Chapter 7 Financial Applications

7.1 Simple Interest

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⟪ **KEY CONCEPTS** ⟫

- Simple interest, I, in dollars, can be calculated by multiplying the principal, P, in dollars, by the annual interest rate, r, expressed as a decimal, and by the time, t, in years.
  $I = Prt$

- The amount, A, of an account earning simple interest is the sum of the principal, P, and the interest, I.
  $A = P + I$

- The amount in an account earning simple interest can be represented using
  - a table of values
  - a partial variation equation
  - a linear graph
  - an arithmetic sequence

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Example

Ester buys a $2700 annual interest Step-Up Canada Savings Bond (CSB). This bond offers increasing interest rates each year for 5 years, as shown in the table. Determine the amount of the bond at the end of the 5 years.

Year	Annual Interest Rate (%)
1	3.5
2	3.7
3	3.9
4	4.1
5	4.9

Solution

To determine the amount, first determine the interest earned each year.
Year 1:
  Substitute $P = 2700$, $r = 0.035$, and $t = 1$ into the formula $I = Prt$.
  $I = Prt$
  $ = 2700 \times 0.035 \times 1$
  $ = 95.50$
The interest earned in year 1 is $95.50.

Year 2:
  Substitute $P = 2700$, $r = 0.037$, and $t = 1$ into the formula $I = Prt$.
  $I = Prt$
    $= 2700 \times 0.037 \times 1$
    $= 99.90$
  The interest earned in year 2 is $99.90.

Year 3:
  Substitute $P = 2700$, $r = 0.039$, and $t = 1$ into the formula $I = Prt$.
  $I = Prt$
    $= 2700 \times 0.039 \times 1$
    $= 105.30$
  The interest earned in year 3 is $105.30.

Year 4:
  Substitute $P = 2700$, $r = 0.041$, and $t = 1$ into the formula $I = Prt$.
  $I = Prt$
    $= 2700 \times 0.041 \times 1$
    $= 110.70$
  The interest earned in year 4 is $110.70.

Year 5:
  Substitute $P = 2700$, $r = 0.049$, and $t = 1$ into the formula $I = Prt$.
  $I = Prt$
    $= 2700 \times 0.049 \times 1$
    $= 132.30$
  The interest earned in year 5 is $132.30.

  Total interest $= \$95.50 + \$99.90 + \$105.30 + \$110.70 + \$132.30$
                 $= \$543.70$
  $A = P + I$
    $= 2700 + 543.70$
    $= 3243.70$
  At the end of 5 years, the amount of the bond is $3243.70.

A Practise

1. Determine the simple interest earned on each investment.

   a) $480 is deposited for 5 years and earns 4.5% per year simple interest.

   b) $950 is deposited for 7 months at 6% per year simple interest.

   c) $600 is invested at 3.75% annual simple interest for 42 weeks.

   d) $1320 is invested at 7.4% per year, simple interest, for 90 days.

2. $500 is deposited into an account that earns 7% simple interest annually.

   a) Determine the amount of the investment after 1, 2, 3, 4, and 5 years.

   b) Identify the first term, a, and the common difference, d, of this arithmetic sequence.

   c) Write an equation to represent the nth term of this sequence. What is the significance of the nth term?

3. The table shows the amount of a simple interest GIC over a period of several years.

Time (years)	Amount ($)
1	1040
2	1080
3	1120
4	1160
5	1200
6	1240

   a) Calculate the first differences. What do these values represent?

   b) What is the principal of this investment? How do you know?

   c) What is the annual rate of simple interest?

4. Refer to the table in question 3.

   a) Develop a linear model to represent the amount in the GIC versus time.

   b) Explain why the model in part a) is a partial variation. Identify the fixed part and the variable part.

   c) How long will it take for this investment to double from its initial value?

★5. IOU Credit Card Company charges an annual simple interest rate of 18% on unpaid account balances. Calculate the amount of interest that the company would charge when a balance of $2100 is paid 23 days late.

6. The graph shows the amount of an investment earning simple interest.
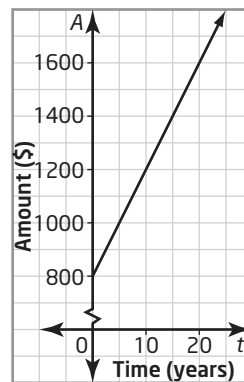
   a) What is the principal?

   b) What is the annual interest rate?

   c) Write an equation to relate the amount to time.

   d) Use your equation to determine how long it will take for the original investment to double.

B Connect and Apply

7. Refer to question 6.

   a) Write an equation to relate the interest to time.

   b) Use your equation to determine how long it will take for the original investment to double. Compare this result with your answer to question 6d).

☆**8.** Katio paid $165 in interest for borrowing a sum of money at an interest rate of $2\frac{3}{4}\%$ for 4 years. How much did he borrow?

9. To save for a new bike, Sasha deposits $320 into a savings account that earns 5.5% per year, simple interest.

a) Write an equation to relate the amount of the investment to time.

b) Graph the function.

c) How long will it take, to the nearest month, for the amount to reach $450?

d) What interest rate is required for the amount to reach $450 in 2 years less than your answer in part c)?

☆**10.** Lorilo borrows $1350 for 8 months to buy a new riding lawn mower. He pays $38.50 in interest on the loan. What was the annual interest rate of Lorilo's loan?

11. Rosalie borrows $3800 at an annual rate of 7.25% simple interest to buy new appliances. She plans to repay the loan in 18 months.

a) What amount must she pay back?

b) How much interest will she pay?

c) How much sooner should Rosalie repay the loan if she wants to pay no more than $280 in interest charges?

