Chapter 2 Transformation of Functions

2.1 Functions and Equivalent Algebraic Expressions

KEY CONCEPTS

- To determine if two expressions are equivalent, simplify both to see if they are algebraically the same.
- Checking several points may suggest that two expressions are equivalent, but it does not prove that they are.
- Rational expressions must be checked for restrictions by determining where the denominator is zero. These restrictions must be stated when the expression is simplified.
- Graphs can suggest whether two functions or expressions are equivalent.

Example

A shipping company designed a scalable box to accommodate different-sized items. The dimensions of the box are $\ell = 3x + 1$, $w = x - 1$, and $h = x + 1$, where x is in metres.

a) Write two equivalent expressions for the volume of the box as a function of x.
b) Write two equivalent expressions for the surface area of the box as a function of x.
c) Determine the domain of the volume and surface area functions.

Solution

a)
$$\begin{aligned} V &= \ell wh \\ V(x) &= (3x + 1)(x - 1)(x + 1) \\ &= (3x + 1)(x^2 - 1) \\ &= 3x^3 + x^2 - 3x - 1 \end{aligned}$$

Two expressions for the volume of the box are $V(x) = (3x + 1)(x - 1)(x + 1)$ and $V(x) = 3x^3 + x^2 - 3x - 1$.

b)
$$\begin{aligned} SA(x) &= 2\ell h + 2\ell w + 2hw \\ &= 2(3x + 1)(x + 1) + 2(3x + 1)(x - 1) + 2(x - 1)(x + 1) \\ &= 2(3x^2 + 4x + 1) + 2(3x^2 - 2x - 1) + 2(x^2 - 1) \\ &= 6x^2 + 8x + 2 + 6x^2 - 4x - 2 + 2x^2 - 2 \\ &= 14x^2 + 4x - 2 \end{aligned}$$

Two expressions for the surface area of the box are $SA(x) = 2(3x + 1)(x + 1) + 2(3x + 1)(x - 1) + 2(x - 1)(x + 1)$ and $SA(x) = 14x^2 + 4x - 2$.

c) None of the dimensions, nor the volume or the surface area, can be negative. The domain for both the volume and the surface area is $\{x \in \mathbb{R}, x > 1\}$.

A Practise

1. **Use Technology** Use a graphing calculator to graph the functions in each pair. Do the functions appear to be equivalent?

   a) $f(x) = 2(x - 4)^2 - (x + 2)(x - 1)$,
   $g(x) = 2x^2 - 17x + 34$

   b) $h(x) = -3(x^2 + 4x - 1) + (2x - 5)^2$,
   $k(x) = x^2 - 32x + 28$

   c) $f(x) = 6(x^2 + 3x - 1) - (4x + 2)(-2x + 3)$,
   $g(x) = 14x^2 - 34x + 10$

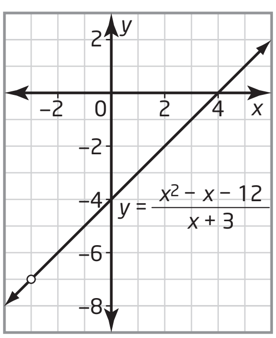
   d) $s(x) = (x + 1)^2 - 4(2 - x)(x + 3)$,
   $t(x) = -(x + 3)(x - 4) + 2(x + 3)^2$

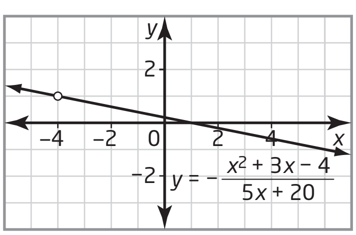
   e) $f(x) = (x - 2)(3x + 1)(x + 4)$,
   $g(x) = 3x^3 + 5x^2 - 5x - 8$

   f) $p(x) = (x^2 + 2x - 3)(x^2 - x + 1)$,
   $q(x) = x^4 + x^3 - 4x^2 + 5x - 3$

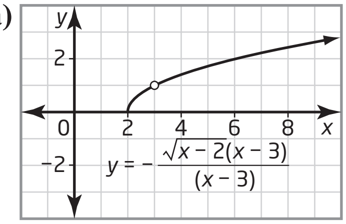
2. Refer to question 1. For those pairs of functions that appear to be equivalent, show algebraically that they are. Otherwise, show that they are not equivalent by substituting a value for x.

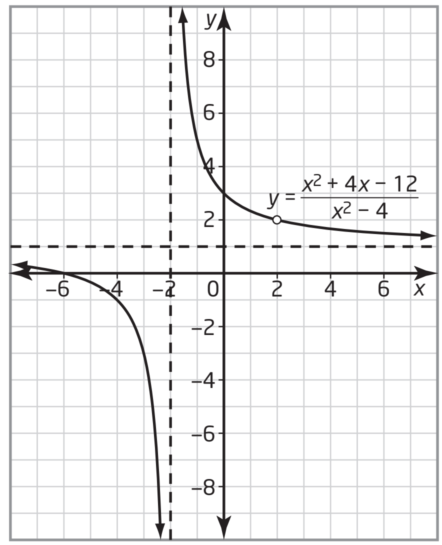
3. State any restrictions for each function.

   a)


   b)


4. State any restrictions for each function.

   a)


   b)


5. Determine if $g(x)$ is the simplified version of $f(x)$. If so, state the restrictions needed. If not, determine the proper simplified version, and then state the restrictions.

   a) $f(x) = \dfrac{x^2 + 12x + 35}{x + 5}$,
   $g(x) = x + 7$

   b) $f(x) = \dfrac{x^2 - 36}{x^2 - 3x - 18}$,
   $g(x) = x + 6$

   c) $f(x) = \dfrac{x^2 + 6x + 5}{x + 5}$,
   $g(x) = 6x$

   d) $f(x) = \dfrac{2x^2 + 5x - 3}{3x^2 + 10x + 3}$,
   $g(x) = \dfrac{2x - 1}{3x + 1}$

6. Simplify each expression. State all restrictions on x.

a) $\frac{x-7}{x^2-4x-21}$

b) $\frac{(x+3)^2(8-2x)}{4x^2-4x-48}$

c) $\frac{2x^2+7x-15}{2x^2+3x-9}$

d) $\frac{x^2-5x-36}{x^2-4x-45}$

e) $\frac{x^2-2x}{x^2-x-2}$

f) $\frac{6x^2-7x+2}{6x^2+2x-4}$

g) $\frac{4x^2+4x+1}{4x^2-1}$

h) $\frac{2x^3-28x^2-102x}{18x-2x^3}$

B Connect and Apply

7. Evaluate each expression for x-values −4, −2, 0, 3, and 10.

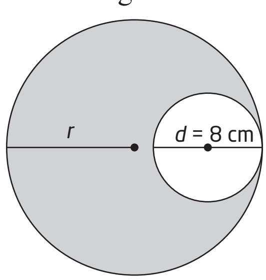
a) $(x-4)(x+8)-(x-9)(x+7)$

b) $\frac{3x^3+27x^2+60x}{x^2+6x+8}$

c) $\frac{x^3-16x}{2x^3-16x^2+32x}$

d) $(x+3)^2+2x(1-x)$

☆8. A circle with diameter 8 cm is removed from a larger circle with radius r cm.

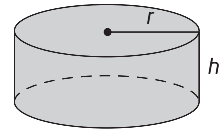


a) Express the area of the shaded region as a function of r.

b) Express the area of the shaded region in factored form.

c) State the domain and range of the area function.

9. A dairy product company has designed a scalable cylindrical container to accommodate the storage of several different-sized products.



The dimensions are $r = 2x + 1$ and $h = x - 2$, where x is in metres.

a) Write two equivalent expressions for the volume of the cylinder as a function of x.

b) Write two equivalent expressions for the surface area of the cylinder as a function of x.

c) Determine the volume and surface area for $x = 2.5$ m.

d) Determine the domain of the volume and surface area functions.

10. A company that makes modular furniture has designed a scalable box to accommodate several different sizes of items. The dimensions are given by $L = 3x + 2$, $W = 2x - 0.5$, and $H = x + 1$, where x is in metres.

a) Express the volume of the box as a function of x.

b) Express the surface area of the box as a functions of x.

c) Determine the volume and surface area for x-values of 1.25 m, 2 m, and 2.5 m.

C Extend

☆11. Determine if $f(x)$ and $g(x)$ are equivalent. Justify your answer.

$f(x) = x^2 + \left[\frac{1}{2}(x-1)(x+1)\right]^2$,

$g(x) = \left[\frac{1}{2}(x^2+1)\right]^2$

12. Without using technology, describe the graph of $f(x) = \frac{(3x+2)(2x^2-7x-4)}{3x^2-10x-8}$. Justify your answer.

2.2 Skills You Need: Operations With Rational Expressions

KEY CONCEPTS

- When multiplying or dividing rational expressions, follow these steps:
  - Factor any polynomials, if possible.
  - When dividing by a rational expression, multiply by the reciprocal of the rational expression.
  - Divide by any common factors.
  - Determine any restrictions.
- When adding or subtracting rational expressions, follow these steps:
  - Factor the denominators.
  - Determine the least common multiple of the denominators.
  - Rewrite the expressions with a common denominator.
  - Add or subtract the numerators.
  - Simplify and state the restrictions.

Example

Simplify and state any restrictions.

a) $\frac{x^2 - 8x + 15}{x - 5} \div \frac{x^2 + x - 12}{6x - 18} \times \frac{3x + 12}{2x - 10}$

b) $\frac{2x - 6}{x^2 - 5x + 6} - \frac{3x - 12}{x^2 - x - 12}$

Solution

a) $\frac{x^2 - 8x + 15}{x - 5} \div \frac{x^2 + x - 12}{6x - 18} \times \frac{3x + 12}{2x - 10}$

Change the division to multiplication.

$= \frac{x^2 - 8x + 15}{x - 5} \times \frac{6x - 18}{x^2 + x - 12} \times \frac{3x + 12}{2x - 10}$

Factor, determine the restrictions, and then simplify.

$= \frac{(x - 5)(x - 3)}{(x - 5)} \times \frac{6(x - 3)}{(x - 3)(x + 4)} \times \frac{3(x + 4)}{2(x - 5)}$

$= \frac{\cancel{(x - 5)}(x - 3)}{\cancel{(x - 5)}} \times \frac{\overset{3}{\cancel{6}}\cancel{(x - 3)}}{\cancel{(x - 3)}\cancel{(x + 4)}} \times \frac{3\cancel{(x + 4)}}{\cancel{2}(x - 5)}$

$= (x - 3) \times 3 \times \frac{3}{(x - 5)}$

$= \frac{9(x - 3)}{(x - 5)}; x \neq -4, 3, 5$

b) $\frac{2x - 6}{x^2 - 5x + 6} - \frac{3x - 12}{x^2 - x - 12}$

Factor and determine the restrictions.

$= \frac{2(x - 3)}{(x - 3)(x - 2)} - \frac{3(x - 4)}{(x - 4)(x + 3)}$

Simplify.

