

   c) $5x^2 + x - 4 = 0$

   d) $\frac{2}{3}x^2 - 4x + 6 = 0$

   e) $\frac{4}{7}x^2 - x + 5 = 0$

   f) $0.8x^2 - 3.20x + 6.2 = 0$

4. Determine the exact values of the x-intercepts of each quadratic function. Use a graphing calculator to check that you have found the correct number of x-intercepts.

   a) $f(x) = 2x^2 + 5x + 1$

   b) $f(x) = x^2 - 6x + 7$

   c) $f(x) = -\frac{1}{2}x^2 + 3x + 6$

   d) $f(x) = \frac{3}{4}x^2 - 5x + 5$

   e) $f(x) = -\frac{5}{8}x^2 + 6x + 2$

5. Use the discriminant to determine the number of roots for each quadratic equation.

   a) $x^2 - 3x + 1 = 0$

   b) $3x^2 - 6x + 3 = 0$

   c) $2x^2 - 5x + 7 = 0$

   d) $-x^2 + 5.5x + 3.25 = 0$

   e) $5x^2 - 10x + 5 = 0$

B Connect and Apply

6. Solve.

   a) $2x^2 - 5x - 12 = 0$

   b) $4x^2 - 81 = 0$

   c) $2x^2 + 5x - 2 = 0$

   d) $\frac{2}{5}x^2 - \frac{3}{5}x = 0$

   e) $0.8x^2 - 2.1x - 3.5 = 0$

   f) $3x^2 - 5x + 2 = 0$

   g) $2x^2 - 10x + 9 = 0$

   h) $4x^2 - x - 3 = 0$

7. Determine the value(s) of k for which the quadratic equation $x^2 + kx + 4 = 0$ will have each number of roots.

   a) two equal real roots

   b) two distinct real roots

   c) no real roots

8. a) Create a table of values for the function $f(x) = -x^2 + 5x + 2$ for the domain $\{-2, -1, 0, 1, 2, 3, 4, 5, 6, 7\}$.

   b) Graph this quadratic function.

   c) On the same set of axes, graph the line $y = 4$.

   d) Use your graph to estimate the x-values of the points of intersection of $y = 4$ and $f(x) = -x^2 + 5x + 2$.

   e) Determine the coordinates of the points of intersection of $f(x) = -x^2 + 5x + 2$ and $y = 4$ algebraically.

9. What value(s) of k, where k is an integer, will allow each quadratic equation to be solved by factoring?

   a) $x^2 + kx + 14 = 0$

   b) $x^2 + kx = 10$

   c) $x^2 - 5x = k$

☆10. The height of a football can be modelled by the function $h(t) = -4.9t^2 + 21.8t + 1.5$, where t is the time, in seconds, since the ball was thrown, and h is the height of the ball, in metres, above the ground. Determine how long the football will be in the air, to the nearest tenth of a second.

11. The function $d = 0.0067v^2 + 0.15v$ can be used to determine the safe stopping distance, d, in metres, for a car given its speed, v, in kilometres per hour. Determine the speed at which a car can be travelling in order to be able to stop in the given distances.

   a) 24 m

   b) 43 m

   c) 82 m

☆12. The sum of two numbers is 24 and the sum of their squares is 306. What are the numbers?

13. A flaming arrow is fired upward from the deck of a ship to mark the beginning of an evening of entertainment and celebration. The flaming arrow hits the water. The height, h, in metres, of the arrow above the water t seconds after it is fired can be modelled by the quadratic function $h(t) = -4.9t^2 + 98t + 8$.

   a) Determine the maximum height of the arrow.

   b) How long does it take the arrow to reach its maximum height?

   c) When does the arrow hit the water?

   d) How high is the deck of the ship above the water?

14. The perimeter of a right triangle is 36.0 cm and the length of the hypotenuse is 15.0 cm. Determine the length of the other two sides.

15. Three pieces of a rod measure 20 cm, 41 cm, and 44 cm. If the same amount is cut off from each piece, the remaining lengths can be formed into a right triangle. Determine the length that should be cut off each piece.