12. Martin took out a loan for $1050 at an annual rate of 9.5% simple interest. When he repaid the loan, the amount was $1848. How long did Martin hold this loan?

13. Arash wants to borrow $4500 to buy a motorcycle. He is considering the following two options:

• Borrow from the bank at 8.5% per year simple interest.

• Borrow from the motorcycle dealership at 8% per year simple interest, plus a $150 administration fee due upon repayment date.

a) Write an equation for each option to relate the amount, A, to time, t, in years.

b) Graph the amount payable versus time for each option on the same set of axes.

c) Which option is the better deal? Explain.

14. After 3 years, Taylor's investment is worth $1041.25. Fours years later the amount has reached $1296.25.

a) How much simple interest is Taylor's investment earning per year?

b) What is the principal?

c) What is the annual simple interest rate per year?

C Extend

15. Abraham's loan payment of $650 is due in 10 months. How much should Abraham invest now, at 7.5% simple interest, to meet the payment?

16. Rebecca's best friend moves to Australia and Rebecca promises to visit her in 5 years. She knows the trip will cost her $3300. How much should Rebecca invest now at a simple interest rate of 7.5% so that she will have $3300 in 5 years?

17. Suppose you invest $1000 at an interest rate of 7.25% per year.

a) Determine the amount in your account after 5 years if the interest is calculated on the sum of the principal plus the interest at the end of each year.

b) How much more interest is earned using the method in part a) than the simple interest on the same amount?

c) Why do you think this type of interest is called *compound* interest?

7.2 Compound Interest

Example

Carmine has the option of the following compounding periods for his $1000 investment. The interest rate is 4% per annum and the time is 6 years.

a) semi-annually

b) quarterly

c) monthly

Which compounding period should he select? Justify your choice.

Solution

The principal is $1000. This means $P = 1000$.

a) When the interest is compounded semi-annually, it is added twice a year.

The semi-annual rate is $\frac{4\%}{2}$, or 2%. This means $i = 0.02$.

In 6 years, there are 6×2, or 12 compounding periods. This means $n = 12$.

Substitute the known values into the compound interest formula.

$A = P(1 + i)^n$

$\quad = 1000(1 + 0.02)^{12}$

$\quad = 1000(1.02)^{12}$

$\quad \doteq 1268.24$

The amount after 6 years is $1268.24.

b) When the interest is compounded quarterly, it is added four times a year.

The quarterly rate is $\frac{4\%}{4}$, or 1%. This means $i = 0.01$.

In 6 years, there are 6×4, or 24 compounding periods. This means $n = 24$.

Substitute the known values into the compound interest formula.

$A = P(1 + i)^n$

$\quad = 1000(1 + 0.01)^{24}$

$\quad = 1000(1.01)^{24}$

$\quad \doteq 1269.73$

The amount after 6 years is $1269.73.

c) When the interest is compounded monthly, it is added 12 times a year.

The monthly rate is $\frac{4\%}{12}$, or $\frac{1}{3}\%$. This means $i \doteq 0.003\ 333\ 3$.

In 6 years, there are 6×12, or 72 compounding periods. This means $n = 72$.

Substitute the known values into the compound interest formula.

$A = P(1 + i)^n$

$\quad = 1000(1 + 0.003\ 333\ 3)^{72}$

$\quad = 1000(1.003\ 333\ 3)^{72}$

$\quad \doteq 1270.74$

The amount after 6 years is $1270.74.

The amounts for each compounding period are
- semi-annually: $1268.24
- quarterly: $1269.73
- monthly: $1270.74

Since the largest amount occurs when the interest is compounded monthly, Carmine should choose this option to invest his $1000.

A Practise

1. $650 is invested for 7 years at 5% interest per year, compounded annually.

a) Determine the amount in the account after 7 years.

b) How much interest was earned?

2. $975 is borrowed at a rate of 8.5% interest per year, compounded annually, for 5 years.

a) Determine the amount to be repaid after 5 years.

b) How much interest was charged?

3. For each compounding condition, determine the interest rate per compounding period, expressed as a decimal.

a) 7% per year, compounded monthly

b) 9% per annum, compounded quarterly

c) 8% annual interest, compounded semi-annually

d) 11% per year, compounded bi-weekly

4. For each compounding condition, determine the number of compounding periods.

  a) quarterly compounding for 4 years

  b) semi-annual compounding for 5 years

  c) monthly compounding for $\frac{2}{3}$ of a year

  d) daily compounding for 3 weeks

  e) annual compounding for 7 years

5. Determine the total number, n, of compounding periods and the interest rate, i, as a decimal, per compounding period for each scenario.

  a) 8.5% per year, compounded annually for 4 years

  b) 7% per annum, compounded quarterly for 5 years

  c) 3.6% per year, compounded monthly for 3 years

  d) 5.5% per annum, compounded semi-annually for 6.5 years

  e) 6.2% per year, compounded daily for 2 years

6. Matteo invests $1300 into a GIC that earns 5.25% interest per year, compounded quarterly, for 4 years.

  a) Determine the amount in the account after 4 years.

  b) How much interest was earned?

  c) Compare this to the amount of interest that would have been earned if simple interest had been earned at the same rate.

7. Moira's chequing account earns 4.8% interest per year, compounded daily. How much interest will she earn if she has $1670 in the account for 40 days?

B Connect and Apply

✩**8.** Richard invested $6800 at 5.2% compounded semi-annually.

  a) Determine the amount of the investment after
    i) 4 years
    ii) 7 years

  b) How much interest was earned on the investment between the 4th year and the 7th year?

9. Kara borrows $1000 for 6 years at $9\frac{1}{4}$% interest per year.

  a) Compare the interest charges under each condition.
    i) simple interest
    ii) annual compounding
    iii) semi-annual compounding
    iv) quarterly compounding
    v) monthly compounding
    vi) daily compounding

  b) Which is the best scenario for Kara? Which is the worst?

  c) Explain the effect of the compounding period on this loan.