$= \frac{2}{(x - 2)} - \frac{3}{(x + 3)}$

Rewrite with a common denominator.

$= \frac{2(x + 3) - 3(x - 2)}{(x - 2)(x + 3)}$

Expand the numerator, and then simplify.

$= \frac{2x + 6 - 3x + 6}{(x - 2)(x + 3)}$

$= \frac{-x + 12}{(x - 2)(x + 3)}; x \neq -3, 2, 3, 4$

A Practise

1. Simplify and state any restrictions.
   a) $\frac{16y}{18x} \times \frac{72y}{4x}$
   b) $\frac{36x^4}{5x^2} \times \frac{80x^3}{12x}$
   c) $\frac{24b^5}{6b} \times \frac{48b^2}{16b^3}$
   d) $\frac{3x}{32y} \div \frac{27x^2}{96y}$
   e) $\frac{44a^3b}{15b} \div \frac{11a^2}{60b}$
   f) $\frac{27p^3q}{18r^2} \div \frac{3p^2q}{36r^4}$

2. Simplify and state any restrictions.
   a) $\frac{12}{x-6} \times \frac{x-6}{3}$
   b) $\frac{x+2}{x} \times \frac{9x}{x+2}$
   c) $\frac{x-8}{x+2} \times \frac{x+2}{x-6}$
   d) $\frac{7x^2}{6x^2+3x} \times \frac{12x+6}{2x+8}$
   e) $\frac{4x-20}{x^2+6x} \times \frac{3x^2}{3x-15}$
   f) $\frac{x^2+12x+32}{x+8} \times \frac{x+1}{x^2+7x+12}$
   g) $\frac{x^2+3x+2}{x^2-1} \times \frac{x-1}{x^2-2x-8}$
   h) $\frac{x^2-2x-8}{x+2} \times \frac{x-3}{x^2+2x-24}$

3. Simplify and state any restrictions.
   a) $\frac{x+1}{x} \div \frac{x+1}{2x}$
   b) $\frac{x}{x-3} \div \frac{1}{x-3}$
   c) $\frac{x+12}{x+10} \div \frac{x+12}{x-5}$
   d) $\frac{x^2+15x}{4x+24} \div \frac{3x}{3x+18}$
   e) $\frac{6x}{8x-72} \div \frac{9x}{2x-18}$
   f) $\frac{x^2+15x+26}{6x^2} \div \frac{x^2-3x-10}{30x^3}$
   g) $\frac{x^2-7x+10}{x^2-4} \div \frac{x^2-4x-5}{3x+6}$
   h) $\frac{2x^2-5x-12}{x^2+x-20} \div \frac{2x^2+5x+3}{x^2+8x+7}$

4. Simplify and state any restrictions.
   a) $\frac{x+2}{6} + \frac{x-2}{4}$
   b) $\frac{x+9}{3} - \frac{3x-4}{7}$
   c) $\frac{5}{7x} - \frac{3}{4x}$
   d) $\frac{4}{ab} + \frac{9}{2b}$
   e) $\frac{11}{12ab^2} - \frac{7}{16a^2}$
   f) $\frac{1+a}{2a} + \frac{a-1}{5a}$
   g) $\frac{2}{x-3} - \frac{5}{x+3}$
   h) $\frac{7}{x+4} + \frac{11}{x-5}$
   i) $\frac{x+9}{x-2} - \frac{x-4}{x+5}$

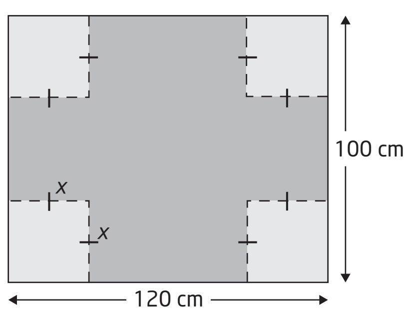
5. Simplify and state any restrictions.
   a) $\frac{4x}{x^2-9x+18} + \frac{2x-1}{x-6}$
   b) $\frac{x-7}{x^2-2x-15} - \frac{3x}{x-5}$
   c) $\frac{2x}{x-2} - \frac{3}{x^2-4}$
   d) $\frac{3x}{x-1} - \frac{2x}{x^2+x-2}$
   e) $\frac{x+3}{x^2+11x+24} - \frac{2x+10}{x^2+11x+30}$
   f) $\frac{x-4}{x^2-8x+16} + \frac{3x+21}{x^2+12x+35}$
   g) $\frac{3x+9}{x^2+5x+6} - \frac{2x-2}{x^2+x-2}$
   h) $\frac{4x^2-20x}{x^2+2x-35} + \frac{3x-6}{x^2-12x+20}$

☆6. David competed in a 40-km dirt bike race. For the first half of the race he rode his bike at an average speed 8 km/h faster than in the second half.
   a) Let x represent David's speed in the first half. Determine a simplified expression in terms of x for the total time he took to complete the race.
   b) If David rode his bike at an average speed of 35 km/h in the first half, how long did it take him to finish the race?

B Connect and Apply

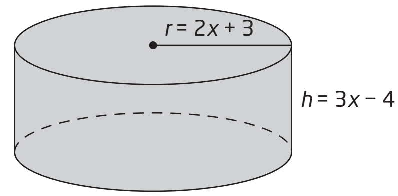
7. In each expression, factor −1 from one of the denominators to identify the common denominator. Then simplify the expression and state the restrictions.
   a) $\frac{1}{x-5} - \frac{1}{5-x}$
   b) $\frac{2x+1}{x-4} + \frac{x-3}{4-x}$
   c) $\frac{a+1}{5-2a} - \frac{a-4}{2a-5}$
   d) $\frac{2b+5}{2b-3} + \frac{9+b}{3-2b}$
   e) $\frac{x+2}{x-2} - \frac{4x}{2-x}$
   f) $\frac{3x+2}{3-4x} + \frac{2x+1}{-3+4x}$
   g) $\frac{6b-5}{2+b} + \frac{b+3}{-2-b}$
   h) $\frac{3c+7}{1-5c} + \frac{-8}{5c-1}$

☆8. Anna wants to make an open-topped box using a rectangular piece of cardboard with dimensions 120 cm by 100 cm. She plans to cut a square of side length x from each corner.



   a) Write an expression for the volume of the box as a function of x.
   b) Write an expression for the surface area of the open-topped box as a function of x.
   c) State the domain for this situation.
   d) Write a simplified expression for the ratio of the volume of the box to the surface area.
   e) Refer to your answer to part d). What are the restrictions on x?

9. Consider a cylinder with height $h = 3x - 4$ and radius $r = 2x + 3$.



   a) Determine the ratio of the volume of the cylinder to its surface area.
   b) Determine any restrictions on x for your answer to part a).

10. **Use Technology**
   a) Use graphing technology to graph $f(x) = \frac{1}{x+4} + \frac{1}{x-4}$.
   b) Rewrite the function using a common denominator. Then, graph the rewritten function.
   c) Compare the graphs. Identify how the restrictions affect the graph.

C Extend

11. Simplify the expression and state any restrictions.

$$\frac{x^2-16}{x^2-x-12} \times \frac{2x^2+6x}{3x^2+9x-12} + \frac{3x^2+7x+4}{2x^2-x-3} \div \frac{-3x^2-x+4}{14x^2-21x}$$

12. If $a = \frac{1}{x}$ and $b = \frac{1}{y}$, write each expression in terms of x and y, in simplified form.
   a) $\frac{a+b}{a-b}$
   b) $\frac{a}{a-b} - \frac{b}{a+b}$

13. If $x = \frac{a+1}{a+2}$, write each expression in terms of a, in simplified form.
   a) $x^2 - 1$
   b) $\frac{x+1}{x+2}$

2.3 Horizontal and Vertical Translations of Functions

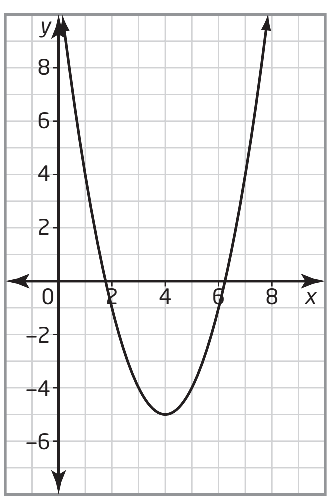
KEY CONCEPTS

- Translations are transformations that cause functions to shift from one place to another without changing shape.
- The graph of $g(x) = f(x) + c$ is a vertical translation of the graph of $f(x)$ by c units. If $c > 0$, the graph moves up c units. If $c < 0$, the graph moves down c units.
- The graph of $g(x) = f(x - d)$ is a horizontal translation of the graph of $f(x)$ by d units. If $d > 0$, the graph moves to the right d units. If $d < 0$, the graph moves to the left d units.
- A sketch of the graph of any transformed function can be created by transforming the related base function.
- In general, the domain and range of a function of the form $g(x) = f(x - d) + c$ can be determined by adding the d-value and the c-value to restrictions on the domain and range, respectively, of the base function.

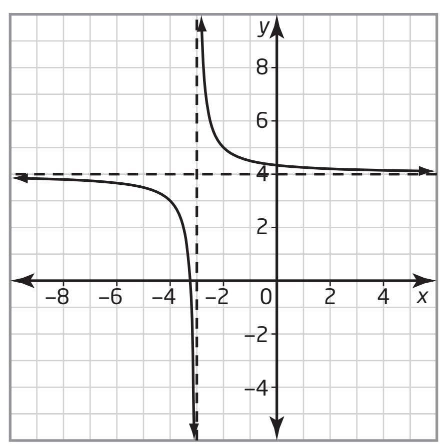
Example

Each graph represents a transformation of a base function, $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, or $f(x) = \frac{1}{x}$. State the base function and the equation of the transformed function.

a)



b)



Solution

a) The base function is $f(x) = x^2$. The vertex of the graph is $(4, -5)$, so $f(x) = x^2$ has been translated 4 units right and 5 units down; $d = 4$ and $c = -5$.
The equation of the transformed function is $g(x) = (x - 4)^2 - 5$.

b) The base function is $f(x) = \frac{1}{x}$, which has asymptotes at $x = 0$ and $y = 0$. The asymptotes of the graph are $x = -3$ and $y = 4$, so $f(x) = \frac{1}{x}$ has been translated 3 units left and 4 units up; $d = -3$ and $c = 4$.
The equation of the transformed function is $g(x) = \frac{1}{x + 3} + 4$.

A Practise

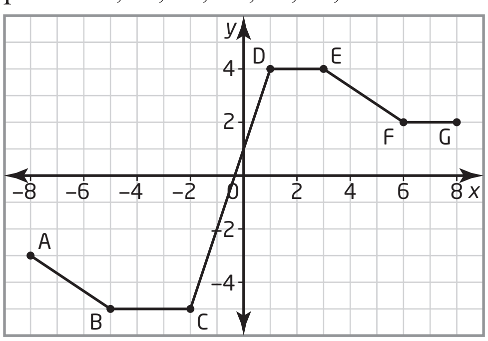
1. **a)** Copy and complete the table of values.

x	$f(x) = \sqrt{x}$	$r(x) = f(x) - 4$	$s(x) = f(x + 5)$
0			
1			
4			
9			

   b) Use the points to graph each function on the same set of axes.

   c) Explain how the points of the translated functions relate to the actual transformation.