16. Find two consecutive whole numbers such that the sum of their squares is 265.

17. A square flower garden is surrounded by a brick walkway that is 1.5 m wide. The area of the walkway is equal to the area of the garden. Determine the dimensions of the flower garden, to the nearest tenth of a metre.

18. The sum of the squares of three consecutive integers is 194. Determine the integers.

19. The outward power, P, in watts, of a 120-V electric generator is given by the relation $P = 120I - 5I^2$, where I is the current, in amperes (A). For what value(s) of I is the outward power 500 W?

C Extend

20. Solve. Give exact answers.

   a) $x^4 - 10x^2 + 9 = 0$

   b) $(2x + 1)^2 + (2x + 3)^2 = 26$

   c) $(x - 1)(x + 1)(x + 3) = x^3$

21. Solve. Give exact answers.

   a) $x - 2 - \dfrac{x}{x + 1} = 0$

   b) $\dfrac{4x}{x^2 - 1} - 3 = \dfrac{2}{x - 1} - \dfrac{3}{x + 1}$

22. The point (1, 2) is on the graph of the quadratic function $f(x) = ax^2 + bx + 1$. Determine the values of a, such that the graph of $f(x)$ intersects the x-axis at two distinct points.

1.6 Determine a Quadratic Equation Given Its Roots

KEY CONCEPTS

- The zeros can be used to find the equation of a family of quadratic functions with the same x-intercepts.
- To determine an individual quadratic function, you also need to be given one other point on the function.

Example

Write the equation of a quadratic function whose only x-intercept is −2 and that passes through (4, −1).

Solution

When a quadratic function has only one x-intercept, then the vertex is on the x-axis.

The vertex of this function is (−2, 0).
The vertex form of the equation is $y = a(x + 2)^2$.

Substitute the point (4, −1) to determine the value of a.

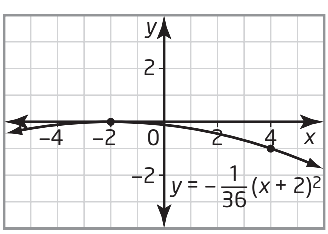
$-1 = a(4 + 2)^2$

$-1 = 36a$

$a = -\frac{1}{36}$

The equation of the quadratic function is $y = -\frac{1}{36}(x + 2)^2$.

Check by graphing the function.



A Practise

1. Determine an equation, in factored form, for a family of quadratic functions with the given roots. Draw a sketch to illustrate each family.

a) $x = 1$ and $x = -4$

b) $x = -3$ and $x = -6$

c) $x = 5$ and $x = -2$

2. Refer to your answers to question 1. Express each equation in standard form.

3. Write the equation for a quadratic function that has the given x-intercepts and that passes through the given point. Express each equation in factored form.

a) x-intercepts: -3 and 4, point: $(1, -24)$

b) x-intercepts: -2 and 5, point: $(-1, 3)$

c) x-intercepts: 0 and $\frac{2}{3}$, point: $(2, -32)$

4. Refer to your answers to question 3. Express each equation in standard form.

5. Write the equation for a quadratic function that has the given zeros and contains the given point. Express each equation in standard form.

a) zeros: $2 \pm \sqrt{5}$, contains $(1, -12)$

b) zeros: $-3 \pm \sqrt{6}$, contains $(-1, 4)$

c) zeros: $4 \pm \sqrt{2}$, contains $(2, 6)$

d) zeros: $-1 \pm \sqrt{7}$, contains $(-2, 3)$

6. Write the equation for the quadratic function that has the given zeros and contains the given point. Express each function in vertex form.

a) zeros: 5 and -1, contains $(3, -24)$

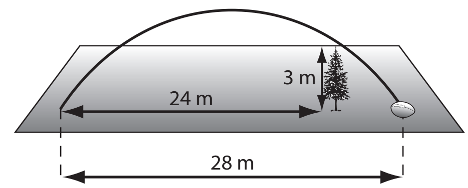
b) zeros: 3 and -4, contains $(2, 12)$

c) zeros: 1 and -3, contains $(-2, -6)$

d) zeros: 2 and -6, contains $(1, 21)$

B Connect and Apply

7. A football is kicked off the ground. After travelling a horizontal distance of 24 m, it just passes over a tree that is 3 m tall before hitting the ground 28 m from where it was kicked.



a) Consider the ground to be the x-axis and assume the vertex lies on the y-axis. Determine the equation of the quadratic function that models the path of the ball.

b) Determine the maximum height of the football.

c) How far has the football travelled horizontally when it reaches its maximum height?

d) Suppose the football is kicked from a starting point at the origin. Develop a new equation of the quadratic function that represents the football.

e) Describe the similarities and differences between the functions found in parts a) and d).

f) Use Technology Use a graphing calculator and compare the solutions.