✩**10.** Isabella has $5000 to invest in a GIC that earns 6% per year, compounded daily. How long will it take, to the nearest month, for Isabella's investment to double?

✩**11.** Meg has $8000 to invest in an RESP (Registered Education Savings Plan) for 7 years.

  a) Which of the following investment options should she choose? Justify your reasoning.
    Option 1: 6% compounded semi-annually
    Option 2: 7.5% simple interest

  b) What type of function is represented by each of the above options? Explain.

12. Krista borrows $12 000 as start-up capital for her new business. She plans to repay the loan in 3 years, at which point she will owe $14 803.80. What rate of interest is Krista being charged, assuming that it is compounded annually?

13. Paolo borrows $650 for 5 years at an interest rate that is compounded quarterly. At the end of the 5 years, he repays $866.87. What annual, quarterly compounded interest rate was Paolo charged?

14. The Rule of 72 states that the number of years required for an investment to double when interest is compounded annually can be estimated by dividing 72 by the annual interest rate.

a) Use the Rule of 72 to determine how many years it will take for an amount to double when invested at each interest rate, compounded annually.
i) 4%
ii) 3%
iii) 6%

b) Verify your results in part a) using the compound interest formula. Is the Rule of 72 exact? Explain.

15. Isam invested $850 in an account that earned 6.75% interest per year, compounded quarterly. When he closed the account, there was $1358.03 in it. How long did Isam invest his money?

16. Maxime has inherited $5000 and would like to invest the money so she can use it to purchase a new car 4 years from now. She has two options:

Option A: invest the money for 4 years at 5.5% compounded annually

Option B: invest the money for 2 years at 3% compounded semi-annually and then, at the end of 2 years, take the amount and invest it for the remaining 2 years at 8% compounded annually

Which option should Maxime select? Justify your answer.

C Extend

17. Describe an investment that could be represented by the function $f(x) = 1200(1.065)^x$.

18. Determine the time it takes to double an investment in an account that pays interest at 4% per annum, compounded quarterly.

19. Mark estimates that the cost of tuition for his first year in university will be $6780. How much should he invest now, at 4.5% compounded quarterly, so that he will have the required amount in 5 years?

20. An investment grows more quickly when the interest is compounded more frequently. There are two types of interest rates that are often available. The *nominal* interest rate is the one usually quoted. The *effective* interest rate is the annual interest rate that produces the amount of interest that is equivalent to the nominal interest rate. For example, $2000 is invested at 12% per annum compounded monthly. The amount of the investment at the end of 1 year is
$A = 2000(1.01)^{12}$
$\doteq 2253.65$
The interest earned is $253.65
The nominal interest rate is 12%.
The effective rate of interest is determined as follows:
$\frac{253.65}{2000} \times 100 \doteq 12.68\%$
The effective rate of interest is 12.68%.

a) Determine the effective rate of interest for the following nominal rates.
i) 6% per annum compounded semi-annually
ii) 8% per annum compounded quarterly

b) Does the principal amount invested influence the effective interest rate? Explain.

7.3 Present Value

KEY CONCEPTS

- Present value refers to the principal that must be invested today to grow to a known future amount under specified interest and time conditions.

- The formula $PV = \dfrac{FV}{(1 + i)^n}$ can be used to calculate the present value, PV, if the future value, FV; number of compounding periods, n; and interest rate, i, as a decimal, per compounding period are known.

Example

Mandy earned \$7500 from her very first summer job and plans to invest it for 10 years. At the end of 10 years she would like to have \$20 000. What rate of interest, to the nearest hundredth of a percent, compounded monthly, does Mandy need to achieve her goal?

Solution

Method 1: Apply Algebraic Techniques

Let i represent the annual interest rate, as a decimal. Then $\dfrac{i}{12}$ represents the monthly rate.

Substitute the known values, $PV = 7500$, $FV = 20\,000$, and $n = 120$, into $PV = \dfrac{FV}{\left(1 + \dfrac{i}{12}\right)^n}$.
Then, rearrange to solve for i.

$$7500 = \frac{20\,000}{\left(1 + \dfrac{i}{12}\right)^{120}}$$

$$\left(1 + \frac{i}{12}\right)^{120} = \frac{20\,000}{7500}$$

$$\left(1 + \frac{i}{12}\right)^{120} = \frac{8}{3}$$

$$1 + \frac{i}{12} = \sqrt[120]{\frac{8}{3}} \qquad \text{Note that } \sqrt[120]{\frac{8}{3}} = \left(\frac{8}{3}\right)^{\frac{1}{120}}.$$

$$\frac{i}{12} = \sqrt[120]{\frac{8}{3}} - 1$$

$$i = 12\left(\sqrt[120]{\frac{8}{3}} - 1\right)$$

$$i \doteq 0.0985$$

The annual rate of interest is approximately 9.85% per annum, compounded monthly.

Method 2: Use a TVM Solver

To access the Time Value of Money (TVM) Solver on a graphing calculator, press APPS , select **1:Finance**, and then select **1:TVM Solver…**.

Enter the values in the fields as shown.

The investment is for 10 years, so N = 10.

Since Mandy must pay $7500 for the investment, PV = −7500.

The amount wanted in 10 years is $20 000, so FV = 20 000.

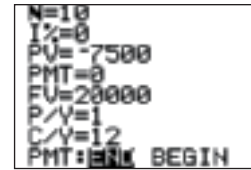Since the investment is one time only, P/Y = 1.