2. Given the graph of $f(x)$, sketch a graph of each function by determining the image points A′, B′, C′, D′, E′, F′, and G′.



   a) $b(x) = f(x) + 6$

   b) $g(x) = f(x) - 4$

   c) $h(x) = f(x - 3)$

   d) $m(x) = f(x + 1)$

   e) $n(x) = f(x - 1) + 9$

   f) $r(x) = f(x + 2) - 7$

3. For each function $g(x)$, identify the base function as one of $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, or $f(x) = \frac{1}{x}$, and describe the transformation in the form $y = f(x - d) + c$ and in words. Then sketch a graph of $g(x)$ and state the domain and range of each function.

   a) $g(x) = x - 7$

   b) $g(x) = x^2 + 3$

   c) $g(x) = \sqrt{x} + 9$

   d) $g(x) = (x - 5)^2$

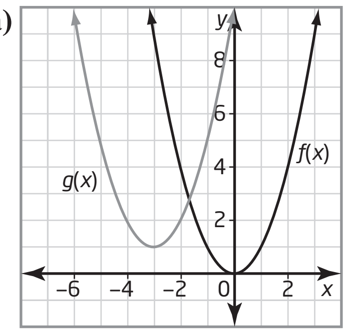
   e) $g(x) = \frac{1}{x} + 2$

   f) $g(x) = \sqrt{x + 3}$

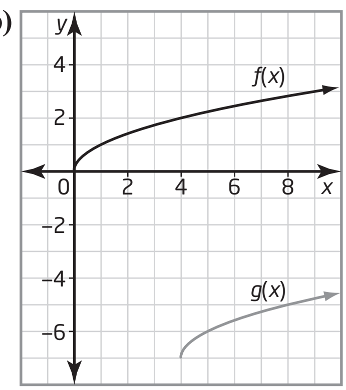
   g) $g(x) = \frac{1}{x - 8}$

4. Use words and function notation to describe the transformation that can be applied to each graph of $f(x)$ to obtain the graph of $g(x)$. State the domain and range of $f(x)$ and $g(x)$.

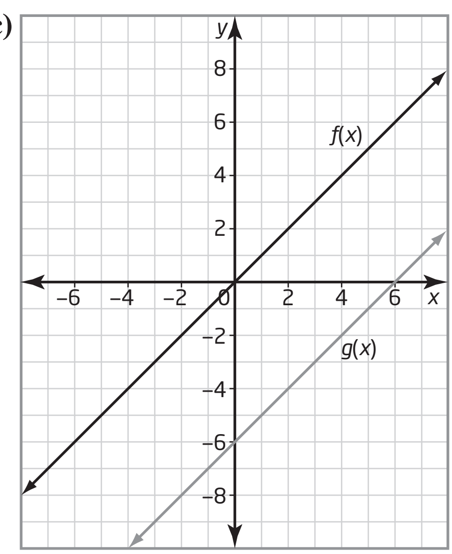
   a)



   b)

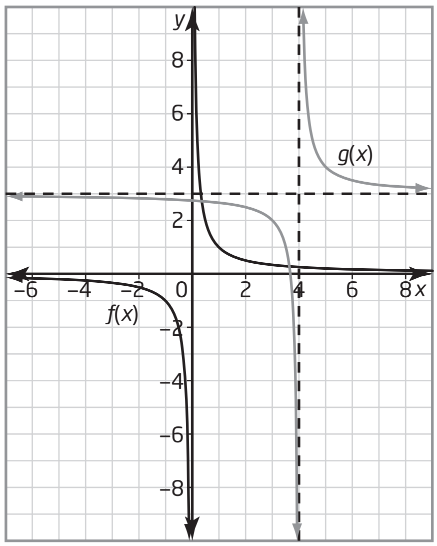


   c)

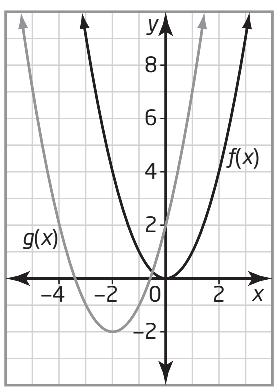


5. Use words and function notation to describe the transformation that can be applied to each graph of $f(x)$ to obtain the graph of $g(x)$. State the domain and range of $f(x)$ and $g(x)$.

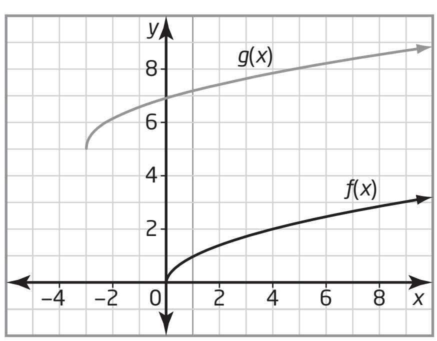
a)



b)



c)



B Connect and Apply

6. Use the base function $f(x) = x$. Write the equation for each transformed function.

   a) $b(x) = f(x + 2)$

   b) $h(x) = f(x) - 5$

   c) $m(x) = f(x) + 9$

   d) $n(x) = f(x - 3) - 7$

   e) $r(x) = f(x + 4) + 6$

   f) $s(x) = f(x + 2) - 8$

   g) $t(x) = f(x - 5) + 1$

7. Repeat question 6 using the base function $f(x) = x^2$.

8. Repeat question 6 using the base function $f(x) = \sqrt{x}$.

9. Repeat question 6 using the base function $f(x) = \frac{1}{x}$.

☆10. Is each statement true or false? Support your answer with an example or a graph.

   a) When transforming a function using translations, a horizontal translation must be applied before a vertical translation.

   b) The function $y = x - 2$ could be a vertical translation of $y = x$ two units downward or a horizontal translation of $y = x$ two units to the right.

☆11. Which of these transformations are equivalent to horizontally shifting $f(x) = x$ five units left? Justify your answer by writing the transformed equation.

   a) vertical translation of 5 units up

   b) vertical translation of 5 units down

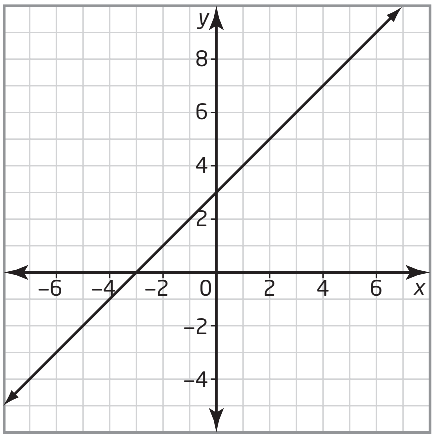
   c) vertical translation of 3 units up and horizontal translation of 2 units right

   d) horizontal translation of 3 units left and vertical translation of 2 units up

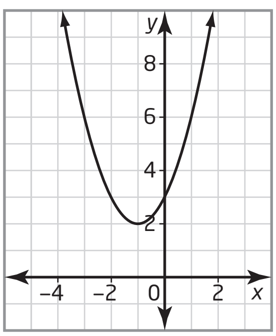
   e) vertical translation of 1 unit up and horizontal translation of 4 units right

12. Each graph represents a transformation of one of the base functions $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, or $f(x) = \frac{1}{x}$. State the base function and the equation of the transformed function.

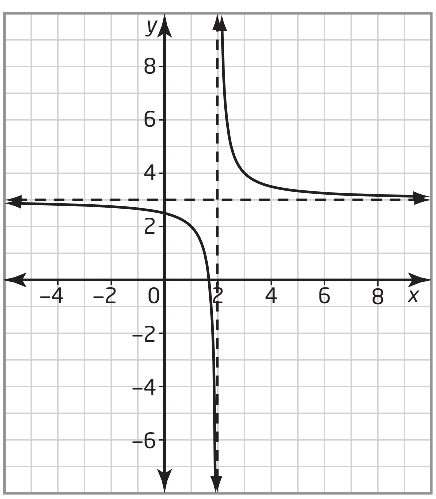
a)



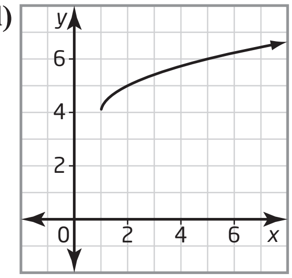
b)



c)



d)



13. The cost, in dollars, to produce x units of a product can be modelled by the function $c(x) = \sqrt{x} + 400$.

a) State the domain and range of the cost function and interpret their meanings.

b) The first 8 units of this product are sold, but are not included in the cost. Write a new function representing the cost of this product.

c) What type of transformation does the change in part b) represent?

d) How does the transformation in part b) affect the domain and range?

C Extend

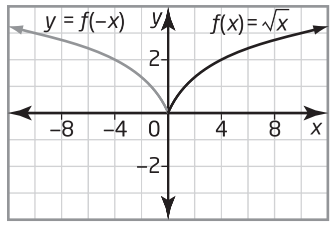
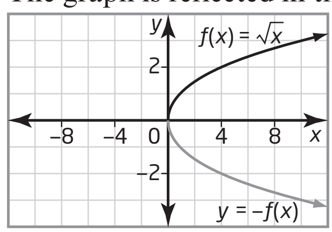
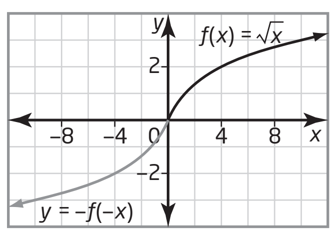
14. a) State the transformed function $g(x) = f(x + 6) + 5$ for each of the base functions $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, and $f(x) = \frac{1}{x}$. Use words to describe how each base function is transformed.

b) Transform the resulting function $g(x)$ in part a) by applying another transformation, $h(x) = g(x - 4) - 3$. Use words to describe how each function $g(x)$ is transformed.

c) Describe a transformation $p(x)$ that can be applied to $f(x)$ that gives the same result as the two transformations applied in parts a) and b). Justify your answer.

2.4 Reflections of Functions

KEY CONCEPTS

Reflection	Numerical Representation	Graphical Representation	Algebraic Representation
$y = f(-x)$	A point (x, y) becomes $(-x, y)$.	The graph is reflected in the y-axis. 	Replace x with $-x$ in the expression.
$y = -f(x)$	A point (x, y) becomes $(x, -y)$.	The graph is reflected in the x-axis. 	Multiply the entire expression by −1.
$y = -f(-x)$	A point (x, y) becomes $(-x, -y)$.	The graph is reflected in one axis and then the other. 	First replace x with $-x$ in the expression and then multiply the entire expression by −1.