8. Find the quadratic function that has only one x-intercept and passes through the given point.

a) x-intercept of 0, and through the point $(4, -3)$

b) x-intercept of 3, and through the point $(-2, 5)$

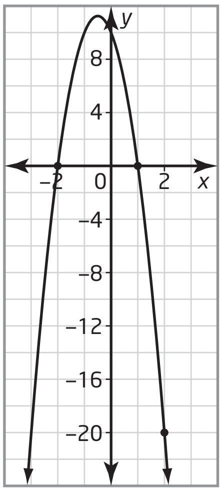
c) x-intercept of -6, and through the point $(-3, -9)$

d) x-intercept of 4, and through the point $(1, -6)$

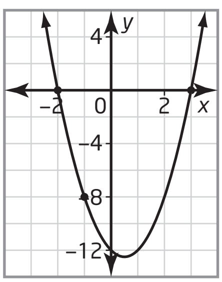
e) x-intercept of -1, and through the point $(0, 5)$

9. Write an equation in standard form for each quadratic function.

a)



b)



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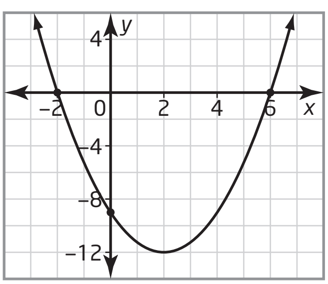


10. Use Technology For each function in question 9, use a graphing calculator to verify your solution by plotting the three points and the quadratic function. Explain how you can use this method to check that your solution is correct.

11. If the function $f(x) = ax^2 + 6x + c$ has no x-intercepts, what is the mathematical relationship between a and c?

☆**12.** The arch of a domed sports arena is in the shape of a parabola. The arch spans a width of 32 m from one side of the arena to the other. The height of the arch is 18 m at a horizontal distance of 8 m from each end of the arch.

a) Sketch the quadratic function so that the vertex of the parabola is on the y-axis and the width is along the x-axis.

b) Use this information to determine the equation that models the arch.

c) Find the maximum height of the arch.

☆**13.** Consider the domed sports arena described in question 12. Suppose that instead of having the vertex on the y-axis, the parabola was positioned with one end of the arch at the origin of the grid.

a) Determine the equation, in factored form and in standard form, of the quadratic function for this orientation.

b) Find the maximum height of the arch and compare the result to the height calculated in question 12.

C Extend

14. Suppose two points are given. Is it always possible to find the equation of a parabola that passes through one of the points and has the other point as its vertex? Explain.

15. a) A family of cubic functions with zeros 2, −1, and 4 is represented by the equation $f(x) = a(x - 2)(x + 1)(x - 4)$. Determine the equation of the member of the family that passes through the point (3, 8). Express your answer in factored form and in expanded form.

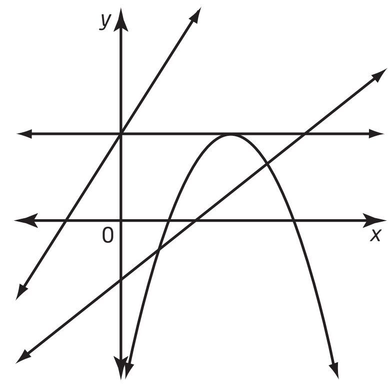
b) Write the equation, in factored form, for a family of cubic functions with zeros $\frac{1}{2}$, 3, and −2.

c) Determine the equation, in expanded form, of the member of the family in part b) that passes through the point (−1, −8).

1.7 Solve Linear-Quadratic Systems

KEY CONCEPTS

- A linear function and a quadratic function may
  - intersect at two points (the line is a secant)
  - intersect at one point (the line is a tangent line)
  - never intersect
- The discriminant can be used to determine which of the above situations occurs.
- The quadratic formula can be used to determine the x-values of actual points of intersection.