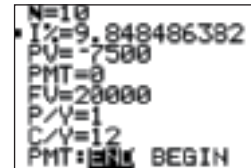The interest is compounded monthly, so C/Y = 12.

The interest is calculated at the end of each compounding period, so select **END**.

To solve for the unknown interest, move the cursor to the **I%** field and press ALPHA ENTER for [SOLVE]. The interest will be calculated.

To achieve her goal Mandy needs an interest rate of approximately 9.85% per annum, compounded monthly.

A Practise

1. Determine the present value of each future amount for the given conditions.

a) In 4 years, an investment earning 6% per year, compounded annually, will have a value of $800.

b) Three years from now, an investment earning 4.2% annual interest, compounded quarterly, will have a value of $1076.86.

2. In 5 years, an investment will be worth $66.21. If interest is earned at a rate of 7% per year, compounded annually, what is the present value of this investment?

3. In 6 years, money invested at 6.5% per annum, compounded semi-annually, will grow to $1614.63.

a) How much money was invested?

b) How much interest will be earned in 6 years?

4. Sam receives a financial gift from his grandparents, which he invests at 9.2% annual interest, compounded quarterly. He is advised that the investment will be worth $3151.68 in 5 years.

a) What is the amount of the gift?

b) How much interest will be earned?

☆**5.** How much money should Tara invest now at a rate of 6.3% per year, compounded monthly, to have $10 000 in 5 years?

B Connect and Apply

6. A bond will be worth $950 when it becomes due in 5 years. If the bond is purchased today for $700 at 6.15% per year, determine how frequently the interest was compounded.

7. Simon invests $2000 at 9.5% per year, compounded semi-annually. When the account is closed, its value will be $3829.89. How long will Simon's money be invested?

★**8.** Paula would like to have $8000 in 6 years to use as a down payment for a house. She is considering two investment options:

Investment A: 5.5% annual interest, compounded quarterly

Investment B: 5.3% annual interest, compounded monthly

  a) Compare the present values of each option.

  b) Which investment is the better choice for Paula? Explain your reasoning.

9. Five years ago, money was invested at 8% per year. Today the investment is worth $6000. Determine the amount originally invested for each compounding period.

  a) annually

  b) semi-annually

  c) quarterly

10. Three and a half years ago, some money was invested in a fund that paid an annual interest rate of 8%, compounded quarterly. Today, the account has a value of $1000. What was the amount of the original deposit?

11. Leah borrows $800 to buy a treadmill. She agrees to repay $875 in a year and a half. What annual rate of interest, compounded monthly, is Leah being charged?

12. Tim wants to have $4450 in 2.5 years to finish his basement.

  a) How much money must he invest today at 6.8% annual interest, compounded quarterly, to have enough money?

  b) Tim only has $3500 today. What interest rate must he obtain to have enough money to finish his basement on time?

13. A 4-L container of milk costs $4.75 in 2009.

  a) What would the price of a 4-L container of milk have been in 1989, assuming an average inflation rate of 3% per year, compounded annually?

  b) How much would a similar volume of milk have cost in 1929?

14. A financial institution is owed $100 000, due in 10 years. The financial institution is willing to sell the debt today discounted at 7.2% per year, compounded semi-annually. What is the value of the debt today?

C Extend

15. Nine years ago Amelia inherited a certain amount of money. For the first 5 years the money was invested at a rate of 10% per annum, compounded semi-annually. For the next 4 years the money was invested at a rate of 12% per annum, compounded quarterly. Now, the investment is worth $20 000. How much money did Amelia inherit?

16. Tate plans to buy a new small car in 3 years. The base price of the model he prefers is currently $21 800. The dealership anticipates that in 3 years the price of a similar new model will have increased by 18%. They also guarantee that he will receive a $6200 trade-in allowance on his current vehicle. How much money should Tate invest today at a rate of 6.4% per year, compounded quarterly, so that he will have enough to purchase the car in 3 years?

17. Dean needs $1000 one year from now to purchase a new laptop and printer. He plans on making two equal deposits, one now and the other in 6 months, into an account that pays 7.5% per year, compounded semi-annually. What is the amount of each deposit?

7.4 Annuities

KEY CONCEPTS

- An annuity is an investment in which regular payments are deposited into an account.

- An ordinary simple annuity is one in which payments are made at the end of every payment period and interest is compounded at the end of the same payment period.

- The amount, A, of an annuity can be calculated using the formula $A = \dfrac{R[(1 + i)^n - 1]}{i}$, where R represents the regular payment; i represents the interest rate per compounding period, as a decimal; and n represents the number of compounding periods.

Example

Clarissa would like to purchase a cottage in no more than 10 years from now. She wants to have at least $50 000 for the down payment. So, she deposits $100 at the end of each week into an investment plan that pays 4.55% per year, compounded weekly. In how many years will Clarissa have enough money to make the down payment?

Solution

Draw a time line to represent this annuity.
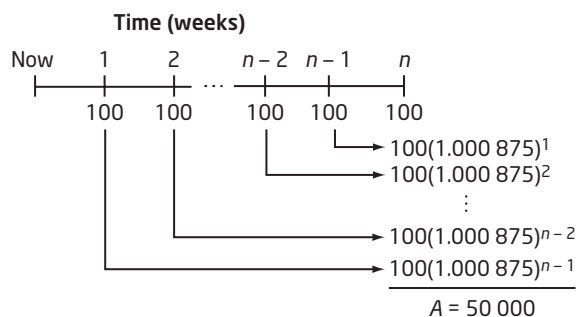Determine the interest per compounding period.

$$i = \frac{0.0455}{52}$$
$$= 0.000\ 875$$
$$n = ?$$
$$R = 100$$

Time (weeks)

Now 1 2 $n-2$ $n-1$ n

100 100 100 100 100

$100(1.000\ 875)^1$
$100(1.000\ 875)^2$
$\vdots$
$100(1.000\ 875)^{n-2}$
$100(1.000\ 875)^{n-1}$

$A = 50\ 000$

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Method 1: Apply Algebraic Techniques

Substitute the known values into the formula $A = \dfrac{R[(1 + i)^n - 1]}{i}$ and solve for n.