Example

Use algebra to determine if $g(x)$ is a reflection of $f(x)$. Check by graphing.

a) $f(x) = x^2 - 5$ $\quad g(x) = -x^2 + 5$

b) $f(x) = \sqrt{x + 8}$ $\quad g(x) = \sqrt{8 - x}$

c) $f(x) = \dfrac{1}{x - 1} + 3$ $\quad g(x) = \dfrac{1}{x - 1} - 3$

Solution

$g(x)$ is a reflection of $f(x)$ if one of these conditions is true:

i) $g(x) = -f(x)$

ii) $g(x) = f(-x)$

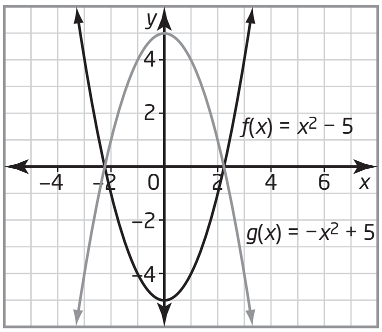
iii) $g(x) = -f(-x)$

a) $f(x) = x^2 - 5$

Check condition i).

$$\begin{aligned} -f(x) &= -(x^2 - 5) \\ &= -x^2 + 5 \\ &= g(x) \end{aligned}$$

Since $g(x) = -f(x)$, $g(x)$ is a reflection of $f(x)$ in the x-axis.



b) $f(x) = \sqrt{x + 8}$

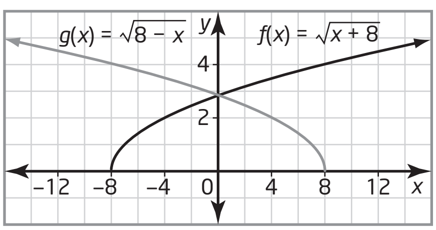
Check condition i).

$$\begin{aligned} -f(x) &= -\sqrt{x + 8} \\ &\neq g(x) \end{aligned}$$

Check condition ii).

$$\begin{aligned} f(-x) &= \sqrt{-x + 8} \\ &= \sqrt{8 - x} \\ &= g(x) \end{aligned}$$

Since $g(x) = f(-x)$, $g(x)$ is a reflection of $f(x)$ in the y-axis.



c) $f(x) = \dfrac{1}{x - 1} + 3$

Check condition i).

$$\begin{aligned} -f(x) &= \frac{-1}{x - 1} - 3 \\ &= \frac{1}{1 - x} - 3 \\ &\neq g(x) \end{aligned}$$

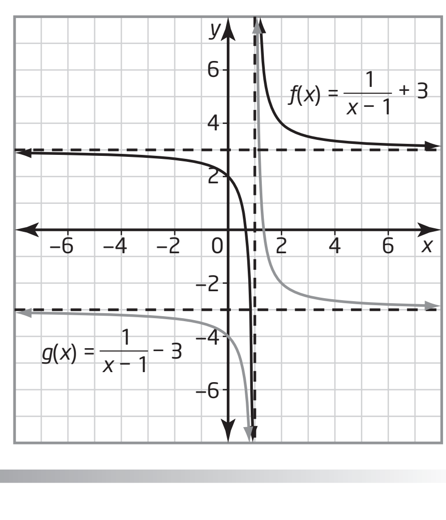
Check condition ii).

$$\begin{aligned} f(-x) &= \frac{1}{-x - 1} + 3 \\ &= \frac{-1}{x + 1} + 3 \\ &\neq g(x) \end{aligned}$$

Check condition iii).

$$\begin{aligned} -f(-x) &= \frac{-1}{-x - 1} - 3 \\ &= \frac{1}{x + 1} - 3 \\ &\neq g(x) \end{aligned}$$

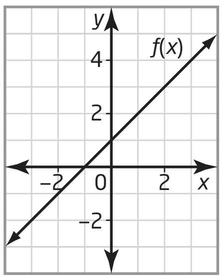
Since $g(x)$ is not equivalent to $f(-x)$, $-f(x)$, or $-f(-x)$, $g(x)$ is not a reflection of $f(x)$. Note that $g(x)$ is a vertical translation of $f(x)$ 6 units down.



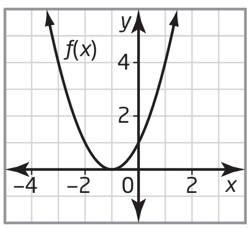
A Practise

1. Copy each graph of $f(x)$. Sketch $g(x)$, which is the reflection of $f(x)$ in the y-axis. State the domain and range of each function.

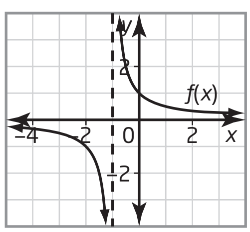
a)



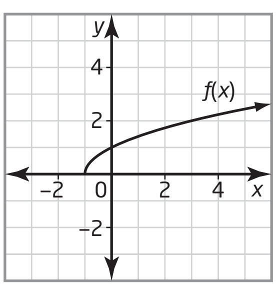
b)



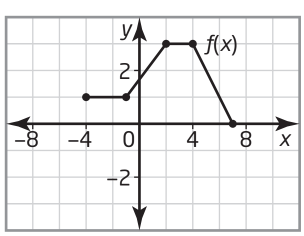
c)



d)



e)



2. For each function in question 1, sketch $h(x)$, which is the reflection of $f(x)$ in the x-axis. State the domain and range of each function.

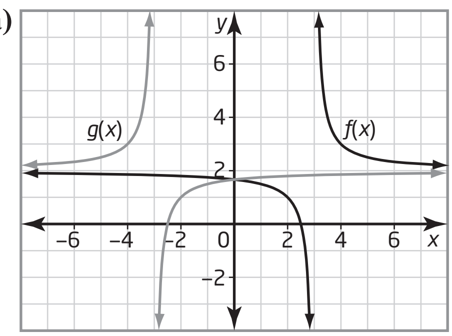
3. For each function $f(x)$ in question 1, sketch a graph of $k(x) = -f(-x)$. State the domain and range of each function.

4. For each function $f(x)$, determine the equation for $g(x)$.

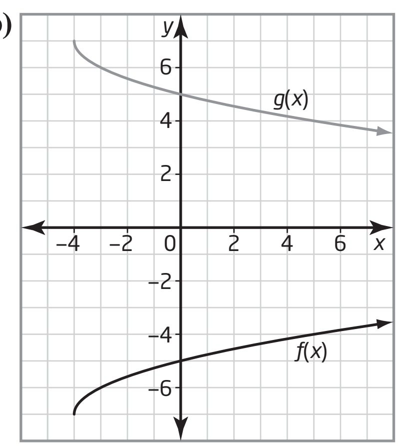
a) $f(x) = \sqrt{x - 21} + 9$	$g(x) = -f(x)$
b) $f(x) = (x + 8)^2 - 17$	$g(x) = f(-x)$
c) $f(x) = (x - 1)^2 + 11$	$g(x) = -f(-x)$
☆**d)** $f(x) = \dfrac{1}{x - 6} + 5$	$g(x) = -f(-x)$
e) $f(x) = -\sqrt{x + 4} + 19$	$g(x) = f(-x)$
f) $f(x) = \dfrac{1}{x - 8} + 3$	$g(x) = -f(x)$
g) $f(x) = x + 18$	$g(x) = -f(-x)$

5. For each graph, describe the reflection that transforms $f(x)$ into $g(x)$.

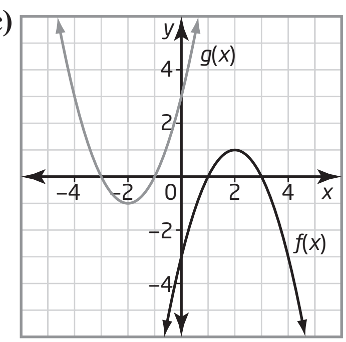
a)



b)



c)



B Connect and Apply

6. **Use Technology** Use graphing technology to graph the function $f(x) = (x + 3)^2 - 1$.
   a) Determine the invariant point(s), if any, when $f(x)$ is reflected in
   i) the x-axis
   ii) the y-axis
   iii) the x-axis and the y-axis
   b) Write the equation of a quadratic function that has an invariant point for all the reflections in part a). Justify your answer.

7. Determine algebraically whether $g(x)$ is a reflection of $f(x)$. Check by graphing.
   a) $f(x) = x^2 + 2$ $\quad g(x) = -x^2 + 2$
   b) $f(x) = \sqrt{x}$ $\quad g(x) = -\sqrt{x}$
   c) $f(x) = \frac{1}{x + 3}$ $\quad g(x) = \frac{1}{3 - x}$
   d) $f(x) = (x - 6)^2 + 2$ $\quad g(x) = -(x + 6)^2 - 2$
   e) $f(x) = \sqrt{x + 12} - 7$ $\quad g(x) = -\sqrt{x + 12} - 7$
   f) $f(x) = \frac{1}{x} - 9$ $\quad g(x) = -\frac{1}{x} - 9$

☆8. **a)** Graph the function $f(x) = (x + 3)^2 - 2$.
   b) Graph $g(x)$, which is the reflection of $f(x)$ in the y-axis. Write the equation of $g(x)$.
   c) Determine a translation that can be applied to $f(x)$ to obtain $g(x)$.
   d) Verify algebraically that the transformations in parts b) and c) are the same.
   e) Predict if the same would be true for reflections in the x-axis. Explain.
   f) Would this work for any other type of function? Explain.

9. **a)** State the base function $f(x)$ that corresponds to each transformed function, $g(x)$.
   i) $g(x) = \frac{1}{x - 9} + 4$
   ii) $g(x) = \sqrt{x + 5} - 7$
   b) Describe the transformations that are applied to the base function to obtain each function in part a).
   c) For each function in part a) write the equations for
   $k(x)$: a reflection in the x-axis,
   $p(x)$: a reflection in the y-axis, and
   $q(x)$: a reflection in the x-axis and in the y-axis.

C Extend

10. **a)** Sketch a graph of $f(x) = \sqrt{x}$ reflected in each given line. Write the equation of the transformed function, $g(x)$.
   i) $x = -1$
   ii) $x = -2$
   iii) $x = -3$
   b) In each case, describe two transformations that give the same result as $g(x)$.
   c) Will the results from part b) be true when $f(x) = \sqrt{x}$ is reflected in the line $x = a$, $\{a \in \mathbb{R}\}$? Explain. Write the corresponding transformed function.

11. **Use Technology**
   a) Given $f(x) = \sqrt{16 - x^2}$, write the equations of $f(-x)$, $-f(x)$, and $-f(-x)$. Which functions are equivalent?
   b) Graph each function. Determine any invariant points.
   c) State the domain and range of each function.

12. **a)** Identify the invariant points when the circle $x^2 + y^2 = 36$ is reflected in
   i) the x-axis
   ii) the y-axis
   iii) the x-axis and the y-axis
   b) Will the results in part a) be true for all circles? Explain.