Example

Consider the parabola $y = -x^2 + 6x + k$ and the line $y = 4x - 3$. Determine the value of k in each case.

a) The line intersects the parabola at two points.
b) The line intersects the parabola at one point.
c) The line does not intersect the parabola.

Solution

At the point(s) of intersection, the y-values are equal. Set the expressions equal and simplify.

$-x^2 + 6x + k = 4x - 3$
$-x^2 + 2x + k + 3 = 0$

Use the discriminant; $a = -1$, $b = 2$, $c = k + 3$.

$$\begin{aligned} b^2 - 4ac &= 2^2 - 4(-1)(k + 3) \\ &= 4 + 4k + 12 \\ &= 16 + 4k \end{aligned}$$

a) Two points of intersection occur when the discriminant is positive.

$$\begin{aligned} 16 + 4k &> 0 \\ 4k &> -16 \\ k &> -4 \end{aligned}$$

b) One point of intersection occurs when the discriminant is zero.

$$\begin{aligned} 16 + 4k &= 0 \\ k &= -4 \end{aligned}$$

c) There are no points of intersection when the discriminant is negative.

$$\begin{aligned} 16 + 4k &< 0 \\ k &< -4 \end{aligned}$$

A Practise

1. Determine algebraically the coordinates of the point(s) of intersection of each pair of functions.

   a) $y = x^2 + 4x + 3$ and $y = 5x + 9$

   b) $y = 6x^2 - 4x - 25$ and $y = 3x - 5$

   c) $y = 2x^2 - x - 1$ and $y = -\frac{3}{2}x + \frac{1}{2}$

   d) $y = -x^2 - 4x + 6$ and $y = x - 8$

2. Verify the solutions to question 1 using a graphing calculator or by substituting into the original equations.

3. Determine the number of points of intersection of each quadratic function with the given linear function.

   a) $y = 3x^2 - x + 1$ and $y = 4x - 5$

   b) $y = 3x^2 - 4x - 1$ and $y = -3x + 1$

   c) $y = x^2 - 6x + 11$ and $y = -2x + 7$

   d) $y = -\frac{3}{4}x^2 + 5x - 3$ and $y = 3x - 2$

4. **Use Technology** Verify your responses to question 3 using a graphing calculator.

5. Given the equation of a parabola and the slope of a line that is tangent to the parabola, determine the y-intercept of the tangent line.

   a) $f(x) = 2x^2 + 2x - 5$, tangent line has slope 1

   b) $f(x) = -x^2 + 4x - 6$, tangent line has slope -2

   c) $f(x) = -\frac{1}{2}x^2 + x - 3$, tangent line has slope -3

   d) $f(x) = -3x^2 + x - 4$, tangent line has slope 13

6. Verify your solutions to question 5 using a graphing calculator or by substituting into the original equations.

B Connect and Apply

7. The path of an underground stream can be modelled by the function $f(x) = 3x^2 - 2x - 28$. Two new houses are being built that require wells to be dug. On the site plan, these houses and their wells lie on a line defined by the equation $y = -5x + 32$. Determine the coordinates of the locations of the two new wells.

☆8. The path of an asteroid is in the shape of a parabolic arch modelled by the function $f(x) = -8x^2 + 720x + 56\,800$. For the period of time that it is in the same area, a space probe is moving along a straight path on the same plane as the asteroid according to the linear equation $y = -960x + 145\,000$. Determine if the paths of the asteroid and the space probe will intersect. Show your work.

9. **Use Technology** Check your solutions to questions 7 and 8 using a graphing calculator.

10. Consider the parabola $y = -2x^2 + 13x + k$ and the line $y = 7x + 3$. Determine the value of k in each case.

    a) The line intersects the parabola at two points.

    b) The line intersects the parabola at one point.

    c) The line does not intersect the parabola.

☆11. Consider the parabola $y = kx^2 + 3x + 10$ and the line $y = -5x + 3$. Determine the value of k in each case.

    a) The line intersects the parabola at two points.

    b) The line intersects the parabola at one point.

    c) The line does not intersect the parabola.