$A = 50\ 000$, $i = 0.000\ 875$, and $R = 100$.

$$50\ 000 = \frac{100[(1 + 0.000\ 875)^n - 1]}{0.000\ 875}$$

$$43.75 = 100[(1.000\ 875)^n - 1]$$

$$0.4375 = (1.000\ 875)^n - 1$$

$$1.4375 = (1.000\ 875)^n$$

Use systematic trial to find the value of n.

Use a table to organize your work. Try $n = 400$ to start.

Estimate, n	$(1.000\ 875)^n$	Analysis
400	$(1.000\ 875)^{400} \doteq 1.418\ 85$	Too low. Try a higher value.
425	$(1.000\ 875)^{425} \doteq 1.450\ 22$	Too high. Try a lower value.
420	$(1.000\ 875)^{420} \doteq 1.443\ 89$	Close, but a little high.
410	$(1.000\ 875)^{410} \doteq 1.431\ 31$	Close, but a little low.
415	$\mathbf{(1.000\ 875)^{415} \doteq 1.437\ 59}$	**That's it.**

Since there are 52 weeks in a year, it will take approximately $415 \div 52$, or 8 years, for Clarissa to have enough money to make the down payment.

Method 2: Use a TVM Solver

Access the TVM Solver on a graphing calculator and enter the values, as shown.

```
N=0
I%=4.55
PV=0
PMT=-100
FV=50000
P/Y=52
C/Y=52
PMT:END BEGIN
```
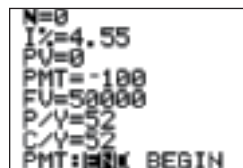
Move the cursor to the **N** field and press [ALPHA] [SOLVE].

```
•N=414.9305619
I%=4.55
PV=0
PMT=-100
FV=50000
P/Y=52
C/Y=52
PMT:END BEGIN
```

Since there are 52 weeks in a year, it will take approximately $415 \div 52$, or 8 years, for Clarissa to have enough money to make the down payment.

A Practise

1. The time line shows an annuity with an annual interest rate of 10%.
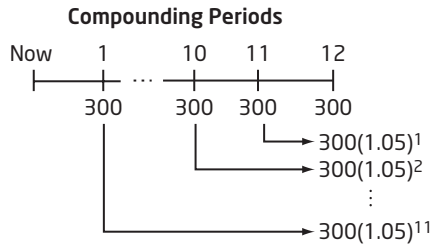
Compounding Periods

a) How often is interest compounded? Justify your answer.

b) What is the duration of the annuity? Justify your answer.

c) Calculate the amount of the annuity.

2. The time line shows an annuity with an annual interest rate of 8%.

Compounding Periods
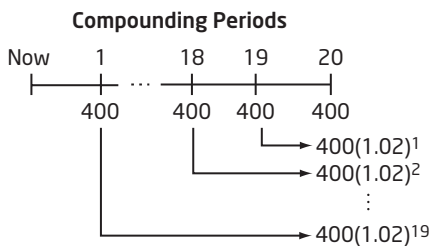
a) How often is interest compounded? Justify your answer.

b) What is the duration of the annuity? Justify your answer.

c) Calculate the amount of the annuity.

3. Anita deposits $1200 in an investment account every 6 months, for 5 years. The plan pays 7.5% annual interest, compounded semi-annually.

a) Draw a time line to represent this annuity.

b) Determine the amount of the annuity at the end of 5 years.

c) Verify the amount in part b) using the formula.

d) How much interest has been earned?

4. At the end of every month, for 2 years, Martino deposits $40 into an account that earns 6% per year, compounded monthly.

a) Draw a time line to represent this annuity.

b) Determine the amount of the annuity.

c) How much interest was earned?

5. How much must be invested every 6 months, for 4 years, in order to achieve an amount of $8000, if interest is earned at a rate of 6% per year, compounded semi-annually?

6. Lucinda wants to have $30 000 in her account 4 years from now to renovate her home. How much must she invest per month if her account earns 5.5% annual interest, compounded monthly?

B Connect and Apply

7. At the end of every quarter, Simon deposits $450 in an account that earns interest that is compounded quarterly. After 1.5 years, Simon will have $2874.48 in the account.

a) How much total interest will have been earned?

b) Determine the annual rate of interest, compounded quarterly.

8. How many years will it take Grace to save $25 000 for the car of her dreams if she deposits $130 each week into an account that pays 7.3% annual interest, compounded weekly?

9. Every 6 months, Allen deposits $250 into an investment account that pays 4.5% per annum, compounded semi-annually.

   a) How long will it take him to save $1586.95?

   b) How much interest is earned in that time period?

★10. Daniel plans to retire in 35 years and at that time he would like to have saved $500 000 in his RRSP (Registered Retirement Savings Plan). Determine the monthly deposits he should make into his RRSP if the rate of interest earned is fixed at 5.6% per year, compounded monthly.

11. Three sisters, Anna, Donella, and Tina, make regular deposits in individual investment accounts that pay 6% per year, compounded monthly. Anna deposits $400 per month for 5 years, Donella deposits $200 per month for 10 years, and Tina deposits $100 per month for 20 years.

    a) How much does each sister deposit in the investment account?

    b) Without making calculations, predict which annuity is greatest, or whether they are all equal.

    c) Determine the amount of each annuity. Was your prediction in part b) accurate?