2.5 Stretches of Functions

KEY CONCEPTS

- Stretches and compressions are transformations that cause functions to change shape.
- The graph of $g(x) = af(x)$, $a > 0$, is a vertical stretch or a vertical compression of the graph of $f(x)$ by a factor of a. If $a > 1$, the graph is vertically stretched by a factor of a. If $0 < a < 1$, the graph is vertically compressed by a factor of a.
- The graph of $g(x) = f(kx)$, $k > 0$, is a horizontal stretch or a horizontal compression of the graph of $f(x)$ by a factor of $\frac{1}{k}$. If $k > 1$, the graph is horizontally compressed by a factor of $\frac{1}{k}$. If $0 < k < 1$, the graph is horizontally stretched by a factor of $\frac{1}{k}$.

Example

In each case, write the equation of the transformed function, $g(x)$, that results from applying the given transformation(s) to the base function $f(x)$. Sketch $f(x)$ and $g(x)$ on the same set of axes.

a) $f(x) = \frac{1}{x}$ is stretched horizontally by a factor of 4

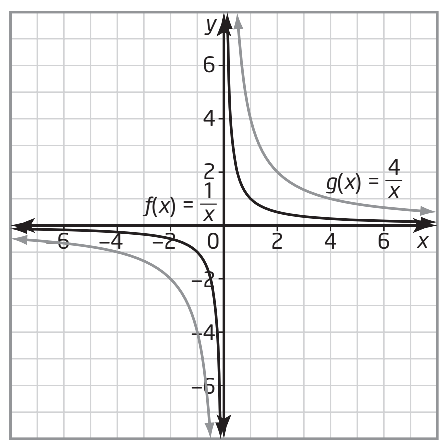
b) $f(x) = \sqrt{x}$ is compressed vertically by a factor of $\frac{1}{5}$

Solution

a) The parameter that corresponds to a horizontal stretch by a factor of 4 is $k = \frac{1}{4}$.

$$g(x) = f\left(\frac{1}{4}x\right) = \frac{1}{\left(\frac{1}{4}x\right)} = \frac{4}{x}$$

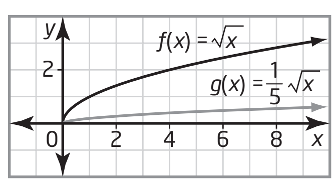
The transformed function is $g(x) = \frac{4}{x}$.



b) The parameter that corresponds to a vertical compression by a factor of $\frac{1}{5}$ is $a = \frac{1}{5}$.

$$g(x) = \frac{1}{5}f(x) = \frac{1}{5}\sqrt{x}$$

The transformed function is $g(x) = \frac{1}{5}\sqrt{x}$.



A Practise

1. a) Copy and complete the table of values.

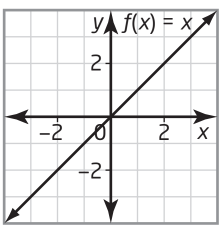
x	$f(x) = x^2$	$g(x) = \frac{3}{4}f(x)$	$h(x) = f\left(\frac{3}{4}x\right)$
−4			
−2			
0			
2			
4			

b) Sketch a graph of the functions on the same set of axes.

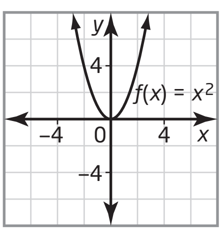
c) Explain how the points on the graphs of $g(x)$ and $h(x)$ relate to the transformation.

2. Given each graph of $f(x)$, graph and label $g(x)$.

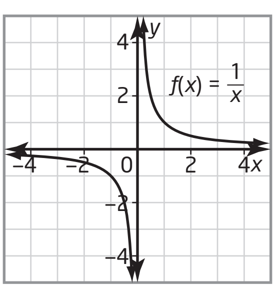
a) $g(x) = 2f(x)$



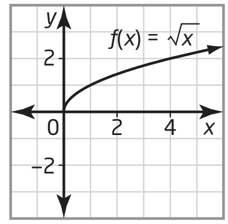
b) $g(x) = f(3x)$



c) $g(x) = f(4x)$



d) $g(x) = f\left(\frac{x}{4}\right)$



3. For each function $g(x)$, identify the value of a or k and describe how the graph of $g(x)$ can be obtained from the graph of $f(x)$.

a) $g(x) = 8f(x)$

b) $g(x) = f(6x)$

c) $g(x) = \frac{2}{3}f(x)$

d) $g(x) = f\left(\frac{1}{9}x\right)$

4. For each function $g(x)$, describe the transformation from a base function of $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, or $f(x) = \frac{1}{x}$. Then sketch a graph of $f(x)$ and $g(x)$ on the same axes.

a) $g(x) = 12x$

b) $g(x) = (4x)^2$

c) $g(x) = \sqrt{\frac{x}{6}}$

d) $g(x) = \frac{7}{x}$

e) $g(x) = \sqrt{25x}$

f) $g(x) = \frac{x}{8}$

B Connect and Apply

5. In each case, write the equation of the transformed function $g(x)$ that results from applying the given transformation to the base function $f(x)$. Sketch $f(x)$ and $g(x)$ on the same set of axes.

a) $f(x) = x^2$ is stretched vertically by a factor of 2

b) $f(x) = \sqrt{x}$ is compressed horizontally by a factor of $\frac{2}{3}$

c) $f(x) = \frac{1}{x}$ is stretched horizontally by a factor of 2

☆**6.** Acceleration due to gravity, a, varies from planet to planet. The distance of the object from the drop location after t seconds is given by $d(t) = \frac{a}{2}t^2$. The table shows the estimated acceleration due to gravity for different planets.

Planet	Acceleration Due to Gravity (m/s^2)
Earth	9.8
Neptune	11.4
Mercury	3.6

a) State the base function, $f(t)$, for $d(t)$.

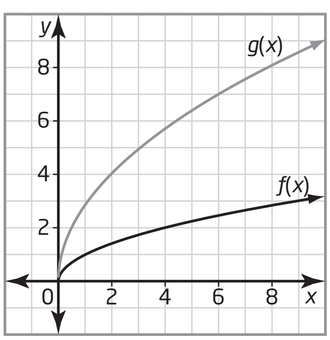
b) Describe the transformation that is applied to $f(t)$ to obtain $d(t)$.

c) Write the equation that represents the distance of an object from the drop location on each planet.

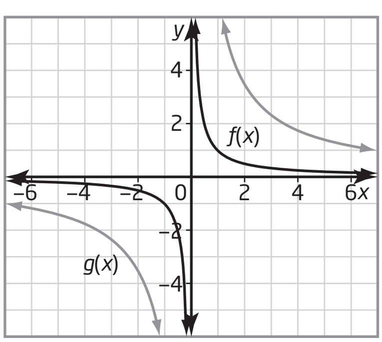
d) Compare the domain and range of each function in part c). What do you notice?

7. Describe the transformation applied to the graph of $f(x)$ to obtain the graph of $g(x)$. Write the equation for $g(x)$.

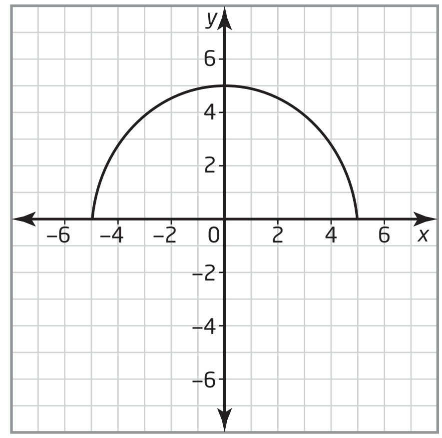
a)



b)



8. Consider the graph of $f(x) = \sqrt{25 - x^2}$.



Use transformations to sketch each graph of $g(x)$. Write the equation of $g(x)$ and state the domain and range.

a) $g(x) = 2f(x)$

b) $g(x) = f(2x)$

c) $g(x) = \frac{1}{2}f(x)$

d) $g(x) = f\left(\frac{1}{2}x\right)$

C Extend

9. a) Use Technology Use technology to graph $f(x) = x^4 - x^2$.

b) If $g(x) = 2f(x)$ and $h(x) = g(2x)$, determine the equations for $g(x)$ and $h(x)$.

c) Without using technology, describe and sketch a graph of $g(x)$ and $h(x)$.

d) Is there a single transformation that will transform $f(x)$ into $h(x)$? Explain.

☆**10.** Describe how the graph of $f(x) = -0.4x^2 + 18$ can be transformed into the graph of $g(x) = -10x^2 + 18$.

2.6 Combinations of Transformations

KEY CONCEPTS

- Stretches, compressions, and reflections can be performed in any order before translations.
- Ensure that the function is written in the form $y = af[k(x - d)] + c$ to identify specific transformations.
- The parameters a, k, d, and c in the function $y = af[k(x - d)] + c$ correspond to the following transformations:
  a corresponds to a vertical stretch or compression and, if $a < 0$, a reflection in the x-axis.
  k corresponds to a horizontal stretch or compression and, if $k < 0$, a reflection in the y-axis.
  d corresponds to a horizontal translation to the right or left.
  c corresponds to a vertical translation up or down.

Example

Transformations are applied to each function $f(x)$. Identify the parameters a, k, d, and c that correspond to the transformations, and then write the equation of the transformed function, $g(x)$.

a) The graph of $f(x) = \sqrt{x}$ is reflected in the y-axis, stretched horizontally by a factor of 3, and then translated 5 units left and 7 units down.

b) The graph of $f(x) = \frac{1}{x}$ is reflected in the x-axis, compressed horizontally by a factor of $\frac{1}{4}$, and then translated 6 units right and 2 units up.

Solution

a) Since the graph of $f(x) = \sqrt{x}$ is reflected in the y-axis and stretched horizontally by a factor of 3, then $k = -\frac{1}{3}$. A translation of 5 units left indicates that $d = -5$. A translation of 7 units down indicates that $c = -7$.

$$g(x) = f\left[-\frac{1}{3}(x + 5)\right] - 7$$
$$= \sqrt{-\frac{1}{3}x + 5} - 7$$

b) Since the graph of $f(x) = \frac{1}{x}$ is reflected in the x-axis, then $a = -1$.