12. A section of a roller coaster has a parabolic shape that can be modelled by the equation $f(x) = -0.017x^2 + 0.2x + 6$. There is a support beam that can be modelled by the equation $y = 0.07x - 1$. Determine the points of intersection of the section of the roller coaster and the support beam, to one decimal place.

13. The line $x = 3$ intersects the quadratic function $y = -x^2 + 16$ at (3, 7). Explain why the line $x = 3$ is not considered a tangent line to the quadratic function.

14. Andrea's boss asked her to determine the safety zone needed for a fireworks display. She must determine where the safety fence needs to be placed on a hill. The fireworks are to be launched from a platform at the base of the hill. Using the top of the launch platform as the origin and having taken some measurements, Andrea developed these equations:
Cross section of the hill: $y = 4x - 18$
Path of the fireworks: $y = -x^2 + 11x$

a) Graph both equations on the same set of axes.

b) Calculate the coordinates of the point where the line of the hill and the path of the fireworks will intersect.

15. The UV index on a sunny day can be modelled by the function $f(x) = -0.15(x - 12.5)^2 + 8.6$, where x represents the time of day on a 24-h clock and $f(x)$ represents the UV index. Between what hours was the UV index more than 8?

16. A movie stunt man jumps from the CN Tower and falls freely for several seconds before releasing his parachute. His height, h, in metres, above the ground t seconds after jumping is modelled by $h(t) = -4.9t^2 + t + 350$ before he releases his parachute and by $h(t) = -4t + 141$ after he releases his parachute.

a) How long after jumping does he release his parachute?

b) What is his height above the ground when he releases his parachute?

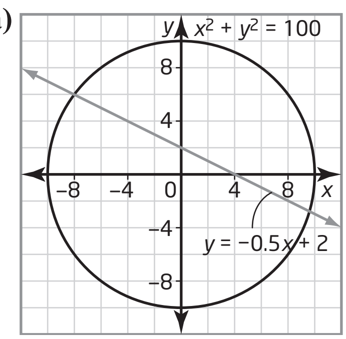
17. A rectangular field has a perimeter of 500 m and an area of 14 400 m^2. Determine the dimensions of the field.

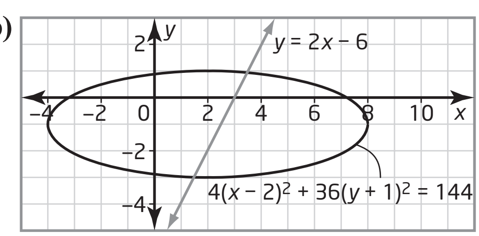
C Extend

18. Determine the equation of the line that passes through the points of intersection of the parabolas $y = x^2 - x - 18$ and $y = -x^2 + 3x - 2$.

19. In how many ways can two parabolas intersect? Support your answer with examples using equations and sketches.

20. You have used substitution to determine the points where a line intersects a quadratic function. This technique can be extended to other curves, such as circles and ellipses. Estimate the points of intersection of the line and the curve in each graph. Verify your answers algebraically.

a) 

b) 

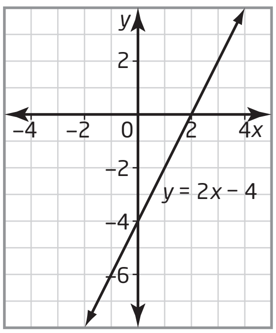
21. Show that the graphs of $2x + 5y = 11$ and $x^2 + y^2 = 4$ do not intersect.

Chapter 1 Review

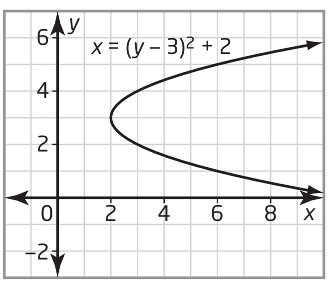
1.1 Functions, Domain, and Range

1. Does each graph represent a function? Justify your answer.

a)



b)



2. The domain and range of some relations are given. Each relation consists of only four points. Is each relation a function? Explain.