12. Mick would like to retire at age 65 and is considering the following investment options:
    Option A: Invest $1000 per month beginning at age 25.
    Option B: Invest $4000 per month beginning at age 45.
    In both cases, the interest is 8% per annum, compounded monthly.

    a) What is the total amount deposited for each option?

    b) Which option should Mick choose? Justify your answer.

★13. Compare the amounts at age 65 that would result from making an annual deposit of $1000 starting at age 20, or from making an annual deposit of $3000 starting at age 50, to an RRSP that earns 6% interest per annum, compounded annually. What is the total of the deposits in each situation?

C Extend

14. a) Gordon deposits $1000 at the *beginning* of each year into an investment account that pays 6% compounded annually. How much will he have saved after 10 years?

    b) How much would Gordon have saved if he had made the deposits at the end of each year instead of the beginning?

    c) The annuity in part a) is an example of an *annuity due* because the payments are made at the beginning of each compounding period. Predict a formula for the amount of an annuity due of R dollars per period after n periods at an interest rate I per period.

15. Austin deposits $50 on the first of each month in an investment account that pays 9% compounded monthly. How much will Austin have in the account after 5 years?

16. Justin makes semi-annual deposits of $150 from November 1, 2003, to May 1, 2009, in an investment account that averages 11.75% compounded semi-annually. If Justin plans on leaving the money in the account for another 4 years, what will be the value of his investment?

17. Corinne deposits $100 at the end of each year in an account that pays 7.5% compounded semi-annually. How much will Corinne have saved after 9 years?

7.5 Present Value of an Annuity

>> **KEY CONCEPTS** >>

- The present value of an annuity is the total amount that can finance a series of regular withdrawals over a specific period of time.

- The present value, PV, of an annuity can be calculated using the formula $PV = \dfrac{R[1-(1+i)^{-n}]}{i}$, where R represents the regular withdrawal; i represents the interest rate per compounding period, as a decimal; and n represents the number of compounding periods.
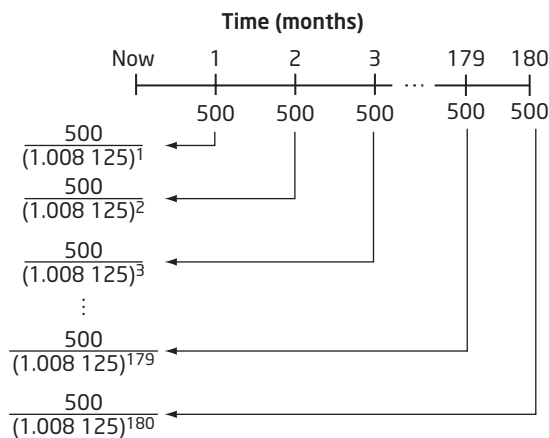
Example

Andrew would like to withdraw $500 per month from a retirement fund for 15 years after he retires. If he can earn $9\frac{3}{4}\%$ annual interest, compounded monthly, on this fund, how much money must Andrew have in this account when he retires?

Solution

Draw a time line to represent this annuity.

Determine the interest per compounding period and the number of compounding periods.

$$i = \frac{0.0975}{12}$$
$$= 0.008\ 125$$
$$n = 15 \times 12$$
$$= 180$$
$$R = 500$$

Time (months)

| Now | 1 | 2 | 3 | 179 | 180 |

$$\frac{500}{(1.008\ 125)^1}$$
$$\frac{500}{(1.008\ 125)^2}$$
$$\frac{500}{(1.008\ 125)^3}$$
$$\vdots$$
$$\frac{500}{(1.008\ 125)^{179}}$$
$$\frac{500}{(1.008\ 125)^{180}}$$
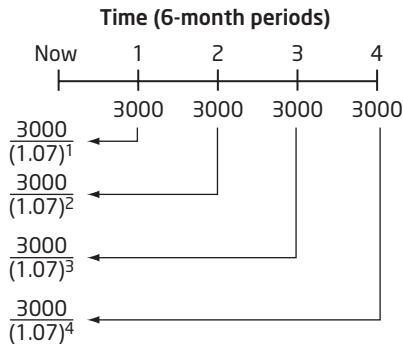
Substitute $R = 500$, $i = 0.008\ 125$, and $n = 180$ into the formula $PV = \dfrac{R[1-(1+i)^{-n}]}{i}$.

$$PV = \frac{500[1-(1+0.008\ 125)^{-180}]}{0.008\ 125}$$
$$= \frac{500[1-(1.008\ 125)^{-180}]}{0.008\ 125}$$
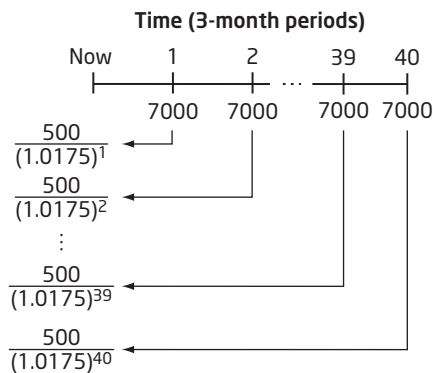$$\doteq 47\ 198.19$$

Therefore, Andrew will need $47 198.19 in his retirement fund.

A Practise

1. The time line shows an annuity from which semi-annual withdrawals are made for 2 years. Interest is compounded semi-annually.

Time (6-month periods)

Now 1 2 3 4

3000 3000 3000 3000

$\dfrac{3000}{(1.07)^1}$

$\dfrac{3000}{(1.07)^2}$

$\dfrac{3000}{(1.07)^3}$

$\dfrac{3000}{(1.07)^4}$

a) What is the annual rate of interest? Justify your answer.

b) How many withdrawals will be made, in total? Justify your answer.

c) Calculate the present value of the annuity.