A horizontal compression by a factor of $\frac{1}{4}$ indicates that $k = 4$. A translation of 6 units right indicates that $d = 6$. A translation of 2 units up indicates that $c = 2$.

$$g(x) = -f[4(x - 6)] + 2$$
$$= -\frac{1}{4(x - 6)} + 2$$

A Practise

1. Compare each transformed equation to $y = af[k(x - d)] + c$ to determine the values of a, k, d, and c. Then describe, in the appropriate order, the transformations that must be applied to a base function $f(x)$ to obtain the transformed function.

   a) $g(x) = 3f(x - 5)$

   b) $g(x) = \frac{1}{4}f(x) + 4$

   c) $g(x) = f(x + 6) + 2$

   d) $g(x) = f\left(\frac{1}{3}x\right) + 7$

   e) $g(x) = f(2x) - 8$

   f) $g(x) = 5f(x) - 3$

2. Repeat question 1 for each transformed function $g(x)$.

   a) $g(x) = 4f(3x) - 2$

   b) $g(x) = -5f(x) + 6$

   c) $g(x) = \frac{1}{3}f(x - 8) + 1$

   d) $g(x) = f(-2x) + 6$

   e) $g(x) = -f\left(\frac{1}{4}x\right) - 1$

   f) $g(x) = \frac{2}{5}f(5x) - 7$

3. Describe, in the appropriate order, the transformations that must be applied to the base function $f(x)$ to obtain the transformed function. Then write the corresponding equation and transform the graph of $f(x)$ to sketch a graph of $g(x)$.

   a) $f(x) = \sqrt{x}$

   $g(x) = 2f(4x)$

   b) $f(x) = \frac{1}{x}$

   $g(x) = -3f(x - 1) + 7$

   c) $f(x) = x^2$

   $g(x) = f\left[\frac{1}{2}(x + 1)\right]$

   d) $f(x) = x$

   $g(x) = -4f(x) - 6$

4. Repeat question 3 for $f(x)$ and the transformed function $g(x)$.

   a) $f(x) = x$

   $g(x) = -\frac{1}{3}f[3(x + 2)] - 4$

   b) $f(x) = x^2$

   $g(x) = -4f[2(x - 1)] + 6$

   c) $f(x) = \sqrt{x}$

   $g(x) = \frac{1}{3}f[2(x + 5)] + 4$

   d) $f(x) = \frac{1}{x}$

   $g(x) = 5f[-(x - 1)] + 2$

5. For each function, identify the base function as one of $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, or $f(x) = \frac{1}{x}$. Sketch the graphs of the base function and the transformed function, and state the domain and range of the functions.

   a) $b(x) = 7x - 2$

   b) $e(x) = 4x^2 - 3$

   c) $h(x) = (3x + 18)^2$

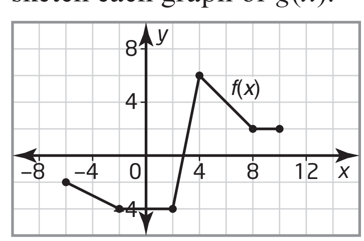
   d) $p(x) = 4\sqrt{x - 3}$

   e) $m(x) = \frac{2}{x + 9}$

   f) $r(x) = \frac{3}{4 - x} + 6$

B Connect and Apply

6. Given the graph of the function $f(x)$, sketch each graph of $g(x)$.



   a) $g(x) = 2f(x + 3)$

   b) $g(x) = f(2x) + 5$

   c) $g(x) = f(4x - 16)$

   d) $g(x) = 3f(-0.5x + 1) - 3$

7. Match each equation with its graph below. Justify your choice.

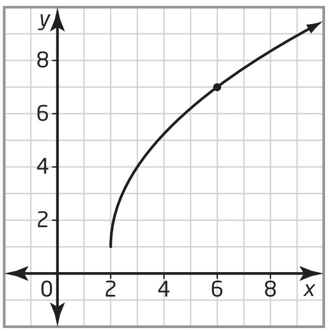
a) $y = \frac{2}{x + 4} - 3$

☆ b) $y = \frac{1}{2}\sqrt{-x + 4} + 3$

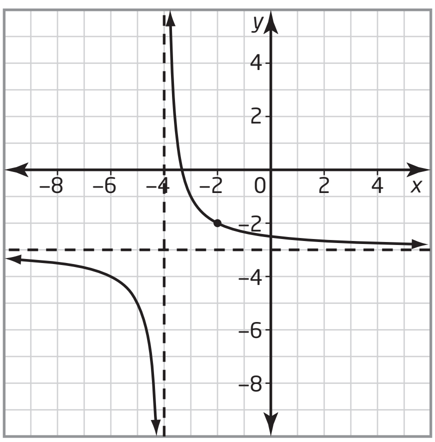
c) $y = 3\sqrt{x - 2} + 1$

d) $y = \left[\frac{1}{4}(x - 5)\right]^2 + 2$

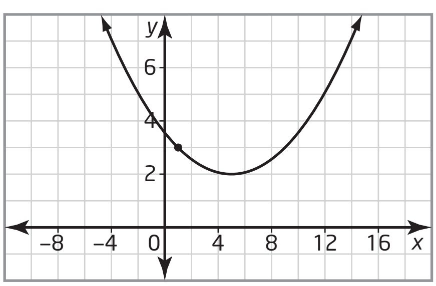
i)



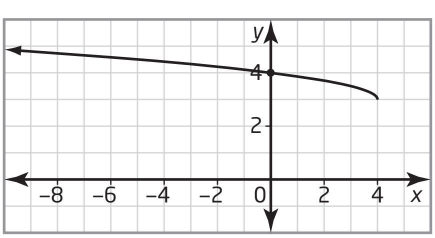
ii)



iii)



iv)



8. Match each equation with its graph below. Justify your choice.

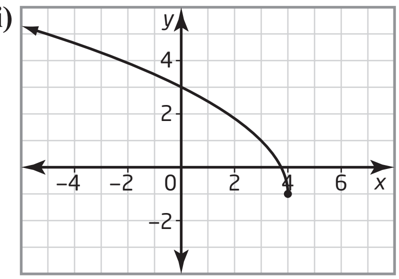
a) $y = -\frac{2}{x + 3} + 1$

b) $y = 2\sqrt{-x + 4} - 1$

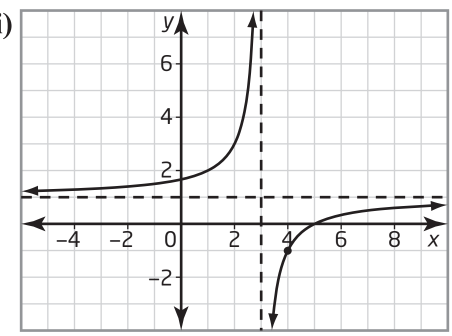
c) $y = \frac{4}{-x + 2} + 1$

d) $y = \frac{1}{2}\sqrt{x + 5} - \frac{5}{2}$

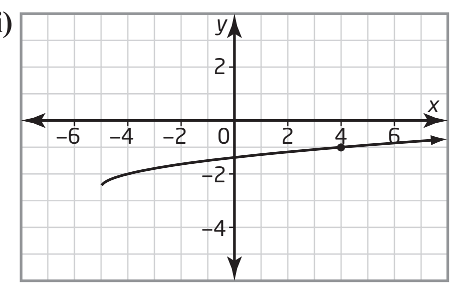
i)



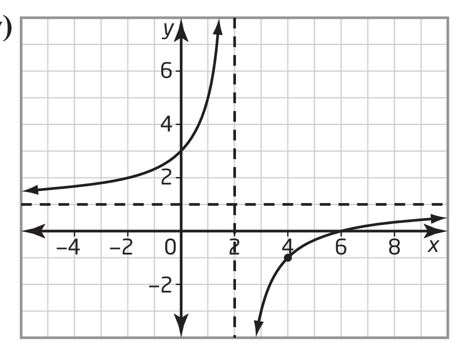
ii)



iii)



iv)



☆**9.** Andrew and David are planning a canoe trip. They want to compare how the travel time for each portion of the trip will vary according to the speed at which they travel. For the first part of the trip, they will travel 24 km across a calm lake. For the second part of the trip, they will travel 18 km up a river whose current flows at 4 km/h. For the last part of the trip, they will travel 36 km down a river that flows at 3 km/h. They used the relationship $t = \frac{d}{s}$, where t is time in hours, d is distance in kilometres, and s is speed in kilometres per hour, to establish that at a speed of s km/h, the time it will take to travel along the lake is $t_1 = \frac{24}{s}$, the time to travel up the river is $t_2 = \frac{18}{s-4}$, and the time taken to travel down the river is $t_3 = \frac{36}{s+3}$.

a) Identify the base function $f(s)$ for this situation.

b) Describe the transformations that must be applied to $f(s)$ to obtain each time function.

c) Graph each function on the same set of axes.

d) How long will each part of the trip take if they paddle their canoe at a constant speed of 6 km/h?

10. The value, in thousands of dollars, of a certain new boat can be modelled by the equation $V(t) = \frac{24}{t+6}$, where t is the time, in years.

a) Sketch a graph of this relation. State the domain and range for thissituation.

b) What was the initial value of the boat?

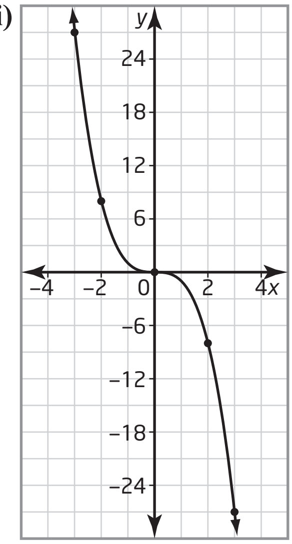
c) What is the projected value of the boat after each time?

i) 1 year
ii) 3 years
iii) 7 years

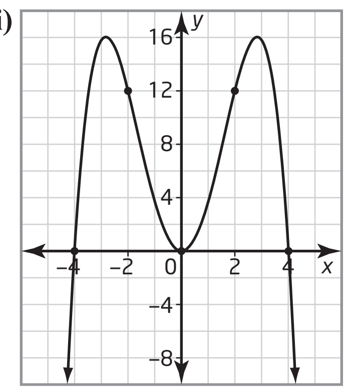
C Extend

11. Given each graph of a base function $f(x)$, sketch the graph of $g(x) = 2f(0.5x - 1.5) + 4$.

i)



ii)



12. a) Given the base function $f(x) = x^4$, use a table of values or a graphing calculator to sketch a graph of $y = f(x)$.

b) Sketch a graph and determine an equation for each transformed function.

i) $g(x) = 4f(-x + 5)$

ii) $h(x) = -f\left(\frac{1}{4}x - 1\right) + 3$

13. The equation of a circle, with centre at the origin and radius r, is given by $x^2 + y^2 = r^2$. Describe the transformations needed to graph each circle, and sketch the circles.

a) $(x - 3)^2 + (y - 5)^2 = 16$
b) $(x + 2)^2 + (y - 7)^2 = 4$

2.7 Inverse of a Function

KEY CONCEPTS

- The inverse of a function $f(x)$ is denoted by $f^{-1}(x)$.
- The inverse of a function can be found by interchanging the x- and y-coordinates of the function.
- The graph of $f^{-1}(x)$ is the graph of $f(x)$ reflected in the line $y = x$.
- The inverse of a function can be found by interchanging x and y in the equation of the function and then solving the new equation for y.
- For algebraic inverses of quadratic functions, the functions must be in vertex form.
- The inverse of a function is not necessarily a function.

Example

Consider the function $f(x) = x^2 + 4x + 1$.

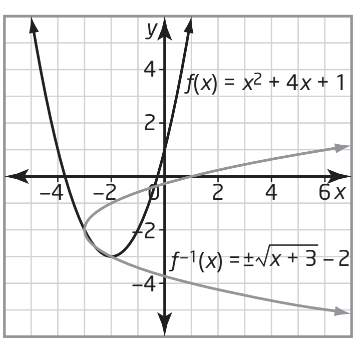
a) Determine the equation of the inverse, $f^{-1}(x)$, of $f(x)$.
b) Graph $f(x)$ and $f^{-1}(x)$ on the same set of axes.
c) Determine if $f^{-1}(x)$ is a function.