a) domain $\{0, 1, 2, 3\}$, range $\{9\}$

b) domain $\{-5\}$, range $\{8, 9, 10, 11\}$

3. Determine the domain and the range of each function.

a) $y = -(x - 3)^2 + 2$

b) $y = \sqrt{4 - 3x}$

4. For the given domain and range, draw one relation that is a function and one that is not. Use the same set of axes.
domain $\{x \in \mathbb{R}, x \geq -4\}$,
range $\{y \in \mathbb{R}, y \geq 1\}$

5. Determine the range of each function for the domain $\{-2, -1, 0, 1, 2\}$.

a) $y = 4x + 3$

b) $y = 2x^2 - 7$

1.2 Functions and Function Notation

6. Find $f(3)$ for each function. Sketch a graph of each function.

a) $f(x) = 7x - 5$

b) $f(x) = -1$

c) $f(x) = 4x^2 - 12x + 9$

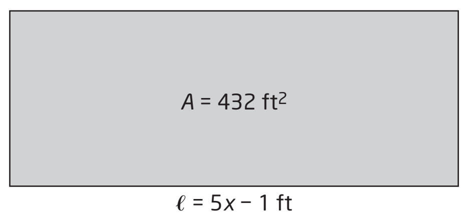
d) $f(x) = 2(x - 3)(x + 1)$

7. Display each set of data in a mapping diagram. State whether each relation is a function. Justify your answer.

a) $\{(-7, 8), (-5, 7), (-3, 6), (-1, 3), (1, 1), (3, 6)\}$

b) $\{(2, 1), (3, 2), (4, 3), (3, 4), (2, 3), (1, 2)\}$

8. Consider this rectangular swimming pool.



a) What does the expression $\frac{432}{5x - 1}$ represent in this situation?

b) State the domain and range for the expression in part a).

c) Does the expression in part a) represent a function? Justify your answer.

d) Determine the width of the pool when the length is 24 ft.

1.3 Maximum or Minimum of a Quadratic Function

9. Determine the vertex of each quadratic function by completing the square. State if the vertex is a minimum or a maximum.

a) $f(x) = -3x^2 - 18x + 2$

b) $f(x) = -4x^2 + 12x + 7$

c) $f(x) = \frac{1}{4}x^2 + 3x + 10$

10. Use partial factoring to determine the vertex of each function. State if the vertex is a minimum or a maximum.

a) $f(x) = 6x^2 - 6x - \frac{3}{2}$

b) $f(x) = \frac{2}{3}x^2 + x + \frac{19}{8}$

11. The monthly profit, $P(x)$, of a sportswear company, in thousands of dollars, is represented by the quadratic function $P(x) = -3x^2 + 18x - 2$, where x is the amount spent on advertising, in thousands of dollars.

a) Determine the company's maximum monthly profit.

b) Determine the amount spent on advertising to achieve the maximum profit.

1.4 Skills You Need: Working With Radicals

12. Express as a mixed radical in simplest form.

a) $\sqrt{147}$

b) $\sqrt{60}$

13. Simplify.

a) $-3\sqrt{7} + 6\sqrt{3} - 8\sqrt{3} + 9\sqrt{7} - 4\sqrt{7}$

b) $7\sqrt{24} + 3\sqrt{28} + 9\sqrt{54} + 6\sqrt{175}$

c) $2\sqrt{112} - 3\sqrt{18} - 2\sqrt{175} - \sqrt{98}$

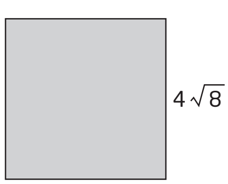
d) $2\sqrt{6}(3\sqrt{6} - 5\sqrt{8})$

e) $8\sqrt{6}(3\sqrt{2} - 4\sqrt{3} - 2\sqrt{6})$

f) $(2\sqrt{5} - 3\sqrt{2})(\sqrt{5} + 2\sqrt{2})$

g) $\frac{1}{2}\sqrt{180} - \frac{6}{7}\sqrt{245} + \frac{2}{3}\sqrt{405}$

14. Find the area of this square.



1.5 Solving Quadratic Equations

15. Solve $18x^2 - 3x - 1 = 0$ by factoring.

16. Use the quadratic formula to solve $2x^2 - 6x + 1 = 0$. Give exact answers.

17. Use the discriminant to determine the number of roots for $1.8x^2 - 2x - 1 = 0$.

18. A farmer enclosed a rectangular field with 400 m of fencing. The area of the field is 9000 m^2. Determine the dimensions of the field.