2. The time line shows an annuity from which quarterly withdrawals are made for 10 years. Interest is compounded quarterly.

Time (3-month periods)

Now 1 2 39 40

7000 7000 7000 7000

$\dfrac{500}{(1.0175)^1}$

$\dfrac{500}{(1.0175)^2}$

$\vdots$

$\dfrac{500}{(1.0175)^{39}}$

$\dfrac{500}{(1.0175)^{40}}$

Calculate the present value of the annuity.

3. $500 is to be withdrawn at the end of every year, for 5 years, from an account that earns 9% interest, compounded annually.

a) Draw a time line to represent this annuity.

b) Determine the present value of the annuity.

4. $300 is to be withdrawn at the end of every month, for 3 years, from an account that earns 6.9% interest, compounded monthly.

a) Draw a time line to represent this annuity.

b) Determine the present value of the annuity.

c) How much interest is earned?

5. An annuity has an initial balance of $7500 in an account that earns 7% interest, compounded quarterly. What amount can be withdrawn at the end of each quarter of the 6.5 years of this annuity?

6. Adam uses $60 000 of savings to purchase an annuity that pays 6.8% interest, compounded semi-annually.

a) What semi-annual withdrawals can Adam make for the next 20 years if the first payment is made 6 months from now?

b) How much interest will the account earn?

B Connect and Apply

☆7. Jessica borrows $12 000 at a rate of 8% per year, compounded monthly, for her school expenses for a year.

a) What are her monthly payments if it takes 5 years to pay back the loan?

b) How much interest is she paying in total?

8. Brendon is converting his RRSP into an income fund. He would like to receive $1500 every 6 months for the next 20 years, starting 6 months from now. The interest rate is 6.25% per year, compounded semi-annually.

a) How much must Brendon convert now to pay for the annuity?

b) How much interest does the annuity earn over the 20 years?

9. A charitable organization is having a contest to raise money. The winner will receive $100 per month for 10 years.

  a) How much money is needed to fund this prize if the organization plans to draw from an account that pays 6% annual interest, compounded monthly?

  b) What is the total amount of money that the winner receives?

  c) How much of the total winnings are earned as interest?

10. A customer needs to borrow $7500 to purchase a used car. The car dealer arranges with a finance company to lend the customer the money at 2.9% per year, compounded monthly for 3 years. What will the customer's monthly payment be?

11. A department store has a five-piece leather living-room set on sale for $2799.95, including taxes. It can be purchased with 24 payments of $129 per month. What annual interest rate compounded monthly is the store charging?

12. An electronics store is promoting the sale of their DVD players by offering the following deal: no money down and 24 equal monthly payments of $23. The interest charged is 10% per year, compounded monthly. Determine the equivalent cash price of the DVD player.

13. An annuity has an initial balance of $4000. Annual withdrawals are made in the amount of $1000 for 5 years, at which point the account balance is zero. What annual rate of interest, compounded annually, was earned over the duration of this annuity?

14. Astil wants to borrow $35 000 to purchase a new vehicle. The car dealership offers loans at 8% compounded monthly for 48 months. Clients who apply for the loan online will receive a 0.5% reduction in the interest rate.

  a) Calculate Astil's monthly payments at each interest rate.

  b) What is the total amount Astil would have to repay for each loan?

  c) How much interest would Astil save if he applies for the loan online?

15. On her 30th birthday, Eileen decides she wants to save enough money so that on her 55th birthday she can purchase an annuity that pays $1000 per month, at a rate of 6% per annum, compounded monthly, until her 85th birthday. How much money should Eileen deposit annually, starting on her 30th birthday, at a rate of 5.5% per annum, compounded annually, so that she can achieve her goal on her 55th birthday?

C Extend

16. Liam just won $1 000 000 in a lottery! He set aside $100 000 for immediate spending money. The remaining amount was invested in an account that earned 6% per annum, compounded monthly. Liam made regular withdrawals of $10 000 per month from this account. Determine the number of years that Liam would be able to withdraw this monthly income.

17. In an ordinary annuity, the first payment is automatically received one period from now. In a *deferred annuity*, the first payment is delayed to a later time. Dana has invested her $25 000 inheritance in an account that earns 8.5% per year, compounded annually. Her plan is that, starting 4 years from now, she will withdraw equal yearly payments from the account, for 10 years. What is the amount of each withdrawal?

Chapter 7 Review

7.1 Simple interest

1. Calculate the missing information in the chart.

Principal, P	Interest Rate, r	Time, t	Simple Interest, I
$627.00	6.5%	2 months	
	9.25%	58 days	$5.72
$270.00		3 years	$64.80
$425.00	$7\frac{1}{2}$%		$12.66
$380.21	$4\frac{3}{4}$%	6 months	$9.03
$178.50	8.6%		$10.30
$3200.00		4.5 months	$138.00

2. MONEY Loan Company charges an annual interest rate of 15.5% on unpaid account balances. Calculate the amount of interest that the company would charge when a balance of $1850 is paid 37 days late.

3. Arianna's financial advisor has recommended that she invest at least 30% of her money in treasury bonds, at a fixed rate of return (interest rate). Following this recommendation, Arianna invests $4000 in a treasury bond for 5 years at a rate of 4.8% per year simple interest. She cannot access this money before the end of the 5 years without paying a financial penalty.

   a) Determine the interest earned over the term of the bond.

   b) Determine the amount of this investment at the end of 5 years.

7.2 Compound Interest

4. Describe the difference between simple interest and compound interest. What type of growth does each represent? Explain.