Solution

a) $y = x^2 + 4x + 1$
$y = (x^2 + 4x + 4) + 1 - 4$
$y = (x + 2)^2 - 3$
Interchange x and y and solve for y.

$$x = (y + 2)^2 - 3$$
$$x + 3 = (y + 2)^2$$
$$\pm\sqrt{x + 3} = y + 2$$
$$\pm\sqrt{x + 3} - 2 = y$$
$$f^{-1}(x) = \pm\sqrt{x + 3} - 2$$

b)



c) Consider $x = 1$.
$f^{-1}(1) = \pm\sqrt{1 + 3} - 2$
$= 0 \text{ or } -4$
The inverse of $f(x)$ is not a function.

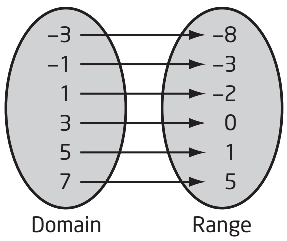
A Practise

1. Write the inverse of each function. Then state the domain and range of the function and of its inverse.

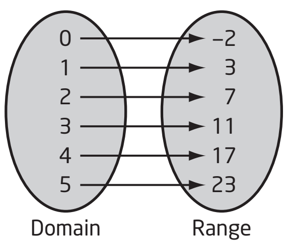
a) {(2, 6), (3, 1), (4, −1), (5, 2)}

b) {(−3, 7), (−2, 5), (−1, −2), (0, −6)}

c)

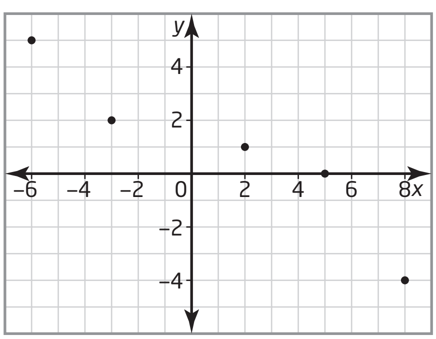


d)

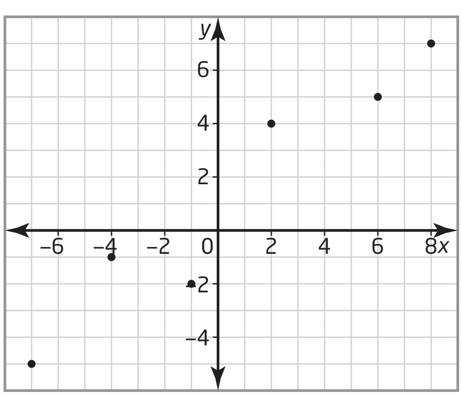


2. Copy each function. Then sketch the inverse of the function, and state the domain and range of the function and of its inverse.

a)

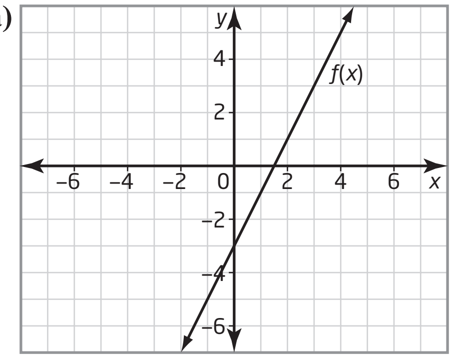


b)

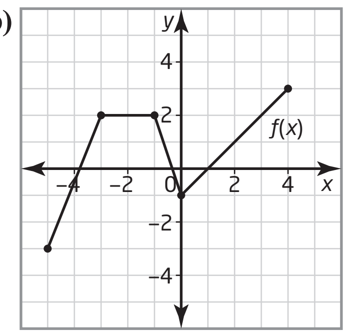


3. Copy each graph of $f(x)$ and then sketch the graph of the inverse of $f(x)$ by reflecting in the line $y = x$. State whether or not the inverse is a function.

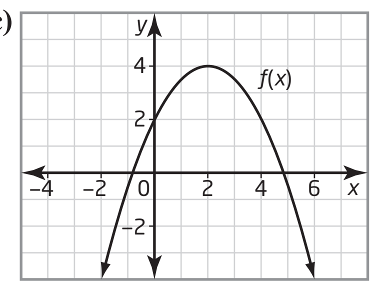
a)



b)



c)



4. Determine an equation for the inverse of each function.

a) $f(x) = 5x$

b) $f(x) = 4x - 3$

c) $f(x) = -x + 7$

d) $f(x) = \dfrac{-2x + 1}{3}$

5. Determine an equation for the inverse of each function.

a) $f(x) = x^2 + 5$

b) $f(x) = 7x^2$

c) $f(x) = (x + 3)^2$

d) $f(x) = \frac{1}{3}x^2 - 4$

6. For each quadratic function, complete the square and then determine the equation of the inverse.

a) $f(x) = x^2 - 4x + 3$

b) $f(x) = -x^2 + 14x - 39$

c) $f(x) = 2x^2 + 16x + 30$

d) $f(x) = -3x^2 - 24x - 100$

7. For each function $f(x)$,

i) determine $f^{-1}(x)$

ii) graph $f(x)$ and its inverse, with or without technology

iii) determine if the inverse of $f(x)$ is a function

a) $f(x) = -4x + 5$

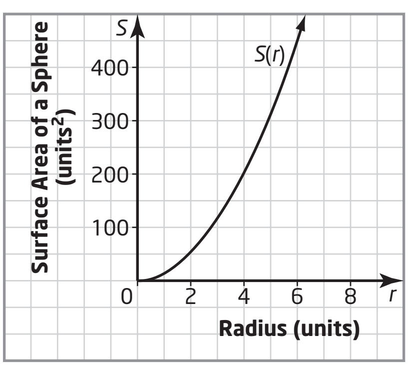
b) $f(x) = \frac{1}{2}x - 6$

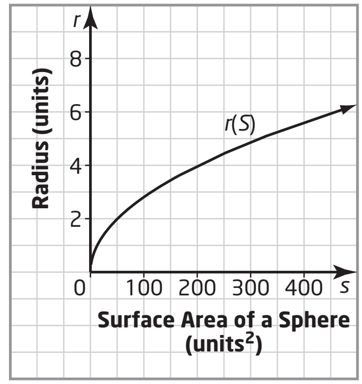
c) $f(x) = (x - 3)^2 + 8$

d) $f(x) = -x^2 + 16x - 61$

B Connect and Apply

8. The relationship between the surface area of a sphere and its radius can be modelled by the function $S(r) = 4\pi r^2$, where S represents the surface area and r represents the radius. The graphs of this function and its inverse are shown.





a) State the domain and range of the function $S(r)$.

b) Determine an equation for the inverse of the function. State its domain and range.

☆9. Aubrey works at an appliance warehouse. She earns \$450 a week, plus a commission of 8% of her sales.

a) Write a function that represents Aubrey's total weekly earnings as a function of her sales.

b) Find the inverse of the function from part a).

c) What does the inverse represent for this situation?

d) One week Aubrey earned \$1025. Calculate her sales for that week.

10. At one point in 2008 the Canadian dollar was worth US\$0.89.

a) Write a function that expresses the value of the U.S. dollar, u, in terms of the Canadian dollar, c.

b) Find the inverse of the equation in part a). Round to the nearest hundredth. What does the inverse represent?

c) Use the inverse to convert US\$250 to Canadian dollars.

☆**11.** If an object is dropped from a height of 100 m, its approximate height, $h(t)$ metres, above the ground t seconds after being dropped is modelled by the function $h(t) = 100 - 4.9t^2$.

a) Graph the function. State the domain and range for this situation.

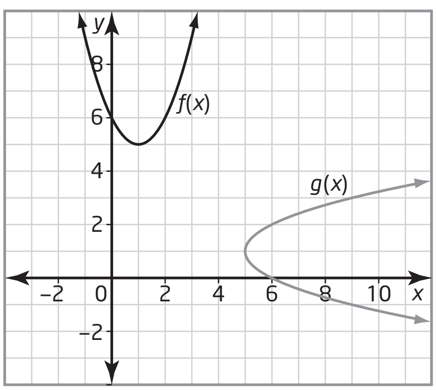
b) Determine the inverse of $h(t)$. State its domain and range.

c) Explain what this inverse represents in terms of the context of the question.

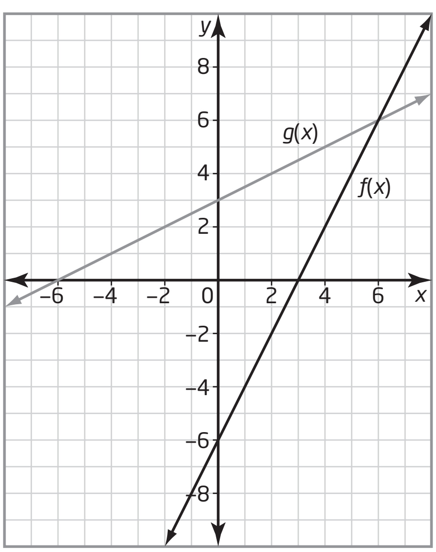
d) How long does it take for the object to hit the ground?

12. Determine whether $f(x)$ and $g(x)$ shown in each graph are inverses of each other. Explain your reasoning.

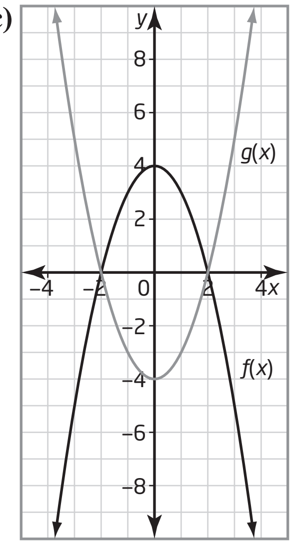
a)



b)



c)



C Extend

13. a) Determine the inverse of the function $f(x) = \sqrt{2x - 5}$.

b) State the domain and range of the function and its inverse.

c) Sketch a graph of the function and its inverse.

14. a) Determine the inverse of the function $f(x) = \dfrac{2}{3x - 1}$.

b) State the domain and range of the function and its inverse.

15. The relationship between the measure of an interior angle, a, in degrees, of a polygon and its number of sides, n, can be modelled by the function $a(n) = 180 - \dfrac{360}{n}$.

a) Determine the inverse of the function.

b) State the domain and range of the function and its inverse.

c) Sketch a graph of the function and its inverse.