1.6 Determine a Quadratic Equation Given Its Roots

19. Write the equation for each quadratic function given the x-intercepts and the coordinates of a point on the parabola. Express the function in standard form.

a) x-intercepts: $\frac{5}{2}$ and $-\frac{3}{4}$, point: (0, 45)

b) x-intercepts: $5 \pm \sqrt{3}$, point: (4, 2)

20. A small rocket is launched. It reaches a maximum height of 120 m and lands 10 m from the launching pad. Assume the rocket follows a parabolic path. Write the equation that describes its height, h metres, as a function of its horizontal distance, x metres, from the launching pad.

21. Write the equation of a quadratic function with only one x-intercept, at -1, that passes through the point (0, 5).

1.7 Solve Linear-Quadratic Systems

22. Determine algebraically the coordinates of the point(s) of intersection of $y = -2x^2 + x - 2$ and $y = 4x - 7$.

23. Determine the number of points of intersection of $y = x^2 - x + 5$ and $y = 5x - 4$.

24. Determine the y-intercept of a line that has slope 7, and that is tangent to $f(x) = 4x^2 - x + 1$.

25. A parachutist jumps from an airplane and immediately opens her parachute. Her altitude in metres, after t seconds, is modelled by the equation $y = -11t + 500$. A second parachutist jumps 5 s later and freefalls for a few seconds. His altitude during this time is modelled by the equation $y = -4.9(t - 4)^2 + 500$. When does he catch up to the first parachutist?

Chapter 1 Math Contest

1. The graph of $y = -x^2 - 5x + 36$ intersects the x-axis at two points, A and B. The length of line segment AB is

   A 9

   B 13

   C 5

   D 36

2. If $f(x) = x^2 + 5x + 3k$ and $f(k) = -16$, $f(2)$ equals

   A -4

   B -16

   C -2

   D 2

3. The minimum distance between the parabolas $y = -5x^2 - 8$ and $y = 7x^2 + 6$ is

   A 14

   B 12

   C 8

   D 6

4. A quadratic function of the form $f(x) = ax^2 + bx + c$ has roots $x = \frac{-3 \pm \sqrt{31}}{2}$. The graph of the function passes through the point $(1, -3)$. What is the equation of the function?

   A $f(x) = 2x^2 - 6x + 11$

   B $f(x) = 2x^2 + 6x - 11$

   C $f(x) = 3x^2 - 2\sqrt{5}x - 4$

   D $f(x) = 2x^2 + 2\sqrt{5}x - 3$

5. The sum, S, and product, P, of the roots of the function $f(x) = -3x^2 + 24x + 477$ are

   A $S = 24$, $P = 477$

   B $S = 24$, $P = 1437$

   C $S = 8$, $P = -159$

   D $S = -8$, $P = 159$

6. If $x^y = 4$, then the value of $x^{3y} - x^{2y}$ is

   A 48

   B 64

   C 4

   D 12

7. If $M = 5^x + 5^{-x}$ and $N = 5^x - 5^{-x}$, then the value of $M^2 - N^2$ is

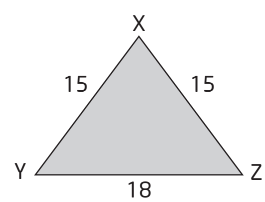
   A $2(5^{2x})$

   B $2(5^{-2x})$

   C 4

   D 0

8. In the given diagram, XY = XZ = 15 and YZ = 18. The value of sin Z is



   A $\frac{3}{5}$

   B $\frac{4}{5}$

   C $\frac{4}{9}$

   D $\frac{4}{3}$

9. The minimum value of $y = (x - a)^2 + (x - b)^2$ occurs when

   A $x = a + b$

   B $x = \frac{a - b}{2}$

   C $x = a - b$

   D $x = \frac{a + b}{2}$

10. A range of values for which $\sin x < \cos x$ is

   A $0° \le x \le 45°$

   B $45° < x \le 90°$

   C $0° \le x < 45°$

   D $45° < x < 90°$