5. Glen purchases a $1500 GIC that earns 6.25% interest each year for 8 years.

   a) Determine the total interest earned at the end of each year under simple interest.

   b) Determine the total interest earned at the end of each year under compound interest.

   c) How much extra interest is earned under compound interest?

   d) Determine the amount of the investment under
      i) simple interest
      ii) compound interest

6. To buy a new laptop for her home business, Brianna borrows $1600 at an interest rate of 3.5% per annum, compounded annually. She plans to pay back the loan in 3 years.

   a) How much will Brianna owe after 3 years?

   b) How much interest will Brianna pay for the loan?

7.3 Present Value

7. How much should Stephen invest today at 9.5% annual interest, compounded semi-annually, so that he will have $2500 in 6 years?

8. Andrew invests $150 in an account in which the interest is compounded semi-annually. In 8 years Andrew will have $275. What is the interest rate?

9. Sarah invests $300 at a rate of 8% per year, compounded monthly. How long will it take for her investment to grow to $1000?

10. Suppose the value of a debt owed to a financial institution today is $35 000. What interest rate is required, compounded semi-annually, so that it will be worth $100 000 in 10 years?

7.4 Annuities

11. Amir deposited $500 in an account at the end of every 3 months for $5\frac{1}{2}$ years. The account paid 6% per year, compounded quarterly.

   a) Determine the amount in the account on the date of the last deposit.

   b) Determine the amount of interest earned in this account.

12. Karrington would like to save $48 000 at the end of 4 years. How much should she deposit at the end of each month into a savings account that pays $3\frac{1}{2}\%$ annual interest, compounded monthly?

13. Nathan would like to have $15 000 in 10 years, and can afford to deposit $500 every 6 months in an investment account. What interest rate, compounded semi-annually, does he need to achieve his goal?

7.5 Present Value of an Annuity

14. Due to the high cost of post-secondary education, many students apply to the Ontario Student Assistance Program (OSAP) for a loan to help pay their tuition fees. Students must begin to make payments to repay their loans 6 months after graduating. Kieran completed his university undergraduate degree 6 months ago. His loan payments of $250 are withdrawn at the end of each month from an account that is earning 6.5% interest, compounded monthly.

   a) How much must Kieran deposit in the account today so that the loan payments can be withdrawn for 1 year?

   b) What is the amount paid for the loan?

   c) How much of this amount is earned in interest?

15. Yasmine's grandparents set up an annuity for her. She will be paid $500 at the end of each month for the next 5 years. The first payment will be made 1 month from now. How much must Yasmine's grandparents deposit to provide for the annuity if the account earns 5.7% per annum, compounded monthly?

16. Chen-Chi has invested $20 000 into an annuity from which she plans to withdraw $1094.41 every 3 months for the next 5 years. If at the end of this time period the balance of the annuity is zero, what annual rate of interest, compounded quarterly, did this account earn?

Chapter 7 Math Contest

1. Determine the values of x and y such that $x^m y^n = \left(\dfrac{2}{3}\right)^{m-n}$ and $x^n y^m = \left(\dfrac{2}{3}\right)^{n-m}$ for all real numbers m, n.

   A $x = \dfrac{2}{3},\ y = \dfrac{2}{3}$

   B $x = \dfrac{2}{3},\ y = \dfrac{3}{2}$

   C $x = \dfrac{3}{2},\ y = \dfrac{2}{3}$

   D $x = \dfrac{3}{2},\ y = \dfrac{3}{2}$

2. Solve the equation $\sqrt{6 - \dfrac{1}{a}} = 6 - \dfrac{1}{a}$.

   A $a = -1$ or $a = 1$

   B $a = \dfrac{1}{6}$ or $a = 1$

   C $a = \dfrac{1}{5}$ or $a = \dfrac{1}{6}$

   D $a = -\dfrac{1}{5}$ or $a = \dfrac{1}{5}$

3. Determine the largest possible value of $ab + bc + cd + ad$ given that the values of a, b, c, and d are 3, 4, 5, and 6, but not necessarily in that order.

   A 80

   B 81

   C 360

   D 77

4. Solve for x given that $(4^{3x-1})(16^{2x+3}) = 64^{5x+2}$.

5. A quadratic equation $f(x)$ satisfies $f(0) = 2$, $f(-1) = 13$, and $f(2) = 4$. Determine the value of $2f\left(\dfrac{1}{2}\right)$.

   A -1

   B $-\dfrac{1}{2}$

   C -3

   D 2

6. Determine the sum, S, and product, P, of the roots of the equation $\dfrac{2}{x-1} + \dfrac{3}{x+2} = 3$.

   A $S = \dfrac{7}{3},\ P = -\dfrac{2}{3}$

   B $S = -\dfrac{2}{3},\ P = -\dfrac{7}{3}$

   C $S = \dfrac{7}{3},\ P = \dfrac{2}{3}$

   D $S = \dfrac{2}{3},\ P = -\dfrac{7}{3}$

7. Determine the value of m so that the roots of $mx^2 - 25x + 12 = 0$ are reciprocals.

8. Determine the real value of x that satisfies the equation $14x + 29\sqrt{x} - 15 = 0$.

9. Determine the values of x such that $13^{x^3 + 4x^2 - 21x} = 1$.

10. State an explicit formula for the general terms of two different geometric sequences such that the sum of the first two terms is 2 and the sum of the first three terms is 3.

11. Determine the smallest positive angle θ, in degrees, such that $\left(8^{\sin^2 \theta}\right)\left(8^{\cos^2 \theta}\right)\left(8^{\tan^2 \theta}\right) = 16$.

12. If θ is an angle such that $\tan \theta + \sec \theta = 2$, then what is the value of $\cos \theta$?

    A 1

    B $\dfrac{4}{5}$

    C $\dfrac{1}{2}$

    D $\dfrac{1}{\sqrt{2}}$