Chapter 2 Review

2.1 Functions and Equivalent Algebraic Expressions

1. Determine if the functions in each pair are equivalent.

a) $f(x) = -4(x^2 + x - 2) + (2x - 3)^2$,
$g(x) = x^2 - 16x + 17$

b) $f(x) = 5(x^2 + 2x - 3) - (2x + 5)(-x + 1)$,
$g(x) = 7x^2 + 13x - 20$

c) $f(x) = (x + 5)^2 - 2(3 - x)(x + 1)$,
$g(x) = -2(-x^2 + 4x - 8) + (x + 4)^2 - 6$

2. State the restrictions on $f(x)$. Then determine whether $g(x)$ is the simplified version of $f(x)$. If not, determine the proper simplified version.

a) $f(x) = \dfrac{8x^2 + 10x - 3}{4x^2 - x}$ $\quad g(x) = \dfrac{2x + 3}{x}$

b) $f(x) = \dfrac{8x^2 - 12x - 8}{4x^2 - 8x}$ $\quad g(x) = 2x + 1$

3. Simplify each expression and state any restrictions on x.

a) $\dfrac{x + 8}{x^2 + 10x + 16}$

b) $\dfrac{8x^2 - 6x - 9}{4x^2 + 27x + 18}$

2.2 Skills You Need: Operations With Rational Expressions

4. Simplify and state any restrictions.

a) $\dfrac{64a^2b}{21ab^2} \times \dfrac{63a^3b^3}{8a^2b}$

b) $\dfrac{54a^3b^2}{81c^2} \div \dfrac{108a^2b}{45c}$

c) $\dfrac{3x - 4}{x + 9} \times \dfrac{2x + 18}{3x - 4}$

d) $\dfrac{x^2 - 5x - 24}{x^2 - 2x - 15} \times \dfrac{x - 5}{x + 4}$

e) $\dfrac{x - 4}{x - 3} \div \dfrac{4 - x}{2x - 6}$

f) $\dfrac{x^2 + 11x + 24}{x^2 + 2x - 3} \div \dfrac{x - 8}{x - 1}$

5. Simplify and state any restrictions.

a) $\dfrac{2}{3x} + \dfrac{3}{5x}$

b) $\dfrac{2 - a}{6ab} + \dfrac{7 + b}{12b^2}$

c) $\dfrac{x + 10}{x - 1} + \dfrac{x - 3}{x + 2}$

d) $\dfrac{8x - 3}{x^2 - 7x + 12} - \dfrac{2x + 1}{x - 4}$

e) $\dfrac{4x + 1}{x + 3} + \dfrac{x - 6}{x^2 - 9}$

f) $\dfrac{5x + 25}{x^2 + 7x + 10} - \dfrac{10x - 20}{x^2 - 4}$

2.3 Horizontal and Vertical Translations of Functions

6. Use the base function $f(x) = x$. Write the equation for each transformed function.

a) $s(x) = f(x + 2) - 8$

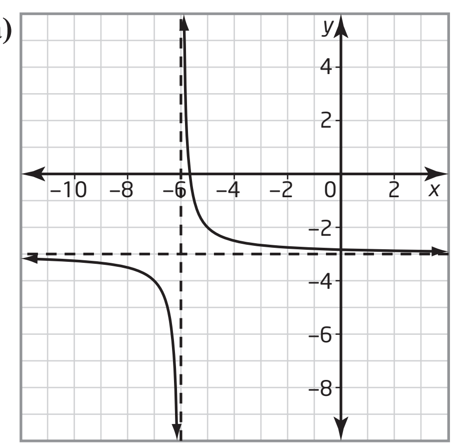
b) $t(x) = f(x - 5) + 1$

7. Repeat question 6 for each base function.

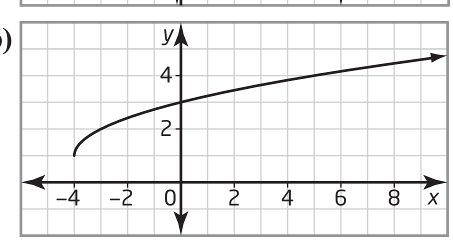
i) $f(x) = x^2$ **ii)** $f(x) = \sqrt{x}$ **iii)** $f(x) = \dfrac{1}{x}$

8. Each graph represents a transformation of one of the base functions $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, or $f(x) = \dfrac{1}{x}$. State the base function and the equation of the transformed function.

a)



b)



2.4 Reflections of Functions

9. Determine if $g(x)$ is a reflection of $f(x)$. Justify your answer.

a) $f(x) = \sqrt{x - 13} + 6$ $\quad g(x) = -\sqrt{13 - x} - 6$

b) $f(x) = x^2 - 2$ $\quad g(x) = x^2 - 2$

c) $f(x) = \sqrt{x + 7}$ $\quad g(x) = \sqrt{x - 7}$

d) $f(x) = \dfrac{1}{x + 15} - 8$ $\quad g(x) = \dfrac{-1}{x + 15} + 8$

10. For each function $f(x)$, determine the equation for $g(x)$.

a) $f(x) = \sqrt{x - 1} + 8$ $\quad g(x) = -f(x)$

b) $f(x) = (x - 3)^2 + 10$ $\quad g(x) = -f(-x)$

c) $f(x) = \dfrac{1}{x + 7} - 2$ $\quad g(x) = -f(-x)$

2.5 Stretches of Functions

11. For each function $g(x)$, identify the value of a or k and describe how the graph of $g(x)$ can be obtained from the graph of $f(x)$.

a) $g(x) = 9f(x)$

b) $g(x) = f(3x)$

c) $g(x) = \frac{2}{5}f(x)$

d) $g(x) = f\left(\frac{1}{7}x\right)$

12. For each function $g(x)$, describe the transformation from a base function of $f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, or $f(x) = \frac{1}{x}$. Then transform the graph of $f(x)$ to sketch a graph of $g(x)$.

a) $g(x) = 13x$

b) $g(x) = (5x)^2$

c) $g(x) = \sqrt{\dfrac{x}{3}}$

d) $g(x) = \dfrac{6}{x}$

2.6 Combinations of Transformations

13. Compare the transformed equation to $y = af[k(x - d)] + c$ to determine the values of the parameters a, k, d, and c. Then describe, in the appropriate order, the transformations that must be applied to a base function $f(x)$ to obtain the transformed function.

a) $g(x) = 7f(x - 1)$

b) $g(x) = \frac{1}{5}f(x) - 3$

c) $g(x) = f(x + 9) + 8$

d) $g(x) = f\left(\frac{1}{2}x\right) + 10$

14. Given each base function $f(x)$, write the equation of the transformed function $g(x)$. Then transform the graph of $f(x)$ to sketch a graph of $g(x)$.

a) $f(x) = \sqrt{x}$ $\quad g(x) = 3f(2x)$

b) $f(x) = \dfrac{1}{x}$ $\quad g(x) = f(x - 4) + 9$

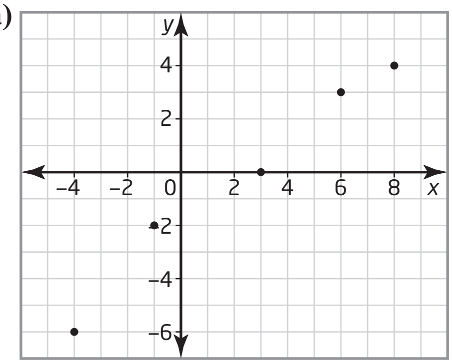
c) $f(x) = x^2$ $\quad g(x) = f\left[\frac{1}{4}(x + 5)\right]$

d) $f(x) = x$ $\quad g(x) = -2f(x - 7)$

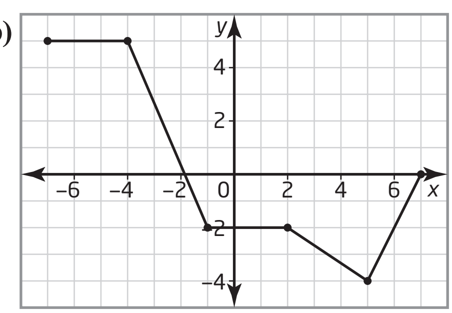
2.7 Inverse of a Function

15. Copy each graph of $f(x)$. Then sketch the inverse of each function, and state the domain and range of the function and of its inverse.

a)



b)



16. Determine an equation for the inverse of each function.

a) $f(x) = -2x + 7$

b) $f(x) = \dfrac{5x - 3}{4}$

c) $f(x) = (x - 3)^2 + 1$

d) $f(x) = -\frac{1}{4}x^2 + 9$

Chapter 2 Math Contest

1. The roots of $8x^2 - 6x = 9$ are x_1 and x_2. What is the value of $(x_1 + 1)(x_2 + 2)$?

A $\frac{3}{2}$ **B** $\frac{5}{8}$ **C** $\frac{7}{8}$ **D** $-\frac{3}{4}$

2. What is the greatest possible integer value for k such that the equation $(1 - 5k)x^2 + 6x - 2 = 0$ has two distinct real roots?

A −2 **B** 0 **C** 1 **D** − 1

3. Given $f(x) = 2^{5x-3}$, what is the product of $f(x)$ and $f(2 - x)$?

A 1024 **B** 16 **C** 256 **D** 8

4. Determine the value of x such that $3^x - 3^{x-2} = \frac{24}{729}$.

A $x = 8$

B $x = 3$

C $x = -4$

D $x = -3$

5. Determine the integer values of x and y so that $3^{x+1} + 3^x = 4^{y+2} - 7(4^y)$.

A $x = 0, y = 0$

B $x = 1, y = 2$

C $x = 2, y = 1$

D $x = -1, y = -2$

6. Consider the function $y = ax^r$. What is the value of r so that $y = 6$ when $x = 4$, and $y = 24$ when $x = 64$?

A $\frac{1}{2}$ **B** $\frac{1}{4}$ **C** 4 **D** 2

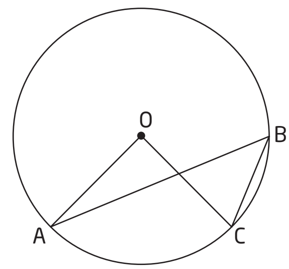
7. If $\cos A = \frac{4}{5}$ such that $\pi < A < 2\pi$, what is the value of tan A?

A $\frac{4}{3}$ **B** $-\frac{4}{3}$ **C** $\frac{3}{4}$ **D** $-\frac{3}{4}$

8. Consider △ABC, with vertices A(2, 2), B(0, 0), and C(4, −2). What is the measure of ∠C to the nearest degree?

A 37° **B** 48° **C** 53° **D** 65°

9. In the diagram, O is the centre of the circle, ∠OAB = 25°, and ∠OCB = 48°. Determine the measure of ∠ABC.



A 48° **B** 63° **C** 23° **D** 55°

10. Determine the sum of four natural numbers a, b, c, and d if $a^3 + b^3 = 1729$ and $c^3 + d^3 = 1729$.

A 24 **B** 32 **C** 28 **D** 36

11. What is the ones digit for the value 8^{1001}?

12. If $x = a + b$ and $y = a - b$, express $\left(\frac{3x - 21y}{6x + 12y}\right)^2 \div \frac{x^2 - 49y^2}{2x^2 + 8xy + 8y^2}$ in terms of a and b, in simplified form.

13. Determine the least number that can be expressed as the sum of two perfect squares in two different ways. Justify your answer.

14. Determine the value of $(1 + 3 + 5 + \cdots + 49) - (2 + 4 + 6 + \cdots + 50)$.

15. How many rectangles are in this diagram?